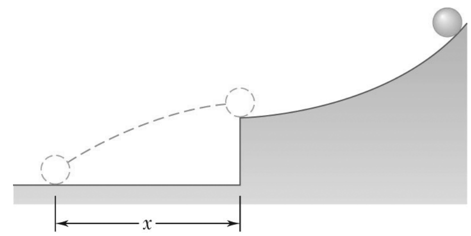
CHAPTER 17

STUDENTS-HUB.com

STUDENTS-HUB.com

PROBLEM 17.CQ1

A round object of mass m and radius r is released from rest at the top of a curved surface and rolls without slipping until it leaves the surface with a horizontal velocity as shown. Will a solid sphere, a solid cylinder or a hoop travel the greatest distance c?



(*a*) A solid sphere

(*b*) A solid cylinder

(*c*) A hoop

(*d*) They will all travel the same distance.

SOLUTION

Answer: (*a*) It has the smallest mass moment of inertia, so it will have the greatest speed at the bottom of the surface.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.CQ2

A solid steel sphere A of radius r and mass m is released from rest and rolls without slipping down an incline as shown. After traveling a distance d the sphere has a speed v. If a solid steel sphere of radius $2r$ is released from rest on the same incline, what will its speed be after rolling a distance d?

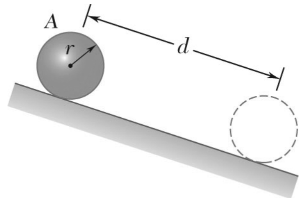
(*a*) $0.25\ v$

(*b*) $0.5\ v$

(*c*) v

(*d*) $2v$

(*e*) $4v$



SOLUTION

Answer: (*c*) Using conservation of energy you can show that the speed after traveling a distance d will be independent of the mass and the radius.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

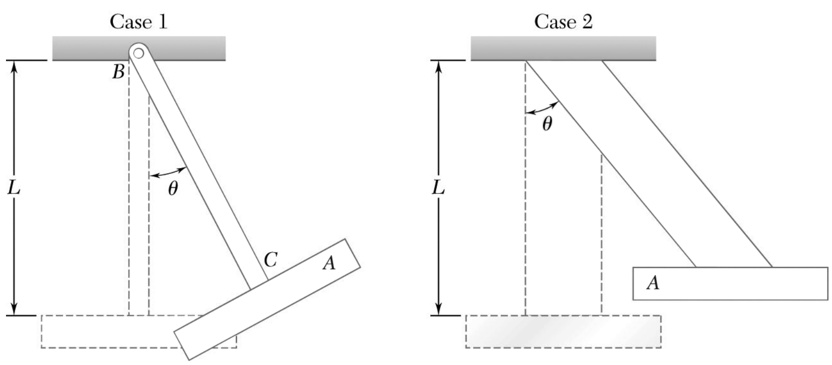
PROBLEM 17.CQ3

Slender bar A is rigidly connected to a massless rod BC in Case 1 and two massless cords in Case 2 as shown. The vertical thickness of bar A is negligible compared to L. In both cases A is released from rest at an angle $\theta = \theta_0$. When $\theta = 0°$ which system will have the larger kinetic energy?

(*a*) Case 1

(*b*) Case 2

(*c*) The kinetic energy will be the same.



SOLUTION

Answer: (*c*)

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.CQ4

In Problem 17.CQ3, how will the speeds of the centers of gravity compare for the two cases when $\theta = 0°$?

(*a*) Case 1 will be larger.

(*b*) Case 2 will be larger.

(*c*) The speeds will be the same.

SOLUTION

Answer: (*b*) Case 1 will also have rotational kinetic energy, so the speed will be smaller.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

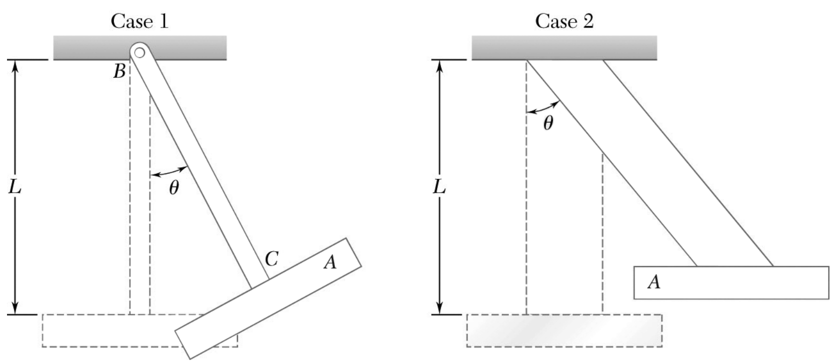
PROBLEM 17.CQ5

Slender bar A is rigidly connected to a massless rod BC in Case 1 and two massless cords in Case 2 as shown. The vertical thickness of bar A is not negligible compared to L. In both cases A is released from rest at an angle $\theta = \theta_0$. When $\theta = \theta°$ which system will have the largest kinetic energy?

(a) Case 1

(b) Case 2

(c) The kinetic energy will be the same.



SOLUTION

Answer: (a) Case 1 will have a greater change in gravitational potential energy, so the kinetic energy will be larger.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.1

The rotor of an electric motor has an angular velocity of 3600 rpm when the load and power are cut off. The 50-kg rotor then coasts to rest after 5000 revolutions. Knowing that the kinetic friction of the rotor produces a couple of magnitude 4 N · m, determine the centroidal radius of gyration of the rotor.

SOLUTION

Angular velocities:
$$\omega_1 = 3600\frac{\text{rev}}{\text{min}}\cdot\frac{1\,\text{min}}{60\text{ s}}\cdot\frac{2\pi\text{ rad}}{\text{rev}} = 120\pi\text{ rad/s}$$
$$\omega_2 = 0$$

Angular displacement: $5000\text{ rev} = 10000\pi\text{ rad}$

Principle of work and energy: $T_1 + U_{1\to2} = T_2$:

$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}\bar{I}(120\pi)^2 = 71.061\times10^3\,\bar{I}$$
$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = 0$$
$$U_{1\to2} = -M\theta = -(4\text{ N}\cdot\text{m})(10000\pi\text{ rad}) = -40000\pi\text{ N}\cdot\text{m}$$
$$71.061\times10^3\bar{I} - 40000\pi = 0$$
$$\bar{I} = 1.76839\text{ kg}\cdot\text{m}^2$$
$$\bar{I} = m\bar{k}^2$$

Centroidal radius of gyration.
$$\bar{k} = \sqrt{\frac{\bar{I}}{m}} = \sqrt{\frac{1.76839\text{ kg}\cdot\text{m}^2}{50\text{ kg}}} = 0.1881\text{ m}$$

$$\bar{k} = 188.1\text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.2

It is known that 1500 revolutions are required for the 6000-lb flywheel to coast to rest from an angular velocity of 300 rpm. Knowing that the radius of gyration of the flywheel is 36 in., determine the average magnitude of the couple due to kinetic friction in the bearings.

SOLUTION

Angular velocity:

$$\omega_0 = 300 \text{ rpm}$$
$$= 10\pi \text{ rad/s}$$
$$\omega_2 = 0$$

Moment of inertia:

$$\bar{I} = m\bar{k}^2 = \frac{6000 \text{ lb}}{32.2 \text{ ft/s}^2}(3 \text{ ft})^2$$
$$= 1677 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Kinetic energy:

$$T_1 = \frac{1}{2}\bar{I}\omega_0^2$$
$$= \frac{1}{2}(1677)(10\pi)^2$$
$$= 827{,}600 \text{ ft} \cdot \text{lb}$$
$$T_2 = 0$$

Work:

$$U_{1\to 2} = -M\theta$$
$$= -M(1500 \text{ rev})(2\pi \text{ rad/rev})$$
$$= -9424.7M$$

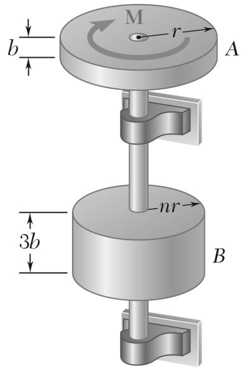
Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$
$$827{,}600 - 9424.7M = 0$$

Average friction couple: $M = 87.81 \text{ lb} \cdot \text{ft}$ $\quad M = 87.8 \text{ lb} \cdot \text{ft}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

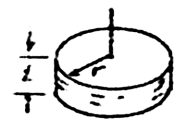
STUDENTS-HUB.com



PROBLEM 17.3

Two disks of the same material are attached to a shaft as shown. Disk *A* has a weight of 30 lb and a radius $r = 5$ in. Disk *B* is three times as thick as disk *A*. Knowing that a couple **M** of magnitude 15 lb · ft is to be applied to disk *A* when the system is at rest, determine the radius *nr* of disk *B* if the angular velocity of the system is to be 600 rpm after 4 revolutions.

SOLUTION



For any disk:

$$m = \rho(\pi r^2 t)$$

$$\bar{I} = \frac{1}{2}mr^2$$

$$= \frac{1}{2}\pi\rho t r^4$$

Moment of inertia.

Disk *A*:

$$I_A = \frac{1}{2}\pi\rho b r^4$$

Disk *B*:

$$I_B = \frac{1}{2}\pi\rho(3b)(nr)^4$$

$$= 3n^4\left[\frac{1}{2}\pi\rho b r^4\right]$$

$$= 3n^4 I_A$$

$$I_{\text{total}} = I_A + I_B = (1 + 3n^4)I_A \quad (1)$$

Angular velocity:

$$\omega_1 = 0$$

$$\omega_2 = 600 \text{ rpm}$$

$$= 20\pi \text{ rad/s}$$

Rotation:

$$\theta = 4 \text{ rev} = 8\pi \text{ rad}$$

Kinetic energy:

$$T_1 = 0$$

$$T_2 = \frac{1}{2}I_{\text{total}}\,\omega_2^2$$

Work:

$$U_{1\to 2} = M\theta$$

$$= (15 \text{ lb}\cdot\text{ft})(8\pi \text{ rad})$$

$$= 376.99 \text{ lb}\cdot\text{ft}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.3 (Continued)

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 376.991 = \frac{1}{2} I_{\text{total}} (20\pi)^2$$

$$I_{\text{total}} = 0.19099 \text{ slug} \cdot \text{ft}^2$$

But, $I_A = \frac{1}{2} m_A r_A^2$

$$= \frac{1}{2}\left(\frac{30 \text{ lb}}{32.2}\right)\left(\frac{5}{12}\text{ft}\right)^2$$

$$= 0.080875 \text{ slug} \cdot \text{ft}^2$$

From (1) $0.19099 = (1 + 3n^4)(0.080875)$

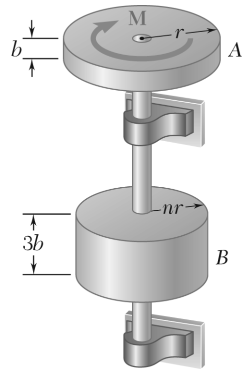
$$n^4 = 0.45383$$

$$n = 0.82078$$

Radius of disk B: $r_B = nr_A = (0.82078)(5 \text{ in.}) = 4.1039 \text{ in.}$ $r_B = 4.10 \text{ in.}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

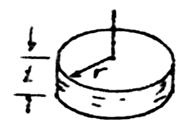
STUDENTS-HUB.com



PROBLEM 17.4

Two disks of the same material are attached to a shaft as shown. Disk A is of radius r and has a thickness b, while disk B is of radius nr and thickness $3b$. A couple **M** of constant magnitude is applied when the system is at rest and is removed after the system has executed 2 revolutions. Determine the value of n which results in the largest final speed for a point on the rim of disk B.

SOLUTION



For any disk:

$$m = \rho(\pi r^2 t)$$

$$\bar{I} = \frac{1}{2}mr^2$$

$$= \frac{1}{2}\pi\rho t r^4$$

Moment of inertia.

Disk A:

$$I_A = \frac{1}{2}\pi\,\rho\,b\,r^4$$

Disk B:

$$I_B = \frac{1}{2}\pi\,\rho(3b)(nr)^4$$

$$= 3n^4\left[\frac{1}{2}\pi\rho b r^4\right]$$

$$= 3n^4 I_A$$

$$I_{\text{total}} = I_A + I_B$$

$$= (1+3n^4)I_A$$

Work-energy.

$$T_1 = 0 \qquad U_{1\to 2} = M\theta = M(4\pi\,\text{rad})$$

$$T_2 = \frac{1}{2}I_{\text{total}}\,\omega_2^2$$

$$T_1 + U_{1\to 2} = T_2: \quad 0 + M(4\pi) = \frac{1}{2}(1+3n^4)I_A\omega_2^2$$

$$\omega_2^2 = \frac{8\pi M}{(1+3n^4)I_A}$$

For Point D on rim of disk B

$$v_D = (nr)\omega_2 \quad \text{or} \quad v_D^2 = n^2r^2\omega_2^2 = \frac{8\pi M r^2}{I_A}\cdot\frac{n^2}{1+3n^4}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.4 (Continued)

Value of n for maximum final speed.

For maximum v_D: $\frac{d}{dn}\left(\frac{n^2}{1+3n^4}\right)=0$

$$\frac{1}{(1+3n^4)^2}[n^2(12n^3)-(1+3n^4)(2n)]=0$$

$$12n^5-2n-6n^5=0$$

$$2n(3n^4-1)=0$$

$n=0$ and $n=\left(\frac{1}{3}\right)^{0.25}=0.7598$ $\qquad n=0.760$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.5

The flywheel of a small punch rotates at 300 rpm. It is known that 1800 ft · lb of work must be done each time a hole is punched. It is desired that the speed of the flywheel after one punching be not less that 90 percent of the original speed of 300 rpm. (*a*) Determine the required moment of inertia of the flywheel. (*b*) If a constant 25-lb · ft couple is applied to the shaft of the flywheel, determine the number of revolutions which must occur between each punching, knowing that the initial velocity is to be 300 rpm at the start of each punching.

SOLUTION

Angular velocities: $\omega_1 = 300 \text{ rpm} = 10\pi \text{ rad/s}$

$$\omega_2 = 0.90\omega_1 = 9\pi \text{ rad/s}$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}\bar{I}(10\pi)^2$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\bar{I}(9\pi)$$

$$U_{1\to 2} = -1800 \text{ ft}\cdot\text{lb}$$

$$\frac{1}{2}\bar{I}(10\pi)^2 - 1800 = \frac{1}{2}\bar{I}(9\pi)^2$$

(*a*) *Required moment of inertia.*

$$\bar{I} = \frac{2(1800)}{\pi^2(100-81)} = 19.198 \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \bar{I} = 19.20 \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

(*b*) *Number of revolution between each punching.*

Definition of work: $U_{2\to 1} = M\theta$:

$$1800 \text{ ft}\cdot\text{lb} = (25 \text{ lb}\cdot\text{ft})\theta$$

$$\theta = 72 \text{ rad} = 11.459 \text{ rev} \qquad \theta = 11.46 \text{ rev} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.6

The flywheel of a punching machine has a mass of 300 kg and a radius of gyration of 600 mm. Each punching operation requires 2500 J of work. (*a*) Knowing that the speed of the flywheel is 300 rpm just before a punching, determine the speed immediately after the punching. (*b*) If a constant 25-N · m couple is applied to the shaft of the flywheel, determine the number of revolutions executed before the speed is again 300 rpm.

SOLUTION

Moment of inertia.

$$I = mk^2 = (300 \text{ kg})(0.6 \text{ m})^2 = 108 \text{ kg}\cdot\text{m}^2$$

Kinetic energy. *Position 1.*

$$\omega_1 = 300 \text{ rpm} = 10\pi \text{ rad/s}$$

$$T_1 = \frac{1}{2}I\omega_1^2 = \frac{1}{2}(108)(10\pi)^2 = 53.296\times 10^3 \text{ J}$$

Position 2.

$$T_2 = \frac{1}{2}I\omega_2^2 = 54\omega_2^2$$

Work.

$$U_{1\to 2} = -2500 \text{ J}$$

Principle of work and energy for punching.

$$T_1 + U_{1\to 2} = T_2: \quad 53.296\times 10^3 - 2500 = 54\omega_2^2$$

(*a*)

$$\omega_2^2 = 940.66$$

$$\omega_2 = 30.67 \text{ rad/s} \qquad \omega_2 = 293 \text{ rpm} \blacktriangleleft$$

Principle of work and energy for speed recovery.

$$T_2 + U_{2\to 1} = T_1$$

$$U_{2\to 1} = 2500 \text{ J}$$

$$M = 25 \text{ N}\cdot\text{m}$$

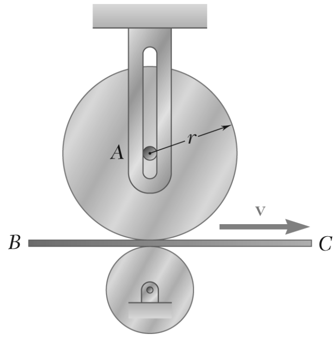
$$U_{2\to 1} = M\theta \quad 2500 = 25\theta \qquad \theta = 100 \text{ rad}$$

(*b*)

$$\theta = 15.92 \text{ rev} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

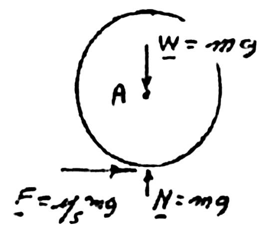
STUDENTS-HUB.com



PROBLEM 17.7

Disk A, of weight 10 lb and radius $r = 6$ in., is at rest when it is placed in contact with belt BC, which moves to the right with a constant speed $v = 40$ ft/s. Knowing that $\mu_k = 0.20$ between the disk and the belt, determine the number of revolutions executed by the disk before it attains a constant angular velocity.

SOLUTION



Work of external friction force on disk A.

Only force doing work is F. Since its moment about A is $M = rF$, we have

$$\begin{aligned} U_{1\to 2} &= M\theta \\ &= rF\theta \\ &= r(\mu_k mg)\theta \end{aligned}$$

Kinetic energy of disk A.

Angular velocity becomes constant when $\quad \omega_2 = \dfrac{v}{r}$

$$\begin{aligned} T_1 &= 0 \\ T_2 &= \frac{1}{2}\bar{I}\omega_2^2 \\ &= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 \\ &= \frac{mv^2}{4} \end{aligned}$$

Principle of work and energy for disk A.

$$T_1 + U_{1-2} = T_2: \quad 0 + r\mu_k mg\theta = \frac{mv^2}{4}$$

Angle change $\quad \theta = \dfrac{v^2}{4r\mu_k g}\,\text{rad} \qquad \theta = \dfrac{v^2}{8\pi r\mu_k g}\,\text{rev}$

Data:

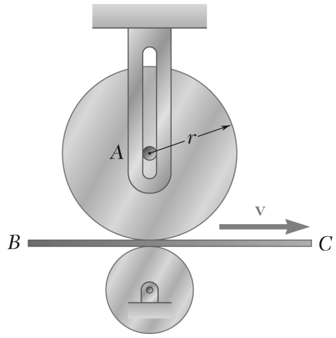
$$\begin{aligned} r &= 0.5\ \text{ft} \\ \mu_k &= 0.20 \\ v &= 40\ \text{ft/s} \end{aligned}$$

$$\theta = \frac{(40\ \text{ft/s})^2}{8\pi(0.5\ \text{ft})(0.20)(32.2\ \text{ft/s}^2)}$$

$\theta = 19.77$ rev ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

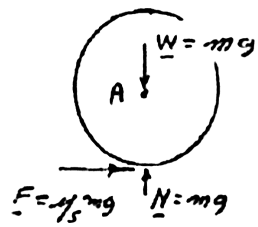
STUDENTS-HUB.com



PROBLEM 17.8

Disk A is of constant thickness and is at rest when it is placed in contact with belt BC, which moves with a constant velocity $\mathbf{v}$. Denoting by μ_k the coefficient of kinetic friction between the disk and the belt, derive an expression for the number of revolutions executed by the disk before it attains a constant angular velocity.

SOLUTION



Work of external friction force on disk A.

Only force doing work is F. Since its moment about A is $M = rF$, we have

$$U_{1\to 2} = M\theta = rF\theta = r(\mu_k mg)\theta$$

Kinetic energy of disk A.

Angular velocity becomes constant when $\omega_2 = \dfrac{v}{r}$

$$T_1 = 0$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{mv^2}{4}$$

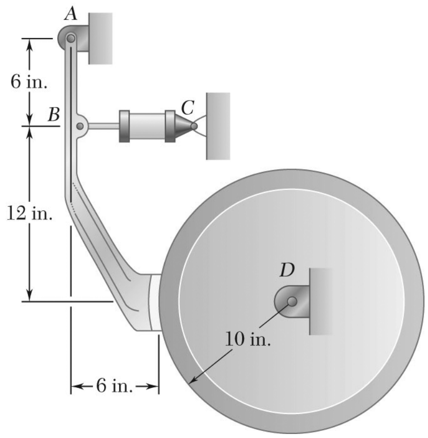
Principle of work and energy for disk A.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + r\mu_k mg\,\theta = \frac{mv^2}{4}$$

Angle change. $\theta = \dfrac{v^2}{4r\mu_k g}$ rad $\qquad \theta = \dfrac{v^2}{8\pi r\mu_k g}$ rev ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.9

The 10-in.-radius brake drum is attached to a larger flywheel which is not shown. The total mass moment of inertia of the flywheel and drum is 16 lb · ft · s^2 and the coefficient of kinetic friction between the drum and the brake shoe is 0.40. Knowing that the initial angular velocity is 240 rpm clockwise, determine the force which must be exerted by the hydraulic cylinder if the system is to stop in 75 revolutions.

SOLUTION

Kinetic energies.

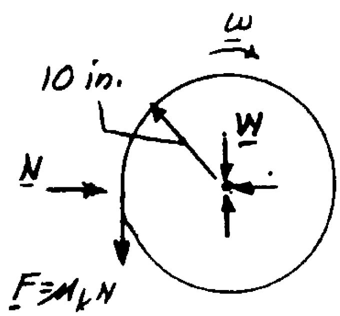
$$\omega_1 = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

$$\bar{I} = 16 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(16)(8\pi)^2 = 5053 \text{ ft} \cdot \text{lb}$$

$$\omega_2 = 0 \qquad T_2 = 0$$

Angular displacement. $\theta = 75 \text{ rev} = 75(2\pi) = 150\pi \text{ rad}$



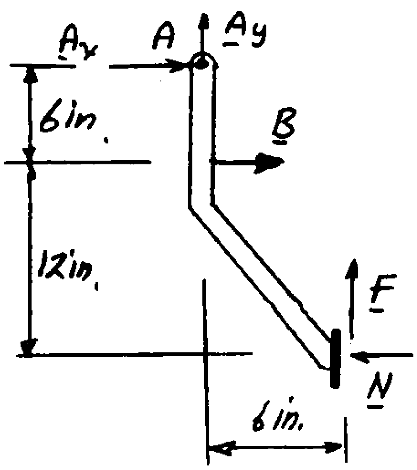
Work. $U_{1-2} = -M\theta = -\left[F\left(\frac{10}{12}\text{ft}\right)\right](150\pi \text{ rad}) = -392.7F$

Principle of work and energy. $T_1 + U_{1-2} = T_2:$

$$5053 - 392.7F = 0 \qquad F = 12.868 \text{ lb}$$

$$F = \mu_k N: \quad 12.868 = (0.40)N \qquad N = 32.17 \text{ lb}$$

Free body brake arm:



$$+\circlearrowleft \Sigma M_A = 0: \quad B(6 \text{ in.}) + F(6 \text{ in.}) - N(18 \text{ in.}) = 0$$

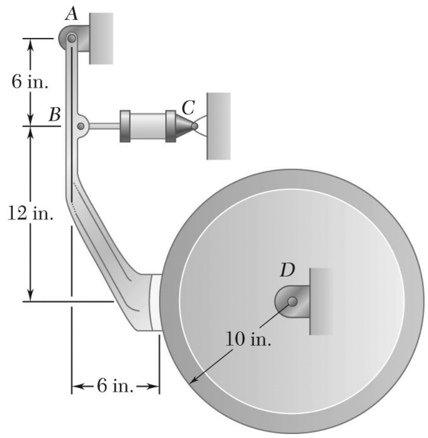
$$B(6 \text{ in.}) + (12.868 \text{ lb})(6 \text{ in.}) - (32.17 \text{ lb})(18 \text{ in.}) = 0$$

$$B = 83.64 \text{ lb}$$

$B = 83.6$ lb ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

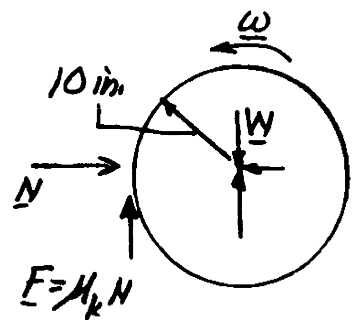


PROBLEM 17.10

Solve Problem 17.9, assuming that the initial angular velocity of the flywheel is 240 rpm counterclockwise.

PROBLEM 17.9 The 10-in.-radius brake drum is attached to a larger flywheel which is not shown. The total mass moment of inertia of the flywheel and drum is $16\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.40. Knowing that the initial angular velocity is 240 rpm clockwise, determine the force which must be exerted by the hydraulic cylinder if the system is to stop in 75 revolutions.

SOLUTION



Kinetic energies.

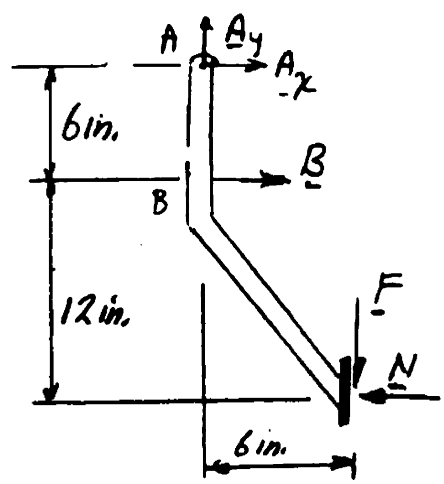
$$\omega_1 = 240\ \text{rpm} = 8\pi\ \text{rad/s}$$

$$\bar{I} = 16\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(16)(8\pi)^2 = 5053\ \text{ft}\cdot\text{lb}$$

$$\omega_2 = 0 \qquad T_2 = 0$$

Angular displacement. $\quad \theta = 75\ \text{rev} = 75(2\pi) = 150\pi\ \text{rad}$



Work. $\quad U_{1-2} = -M\theta = -\left[F\left(\frac{10}{12}\text{ft}\right)\right](150\pi\ \text{rad}) = -392.7F$

Principle of work and energy. $\quad T_1 + U_{1-2} = T_2:$

$$5053 - 392.7F = 0 \qquad F = 12.868\ \text{lb}$$

$$F = \mu_k N: \quad 12.868 = (0.40)N \qquad N = 32.17\ \text{lb}$$

Free body brake arm:

$$+\circlearrowleft \Sigma M_A = 0: \quad B(6\ \text{in.}) - F(6\ \text{in.}) - N(18\ \text{in.}) = 0$$

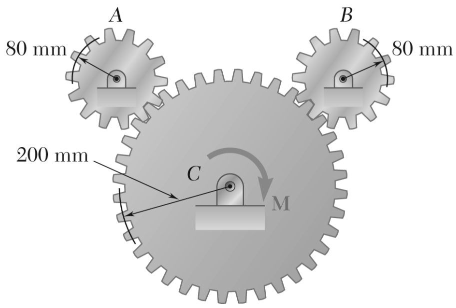
$$B(6\ \text{in.}) - (12.868\ \text{lb})(6\ \text{in.}) - (32.17\ \text{lb})(18\ \text{in.}) = 0$$

$$B = 109.37\ \text{lb}$$

$B = 109.4\ \text{lb}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.11

Each of the gears A and B has a mass of 2.4 kg and a radius of gyration of 60 mm, while gear C has a mass of 12 kg and a radius of gyration of 150 mm. A couple **M** of constant magnitude 10 N · m is applied to gear C. Determine (a) the number of revolutions of gear C required for its angular velocity to increase from 100 to 450 rpm, (b) the corresponding tangential force acting on gear A.

SOLUTION

Moments of inertia.

Gears A and B: $$I_A = I_B = mk^2 = (2.4)(0.06)^2 = 8.64\times10^{-3}\,\text{kg}\cdot\text{m}^2$$

Gear C: $$I_C = (12)(0.15)^2 = 270\times10^{-3}\,\text{kg}\cdot\text{m}^2$$

Kinematics. $$r_A\omega_A = r_B\omega_B = r_C\omega_C$$

$$\omega_A = \omega_B = \frac{200}{80}\omega_C = 2.5\omega_C$$

$$\theta_A = \theta_B = 2.5\theta_C$$

Kinetic energy. $$T = \frac{1}{2}I\omega^2:$$

Position 1. $$\omega_C = 100\text{ rpm} = \frac{10}{3}\pi\text{ rad/s}$$

$$\omega_A = \omega_B = 250\text{ rpm} = \frac{25}{3}\pi\text{ rad/s}$$

Gear A: $$(T_1)_A = \frac{1}{2}(8.64\times10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$$

Gear B: $$(T_1)_B = \frac{1}{2}(8.64\times10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$$

Gear C: $$(T_1)_C = \frac{1}{2}(270\times10^{-3})\left(\frac{10\pi}{3}\right)^2 = 14.8044\text{ J}$$

System: $$T_1 = (T_1)_A + (T_1)_B + (T_1)_C = 20.726\text{ J}$$

Position 2. $$\omega_C = 450\text{ rpm} = 15\pi\text{ rad/s}$$

$$\omega_A = \omega_B = 37.5\pi\text{ rad/s}$$

Gear A: $$(T_2)_A = \frac{1}{2}(8.64\times10^{-3})(37.5\pi)^2 = 59.957\text{ J}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.11 (Continued)

Gear B: $(T_2)_B = \frac{1}{2}(8.64\times 10^{-3})(37.5\pi)^2 = 59.957\text{ J}$

Gear C: $(T_2)_C = \frac{1}{2}(270\times 10^{-3})(15\pi)^2 = 299.789\text{ J}$

System: $T_2 = (T_2)_A + (T_2)_B + (T_2)_C = 419.7\text{ J}$

Work of couple. $U_{1\to 2} = M\theta_C = 10\theta_C$

Principle of work and energy for system.

$$T_1 + U_{1\to 2} = T_2: \quad 20.726 + 10\theta_C = 419.7$$

$$\theta_C = 39.898\text{ radians}$$

(*a*) Rotation of gear C. $\theta_C = 6.35\text{ rev}$ ◀

Rotation of gear A. $\theta_A = (2.5)(39.898)$
$= 99.744\text{ radians}$

Principle of work and energy for gear A.

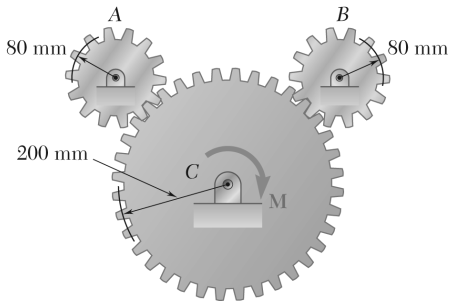
$$(T_1)_A + M_A\theta_A = (T_2)_A: \quad 2.9609 + M_A(99.744) = 59.957$$

$$M_A = 0.57142\text{ N}\cdot\text{m}$$

(*b*) Tangential force on gear A. $F_t = \frac{M_A}{r_A} = \frac{0.57142}{0.08}$ $F_t = 7.14\text{ N}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.12

Solve Problem 17.11, assuming that the 10-N · m couple is applied to gear *B*.

PROBLEM 17.11 Each of the gears *A* and *B* has a mass of 2.4 kg and a radius of gyration of 60 mm, while gear *C* has a mass of 12 kg and a radius of gyration of 150 mm. A couple **M** of constant magnitude 10 N · m is applied to gear *C*. Determine (*a*) the number of revolutions of gear *C* required for its angular velocity to increase from 100 to 450 rpm, (*b*) the corresponding tangential force acting on gear *A*.

SOLUTION

Moments of inertia.

Gears *A* and *B*: $I_A = I_B = mk^2 = (2.4)(0.06)^2 = 8.64\times 10^{-3}\,\text{kg}\cdot\text{m}^2$

Gear *C*: $I_C = (12)(0.15)^2 = 270\times 10^{-3}\,\text{kg}\cdot\text{m}^2$

Kinematics.

$$r_A\omega_A = r_B\omega_B = r_C\omega_C$$

$$\omega_A = \omega_B = \frac{200}{80}\omega_C = 2.5\omega_C$$

$$\theta_A = \theta_B = 2.5\theta_C$$

Kinetic energy. $T = \frac{1}{2}I\omega^2$:

Position 1.

$$\omega_C = 100\text{ rpm} = \frac{10}{3}\pi\text{ rad/s}$$

$$\omega_A = \omega_B = 250\text{ rpm} = \frac{25}{3}\pi\text{ rad/s}$$

Gear *A*: $(T_1)_A = \frac{1}{2}(8.64\times 10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$

Gear *B*: $(T_1)_B = \frac{1}{2}(8.64\times 10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$

Gear *C*: $(T_1)_C = \frac{1}{2}(270\times 10^{-3})\left(\frac{10\pi}{3}\right)^2 = 14.8044\text{ J}$

System: $T_1 = (T_1)_A + (T_1)_B + (T_1)_C = 20.726\text{ J}$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Position 2. $\omega_C = 450 \text{ rpm} = 15\pi \text{ rad/s}$

$\omega_A = \omega_B = 37.5\pi \text{ rad/s}$

Gear A: $$(T_2)_A = \frac{1}{2}(8.64\times10^{-3})(37.5\pi)^2 = 59.957 \text{ J}$$

Gear B: $$(T_2)_B = \frac{1}{2}(8.64\times10^{-3})(37.5\pi)^2 = 59.957 \text{ J}$$

Gear C: $$(T_2)_C = \frac{1}{2}(270\times10^{-3})(15\pi)^2 = 299.789 \text{ J}$$

System: $$T_2 = (T_2)_A + (T_2)_B + (T_2)_C = 419.7 \text{ J}$$

Work of couple. $$U_{1\to2} = M\theta_B = 10\theta_B$$

Principle of work and energy for system.

$$T_1 + U_{1\to2} = T_2: \quad 20.726 + 10\theta_B = 419.7$$

$$\theta_B = 39.898 \text{ radians}$$

(*a*) Rotation of gear C. $$\theta_C = \frac{39.898}{2.5} = 15.959 \text{ radians}$$ $\theta_C = 2.54 \text{ rev}$ ◀

Rotation of gear A. $$\theta_A = \theta_B = 39.898 \text{ radians}$$

Principle of work and energy for gear A.

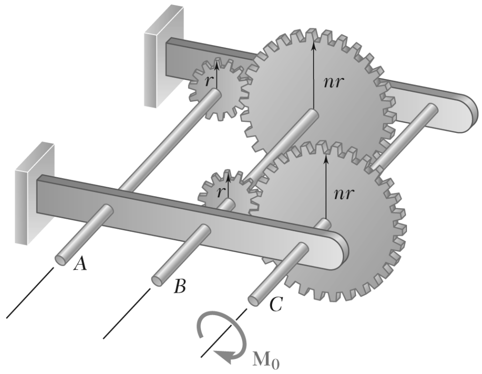
$$(T_1)_A + M_A\theta_A = (T_2)_A: \quad 2.9609 + M_A(39.898) = 59.957$$

$$M_A = 1.4285 \text{ N}\cdot\text{m}$$

(*b*) Tangential force on gear A. $$F_t = \frac{M_A}{r_A} = \frac{1.4285}{0.08}$$ $F_t = 17.86 \text{ N}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.13

The gear train shown consists of four gears of the same thickness and of the same material; two gears are of radius r, and the other two are of radius nr. The system is at rest when the couple $\mathbf{M}_0$ is applied to shaft C. Denoting by I_0 the moment of inertia of a gear of radius r, determine the angular velocity of shaft A if the couple $\mathbf{M}_0$ is applied for one revolution of shaft C.

SOLUTION

Mass and moment of inertia:

For a disk of radius r and thickness t: $\quad m = \rho(\pi r^2)t = \rho\pi t r^2$

$$\bar{I}_0 = \frac{1}{2}mr^2 = \frac{1}{2}(\rho\pi t r^2)r^2 = \frac{1}{2}\rho\pi t r^4$$

For a disk of radius nr and thickness t, $\quad \bar{I} = \frac{1}{2}\rho\pi t(nr)^4 \qquad \bar{I} = n^4\bar{I}_0$

Kinematics: If for shaft A we have ω_A

Then, for shaft B we have $\omega_B = \omega_A/n$

And, for shaft C we have $\omega_C = \omega_A/n^2$

Principle of work-energy:

Couple M_0 applied to shaft C for one revolution. $\theta = 2\pi$ radians, $\quad T_1 = 0,$

$$U_{1-2} = M_0\theta = M_0(2\pi \text{ radians}) = 2\pi M_0$$

$$T_2 = \frac{1}{2}(\bar{I}_{\text{shaft }A})w_A^2 + \frac{1}{2}(\bar{I}_{\text{shaft }B})\omega_B^2 + \frac{1}{2}(\bar{I}_{\text{shaft }C})\omega_C^2$$

$$= \frac{1}{2}\bar{I}_0 w_A^2 + \frac{1}{2}(\bar{I}_0 + n^4\bar{I}_0)\left(\frac{\omega_A}{n}\right)^2 + \frac{1}{2}(n^4\bar{I}_0)\left(\frac{\omega_A}{n^2}\right)^2$$

$$= \frac{1}{2}\bar{I}_0\omega_A^2\left(n^2 + 2 + \frac{1}{n^2}\right)$$

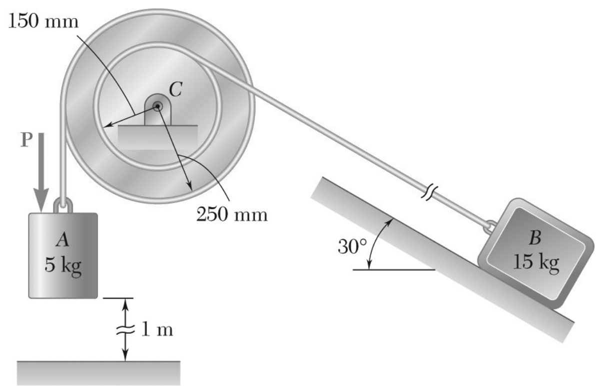
$$= \frac{1}{2}\bar{I}_0\omega_A^2\left(n + \frac{1}{n}\right)^2$$

$$T_1 + U_{1-2} = T_2: \quad 0 + 2\pi M_0 = \frac{1}{2}\bar{I}_0\omega_A^2\left(n + \frac{1}{n}\right)^2$$

Angular velocity. $\quad \omega_A^2 = \dfrac{4\pi M_0}{\bar{I}_0}\dfrac{1}{\left(n+\frac{1}{n}\right)^2} \qquad\qquad \omega_A = \dfrac{2n}{n^2+1}\sqrt{\dfrac{\pi M_0}{\bar{I}_0}}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.14

The double pulley shown has a mass of 15 kg and a centroidal radius of gyration of 160 mm. Cylinder *A* and block *B* are attached to cords that are wrapped on the pulleys as shown. The coefficient of kinetic friction between block *B* and the surface is 0.2. Knowing that the system is at rest in the position shown when a constant force $P = 200$ N is applied to cylinder *A*, determine (*a*) the velocity of cylinder *A* as it strikes the ground, (*b*) the total distance that block *B* moves before coming to rest.

SOLUTION

Kinematics. Let r_A be the radius of the outer pulley and r_B that of the inner pulley.

$$v_A = r_A\omega_C \qquad v_B = r_B\omega_C = \frac{r_B}{r_A}v_A$$

$$s_A = r_A\theta_C \qquad s_B = \frac{r_B}{r_A}s_A$$

Use the principle of work and energy with position 1 being the initial rest position and position 2 being when cylinder *A* strikes the ground.

$$T_1 + U_{1\to 2} = T_2:$$

where $$T_1 = 0$$

and $$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}\bar{I}_C\omega_C^2$$

with $\quad m_A = 5\text{ kg}, \quad m_B = 15\text{ kg}, \quad \bar{I}_C = m_C\bar{k}_C^2 = (15\text{ kg})(0.160\text{ m})^2 = 0.384\text{ kg}\cdot\text{m}^2$

$$\begin{aligned}T_2 &= \frac{1}{2}\left[m_A + \frac{m_B r_B^2}{r_A^2} + \frac{\bar{I}_C}{r_A^2}\right]v_A^2 \\ &= \frac{1}{2}\left[5\text{ kg} + \frac{(15\text{ kg})(0.150\text{ m})^2}{(0.250\text{ m})^2} + \frac{0.384\text{ kg}\cdot\text{m}^2}{(0.250\text{ m})^2}\right]v_A^2 \\ &= (8.272\text{ kg})v_A^2\end{aligned}$$

Principle of work and energy applied to the system consisting of blocks *A* and *B* and the double pulley *C*.

Work. $$U_{1\to 2} = Ps_A + m_A g\, s_A - F_F s_B - m_B g\, s_B \sin 30°$$

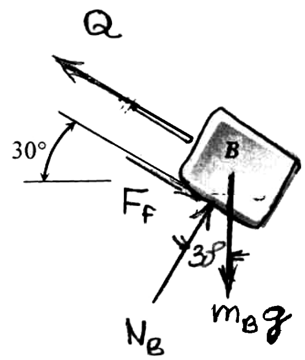
where $$s_A = 1\text{ m}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

and $$s_B = \frac{r_B}{r_A} s_A = \frac{0.150 \text{ m}}{0.250 \text{ m}}(1 \text{ m}) = 0.6 \text{ m}$$

To find F_f use the free body diagram of block B.



$$\measuredangle 60° \; \Sigma F = 0: \quad N_B - m_B g \cos 30° = 0$$

$$N_B = m_B g \cos 30° = (15 \text{ kg})(9.81 \text{ m/s})\cos 30° = 127.44 \text{ N}$$

$$F_f = \mu_k N_B = (0.2)(127.44 \text{ N}) = 25.487 \text{ N}$$

$$\begin{aligned} U_{1\to 2} &= (200 \text{ N})(1 \text{ m}) + (5 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) \\ &\quad - (25.487 \text{ N})(0.6 \text{ m}) - (15 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m})\sin 30° \\ &= 189.613 \text{ J} \end{aligned}$$

Work-energy: $0 + 189.613 \text{ J} = (8.272 \text{ kg})v_A^2$

(*a*) *Velocity of A*. $v_A = 4.7877$ m/s $\qquad \mathbf{v}_A = 4.79 \text{ m/s} \uparrow$ ◀

when the cylinder strikes the ground,

$$v_B = \frac{r_B}{r_A} v_A = \frac{0.150 \text{ m}}{0.250 \text{ m}}(4.7877 \text{ m/s}) = 2.8726 \text{ m/s}$$

$$\omega_C = \frac{v_A}{r_A} = \frac{4.7877 \text{ m/s}}{0.250 \text{ m}} = 19.1508 \text{ rad/s}$$

After the cylinder strikes the ground use the principle of work and energy applied to a system consisting of block B and double pulley C.

Let T_3 be its kinetic energy when A strikes the ground.

$$\begin{aligned} T_3 &= \frac{1}{2} m_B v_B^2 + \frac{1}{2} \bar{I}_C \omega_C^2 \\ &= \frac{1}{2}(15 \text{ kg})(2.8726 \text{ m/s})^2 + \frac{1}{2}(0.384 \text{ kg} \cdot \text{m}^2)(19.1508 \text{ rad/s})^2 \\ &= 132.305 \text{ J} \end{aligned}$$

When the system comes to rest, $T_4 = 0$

$$\begin{aligned} U_{3\to 4} &= -(25.487 \text{ N})s'_B - (15 \text{ kg})(9.81 \text{ m/s}^2)(s'_B \sin 30°) \\ &= -(99.062 \text{ N})s'_B \end{aligned}$$

where s'_B is the additional travel of block B.

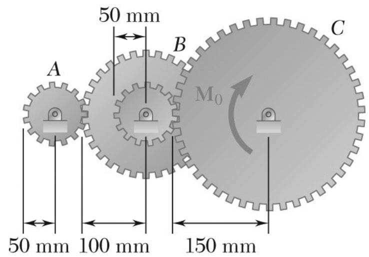
$$T_3 + U_{3\to 4} = T_4: \quad 132.305 \text{ J} - (99.062 \text{ N})s'_B = 0$$

$$s'_B = 1.3356 \text{ m}$$

(*b*) *Total distance:* $\qquad s_B + s'_B = 1.936 \text{ m}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.15

Gear A has a mass of 1 kg and a radius of gyration of 30 mm; gear B has a mass of 4 kg and a radius of gyration of 75 mm; gear C has a mass of 9 kg and a radius of gyration of 100 mm. The system is at rest when a couple $\mathbf{M}_0$ of constant magnitude 4 N · m is applied to gear C. Assuming that no slipping occurs between the gears, determine the number of revolutions required for disk A to reach an angular velocity of 300 rpm.

SOLUTION

Moments of inertia: $\bar{I} = mk^2$

Gear A: $I_A = (1\text{ kg})(0.030\text{ m})^2 = 0.9\times10^{-3}\text{ kg}\cdot\text{m}^2$

Gear B: $I_B = (4\text{ kg})(0.075\text{ m})^2 = 22.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

Gear C: $I_C = (9\text{ kg})(0.100\text{ m})^2 = 90\times10^{-3}\text{ kg}\cdot\text{m}^2$

Let r_A be the radius of gear A, r_1 the outer radius of gear B, r_2 the inner radius of gear B, and r_C the radius of gear C.

$$r_A = 50\text{ mm},\quad r_1 = 100\text{ mm},\quad r_2 = 50\text{ mm},\quad r_C = 150\text{ mm}$$

At the contact point between gears A and B,

$$r_1\omega_B = r_A\omega_A:\quad \omega_B = \frac{r_A}{r_1}\omega_A = 0.5\omega_A$$

At the contact point between gear B and C.

$$r_C\omega_C = r_2\omega_B:\quad \omega_C = \frac{r_2}{r_C}\omega_B = 0.33333\omega_B$$

$$\omega_C = 0.16667\omega_A$$

Kinetic energy:

$$T = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 + \frac{1}{2}I_C\omega_C^2$$

$$T = \frac{1}{2}[0.9\times10^{-3}\omega_A^2 + (22.5\times10^{-3})(0.5\omega_A)^2 + (90\times10^{-3})(0.16667\omega_A)^2]$$

$$= (4.5125\times10^{-3}\text{ kg}\cdot\text{m}^2)\omega_A^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.15 (Continued)

Use the principle of work and energy applied to the system of all three gears with position 1 being the initial rest position and position 2 being when $\omega_A = 300$ rpm.

$$\omega_A = \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} \cdot 300 \frac{\text{rev}}{\text{min}} = 31.416 \text{ rad/s}$$

$$T_1 = 0$$

$$T_2 = (4.5125 \times 10^{-3} \text{ kg} \cdot \text{m}^2)(31.416 \text{ rad/s})^2 = 4.4565 \text{ J}$$

$$U_{1\to2} = M\theta_C = (4 \text{ N} \cdot \text{m})\theta_C$$

Principle of work and energy.

$$T_1 + U_{1\to2} = T_2: \quad 0 + 4.4565 \text{ J} = 4(\text{N} \cdot \text{m})\theta_C$$

$$\theta_C = 1.11413 \text{ rad}$$

$$\theta_A = \frac{\theta_C}{0.16667} = 6\theta_C = 6.6848 \text{ rad}$$

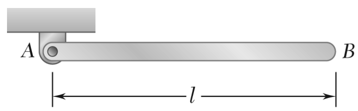
$$\theta_A = \frac{6.6848 \text{ rad}}{2\pi \text{ rad/rev}}$$

$\theta_A = 1.063$ rev ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

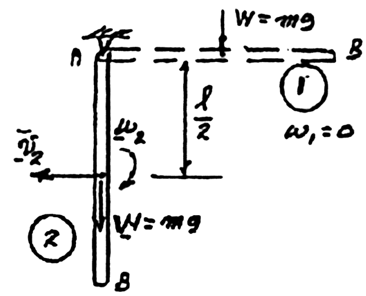
STUDENTS-HUB.com

PROBLEM 17.16



A slender rod of length l and weight W is pivoted at one end as shown. It is released from rest in a horizontal position and swings freely. (*a*) Determine the angular velocity of the rod as it passes through a vertical position and determine the corresponding reaction at the pivot, (*b*) Solve part *a* for $W = 1.8$ lb and $l = 3$ ft.

SOLUTION



Position 1:

$$v_1 = 0$$
$$\omega_1 = 0$$
$$T_1 = 0$$
$$\bar{v}_2 = \frac{l}{2}\omega_2$$

Position 2:

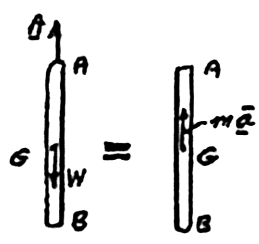
$$T_2 = \frac{1}{2}m\bar{r}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}m\left(\frac{l}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_2^2$$
$$T_2 = \frac{1}{6}ml^2\omega_2^2$$

Work: $$U_{1\to 2} = mg\frac{l}{2}$$

Principle of work and energy: $$T_1 + U_{1\to 2} = T_2$$
$$0 + mg\frac{l}{2} = \frac{1}{6}ml^2\omega_2^2$$

(*a*) Expressions for angular velocity and reactions.

$$\omega_2^2 = \frac{3g}{l} \qquad \boldsymbol{\omega}_2 = \sqrt{\frac{3g}{l}} \circlearrowright \blacktriangleleft$$



$$\bar{a} = \frac{l}{2}\omega_2^2 = \frac{l}{2}\cdot\frac{3g}{l} = \frac{3}{2}g$$

$$+\uparrow \Sigma F = \Sigma(F)_{\text{eff}}: \quad A - W = m\bar{a}$$
$$A - mg = m\frac{3}{2}g$$
$$A = \frac{5}{2}mg \qquad \mathbf{A} = \frac{5}{2}W\uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.16 (Continued)

(*b*) Application of data:

$$W = 1.8 \text{ lb}, \quad l = 3 \text{ ft}$$

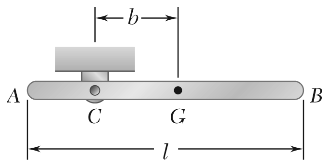
$$\omega_2^2 = \frac{3g}{l} = \frac{3g}{3} = 32.2 \text{ rad}^2/\text{s}^2 \qquad \boldsymbol{\omega}_2 = 5.67 \text{ rad/s} \circlearrowright \blacktriangleleft$$

$$A = \frac{5}{2}W = \frac{5}{2}(1.8 \text{ lb}) \qquad \mathbf{A} = 4.5 \text{ lb} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

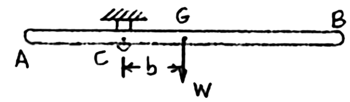
STUDENTS-HUB.com

PROBLEM 17.17



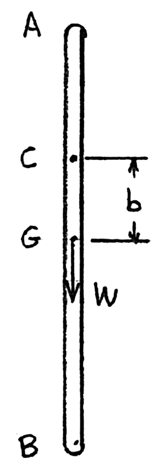
A slender rod of length l is pivoted about a Point C located at a distance b from its center G. It is released from rest in a horizontal position and swings freely. Determine (*a*) the distance b for which the angular velocity of the rod as it passes through a vertical position is maximum, (*b*) the corresponding values of its angular velocity and of the reaction at C.

SOLUTION



Position 1. $\quad \bar{v} = 0, \quad \omega = 0 \quad T_1 = 0$

Elevation: $\quad h = 0 \quad V_1 = mgh = 0$



Position 2. $\quad \bar{v}_2 = b\omega_2$

$$I = \frac{1}{12}ml^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(b^2 + \frac{1}{12}l^2\right)\omega_2^2$$

Elevation: $\quad h = -b \quad V_2 = -mgb$

Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{2}m\left(b^2 + \frac{1}{12}l^2\right)\omega_2^2 - mgb$$

$$\omega_2^2 = \frac{2gb}{b^2 + \frac{1}{12}l^2}$$

(*a*) Value of b for maximum ω_2.

$$\frac{d}{db}\left(\frac{b}{b^2 + \frac{1}{12}l^2}\right) = \frac{\left(b^2 + \frac{1}{12}l^2\right) - b(2b)}{\left(b^2 + \frac{1}{12}l^2\right)^2} = 0 \qquad b^2 = \frac{1}{12}l^2 \qquad b = \frac{l}{\sqrt{12}} \blacktriangleleft$$

(*b*) Angular velocity.

$$\omega_2^2 = \frac{2g\frac{l}{\sqrt{12}}}{\frac{l^2}{12} + \frac{l^2}{12}}$$

$$= \sqrt{12}\frac{g}{l}$$

$$\omega_2 = 12^{1/4}\sqrt{\frac{g}{l}} \qquad \omega_2 = 1.861\sqrt{\frac{g}{l}} \blacktriangleleft$$

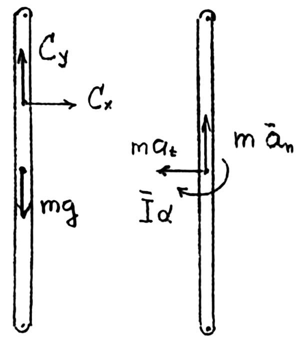
PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.17 (Continued)

Reaction at C.

$$a_n = b\omega_2^2$$
$$= \frac{l}{\sqrt{12}}\sqrt{12}\frac{g}{l}$$
$$= g$$



$$+\uparrow \Sigma F_y = ma_n: \quad C_y - mg = mg$$
$$C_y = 2mg$$

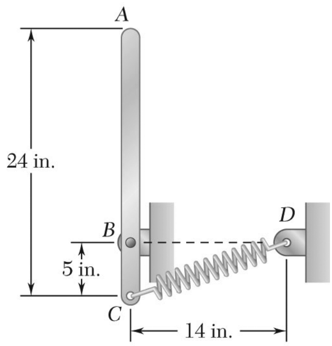
$$+\circlearrowright \Sigma M_C = mba_t + \bar{I}\alpha: \quad 0 = (mb^2 + \bar{I})\alpha$$
$$\alpha = 0, \quad a_t = 0$$

$$\xrightarrow{+} \Sigma F_x = ma_t: \quad C_x = -ma_t = 0$$

$$\mathbf{C} = 2mg \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

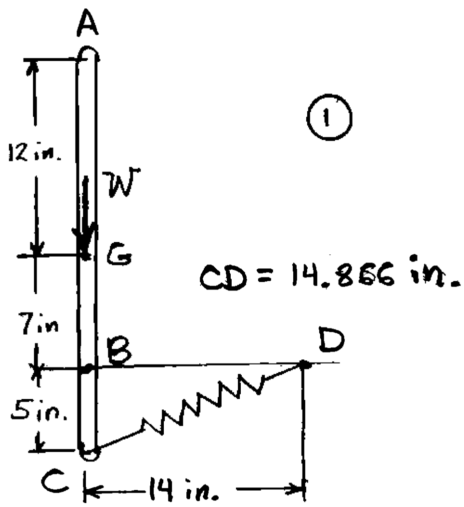
STUDENTS-HUB.com

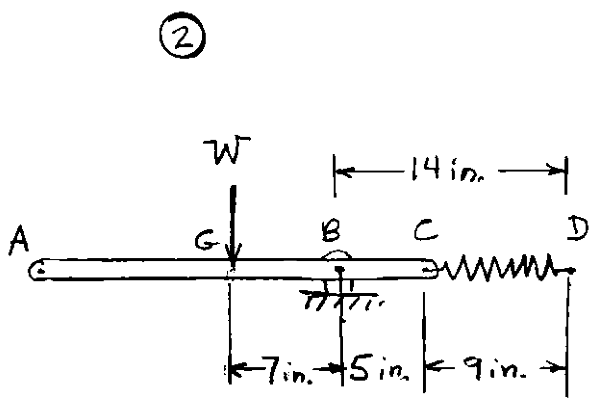


PROBLEM 17.18

A slender 9 lb rod can rotate in a vertical plane about a pivot at *B*. A spring of constant $k = 30$ lb/ft and of unstretched length 6 in. is attached to the rod as shown. Knowing that the rod is released from rest in the position shown, determine its angular velocity after it has rotated through 90°.

SOLUTION





Position 1:

Spring: $x_1 = CD - (\overbrace{6\text{ in.}}^{\text{Unstretched Length}}) = 14.866 - 6 = 8.8661\text{ in.} = 0.73884\text{ ft}$

$$V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(30\text{ lb/ft})(0.73884)^2 = 8.1882\text{ lb}\cdot\text{ft}$$

Gravity: $V_g = Wh = (9\text{ lb})\left(\frac{7}{12}\text{ft}\right) = 5.25\text{ lb}\cdot\text{ft}$

$$V_1 = V_e + V_g = 8.1882\text{ lb}\cdot\text{ft} + 5.25\text{ lb}\cdot\text{ft} = 13.438\text{ lb}\cdot\text{ft}$$

Kinetic energy: $T_1 = 0$

Position 2:

Spring: $x_2 = 9\text{ in.} - 6\text{ in.} = 3\text{ in.} = 0.25\text{ ft}$

$$V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(30\text{ lb/ft})(0.25\text{ ft})^2 = 0.9375\text{ lb}\cdot\text{ft}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.18 (Continued)

Gravity: $V_g = Wh = 0$

$$V_2 = V_e + V_g$$
$$= 0.9375 \text{ lb} \cdot \text{ft}$$

Kinetic energy: $\bar{v}_2 = r\omega_2 = \left(\frac{7}{12}\text{ ft}\right)\omega_2$

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}\left(\frac{9 \text{ lb}}{32.2}\right)(2 \text{ ft})^2 = 0.093168 \text{ slug} \cdot \text{ft}^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}\left(\frac{9 \text{ lb}}{32.2}\right)\left(\left(\frac{7}{12}\text{ ft}\right)\omega_2\right)^2 + \frac{1}{2}(0.093168)\omega_2^2$$

$$T_2 = 0.094138\omega_2^2$$

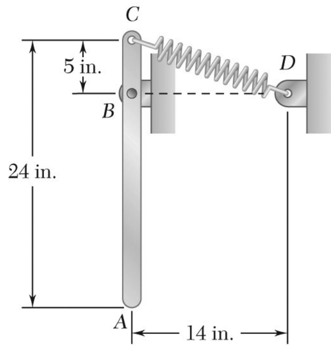
Conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 13.438 = 0.094138\,\omega_2^2 + 0.9375$$
$$\omega_2^2 = 132.79$$
$$\omega_2 = 11.524 \text{ rad/s}$$

$\boldsymbol{\omega}_2 = 11.52 \text{ rad/s} \circlearrowright$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

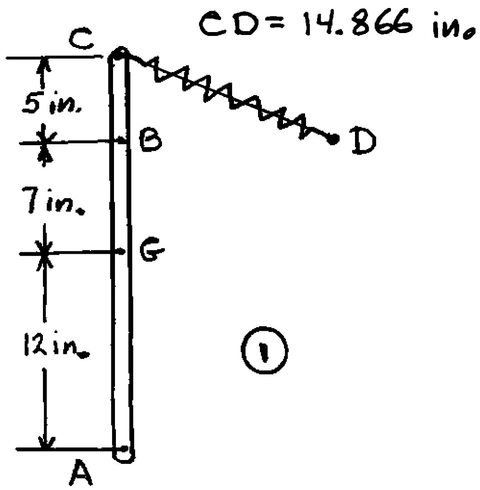
STUDENTS-HUB.com

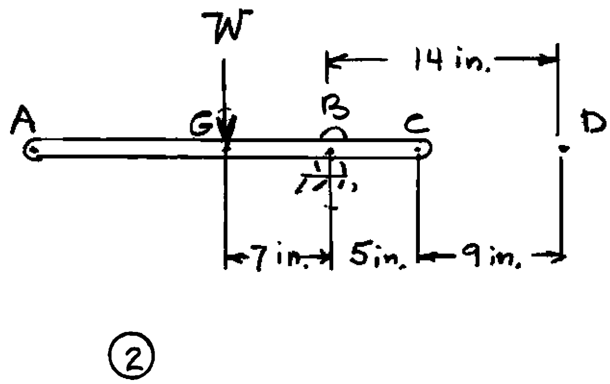


PROBLEM 17.19

A slender 9 lb rod can rotate in a vertical plane about a pivot at B. A spring of constant k = 30 lb/ft and of unstretched length 6 in. is attached to the rod as shown. Knowing that the rod is released from rest in the position shown, determine its angular velocity after it has rotated through 90°.

SOLUTION





Position 1: $CD = \sqrt{14^2 + 5^2} = 14.866 \text{ in.}$

Spring: $x_1 = CD - (\overbrace{6 \text{ in.}}^{\text{Unstretched Length}}) = 14.866 - 6 = 8.8661 \text{ in.} = 0.73884 \text{ ft}$

$$V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(30 \text{ lb/ft})(0.73884)^2 = 8.1882 \text{ lb} \cdot \text{ft}$$

Gravity: $V_g = Wh = 9 \text{ lb} = \left(-\frac{7}{12}\text{ft}\right) = -5.25 \text{ lb} \cdot \text{ft}$

$$V_1 = V_e + V_g = 8.1882 \text{ lb} \cdot \text{ft} - 5.25 \text{ lb} \cdot \text{ft} = 2.9382 \text{ lb} \cdot \text{ft}$$

Kinetic energy: $T_1 = 0$

Position 2:

Spring: $x_2 = 9 \text{ in.} - 6 \text{ in.} = 3 \text{ in.} = 0.25 \text{ ft}$

$$V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(30 \text{ lb/ft})(0.25 \text{ ft})^2 = 0.9375 \text{ lb} \cdot \text{ft}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.19 (Continued)

Gravity:
$$V_g = Wh = 0$$
$$V_2 = V_e + V_g$$
$$= 0.9375 \text{ lb} \cdot \text{ft}$$

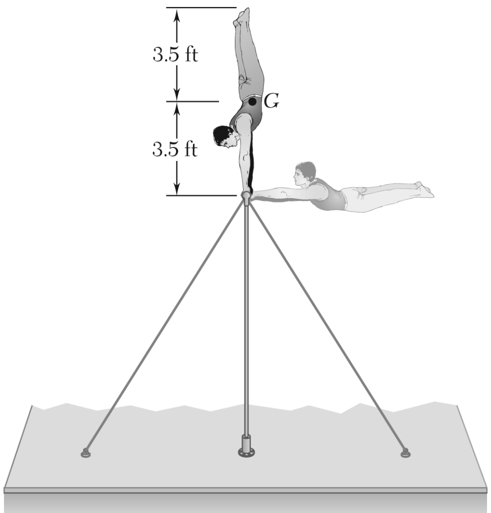
Kinetic energy:
$$\bar{v}_2 = r\omega_2 = \left(\frac{7}{12}\text{ ft}\right)\omega_2$$
$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}\left(\frac{9 \text{ lb}}{32.2}\right)(2 \text{ ft})^2 = 0.093168 \text{ slug} \cdot \text{ft}^2$$
$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}\left(\frac{9 \text{ lb}}{32.2}\right)\left(\left(\frac{7}{12}\text{ ft}\right)\omega_2\right)^2 + \frac{1}{2}(0.093168)\omega_2^2$$
$$T_2 = 0.094138\omega_2^2$$

Conservation of energy:
$$T_1 + V_1 = T_2 + V_2$$
$$0 + 2.9382 = 0.094138\omega_2^2 + 0.9375$$
$$\omega_2^2 = 21.253$$
$$\omega_2 = 4.6101 \text{ rad/s}$$

$\boldsymbol{\omega}_2 = 4.61 \text{ rad/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.20

A 160-lb gymnast is executing a series of full-circle swings on the horizontal bar. In the position shown he has a small and negligible clockwise angular velocity and will maintain his body straight and rigid as he swings downward. Assuming that during the swing the centroidal radius of gyration of his body is 1.5 ft, determine his angular velocity and the force exerted on his hands after he has rotated through (*a*) 90°, (*b*) 180°.

SOLUTION

Position 1. (Directly above the bar).

Elevation: $h_1 = 3.5 \text{ ft}$

Potential energy: $V_1 = Wh_1 = (160 \text{ lb})(3.5 \text{ ft}) = 560 \text{ ft} \cdot \text{lb}$

Speeds: $\omega_1 = 0, \ \ \bar{v}_1 = 0$

Kinetic energy: $T_1 = 0$

(*a*) *Position 2. (Body at level of bar after rotating* 90°).

Elevation: $h_2 = 0.$

Potential energy: $V_2 = 0$

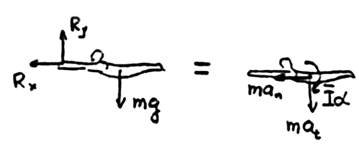
Speeds: $\bar{v}_2 = 3.5\omega_2.$

Kinetic energy:

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}mk^2\omega_2^2$$

$$T_2 = \frac{1}{2}\left(\frac{160}{32.2}\right)(3.5\omega_2)^2 + \frac{1}{2}\left(\frac{160}{32.2}\right)(1.5)^2\omega_2^2$$

$$= 36.025\omega_2^2$$



Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 560 = 36.025\omega_2^2$$

$$\omega_2^2 = 15.545$$

$\omega_2 = 3.94 \text{ rad/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

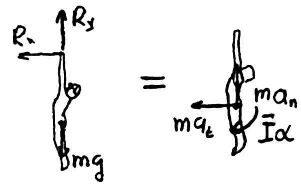
Kinematics: $\bar{a}_t = 3.5\alpha$

$$\bar{a}_n = 3.5\omega_2^2 = (3.5)(15.545) = 54.407 \text{ ft/s}^2 \longleftarrow$$

$$+\circlearrowright \Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad (3.5)(160) = \left(\frac{160}{32.2}\right)(3.5)(3.5\alpha) + \left(\frac{160}{32.2}\right)(1.5)^2\alpha$$

$$\alpha = 7.7724 \text{ rad/s}^2 \qquad \bar{a}_t = 27.203 \text{ ft/s}^2 \downarrow$$

$$\overset{+}{\longleftarrow} \Sigma F_x = ma_n: \qquad R_x = \left(\frac{160}{32.2}\right)(54.407) = 270.35 \text{ lb} \longleftarrow$$



$$+\uparrow \Sigma F_y = -ma_t: \qquad R_y - 160 = -\left(\frac{160}{32.2}\right)(27.203)\uparrow$$

$$R_y = 24.83 \text{ lb}\uparrow$$

$\mathbf{R} = 271 \text{ lb} \measuredangle 5.25°$ ◀

(*b*) *Position 3*. (*Directly below bar after rotating* 180°).

Elevation: $h_3 = -3.5$ ft.

Potential energy: $V_3 = Wh_3 = (160)(-3.5) = -560 \text{ ft} \cdot \text{lb}$

Speeds: $\bar{v}_3 = 3.5\omega_3$.

Kinetic energy: $T_3 = 36.025\omega_3^2$

Principle of conservation of energy.

$$T_1 + V_1 = T_3 + V_3: \quad 0 + 560 = 36.025\omega_3^2 - 560$$

$$\omega_3^2 = 31.09$$

$\omega_3 = 5.58 \text{ rad/s} \circlearrowright$ ◀

Kinematics: $a_n = (3.5)(31.09) = 108.81 \text{ ft/s}^2 \uparrow$

From $\Sigma M_0 = \Sigma(M_0)_{\text{eff}}$ and $\Sigma F_x = 0$,

$$\alpha = 0, \qquad a_t = 0 \qquad R_x = 0$$

$$+\uparrow \Sigma F_y = ma_n: \quad R_y - 160 = \left(\frac{160}{32.2}\right)(108.81)$$

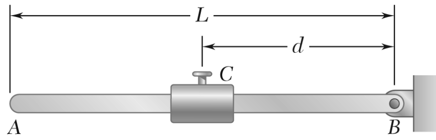
$$R_y = 700.62 \text{ lb}$$

$\mathbf{R} = 701 \text{ lb}\uparrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

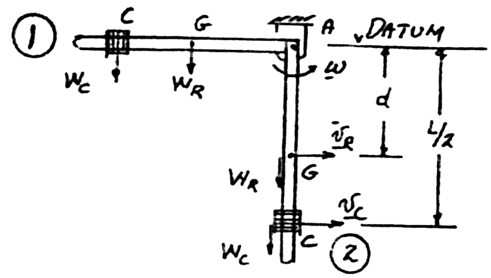
STUDENTS-HUB.com

PROBLEM 17.21



A collar with a mass of 1 kg is rigidly attached at a distance $d = 300$ mm from the end of a uniform slender rod AB. The rod has a mass of 3 kg and is of length $L = 600$ mm. Knowing that the rod is released from rest in the position shown, determine the angular velocity of the rod after it has rotated through 90°.

SOLUTION



Kinematics.

<u>Rod</u> $$v_R = \frac{L}{2}\omega$$

<u>Collar</u> $$v_C = d\omega$$

Position 1. $$\omega = 0$$
$$T_1 = 0 \quad V_1 = 0$$

Position 2.
$$T_2 = \frac{1}{2}m_R\bar{v}_R^2 + \frac{1}{2}\bar{I}_R\omega^2 + \frac{1}{2}m_C v_C^2$$
$$= \frac{1}{2}m_R\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_R L^2\right)\omega^2 + \frac{1}{2}m_C d^2\omega^2$$
$$= \frac{1}{6}m_R L^2\omega^2 + \frac{1}{2}m_C d^2\omega^2$$
$$V_2 = -W_C d - W_R\frac{L}{2}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \left(\frac{1}{6}m_R L^2 + \frac{1}{2}m_C d^2\right)\omega^2 - W_C d - W_R\frac{L}{2}$$

$$\omega^2 = \frac{3(2W_C d + W_C L)}{3m_C d^2 + m_R L^2} = \frac{3g(2m_C d + m_R L)}{3m_C d^2 + m_R L^2} \qquad (1)$$

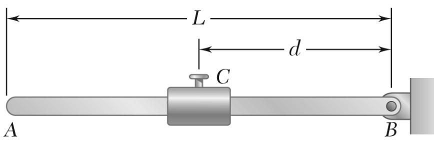
Data: $$m_C = 1\text{ kg}, \quad d = 0.3\text{ m}, \quad m_R = 3\text{ kg}, \quad L = 0.6\text{ m}$$

From Eq. (1), $$\omega^2 = 3(9.81)\left[\frac{(2)(1)(0.3) + 3(0.6)}{3(1)(0.3)^2 + 3(0.6)^2}\right]$$
$$= 52.32$$

$\bar{\omega} = 7.23$ rad/s ↻ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

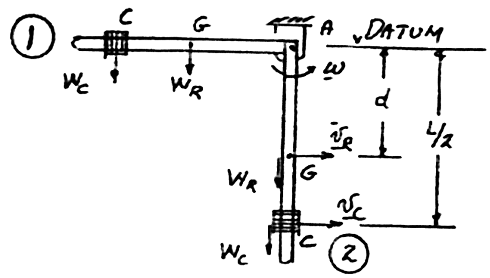
STUDENTS-HUB.com



PROBLEM 17.22

A collar with a mass of 1 kg is rigidly attached to a slender rod *AB* of mass 3 kg and length $L = 600$ mm. The rod is released from rest in the position shown. Determine the distance *d* for which the angular velocity of the rod is maximum after it has rotated 90°.

SOLUTION



Kinematics.

<u>Rod</u> $$v_R = \frac{L}{2}\omega$$

<u>Collar</u> $$v_C = d\omega$$

Position 1. $$\omega = 0$$

$$T_1 = 0 \quad V_1 = 0$$

Position 2. $$T_2 = \frac{1}{2}m_R\bar{v}_R^2 + \frac{1}{2}\bar{I}_R\omega^2 + \frac{1}{2}m_C v_C^2$$

$$= \frac{1}{2}m_R\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_R L^2\right)\omega^2 + \frac{1}{2}m_C d^2\omega^2$$

$$= \frac{1}{6}m_R L^2\omega^2 + \frac{1}{2}m_C d^2\omega^2$$

$$V_2 = -W_C d - W_R\frac{L}{2}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \left(\frac{1}{6}m_R L^2 + \frac{1}{2}m_C d^2\right)\omega^2 - W_C d - W_R\frac{L}{2}$$

$$\omega^2 = \frac{3(2W_C d + W_C L)}{3m_C d^2 + m_R L^2} = \frac{3g(2m_C d + m_R L)}{3m_C d^2 + m_R L^2} \tag{1}$$

Let $$x = \frac{d}{L}.$$

$$\omega^2 = \frac{3g}{L}\cdot\frac{2x + \frac{m_R}{m_C}}{3x^2 + \frac{m_R}{m_C}}$$

Data: $$m_C = 1\text{ kg}, \quad m_R = 3\text{ kg}$$

$$\frac{L\omega^2}{3g} = \frac{2x+3}{3x^2+3}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.22 (Continued)

$L\omega^2/3g$ is maximum. Set its derivative with respect to x equal to zero.

$$\frac{d}{dx}\left(\frac{L\omega^2}{3g}\right) = \frac{(3x^2+3)(2)-(2x+3)(6x)}{(3x^2+3)^2} = 0$$

$$-6x^2 - 18x + 6 = 0$$

Solving the quadratic equation

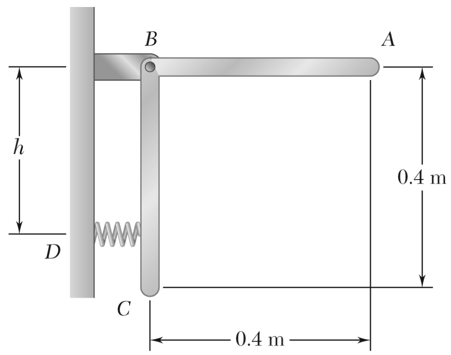
$$x = -3.30 \quad \text{and} \quad x = 0.30278$$

$$\begin{aligned} d &= 0.30278L \\ &= (0.30278)(0.6) \\ &= 0.1817 \text{ m} \end{aligned}$$

$d = 181.7$ mm ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

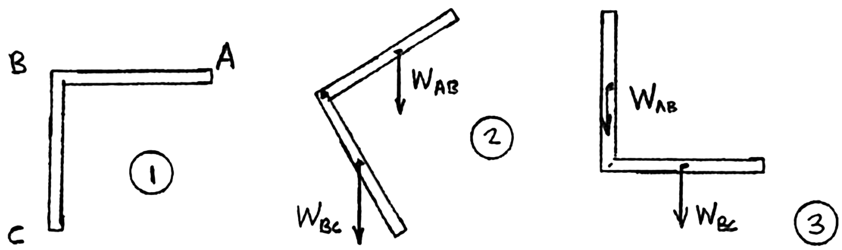


PROBLEM 17.23

Two identical slender rods *AB* and *BC* are welded together to form an L-shaped assembly. The assembly is pressed against a spring at *D* and released from the position shown. Knowing that the maximum angle of rotation of the assembly in its subsequent motion is 90° counterclockwise, determine the magnitude of the angular velocity of the assembly as it passes through the position where rod *AB* forms an angle of 30° with the horizontal.

SOLUTION

Moment of inertia about *B*. $$I_B = \frac{1}{3}m_{AB}l^2 + \frac{1}{3}m_{BC}l^2$$



Position 2.

$$\theta = 30°$$

$$V_2 = W_{AB}(h_{AB})_2 + W_{BC}(h_{BC})_2$$

$$= W_{AB}\frac{l}{2}\sin 30° + W_{BC}\left(-\frac{l}{2}\cos 30°\right)$$

$$T_2 = \frac{1}{2}I_B\omega_2^2 = \frac{1}{6}(m_{AB} + m_{BC})l^2\omega_2^2$$

Position 3.

$$\theta = 90°$$

$$V_3 = W_{AB}\frac{l}{2} \quad T_3 = 0$$

Conservation of energy.

$$T_2 + V_2 = T_3 + V_3:$$

$$\frac{1}{6}(m_{AB} + m_{BC})l^2\omega_2^2 + W_{AB}\frac{l}{2}\sin 30° - W_{BC}\frac{l}{2}\cos 30° = 0 + W_{AB}\frac{l}{2}$$

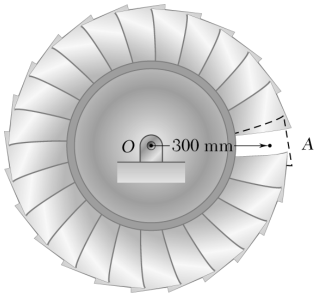
$$\omega_2^2 = \frac{3}{l}\cdot\frac{W_{AB}(1-\sin 30°) + W_{BC}\cos 30°}{m_{AB} + m_{BC}}$$

$$= \frac{3}{2}\frac{g}{l}[1 - \sin 30° + \cos 30°]$$

$$= 2.049\frac{g}{l} = 2.049\frac{9.81}{0.4} = 50.25 \qquad \omega_2 = 7.09 \text{ rad/s} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.24

The 30-kg turbine disk has a centroidal radius of gyration of 175 mm and is rotating clockwise at a constant rate of 60 rpm when a small blade of weight 0.5 N at Point A becomes loose and is thrown off. Neglecting friction, determine the change in the angular velocity of the turbine disk after it has rotated through (*a*) 90°, (*b*) 270°.

SOLUTION

Mass of blade. $m_A = 51 \text{ grams} = 0.051 \text{ kg}$

Weight of blade. $m_A g = (0.051)(9.81) = 0.5 \text{ N}$

Moment of inertia about O. $I_O = mk^2 - m_A r_2^2 = 30(0.175)^2 - 51\times10^{-3}(0.3)^2 = 0.91416 \text{ kg}\cdot\text{m}^2$

Location of mass center for the position shown.

$$(m - m_A)\overline{x} = -m_A r_A \qquad \overline{x} = -\frac{m_A r_A}{m - m_A}$$

Position 1. $\theta = 0°, \quad \omega_1 = 60 \text{ rpm} = 2\pi \text{ rad/s}$

Kinetic energy: $T_1 = \frac{1}{2} I_O \omega_1^2$

Center of gravity lies at the level of Point O. $h_1 = 0$

Potential energy: $V_1 = (mg - m_A g)h_1 = 0$

(*a*) *Position 2.* $\theta = 90°$

Kinetic energy: $T_2 = \frac{1}{2} I_O \omega_2^2$

Center of gravity lies a distance $\dfrac{m_A r_A}{m - m_A}$ above Point O.

$$h_2 = \frac{m_A r_A}{m - m_A}$$

Potential energy: $V_2 = (mg - m_A g)h_2 = m_A g r_A = (0.5)(0.3) = 0.150 \text{ N}\cdot\text{m}$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.24 (Continued)

Conservation of energy. $T_1 + V_1 = T_2 + V_2:$

$$\frac{1}{2}I_O\omega_1^2 + 0 = \frac{1}{2}I_O\omega_2^2 + V_2$$

$$\omega_2^2 = \omega_1^2 - \frac{2V_2}{I_O} - (2\pi)^2 - \frac{(2)(0.15)}{0.91416} \qquad \omega_2 = 6.257016 \text{ rad/s}$$

$$\Delta\omega = \omega_2 - \omega_1 = 6.257016 - 2\pi = -0.02617 \text{ rad/s}$$

$\Delta\omega = -0.250$ rpm ◀

(*b*) *Position 3.* $\theta = 270°$

Kinetic energy: $T_3 = \frac{1}{2}I_O\omega_3^2$

Center of gravity lies a distance $\frac{m_A r_A}{m - m_A}$ below Point O.

$$h_3 = -\frac{m_A r_A}{m - m_A}$$

Potential energy: $V_3 = (mg - m_A g)h_3 = -m_A g r_A = -(0.5)(0.3) = -0.15 \text{ N}\cdot\text{m}$

Conservation of energy. $T_1 + V_1 = T_3 + V_3:$

$$\frac{1}{2}I_O\omega_1^2 + 0 = \frac{1}{2}I_O\omega_3^2 + V_3$$

$$\omega_3^2 = \omega_1^2 - \frac{2V_3}{I_O} = (2\pi)^2 - \frac{(2)(-0.15)}{0.91416}$$

$$\omega_3 = 6.309246 \text{ rad/s}$$

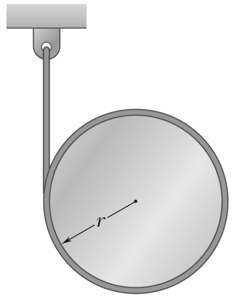
$$\Delta\omega = \omega_3 - \omega_1 = 6.309246 - 2\pi = 0.026061 \text{ rad/s}$$

$\Delta\omega = 0.249$ rpm ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

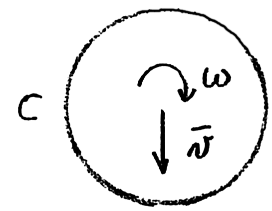
STUDENTS-HUB.com

PROBLEM 17.25



A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward a distance s.

SOLUTION



Point C is the instantaneous center.

$$\bar{v} = r\omega \qquad \omega = \frac{\bar{v}}{r}$$

Position 1. At rest. $\quad T_1 = 0$

Position 2. Cylinder has fallen through distance s.

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\bar{v}}{r}\right)^2$$

$$= \frac{3}{4}m\bar{v}^2$$

Work. $\quad U_{1\to 2} = mgs$

Principle of work and energy.

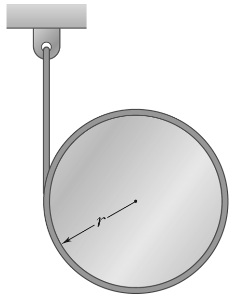
$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgs = \frac{3}{4}m\bar{v}^2$$

$$\bar{v}^2 = \frac{4gs}{3}$$

$$\bar{\mathbf{v}} = \sqrt{\frac{4gs}{3}} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



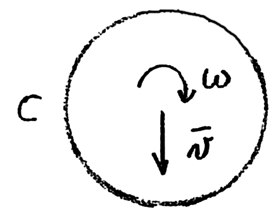
PROBLEM 17.26

Solve Problem 17.25, assuming that the cylinder is replaced by a thin-walled pipe of radius r and mass m.

PROBLEM 17.25 A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward a distance s.

SOLUTION

Point C is the instantaneous center.



$$\bar{v} = r\omega \qquad \omega = \frac{\bar{v}}{r}$$

Position 1. At rest. $\quad T_1 = 0$

Position 2. Cylinder has fallen through distance s.

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}(mr^2)\left(\frac{\bar{v}}{r}\right)^2$$

$$= m\bar{v}^2$$

Work. $\quad U_{1\to 2} = mgs$

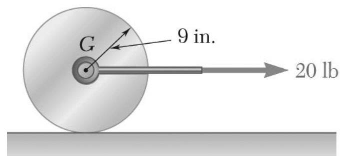
Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgs = m\bar{v}^2$$

$$\bar{v}^2 = gs \qquad \bar{\mathbf{v}} = \sqrt{gs}\downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.27

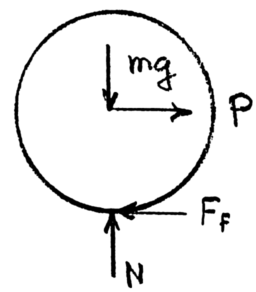
A 45-lb uniform cylindrical roller, initially at rest, is acted upon by a 20-lb force as shown. Knowing that the body rolls without slipping, determine (*a*) the velocity of its center *G* after it has moved 5 ft, (*b*) the friction force required to prevent slipping.

SOLUTION

Since the cylinder rolls without slipping, the point of contact with the ground is the instantaneous center.

Kinematics: $\bar{v} = r\omega$

Position 1. At rest. $T_1 = 0$



Position 2. $s = 5 \text{ ft} \quad v_G = \bar{v} \quad \omega = \dfrac{v_G}{r}$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv_G^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_G}{r}\right)^2$$

$$= \frac{3}{4}mv_G^2 = \frac{3}{4}\left(\frac{45}{32.2}\right)v_G^2 = 1.04815v_G^2$$

Work: $U_{1\to 2} = Ps = (20)(5) = 100 \text{ lb}\cdot\text{ft}. \quad F_f$ does no work.

(*a*) Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + 100 = 1.0481v_G^2$$

$$v_G^2 = 95.407$$

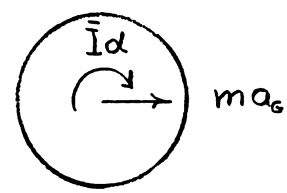
$\mathbf{v}_G = 9.77 \text{ ft/s} \longrightarrow$ ◀

(*b*) Since the forces are constant, $a_G = \bar{a} = \text{constant}$

$$a_G = \frac{v_G^2}{2s}$$

$$= \frac{95.407}{(2)(5)}$$

$$= 9.5407 \text{ ft/s}^2$$



$$\xrightarrow{+} \Sigma F_x = m\bar{a}: \quad P - F_f = m\bar{a}$$

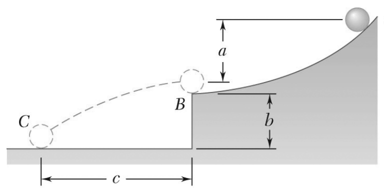
$$F_f = P - m\bar{a}$$

$$= 20 - \left(\frac{45}{32.2}\right)(9.5407)$$

$\mathbf{F}_f = 6.67 \text{ lb} \longleftarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

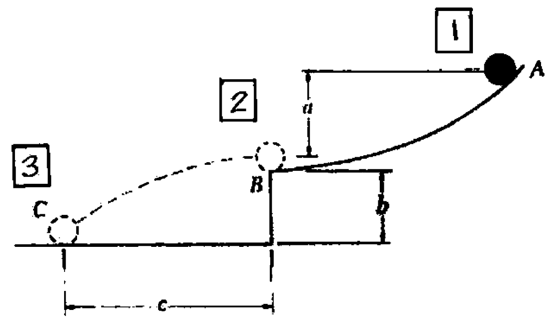
STUDENTS-HUB.com



PROBLEM 17.28

A small sphere of mass m and radius r is released from rest at A and rolls without sliding on the curved surface to Point B where it leaves the surface with a horizontal velocity. Knowing that a = 1.5 m and b = 1.2 m, determine (a) the speed of the sphere as it strikes the ground at C, (b) the corresponding distance c.

SOLUTION



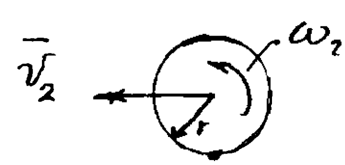
Work: $U_{1\to 2} = mga$

Kinetic energy: $T_1 = 0$

Rolling motion at position 2. $\bar{v}_2 = r\omega$ or $60\omega = \dfrac{v}{r}$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{7}{10}m\bar{v}^2$$



Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mga = \frac{7}{10}m\bar{v}^2$$

$$\bar{v}^2 = \frac{10ga}{7} = \frac{(10)(9.81\text{ m/s}^2)(1.5\text{ m})}{7} = 21.021\text{ m/s}^2$$

$$\bar{v} = 4.5849\text{ m/s}$$

For path B to C the motion is projectile motion. Let $t = 0$ at Point B. Let $y = 0$ at Point C.

Vertical motion: $v_y = (v_y)_0 - gt = -gt$

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

At Point C, $0 = b + 0 - \dfrac{1}{2}gt_C^2$

$$t_C = \sqrt{\frac{2b}{g}} = \sqrt{\frac{(2)(1.2\text{ m})}{9.81\text{ m/s}^2}} = 0.49462\text{ s}$$

$$(v_y)_C = -gt_C = -(9.81\text{ m/s}^2)(0.49462\text{ s}) = -4.8522\text{ m/s}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.28 (Continued)

Horizontal motion: Let the x coordinate point to the left with origin below B.

$$v_x = (v_x)_B = \overline{v} = 4.5849 \text{ m/s}$$

(*a*) *Speed at C.*

$$v_C = \sqrt{(v_x)_C^2 + (v_y)_C^2}$$

$$v_C = \sqrt{(4.5849)^2 + (4.8522)^2}$$

$$v_C = 6.68 \text{ m/s} \blacktriangleleft$$

(*b*) *Distance c.*

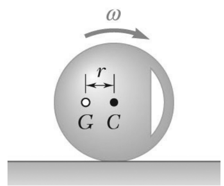
$$c = v_x t_C$$

$$c = (4.5849 \text{ m/s})(0.49462 \text{ s}) \qquad c = 2.27 \text{ m} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

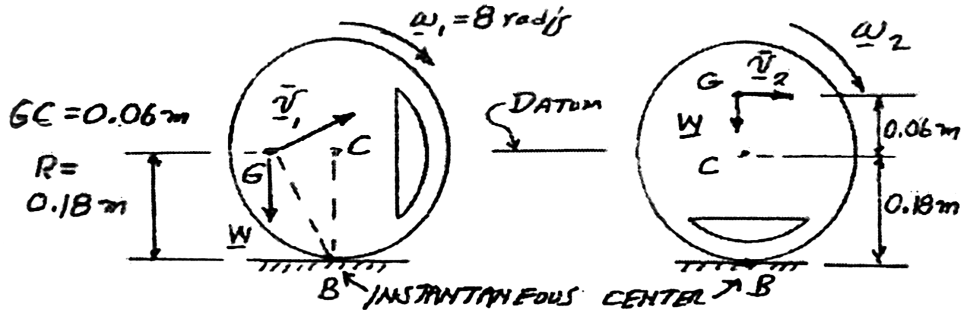
STUDENTS-HUB.com

PROBLEM 17.29



The mass center G of a 3-kg wheel of radius $R = 180$ mm is located at a distance r = 60 mm from its geometric center C. The centroidal radius of gyration of the wheel is $\bar{k} = 90$ mm. As the wheel rolls without sliding, its angular velocity is observed to vary. Knowing that $\omega = 8$ rad/s in the position shown, determine (*a*) the angular velocity of the wheel when the mass center G is directly above the geometric center C, (*b*) the reaction at the horizontal surface at the same instant.

SOLUTION



$$\bar{v}_1 = (BG)\omega_1$$
$$= \sqrt{(0.18)^2 + (0.06)^2}\,(8)$$
$$= 8\sqrt{0.036}\ \text{m/s}$$
$$\bar{v}_2 = 0.24\omega_2$$
$$m = 3\ \text{kg}$$
$$\bar{k} = 0.09\ \text{m}$$

Position 1.

$$V_1 = 0$$
$$T_1 = \frac{1}{2}m\bar{v}_1^2 + \frac{1}{2}\bar{I}\omega_1^2$$
$$= \frac{1}{2}(3)(8\sqrt{0.036})^2 + \frac{1}{2}(3)(0.09)^2(8)^2$$
$$= 4.2336\ \text{J}$$

Position 2.

$$V_2 = Wh$$
$$= mgh$$
$$= (3)(9.81)(0.06)$$
$$= 1.7658\ \text{J}$$
$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}(3)(0.24\omega_2)^2 + \frac{1}{2}(3)(0.09)^2\omega_2^2$$
$$= 0.09855\omega_2^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.29 (Continued)

(*a*) Conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

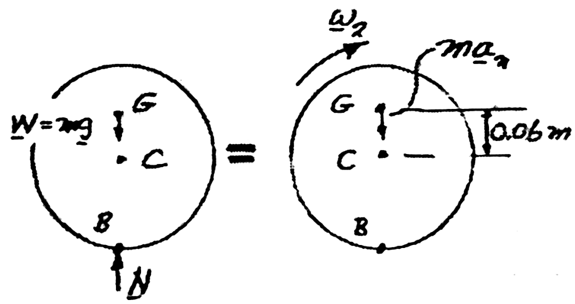
$$4.2336 \text{ J} + 0 = 0.09855\omega_2^2 + 1.7658 \text{ J}$$

$$\omega_2^2 = 25.041$$

$$\omega_2 = 5.004 \text{ rad/s}$$

$\boldsymbol{\omega}_2 = 5.00$ rad/s ◀

(*b*) Reaction at *B*.



$$ma_n = m(CG)\omega_2^2$$

$$= (3 \text{ kg})(0.06 \text{ m})(5.00 \text{ rad/s})^2$$

$$= 4.5 \text{ N}\downarrow$$

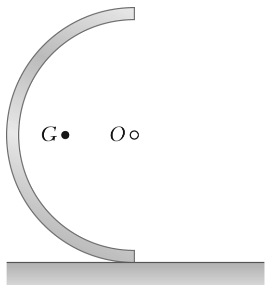
$$+\uparrow \Sigma F_y = ma_y: \quad N - mg = -ma_n$$

$$N - (3)(9.81) = -4.5$$

$\mathbf{N} = 24.9 \text{ N}\uparrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

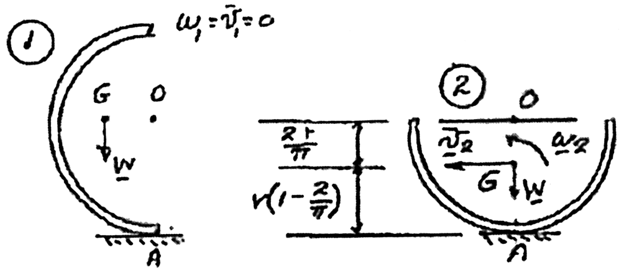


PROBLEM 17.30

A half section of pipe of mass m and radius r is released from rest in the position shown. Knowing that the pipe rolls without sliding, determine (*a*) its angular velocity after it has rolled through 90°, (*b*) the reaction at the horizontal surface at the same instant. [*Hint:* Note that $GO = 2r/\pi$ and that, by the parallel-axis theorem, $\bar{I} = mr^2 - m(GO)^2$.]

SOLUTION

Position 1. $\quad \omega_1 = 0 \quad v_1 = 0 \quad T_1 = 0$



Position 2. Kinematics: $\quad \bar{v}_2 = (AG)\omega_2 = r\left(1 - \dfrac{2}{\pi}\right)\omega_2$

Moment of inertia: $\quad \bar{I} = mr^2 - m(0.6)^2 = mr^2 - m\left(\dfrac{2r}{\pi}\right)^2 = mr^2\left(1 - \dfrac{4}{\pi^2}\right)$

Kinetic energy:

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\,\omega_2^2$$

$$= \frac{1}{2}m\left(1 - \frac{2}{\pi}\right)^2 r^2\omega_2^2 + \frac{1}{2}mr^2\left(1 - \frac{4}{\pi^2}\right)\omega_2^2$$

$$= \frac{1}{2}mr^2\left[\left(1 - \frac{4}{\pi} + \frac{4}{\pi^2}\right) + \left(1 - \frac{4}{\pi^2}\right)\right]$$

$$= \frac{1}{2}mr^2\left(2 - \frac{4}{\pi}\right)$$

Work: $\quad U_{1\to2} = W(OG) = mg\dfrac{2r}{\pi} = \dfrac{2}{\pi}mgr$

Principle of work and energy:

$$T_1 + U_{1\to2} = T_2$$

$$0 + mg\frac{2r}{\pi} = \frac{1}{2}mr^2\left(2 - \frac{4}{\pi}\right)\omega_2^2$$

$$\omega_2^2 = \frac{2}{\pi\left(1 - \frac{2}{\pi}\right)} \cdot \frac{g}{r} = 1.7519\frac{g}{r}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

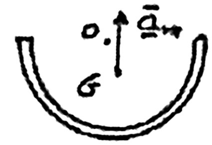
STUDENTS-HUB.com

PROBLEM 17.30 (Continued)

(*a*) Angular velocity. $\omega_2 = 1.324\sqrt{\frac{g}{r}}$ ◀

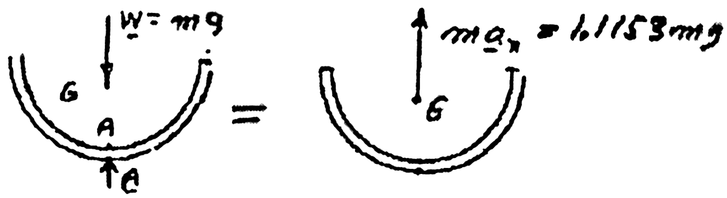
(*b*) Reaction at *A*.

Kinematics: Since *O* moves horizontally, $(a_0)_y = 0$



$$a_n = (0.6)\omega_2^2$$
$$= \frac{2r}{\pi}\left(1.7519\frac{g}{r}\right)$$
$$= 1.1153g \uparrow$$

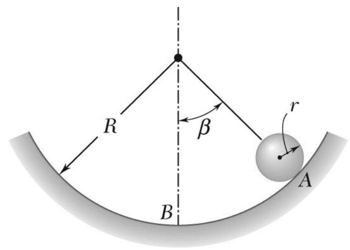
Kinetics:



$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A - mg = 1.1153mg$ $\mathbf{A} = 2.12mg \uparrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

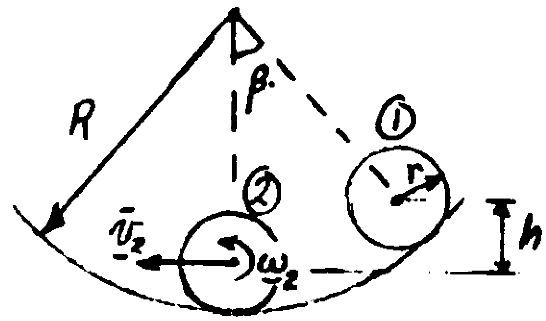


PROBLEM 17.31

A sphere of mass m and radius r rolls without slipping inside a curved surface of radius R. Knowing that the sphere is released from rest in the position shown, derive an expression (a) for the linear velocity of the sphere as it passes through B, (b) for the magnitude of the vertical reaction at that instant.

SOLUTION

Kinematics: The sphere rolls without slipping.



$$\bar{v} = r\omega \quad \omega = \frac{\bar{v}}{r}$$

Kinetic energy.

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{\bar{v}}{r}\right)^2$$

$$T = \frac{7}{10}m\bar{v}^2$$

$$T_1 = 0 \quad T_2 = \frac{7}{10}m\bar{v}_2^2$$

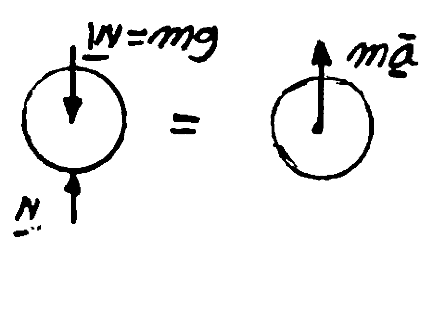
Work. $$U_{1-2} = mgh = mg(R-r)(1-\cos\beta)$$

Principle of work and energy. $T_1 + U_{1-2} = T_2$:

$$0 + mg(R-r)(1-\cos\beta) = \frac{7}{10}m\bar{v}_2^2$$

(a) *Linear velocity at B.* $$\bar{v}_2 = \sqrt{\frac{10}{7}g(R-r)(1-\cos\beta)} \blacktriangleleft$$

Free body diagram when $\beta = 0$.



$\xrightarrow{+} \Sigma F = ma_t$: $\quad a_t = 0$

$+\circlearrowleft \Sigma M_G = \bar{I}\alpha$: $\quad \alpha = 0$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

The sphere rolls so that its mass center moves on a circle of radius $\rho = R - r$.

$$\bar{a} = a_n = \frac{\bar{v}_2^2}{R-r} \uparrow$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - mg = m\bar{a}$$

$$N - mg = m\left(\frac{1}{R-r}\right)\left[\frac{10}{7}g(R-r)(1-\cos\beta)\right]$$

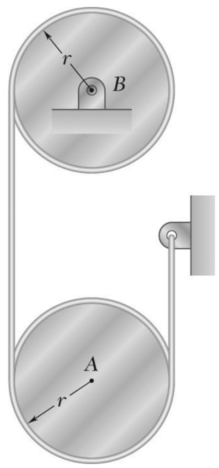
$$N = mg\left[1 + \frac{10}{7}(1-\cos\beta)\right]$$

(*b*) *Vertical reaction.* $N = \frac{1}{7}mg[17 - 10\cos\beta] \uparrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

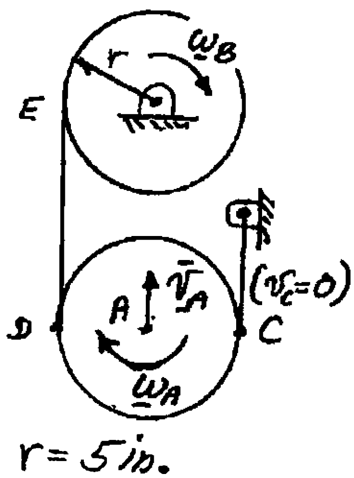
PROBLEM 17.32



Two uniform cylinders, each of weight $W = 14$ lb and radius $r = 5$ in., are connected by a belt as shown. Knowing that at the instant shown the angular velocity of cylinder B is 30 rad/s clockwise, determine (*a*) the distance through which cylinder A will rise before the angular velocity of cylinder B is reduced to 5 rad/s, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION

Kinematics.



$$v_D = v_E = r\omega_B$$

Point C is the instantaneous center of cylinder A.

$$\omega_A = \frac{v_D}{cd} = \frac{r\omega_B}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

$$v_D = 2\bar{v}_A$$

Kinetic energy of the system.

$$T = \frac{1}{2}m\bar{v}_A^2 + \frac{1}{2}\bar{I}\omega_A^2 + \frac{1}{2}\bar{I}\omega_B^2$$

$$T = \frac{1}{2}m\left(\frac{r}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{1}{2}\omega_B\right)^2 + \left(\frac{1}{2}mr^2\right)\omega_B^2$$

$$T = \frac{7}{16}mr^2\omega_B^2 \qquad (1)$$

Position 1: $(\omega_B)_1 = 30$ rad/s ↻

Position 2: $(\omega_B)_2 = 5$ rad/s ↻

Work. For the system considered, the only force which does work is the weight of disk A.

$$U_{1-2} = -Wh = -mgh$$

where h is the rise of cylinder A.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.32 (Continued)

Principle of work and energy.

$$T_1 + U_{1-2} = T_2: \quad \frac{7}{16}mr^2(\omega_B)_1^2 - mgh = \frac{7}{16}mr^2(\omega_B)_2^2$$

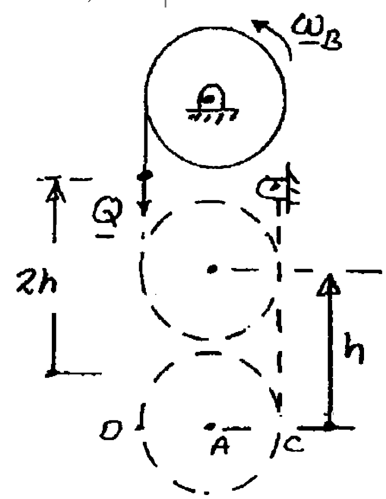
$$h = \frac{7}{16}\frac{r^2}{g}[(\omega_B)_1^2 - (\omega_B)_2^2] \qquad (2)$$

$$h = \frac{7}{16}\left(\frac{5}{12}\text{ft}\right)^2 \frac{1}{32.2\text{ ft/s}^2}[(30\text{ rad/s})^2 - (5\text{ rad/s})^2] = 2.064\text{ ft}$$

(*a*) *Rise of cylinder A.* $h = 2.06$ ft ◀

(*b*) *Tension in cord DE.* Let Q be its value.

Recall that $v_D = 2v_A$ thus D moves twice the distance that A moves, i.e $2h\uparrow$



$$T_1 = \frac{1}{2}\bar{I}(\omega_B)_1^2$$

$$T_2 = \frac{1}{2}\bar{I}(\omega_B)_2^2$$

$$U_{1-2} = -Q(2h)$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}\bar{I}(\omega_B)_1^2 - 2Qh = \frac{1}{2}\bar{I}(\omega_B)_2^2$$

$$Qh = \frac{1}{4}\bar{I}[(\omega_B)_1^2 - (\omega_B)_2^2] \qquad (3)$$

Divide Equation (3) by Equation (2):

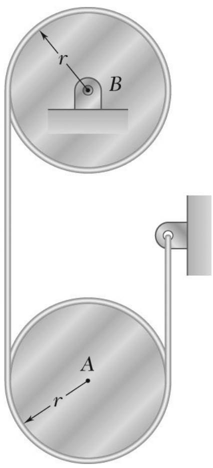
$$Q = \frac{1}{4}\bar{I}\frac{16\text{ g}}{7r^2} = \frac{1}{4}\left(\frac{1}{2}mr^2\right)\frac{16\text{ g}}{7r^2} = \frac{2}{7}mg = \frac{2}{7}W \qquad (4)$$

$$Q = \frac{2}{7}(14\text{ lb})$$

Tension $= Q = 4.00$ lb ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.33

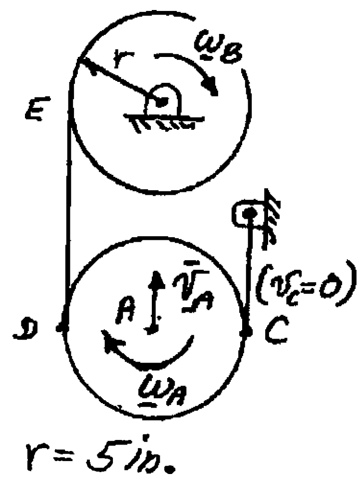
Two uniform cylinders, each of weight $W = 14$ lb and radius $r = 5$ in., are connected by a belt as shown. If the system is released from rest, determine (*a*) the velocity of the center of cylinder *A* after it has moved through 3 ft, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION

Kinematics.

$$v_D = v_E = r\omega_B$$

Point *C* is the instantaneous center of cylinder *A*.



$$\omega_A = \frac{v_D}{CD} = \frac{r\omega_B}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

$$v_D = 2\bar{v}_A$$

Kinetic energy of the system.

$$T = \frac{1}{2}m\bar{v}_A^2 + \frac{1}{2}\bar{I}\,\omega_A^2 + \frac{1}{2}\bar{I}\,\omega_B^2$$

$$T = \frac{1}{2}m\left(\frac{r}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{1}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_B^2$$

$$T = \frac{7}{16}mr^2\omega_B^2 \qquad (1)$$

Position 1: At rest $T_1 = 0$

Position 2: Center of cylinder *C* has moved 3 ft$\downarrow$.

Work. For the system considered, the only force which does work is the weight of disk *A*.

$$U_{1-2} = Wh = (14\text{ lb})(3\text{ ft}) = 42\text{ ft}\cdot\text{lb}$$

where *h* is the distance that cylinder *A* falls.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.33 (Continued)

Principle of work and energy:

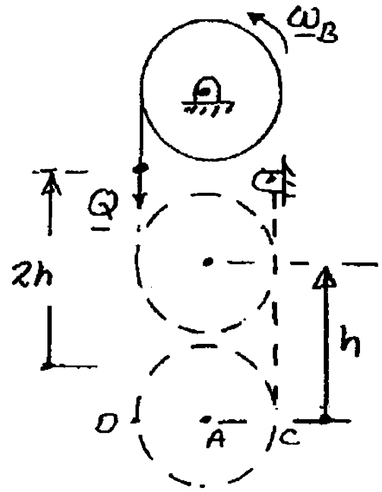
$$T_1 + U_{1\to 2} = T_2: \quad 0 + 42\text{ ft}\cdot\text{lb} = \frac{7}{16}\frac{14\text{ lb}}{32.2\text{ ft/s}^2}\left(\frac{5}{12}\text{ft}\right)^2(\omega_B)_2^2$$

$$(\omega_B)_2 = 35.662\text{ rad/s}$$

(*a*) *Velocity of A.* $\quad \bar{v}_A = \frac{1}{2}r\omega_B = \frac{1}{2}\left(\frac{5}{12}\text{ft}\right)(35.66\text{ rad/s})$ $\quad \bar{\mathbf{v}}_A = 7.43\text{ ft/s}\downarrow$ ◀

(*b*) *Tension in cord DE.* Let Q be its value.

Recall that $v_D = 2v_A$ thus D moves twice the distance that A moves, i.e $2h\downarrow$



$$T_1 = 0$$

$$T_2 = \frac{1}{2}\bar{I}(\omega_B)_2^2$$

$$U_{1-2} = Q(2h)$$

$$T_1 + U_{1-2} = T_2$$

$$0 + Qh = \frac{1}{2}\left(\frac{1}{2}\frac{W}{g}r^2\right)\omega_B^2$$

$$Q = \frac{1}{4}\frac{W}{g}r^2\frac{\omega_B^2}{h}$$

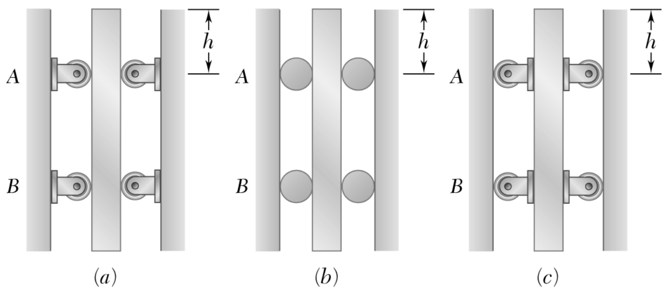
$$= \frac{1}{4}\frac{14\text{ lb}}{32.2\text{ ft/s}^2}\left(\frac{5}{12}\text{ft}\right)^2\frac{(35.662\text{ rad/s})^2}{6\text{ ft}}$$

$$= 4.00\text{ lb}$$

$Q = 4.00$ lb. ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.34

A bar of mass $m = 5$ kg is held as shown between four disks each of mass $m' = 2$ kg and radius $r = 75$ mm. Knowing that the forces exerted on the disks are sufficient to prevent slipping and that the bar is released from rest, for each of the cases shown determine the velocity of the bar after it has moved through the distance h.

SOLUTION

Let $\mathbf{v}$ be the velocity of the bar $(\mathbf{v} = v\downarrow)$, $\overline{\mathbf{v}}'$ be the velocity of the mass center G of the upper left disk, $(\overline{\mathbf{v}}' = \overline{v}'\downarrow)$ and $\boldsymbol{\omega}$ be its angular velocity.

For all three arrangements, the magnitudes of mass center velocities are the same for all disks. Likewise, the angular speeds are the same for all disks.

Moment of inertia of one disk. $\quad \overline{I} = \frac{1}{2}m'r^2$

Kinetic energy. $\quad T = \frac{1}{2}mv^2 + 4\left[\frac{1}{2}m'(\overline{v}')^2 + \frac{1}{2}\overline{I}\omega^2\right]$

$$T = \frac{1}{2}(5)v^2 + 4\left[\frac{1}{2}(2)(\overline{v}')^2 + \frac{1}{2}\left(\frac{1}{2}\right)(2)r^2\omega^2\right]$$

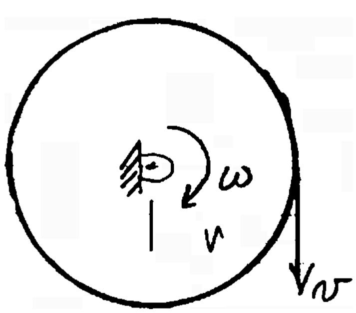
$$= 2.5v^2 + 4(\overline{v}')^2 + 2r^2\omega^2$$

Position 1. Initial at rest position. $\quad T_1 = 0$

Position 2. Bar has moved down a distance h. All the disks move down a distance h'.

Work $\quad U_{1\to 2} = mgh + 4m'gh' = 5gh + 8gh'$

Kinematics and kinetic energy for case (*a*).



The mass center of each disk is not moving.

$$\overline{v}' = 0, \qquad h' = 0$$

$$\omega = \frac{v}{r} \qquad r\omega = v$$

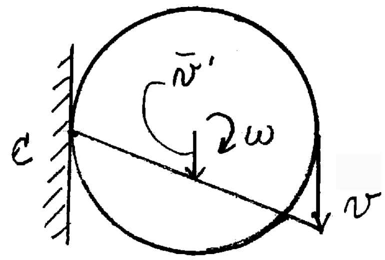
$$T_{2a} = 2.5v^2 + 0 + 2v^2 = 4.5v^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.34 (Continued)

Kinematics and kinetic energy for case (*b*).



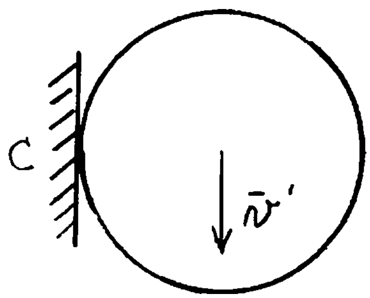
The instantaneous center C of a typical disk lies at its point of contact with the fixed wall.

$$\omega = \frac{v}{2r}$$

$$\bar{v}' = r\omega = \frac{1}{2}v, \qquad h' = \frac{1}{2}h$$

$$T_{2b} = 2.5v^2 + (4)\left(\frac{1}{2}v\right)^2 + 2\left(\frac{1}{2}v\right)^2 = 4.0v^2$$

Kinematics and kinetic energy for case (*c*).



The mass center of each disk moves with the bar.

$$\bar{v}' = v, \qquad h'' = h$$

The instantaneous center C of a typical disk lies at its point of contact with the fixed wall.

$$\bar{v}' = r\omega = v,$$

$$T_{2c} = 2.5v^2 + (4)v^2 + 2v^2 = 8.5v^2$$

Principle of Work and Energy. $T_1 + U_{1\to 2} = T_2$

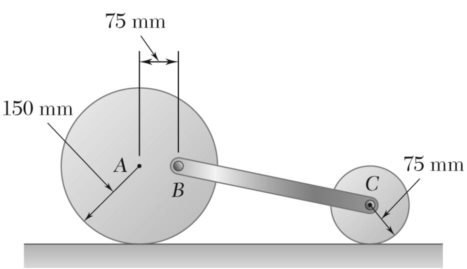
(*a*) $0 + 5gh + 0 = 4.5v^2$ $\qquad v = 1.054\sqrt{gh}$ ◀

(*b*) $0 + 5gh + 4gh = 4.0v^2$ $\qquad v = 1.500\sqrt{gh}$ ◀

(*c*) $0 + 5gh + 8gh = 8.5v^2$ $\qquad v = 1.237\sqrt{gh}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

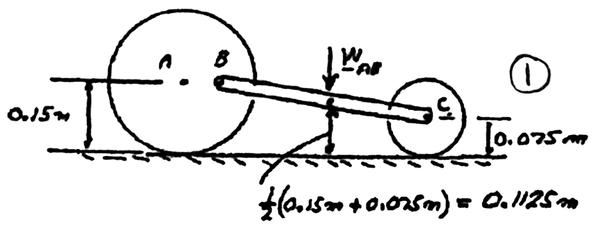


PROBLEM 17.35

The 5-kg rod BC is attached by pins to two uniform disks as shown. The mass of the 150-mm-radius disk is 6 kg and that of the 75-mm-radius disk is 1.5 kg. Knowing that the system is released from rest in the position shown, determine the velocity of the rod after disk A has rotated through 90°.

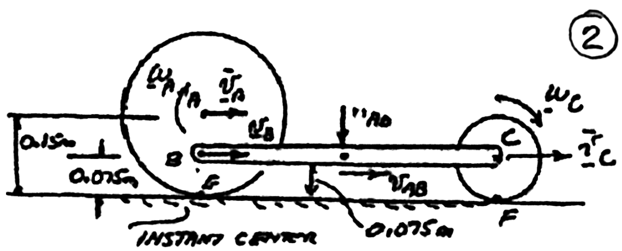
SOLUTION

Position 1. $T_1 = 0$



Position 2.

Kinematics.



$$v_B = v_{AB} \qquad \omega_A = \frac{v_B}{BE} = \frac{v_{AB}}{0.075\text{ m}} \qquad \bar{v}_A = 2v_B = 2v_{AB}$$

$$\bar{v}_C = v_{AB} \qquad \omega_C = \frac{v_C}{CF} = \frac{v_{AB}}{0.075\text{ m}} \qquad \omega_{AB} = 0$$

Kinetic energy.

$$T_2 = \frac{1}{2}m_A\bar{v}_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}m_{AB}v_{AB}^2 + \frac{1}{2}m_B\bar{v}_B^2 + \frac{1}{2}\bar{I}_B\omega_B^2$$

$$= \frac{1}{2}\left[(6\text{ ft/s})(2v_{AB})^2 + \frac{1}{2}(6\text{ kg})(0.15\text{ m})^2\left(\frac{v_{AB}}{0.075}\right)^2 + (5\text{ kg})v_{AB}^2\right.$$

$$\left. + (1.5\text{ kg})(v_{AB})^2 + \frac{1}{2}(1.5\text{ kg})(0.075\text{ m})^2\left(\frac{v_{AB}}{0.075}\right)^2\right]$$

$$= \frac{1}{2}[24 + 12 + 5 + 1.5 + 0.75]v_{AB}^2$$

$$T_2 = 21.625\, v_{AB}^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.35 (Continued)

Work:

$$U_{1\to 2} = W_{AB}(0.1125\text{ m} - 0.075\text{ m})$$
$$= (5\text{ kg})(9.81)(0.0375\text{ m})$$
$$U_{1\to 2} = 1.8394\text{ J}$$

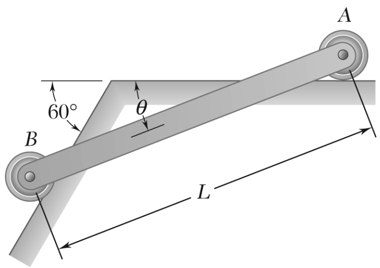
Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$
$$0 + 1.8394\text{ J} = 21.625\, v_{AB}^2$$
$$v_{AB}^2 = 0.08506$$
$$v_{AB} = 0.2916\text{ m/s}$$

Velocity of the rod. $\mathbf{v}_{AB} = 292\text{ mm/s} \longrightarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

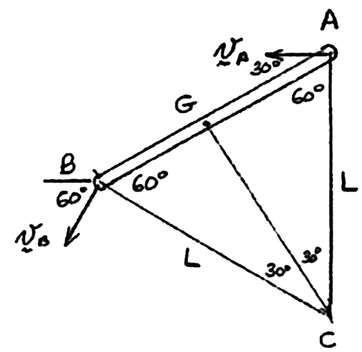
STUDENTS-HUB.com



PROBLEM 17.36

The motion of the uniform rod AB is guided by small wheels of negligible mass that roll on the surface shown. If the rod is released from rest when $\theta = 0$, determine the velocities of A and B when $\theta = 30°$.

SOLUTION



Position 1.

$$\theta = 0$$
$$v_A = v_B = 0$$
$$\omega = 0$$
$$T_1 = 0$$
$$V_1 = 0$$

Position 2. $\qquad \theta = 30°$

Kinematics. Locate the instantaneous center C. Triangle ABC is equilateral.

$$v_A = v_B = L\omega$$
$$v_G = L\omega\cos 30°$$

Moment of inertia. $\qquad \bar{I} = \frac{1}{12}ml^2$

Kinetic energy. $\qquad T_2 = \frac{1}{2}mv_G^2 + \frac{1}{2}\bar{I}\omega^2: \quad T_2 = \frac{1}{2}m(L\omega\cos 30°)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2$

$$= \frac{5}{12}ml^2\omega^2$$

Potential energy. $\qquad V_2 = -mg\frac{L}{2}\sin 30° = -\frac{1}{4}mgL$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{5}{12}mL^2\omega^2 - \frac{1}{4}mgL$$

$$\omega^2 = 0.6\frac{g}{L}$$

$$\omega = 0.775\sqrt{\frac{g}{L}}$$

$$v_A = 0.775\sqrt{gL}$$

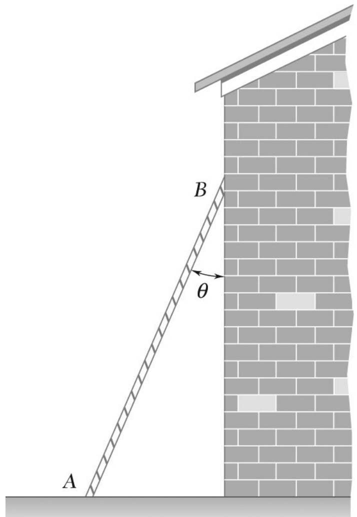
$$v_B = 0.775\sqrt{gL}$$

$\mathbf{v}_A = 0.775\sqrt{gL} \longleftarrow$ ◀

$\mathbf{v}_B = 0.775\sqrt{gL} \swarrow 60°$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

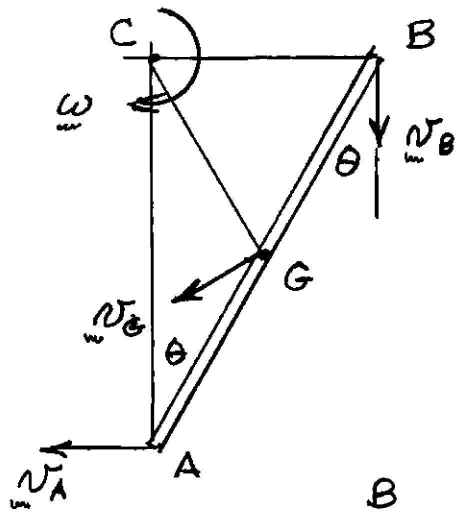
STUDENTS-HUB.com



PROBLEM 17.37

A 5-m long ladder has a mass of 15 kg and is placed against a house at an angle $\theta = 20°$. Knowing that the ladder is released from rest, determine the angular velocity of the ladder and the velocity of A when $\theta = 45°$. Assume the ladder can slide freely on the horizontal ground and on the vertical wall.

SOLUTION



Kinematics:

Let $\mathbf{v}_A = v_A \longleftarrow$, $\mathbf{v}_B = v_B \downarrow$, and $\boldsymbol{\omega} = \omega$ ↻. Locate the instantaneous center C by drawing AC perpendicular to $\mathbf{v}_A$ and BC perpendicular to $\mathbf{v}_B$. Triangle GCB is isosceles.

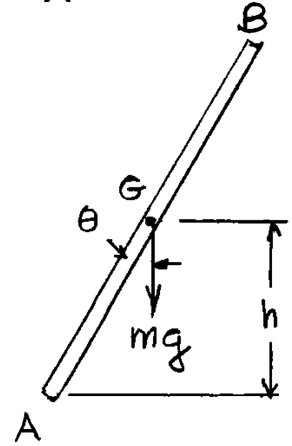
$GA = GB = GC = L/2$. The velocity of the mass center G is

$$\bar{v} = v_G = L\omega/2$$

Kinetic energy:

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}\left(\bar{I} + \frac{1}{4}mL^2\right)\omega^2$$



Since the ladder can slide freely, the friction forces at A and B are zero.

Use the principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2:$$

Potential energy: Use the ground as the datum.

$$V = mgh$$

where $$h = \frac{L}{2}\cos\theta$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.37 (Continued)

Position 1. $\theta = 20°$; rest $(T_1 = 0)$

Position 2. $\theta = 45°$; $\omega = ?$

$$0 + mg\frac{L}{2}\cos 20° = \frac{1}{2}\left(\bar{I} + \frac{1}{4}mL^2\right)\omega^2 + mg\frac{L}{2}\cos 45°$$

Data: $m = 15\text{ kg}.\quad L = 5\text{ m}\quad g = 9.81\text{ m/s}^2$

Assume $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(15\text{ kg})(5\text{ m})^2 = 31.25\text{ kg}\cdot\text{m}^2$

$$\bar{I} + \frac{1}{4}mL^2 = 31.25\text{ kg}\cdot\text{m}^2 + \frac{1}{4}(15\text{ kg})(5\text{ m})^2 = 125\text{ kg}\cdot\text{m}^2$$

$$(15\text{ kg})(9.81\text{ m/s}^2)(2.5\text{ m})(\cos 20° - \cos 45°) = \frac{1}{2}(125\text{ kg}\cdot\text{m}^2)\omega^2$$

$$\omega^2 = 1.3690\text{ rad}^2/\text{s}^2 \qquad \omega = 1.17004\text{ rad/s}$$

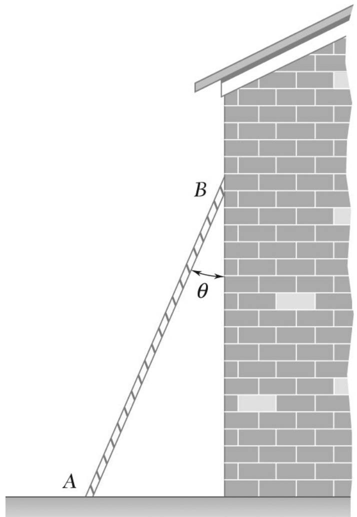
Angular velocity. $\boldsymbol{\omega} = 1.170\text{ rad/s}$ ↻ ◀

Velocity of end A.

$v_A = \omega L\cos\theta = (1.17004\text{ rad/s})(5\text{ m})\cos 30°$ $\mathbf{v}_A = 5.07\text{ m/s} \leftarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

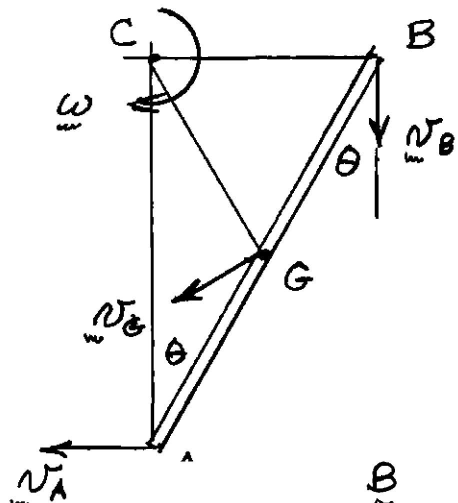
STUDENTS-HUB.com



PROBLEM 17.38

A long ladder of length l, mass m, and centroidal mass moment of inertia $\bar{I}$ is placed against a house at an angle $\theta = \theta_0$. Knowing that the ladder is released from rest, determine the angular velocity of the ladder when $\theta = \theta_2$. Assume the ladder can slide freely on the horizontal ground and on the vertical wall.

SOLUTION



Kinematics:

Let $\mathbf{v}_A = v_A \longleftarrow$, $\mathbf{v}_B = v_B \downarrow$, and $\boldsymbol{\omega} = \omega \circlearrowright$. Locate the instantaneous center C by drawing AC perpendicular to $\mathbf{v}_A$ and BC perpendicular to $\mathbf{v}_B$. Triangle GCB is isosceles.

$GA = GB = GC = L/2$. The velocity of the mass center G is

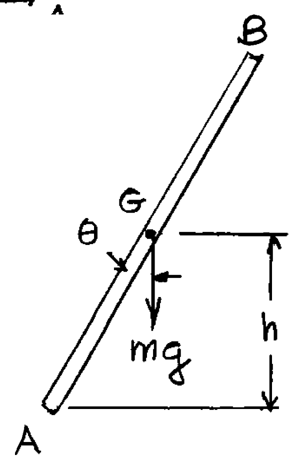
$$\bar{v} = v_G = L\omega/2$$

Kinetic energy:

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\,\omega^2$$

$$= \frac{1}{2}\left(\bar{I} + \frac{1}{4}mL^2\right)\omega^2$$

Since the ladder can slide freely, the friction forces at A and B are zero.



Use the principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2:$$

Potential energy: Use the ground as the datum.

$$V = mgh$$

where $$h = \frac{L}{2}\cos\theta$$

Position 1. $\theta = \theta_0$; rest $(T_1 = 0)$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.38 (Continued)

Position 2. $\theta = \theta_2; \quad \omega = ?$

$$0 + mg\frac{L}{2}\cos\theta = \frac{1}{2}\left(\bar{I} + \frac{1}{4}mL^2\right)\omega^2 + mg\frac{L}{2}\cos\theta_2$$

Assume $\bar{I} = \frac{1}{12}mL^2$

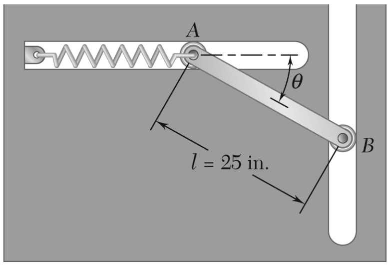
$$\bar{I} + \frac{1}{4}mL^2 = \frac{1}{3}mL^2$$

$$\omega^2 = \frac{3g}{L}(\cos\theta_0 - \cos\theta_2)$$

Angular velocity. $\boldsymbol{\omega} = \sqrt{3g(\cos\theta_0 - \cos\theta_2)/L}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

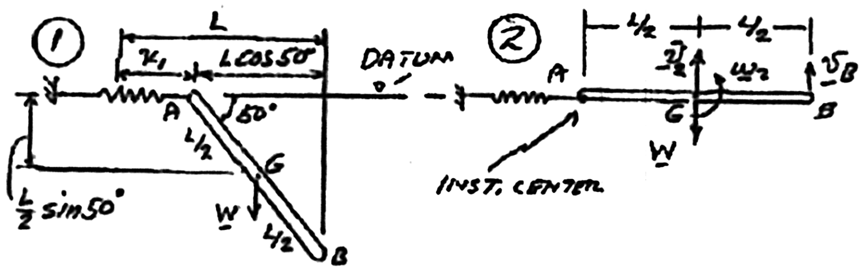
STUDENTS-HUB.com



PROBLEM 17.39

The ends of a 9-lb rod AB are constrained to move along slots cut in a vertical plate as shown. A spring of constant $k = 3$ lb/in. is attached to end A in such a way that its tension is zero when $\theta = 0$. If the rod is released from rest when $\theta = 50°$, determine the angular velocity of the rod and the velocity of end B when $\theta = 0$.

SOLUTION



$$\bar{v}_2 = \frac{L}{2}\omega_2$$

$$v_B = L\omega_2$$

$$x_1 = L - L\cos 50°$$
$$= (25 \text{ in.})(1 - \cos 50°)$$
$$= 8.9303 \text{ in.}$$

Position 1.

$$V_1 = -W\frac{L}{2}\sin 50° + \frac{1}{2}kx_1^2$$

$$V_1 = -(9 \text{ lb})\left(\frac{25 \text{ in.}}{2}\right)\sin 50° + \frac{1}{2}(3 \text{ lb/in.})(8.9303 \text{ in.})^2$$
$$= -86.18 + 119.63$$
$$= 33.45 \text{ in.}\cdot\text{lb}$$
$$= 2.787 \text{ ft}\cdot\text{lb.}$$

$$T_1 = 0$$

Position 2.

$$V_2 = (V_g)_2 + (V_e)_2 = 0$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$
$$= \frac{1}{6}mL^2\omega_2^2 = \frac{1}{6}\left(\frac{9 \text{ lb}}{32.2}\right)\left(\frac{25 \text{ in.}}{12}\right)^2\omega_2^2 = 0.2022\omega_2^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.39 (Continued)

Conservation of energy:
$$T_1 + V_1 = T_2 + V_2$$
$$0 + 2.787 \text{ ft} \cdot \text{lb} = 0.2022\omega_2^2$$
$$\omega_2^2 = 13.7849$$
$$\omega_2 = 3.713 \text{ rad/s}$$

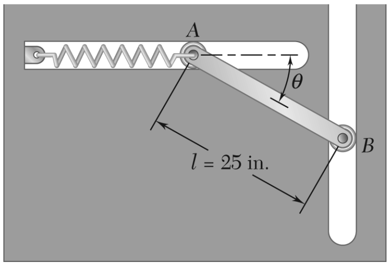
$$\boldsymbol{\omega}_2 = 3.71 \text{ rad/s} \circlearrowright \blacktriangleleft$$

Velocity of B:
$$v_B = L\omega_2 = \left(\frac{25 \text{ in.}}{12}\right)(3.713 \text{ rad/s})$$
$$= 7.735 \text{ ft/s}$$

$$\mathbf{v}_B = 7.74 \text{ ft/s} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

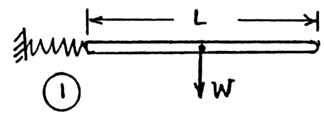
STUDENTS-HUB.com



PROBLEM 17.40

The ends of a 9-lb rod AB are constrained to move along slots cut in a vertical plane as shown. A spring of constant $k = 3$ lb/in. is attached to end A in such a way that its tension is zero when $\theta = 0$. If the rod is released from rest when $\theta = 0$, determine the angular velocity of the rod and the velocity of end B when $\theta = 30°$.

SOLUTION



Moment of inertia. Rod. $\quad \bar{I} = \frac{1}{12}mL^2$

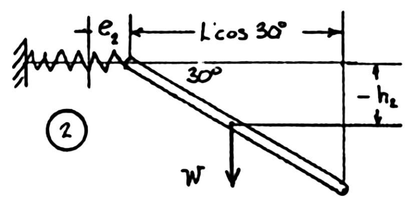
Position 1. $\quad \theta_1 = 0 \quad \bar{v}_1 = 0 \quad \omega_1 = 0$

$h_1 =$ elevation above slot. $\quad h_1 = 0$

$e_1 =$ elongation of spring. $\quad e_1 = 0$

$$T_1 = \frac{1}{2}m\bar{v}_1^2 + \frac{1}{2}\bar{I}\omega_1^2 = 0$$

$$V_1 = \frac{1}{2}ke_1^2 + Wh_1 = 0$$



Position 2. $\quad \theta = 30°$

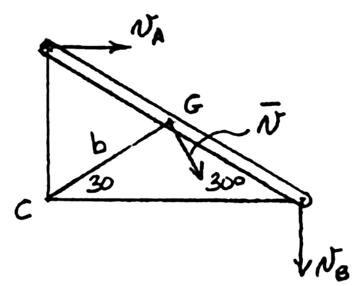
$$e_2 + L\cos 30° = L$$

$$e_2 = L(1 - \cos 30°)$$

$$h_2 = -\frac{L}{2}\sin 30° = -\frac{1}{4}L$$

$$V_2 = \frac{1}{2}ke_2^2 + Wh_2 = \frac{1}{2}kL^2(1-\cos 30°)^2 - \frac{1}{4}WL$$

Kinematics. Velocities at A and B are directed as shown. Point C is the instantaneous center of rotation. From geometry, $b = \frac{L}{2}$.



$$\bar{v} = b\omega = \frac{L}{2}\omega$$

$$v_B = (L\cos 30°)\omega$$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)$$

$$= \frac{1}{6}\frac{W}{g}L^2\omega^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{6}\frac{W}{g}L^2\omega^2 + \frac{1}{2}kL^2(1 - \cos 30°)^2 - \frac{1}{4}WL$$

$$\omega^2 = \frac{3g}{2L} - \frac{3\,kg}{W}(1 - \cos 30°)^2$$

Data: $W = 9$ lb

$g = 32.2$ ft/s^2

$L = 25$ in. $= 2.0833$ ft

$k = 3$ lb/in. $= 36$ lb/ft

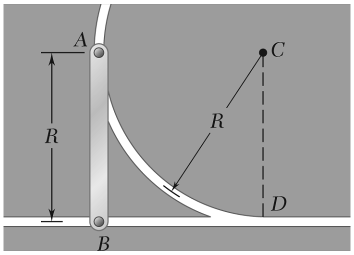
$$\omega^2 = \frac{(3)(32.2)}{(2)(2.0833)} - \frac{(3)(36)(32.2)(1 - \cos 30°)^2}{9}$$

$= 16.2484$ $\qquad$ $\boldsymbol{\omega} = 4.03$ rad/s $\circlearrowright$ ◀

$v_B = (2.0833)(\cos 30°)(4.03)$ $\qquad$ $\mathbf{v}_B = 7.27$ ft/s $\downarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.41

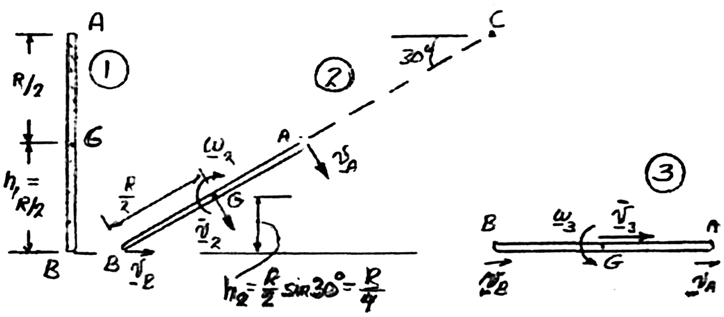
The motion of a slender rod of length R is guided by pins at A and B which slide freely in slots cut in a vertical plate as shown. If end B is moved slightly to the left and then released, determine the angular velocity of the rod and the velocity of its mass center (*a*) at the instant when the velocity of end B is zero, (*b*) as end B passes through Point D.

SOLUTION

The rod AB moves from *Position 1*, where it is nearly vertical, to *Position 2*, where $\mathbf{v}_B = 0$.

In *Position 2*, $\mathbf{v}_A$ is perpendicular to both CA and AB, so CAB is a straight line of length $2L$ and slope angle 30°.

In *Position 3* the end B passes through Point D.



Position 1: $$T_1 = 0 \qquad V_1 = Wh = mg\frac{R}{2}$$

Position 2: Since instantaneous center is at B,

$$v_2 = \frac{1}{2}R\omega_2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(\frac{1}{2}R\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mR^2\right)\omega_2^2$$

$$= \frac{1}{6}mR^2\omega_2^2$$

$$V_2 = Wh_2 = mg\frac{R}{4}$$

Position 3: $$V_3 = 0$$

Since both $\mathbf{v}_A$ and $\mathbf{v}_B$ are horizontal, $\omega_3 = 0$ (1)

$$T_3 = \frac{1}{2}m\bar{v}_2^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.41 (Continued)

(*a*) From 1 to 2: Conservation of energy

$$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{2}mgR = \frac{1}{6}mR^2\omega_2^2 + \frac{1}{4}mgR$$

$$\omega_2^2 = \frac{3}{2}\frac{g}{R}$$

$$\omega_2 = \sqrt{\frac{3}{2}\frac{g}{R}} \qquad \boldsymbol{\omega}_2 = 1.225\sqrt{\frac{g}{R}} \circlearrowright \blacktriangleleft$$

$$\bar{v}_2 = \frac{1}{2}R\omega_2 = \frac{1}{2}\sqrt{\frac{3}{2}gR} = \sqrt{\frac{3}{8}gR} \qquad \mathbf{v}_R = 0.612\sqrt{gR} \searrow 60° \blacktriangleleft$$

(*b*) From 1 to 3: Conservation of energy

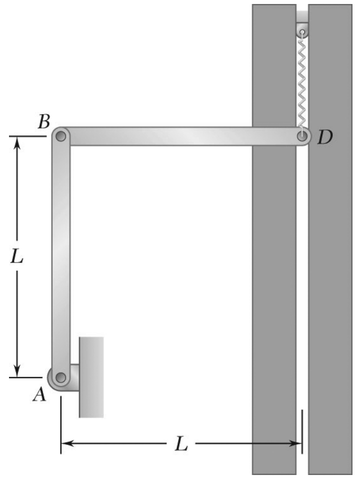
From Eq. (1) we have $\qquad \boldsymbol{\omega}_3 = 0 \blacktriangleleft$

$$T_1 + V_1 = T_3 + V_3: \quad 0 + \frac{1}{2}mgR = \frac{1}{2}m\bar{v}_3^2$$

$$\bar{v}_3^2 = gR \qquad \mathbf{v}_3 = \sqrt{gR} \longrightarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.42

Each of the two rods shown is of length $L = 1$ m and has a mass of 5 kg. Point D is connected to a spring of constant $k = 20$ N/m and is constrained to move along a vertical slot. Knowing that the system is released from rest when rod BD is horizontal and the spring connected to Point D is initially unstretched, determine the velocity of Point D when it is directly to the right of Point A.

SOLUTION

Moments of inertia. $\qquad \bar{I} = \frac{1}{12}mL^2, \quad I_A = \frac{1}{3}mL^2$

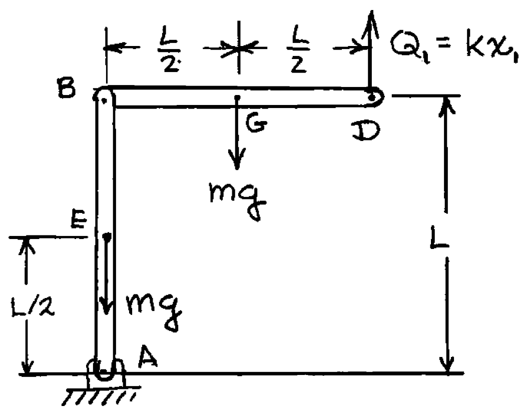
Use the principle of conservation of energy applied to the system consisting of both rods. Use the level at A as the datum for the potential energy of each rod.

Position 1. (no motion)

$$T_1 = 0$$

$$V_1 = mg\left(\frac{1}{2}L\right) + mgL + \frac{1}{2}kx_1^2$$

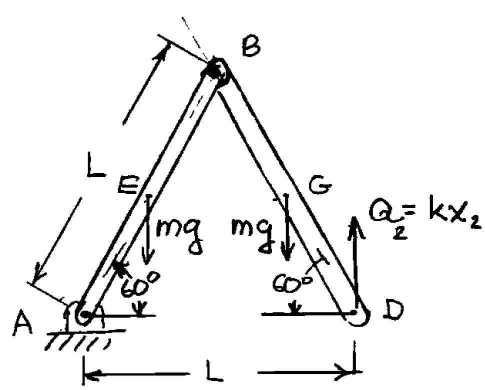
$$= \frac{3}{2}mgL + \frac{1}{2}kx_1^2$$



Position 2.

$$V_2 = mg\frac{L}{2}\sin 60° + mg\frac{L}{2}\sin 60°$$

$$= \frac{\sqrt{3}}{2}mgL + \frac{1}{2}kx_2^2$$



PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

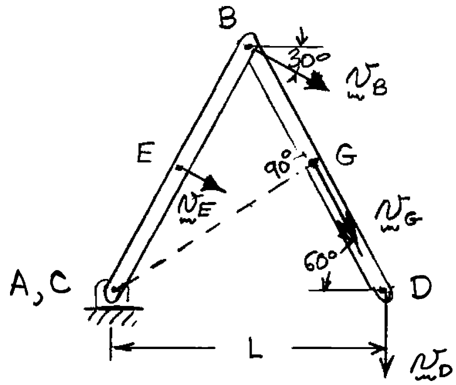
STUDENTS-HUB.com

Kinematics.

$$\boldsymbol{\omega}_{AB} = \omega_{AB} \circlearrowright$$

$$v_B = L\omega_{AB} \quad \mathbf{v}_B = L\omega_{AB} \searrow 30^\circ$$

$$\mathbf{v}_D = v_D \downarrow$$



Locate the instantaneous center C of rod BD by drawing BC perpendicular to $\mathbf{v}_B$ and DC perpendicular to $\mathbf{v}_D$. Point C coincides with Point A in position 2.

Let $$\boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowright$$

$$\omega_{BD} = \frac{v_B}{L} = \omega_{AB}$$

$$v_E = \frac{L}{2}\omega_{AB}$$

$$v_G = (L\sin 60^\circ)\omega_{BD} = \frac{\sqrt{3}}{2}L\omega_{AB}$$

$$v_D = L\omega_{BD} = L\omega_{AB} \qquad (1)$$

$$T_2 = \frac{1}{2}I_A\omega_{AB}^2 + \frac{1}{2}\bar{I}\,\omega_{BD}^2 + \frac{1}{2}mv_G^2$$

$$= \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_{AB}^2 + \frac{1}{2}m\left(\frac{\sqrt{3}}{2}\omega_{AB}\right)^2$$

$$= \left(\frac{1}{6} + \frac{1}{24} + \frac{3}{8}\right)mL^2\omega_{AB}^2 = \frac{7}{12}mL^2\omega_{AB}^2$$

Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{3}{2}mgL + \frac{1}{2}kx_1^2 = \frac{7}{12}mL^2\omega_{AB}^2 + \frac{\sqrt{3}}{2}mgL + \frac{1}{2}kx_2^2$$

$$\frac{7}{12}mL^2\omega_{AB}^2 = \left(\frac{3}{2} - \frac{\sqrt{3}}{2}\right)mgL - \frac{1}{2}k(x_2^2 - x_1^2) \qquad (2)$$

Data: $m = 5\text{ kg}, \quad L = 1\text{ m}, \quad g = 9.81\text{ m/s}^2$

$k = 20\text{ N}\cdot\text{m}, \quad x_1 = 0, \quad x_2 = L = 1\text{ m}$

$$\left(\frac{3}{2} - \frac{\sqrt{3}}{2}\right)mgL = (0.63397)(5\text{ kg})(9.81\text{ m/s}^2)(1\text{ m}) = 31.096\text{ J}$$

$$-\frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(20\text{ N/m})(1\text{ m})^2 = -10\text{ J}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

By Eq. (2), $$\frac{7}{12}mL^2\omega_{AB}^2 = \left(\frac{35}{12}\text{kg}\cdot\text{m}^2\right)\omega_{AB}^2 = 21.096\text{ J}$$

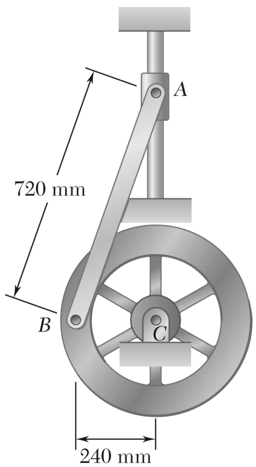
$$\omega_{AB}^2 = 7.2329\text{ rad}^2/\text{s}^2 \quad \omega_{AB} = 2.6894\text{ rad/s}$$

By Eq. (1), $$v_D = (1\text{ m})(2.6894\text{ rad/s})$$ $\mathbf{v}_D = 2.69\text{ m/s}\downarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

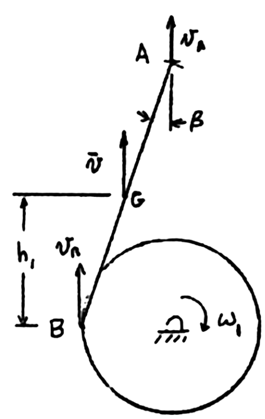
PROBLEM 17.43



The 4-kg rod AB is attached to a collar of negligible mass at A and to a flywheel at B. The flywheel has a mass of 16 kg and a radius of gyration of 180 mm. Knowing that in the position shown the angular velocity of the flywheel is 60 rpm clockwise, determine the velocity of the flywheel when Point B is directly below C.

SOLUTION

Moments of inertia.



Rod AB:
$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2 = \frac{1}{12}(4 \text{ kg})(0.72 \text{ m})^2 = 0.1728 \text{ kg} \cdot \text{m}^2$$

Flywheel:
$$I_C = m\bar{k}^2 = (16 \text{ kg})(0.18 \text{ m})^2 = 0.5184 \text{ kg} \cdot \text{m}^2$$

Position 1. As shown.
$$\boldsymbol{\omega} = \omega_1 \curvearrowright$$

$$\sin\beta = \frac{0.24}{0.72} \qquad \beta = 19.471°$$

$$h_1 = \frac{1}{2}(0.72)\cos\beta = 0.33941 \text{ m}$$

$$V_1 = W_{AB}h_1 = (4)(9.81)(0.33941) = 13.3185 \text{ J}$$

Kinematics.
$$v_B = r\omega_1 = 0.24\omega_1$$

Bar AB is in translation.
$$\omega_{AB} = 0, \quad \bar{v} = v_B$$

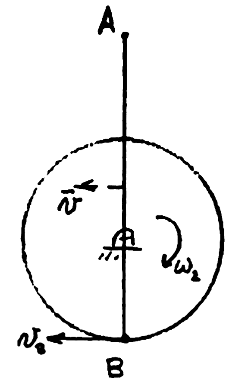
$$T_1 = \frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}I_C\omega_1^2 = \frac{1}{2}(4)(0.24\omega_1)^2 + 0 + \frac{1}{2}(0.5184)\omega_1^2 = 0.3744\omega_1^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.43 (Continued)

Position 2. Point B is directly below C.



$$h_2 = \frac{1}{2}L_{AB} - r$$
$$= \frac{1}{2}(0.72) - 0.24$$
$$= 0.12 \text{ m}$$
$$V_2 = W_{AB}h_2$$
$$= (4)(9.81)(0.12)$$
$$= 4.7088 \text{ J}$$

Kinematics. $\quad v_B = r\omega_2 = 0.24\omega_2$

$$\omega_{AB} = \frac{v_B}{0.72} = 0.33333\omega_2$$
$$\bar{v} = \frac{1}{2}v_B = 0.12\omega_2$$
$$T_2 = \frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}I_C\omega_2^2$$
$$= \frac{1}{2}(4)(0.12\omega_2)^2 + \frac{1}{2}(0.1728)(0.33333\omega_2)^2 + \frac{1}{2}(0.5184)\omega_2^2$$
$$= 0.2976\omega_2^2$$

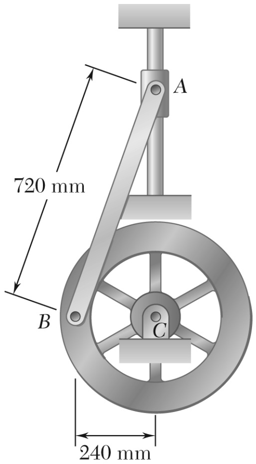
Conservation of energy. $\quad T_1 + V_1 = T_2 + V_2: \quad 0.3744\omega_1^2 + 13.3185 = 0.2976\omega_2^2 + 4.7088 \qquad (1)$

Angular speed data: $\quad \omega_1 = 60 \text{ rpm} = 2\pi \text{ rad/s}$

Solving Equation (1) for ω_2, $\quad \omega_2 = 8.8655 \text{ rad/s} \qquad \boldsymbol{\omega}_2 = 84.7 \text{ rpm} \circlearrowright$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



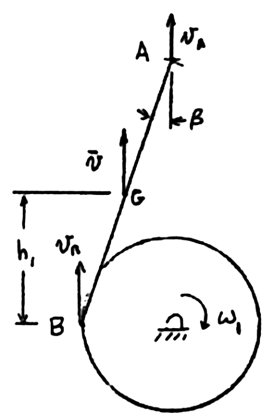
PROBLEM 17.44

If in Problem 17.43 the angular velocity of the flywheel is to be the same in the position shown and when Point B is directly above C, determine the required value of its angular velocity in the position shown.

PROBLEM 17.43 The 4-kg rod AB is attached to a collar of negligible mass at A and to a flywheel at B. The flywheel has a mass of 16 kg and a radius of gyration of 180 mm. Knowing that in the position shown the angular velocity of the flywheel is 60 rpm clockwise, determine the velocity of the flywheel when Point B is directly below C.

SOLUTION

Moments of inertia.



Rod AB:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2$$
$$= \frac{1}{12}(4\text{ kg})(0.72\text{ m})^2$$
$$= 0.1728\text{ kg}\cdot\text{m}^2$$

Flywheel:

$$I_C = m\bar{k}^2$$
$$= (16\text{ kg})(0.18\text{ m})^2$$
$$= 0.5184\text{ kg}\cdot\text{m}^2$$

Position 1. As shown.

$$\boldsymbol{\omega} = \omega_1 \circlearrowright$$

$$\sin\beta = \frac{0.24}{0.72} \qquad \beta = 19.471°$$

$$h_1 = \frac{1}{2}(0.72)\cos\beta = 0.33941\text{ m}$$

$$V_1 = W_{AB} h_1$$
$$= (4)(9.81)(0.33941)$$
$$= 13.3185\text{ J}$$

Kinematics.

$$v_B = r\omega_1 = 0.24\omega_1$$

Bar AB is in translation.

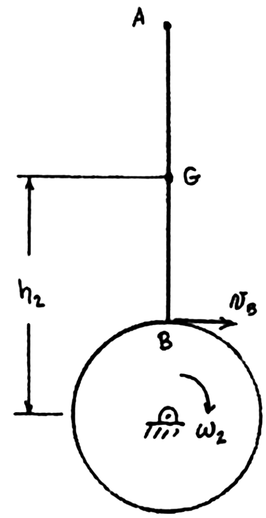
$$\omega_{AB} = 0, \quad \bar{v} = v_B$$

$$T_1 = \frac{1}{2} m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2} I_C\omega_1^2$$
$$= \frac{1}{2}(4)(0.24\omega_1)^2 + 0 + \frac{1}{2}(0.5184)\omega_1^2$$
$$= 0.3744\omega_1^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Position 2. Point *B* is directly above *C*.



$$h_2 = \frac{1}{2}L_{AB} + r$$
$$= \frac{1}{2}(0.72) + 0.24$$
$$= 0.6 \text{ m}$$
$$V_2 = W_{AB}h_2$$
$$= (4)(9.81)(0.6)$$
$$= 23.544 \text{ J}$$

Kinematics. $\quad v_B = r\omega_2 = 0.24\omega_2$

$$\omega_{AB} = \frac{v_B}{0.72} = 0.33333\omega_2$$
$$\bar{v} = \frac{1}{2}v_B = 0.12\omega_2$$
$$T_2 = \frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}I_C\omega_2^2$$
$$= \frac{1}{2}(4)(0.12\omega_2)^2 + \frac{1}{2}(0.1728)(0.33333\omega_2)^2 + \frac{1}{2}(0.5184)\omega_2^2$$
$$= 0.2976\omega_2^2$$

Conservation of energy. $\quad T_1 + V_1 = T_2 + V_2: \quad 0.3744\omega_1^2 + 13.3135 = 0.2976\omega_2^2 + 23.544$

Angular speed data: $\quad \omega_2 = \omega_1$

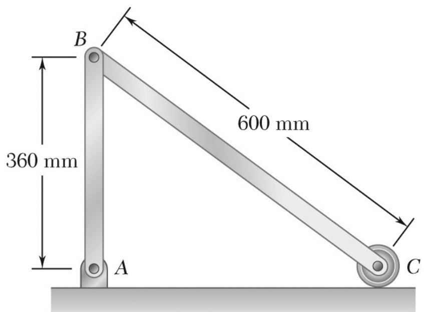
Then, $\quad 0.0760\omega_1^2 = +0.4105$

$$\omega_1 = 11.602 \text{ rad/s}$$

$\omega_1 = 110.8 \text{ rpm}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.45

The uniform rods AB and BC of masses 2.4 kg and 4 kg, respectively, and the small wheel at C is of negligible mass. If the wheel is moved slightly to the right and then released, determine the velocity of pin B after rod AB has rotated through 90°.

SOLUTION

Moments of inertia.

Rod AB: $$I_A = \frac{1}{3} m_{AB} L_{AB}^2 = \frac{1}{3}(2.4)(0.360)^2 = 0.10368 \text{ kg}\cdot\text{m}^2$$

Rod BC: $$\bar{I} = \frac{1}{12} m_{BC} L_{BC}^2 = \frac{1}{12}(4)(0.600)^2 = 0.1200 \text{ kg}\cdot\text{m}^2$$

Position 1. As shown with bar AB vertical. Point G is the midpoint of BC.

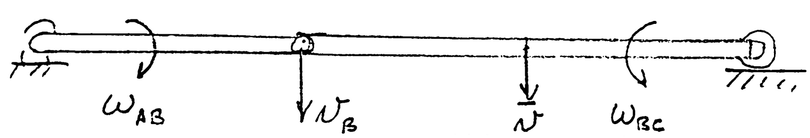
$$V_1 = m_{AB} g h_{AB} + m_{BC} g h_{BC} = (2.4)(9.81)(0.180) + (4)(9.81)(0.180) = 11.3011 \text{ J}$$

Rod BC is at rest. $$\omega_{BC} = 0$$

$$\bar{v} = v_G = v_B = v_C = 0 \qquad \omega_{AB} = \frac{v_B}{L_{AB}} = 0$$

$$T_1 = 0$$

Position 2. Rod AB is horizontal.



$$V_2 = 0$$

Kinematics. $$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{v_B}{0.360} \qquad \omega_{BC} = \frac{v_B}{L_{BC}} = \frac{v_B}{0.600} \qquad \bar{v} = \frac{1}{2} v_B$$

$$T_2 = \frac{1}{2} I_A \omega_{AB}^2 + \frac{1}{2} m_{BC} \bar{v}^2 + \frac{1}{2} \bar{I} \omega_{BC}^2$$

$$= \frac{1}{2}(0.10368)\left(\frac{v_B}{0.360}\right)^2 + \frac{1}{2}(4)\left(\frac{1}{2} v_B\right)^2 + \frac{1}{2}(0.1200)\left(\frac{v_B}{0.600}\right)^2$$

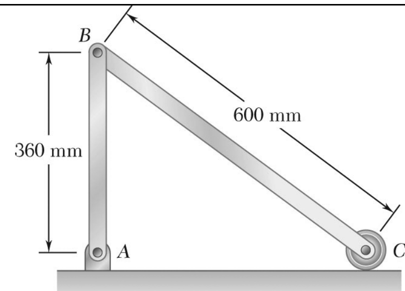
$$= 1.06667 v_B^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $\quad 0 + 11.3011 = 1.06667 v_B^2$

$$v_B = 3.25 \text{ m/s} \qquad \mathbf{v}_B = 3.25 \text{ m/s} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.46

The uniform rods AB and BC of masses 2.4 kg and 4 kg, respectively, and the small wheel at C is of negligible mass. Knowing that in the position shown the velocity of wheel C is 2 m/s to the right, determine the velocity of pin B after rod AB has rotated through 90°.

SOLUTION

Moments of inertia.

Rod AB: $$I_A = \frac{1}{3}m_{AB}L_{AB}^2 = \frac{1}{3}(2.4)(0.36)^2 = 0.10368 \text{ kg}\cdot\text{m}^2$$

Rod BC: $$\bar{I} = \frac{1}{12}m_{BC}L_{BC^2} = \frac{1}{12}(4)(0.600)^2 = 0.1200 \text{ kg}\cdot\text{m}^2$$

Position 1. As shown with rod AB vertical. Point G is the midpoint of BC.

$$\begin{aligned} V_1 &= W_{AB}h_{AB} + W_{BC}h_{BC} \\ &= (2.4)(9.81)(0.180) + (4)(9.81)(0.180) \\ &= 11.301 \text{ J} \end{aligned}$$

Kinematics: At the instant shown in Position 1,

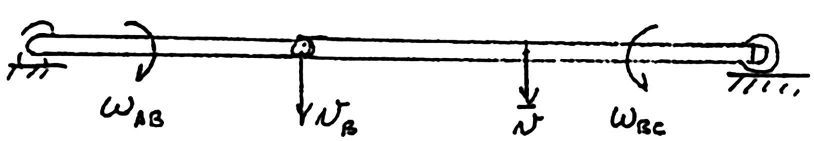
$$\omega_{BC} = 0$$

$$\bar{v} = v_G = v_B = v_C = 2 \text{ m/s}$$

$$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{2}{0.36} = 5.5556 \text{ rad/s}$$

$$\begin{aligned} T_1 &= \frac{1}{2}I_A\omega_{AB}^2 + \frac{1}{2}m_{BC}\bar{v}^2 + \frac{1}{2}I\omega_{BC}^2 \\ &= \frac{1}{2}(0.10368)(5.5556)^2 + \frac{1}{2}(4)(2)^2 + 0 \\ &= 9.6 \text{ J} \end{aligned}$$

Position 2. Rod AB is horizontal.



$$V_2 = 0$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.46 (Continued)

Kinematics.

$$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{v_B}{0.36}$$

$$\omega_{BC} = \frac{v_B}{L_{BC}} = \frac{v_B}{0.60}$$

$$\overline{v} = \frac{1}{2} v_B$$

$$T_2 = \frac{1}{2} I_A \omega_{AB}^2 + \frac{1}{2} m_{BC} \overline{v}^2 + \frac{1}{2} \overline{I} \omega_{BC}^2$$

$$= \frac{1}{2}(0.10368)\left(\frac{v_B}{0.36}\right)^2 + \frac{1}{2}(4)\left(\frac{1}{2} v_B\right)^2 + \frac{1}{2}(0.12)\left(\frac{v_B}{0.60}\right)^2$$

$$= 1.0667 v_B^2$$

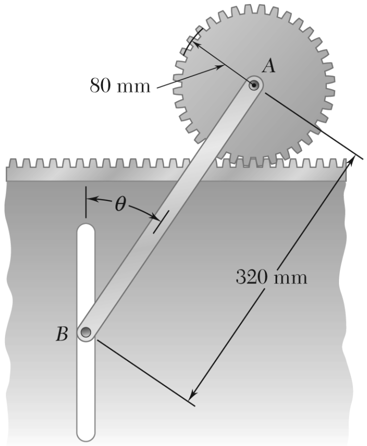
Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $9.6 + 11.301 = 1.0667 v_B^2 + 0$

$$v_B = 4.4266 \text{ m/s}$$

$\mathbf{v}_B = 4.43 \text{ m/s} \downarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

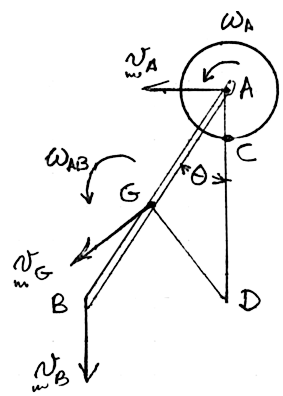
STUDENTS-HUB.com



PROBLEM 17.47

The 80-mm-radius gear shown has a mass of 5 kg and a centroidal radius of gyration of 60 mm. The 4-kg rod AB is attached to the center of the gear and to a pin at B that slides freely in a vertical slot. Knowing that the system is released from rest when $\theta = 60°$, determine the velocity of the center of the gear when $\theta = 20°$.

SOLUTION



Kinematics.

$$\mathbf{v}_A = v_A \longleftarrow$$

$$\mathbf{v}_B = v_B \downarrow$$

Point D is the instantaneous center of rod AB.

$$\omega_{AB} = \frac{v_A}{L\cos\theta}$$

$$v_B = (L\sin\theta)\omega_{AB} = v_A \tan\theta$$

$$v_G = \frac{L}{2}\omega_{AB} = \frac{v_A}{2\cos\theta}$$

Gear A effectively rolls without slipping, with Point C being the contact point.

$$v_C = 0$$

Angular velocity of gear $\quad \omega_A = \dfrac{v_A}{r}.$

Potential energy: Use the level of the center of gear A as the datum.

$$V = -W_{AB}\left(\frac{L}{2}\cos\theta\right) = -\frac{1}{2}m_{AB}gL\cos\theta$$

Kinetic energy: $\quad T = \dfrac{1}{2}m_A v_A^2 + \dfrac{1}{2}I_A\omega_A^2 + \dfrac{1}{2}m_{AB}v_\theta^2 + \dfrac{1}{2}\bar{I}_{AB}\omega_{AB}^2$

Masses and moments of inertia: $\quad m_A = 5 \text{ kg}, \quad m_{AB} = 4 \text{ kg}$

$$I_A = m_A k^2 = (5)(0.060)^2 = 0.018 \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{AB} = \frac{1}{12}m_{AB}L^2 = \frac{1}{12}(4)(0.320)^2 = 0.03413 \text{ kg}\cdot\text{m}^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

Position 1:

$$\theta = 60°$$

$$v_A = 0 \quad T_1 = 0$$

$$V_1 = -\frac{1}{2}(4)(9.81)(0.320)\cos 60°$$

$$= -3.1392 \text{ J}$$

Position 2:

$$\theta = 20° \quad v_A = ?$$

$$T_2 = \frac{1}{2}(5)v_A^2 + \frac{1}{2}(0.018)\left(\frac{v_A}{0.080}\right)^2 + \frac{1}{2}(4)\left(\frac{v_A}{2\cos 20°}\right)^2$$

$$+\frac{1}{2}(0.03413)\left(\frac{v_A}{0.320\cos 20°}\right)^2$$

$$= (2.5 + 1.40625 + 0.56624 + 0.18875)v_A^2$$

$$= 4.66124 v_A^2$$

$$V_2 = -\frac{1}{2}(4)(9.81)(0.320)\cos 20°$$

$$= -5.8998 \text{ J}$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0 - 3.1392 = 4.66124 v_A^2 - 5.8998$$

$$v_A^2 = 0.59225 \text{ m}^2/\text{s}^2$$

$$v_A = 0.770 \text{ m/s}$$

$\mathbf{v}_A = 0.770 \text{ m/s} \longleftarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.48

Knowing that the maximum allowable couple that can be applied to a shaft is 15.5 kip · in., determine the maximum horsepower that can be transmitted by the shaft at (*a*) 180 rpm, (*b*) 480 rpm.

SOLUTION

$$M = 15.5 \text{ kip} \cdot \text{in.}$$
$$= 1.2917 \text{ kip} \cdot \text{ft}$$
$$= 1291.7 \text{ lb} \cdot \text{ft}$$

(*a*)

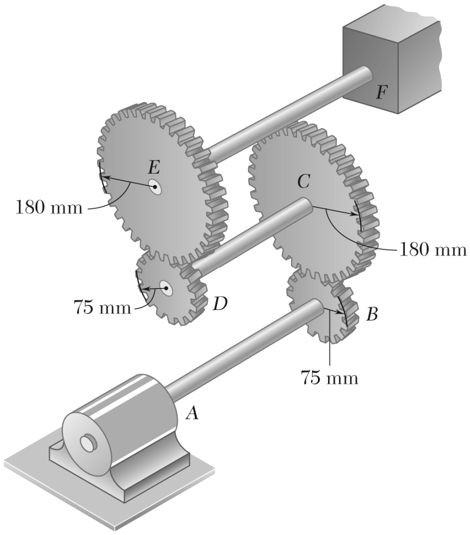
$$\omega = 180 \text{ rpm}$$
$$= 6\pi \text{ rad/s}$$
$$\text{Power} = M\omega$$
$$= (1291.7 \text{ lb} \cdot \text{ft})(6\pi \text{ rad/s})$$
$$= 24348 \text{ ft} \cdot \text{lb/s}$$
$$\text{Horsepower} = \frac{24348}{550}$$
$$= 44.3 \text{ hp} \quad \blacktriangleleft$$

(*b*)

$$\omega = 480 \text{ rpm}$$
$$= 16\pi \text{ rad/s}$$
$$\text{Power} = M\omega$$
$$= (1291.7 \text{ lb} \cdot \text{ft})(16\pi \text{ rad/s})$$
$$= 64930 \text{ ft} \cdot \text{lb/s}$$
$$\text{Horsepower} = \frac{64930}{550}$$
$$= 118.1 \text{ hp} \quad \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.49

Three shafts and four gears are used to form a gear train which will transmit 7.5 kW from the motor at *A* to a machine tool at *F*. (Bearings for the shafts are omitted from the sketch.) Knowing that the frequency of the motor is 30 Hz, determine the magnitude of the couple which is applied to shaft (*a*) *AB*, (*b*) *CD*, (*c*) *EF*.

SOLUTION

Kinematics.

$$\omega_{AB} = 30 \text{ Hz}$$
$$= 30(2\pi)\text{rad/s}$$
$$= 60\pi \text{ rad/s}$$

Gears *B* and *C*.

$$r_B = 75 \text{ mm}$$
$$r_C = 180 \text{ mm}$$

$$r_B\omega_{AB} = r_C\omega_{CD}: \quad (75 \text{ mm})(60\pi \text{ rad/s}) = (180 \text{ mm})(\omega_{CD})$$

Gears *D* and *E*.

$$\omega_{CD} = 25\pi \text{ rad/s}$$
$$r_D = 75 \text{ mm}$$
$$r_E = 180 \text{ mm}$$

$$r_D\omega_{CD} = r_E\omega_{EF}: \quad (75 \text{ mm})(25\pi \text{ rad/s}) = (180 \text{ mm})(\omega_{EF})$$

$$\omega_{EF} = 10.4167\pi \text{ rad/s}$$

$$\text{Power} = 7.5 \text{ kW}$$

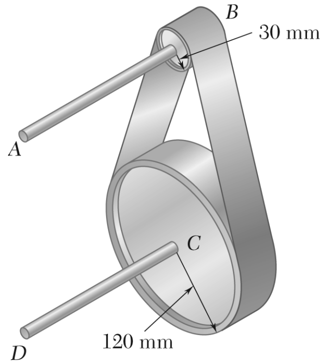
(*a*) Shaft *AB*. $\text{Power} = M_{AB}\omega_{AB}: \quad 7500 \text{ W} = M_{AB}(60\pi \text{ rad/s})$ $\quad M_{AB} = 39.8 \text{ N}\cdot\text{m}$ ◀

(*b*) Shaft *CD*. $\text{Power} = M_{CD}\omega_{CD}: \quad 7500 \text{ W} = M_{CD}(25\pi \text{ rad/s})$ $\quad M_{CD} = 95.5 \text{ N}\cdot\text{m}$ ◀

(*c*) Shaft *EF*. $\text{Power} = M_{EF}\omega_{EF}: \quad 7500 \text{ W} = M_{EF}(10.4167\pi \text{ rad/s})$ $\quad M_{EF} = 229 \text{ N}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.50

The shaft-disk-belt arrangement shown is used to transmit 2.4 kW from Point A to Point D. Knowing that the maximum allowable couples that can be applied to shafts AB and CD are $25\ \text{N}\cdot\text{m}$ and $80\ \text{N}\cdot\text{m}$, respectively, determine the required minimum speed of shaft AB.

SOLUTION

Power.

$$2.4\ \text{kW} = 2400\ \text{W}$$

$$M_{AB} < 25\ \text{N}\cdot\text{m}$$

$$P = M_{AB}\omega_{AB}$$

$$\min \omega_{AB} = \frac{P}{\max M_{AB}} = \frac{2400}{25} = 96\ \text{rad/s}$$

$$M_{CD} < 80\ \text{N}\cdot\text{m}$$

$$P = M_{CD}\omega_{CD}$$

$$\min \omega_{CD} = \frac{P}{\max M_{CD}} = \frac{2400}{80} = 30\ \text{rad/s}$$

Kinematics.

$$r_A\omega_{AB} = r_C\omega_{CD}$$

$$\min \omega_{AB} = \frac{r_C}{r_A}(\min \omega_{CD})$$

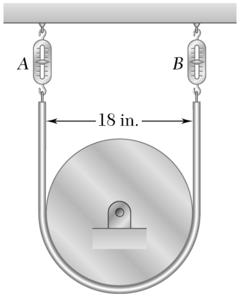
$$= \left(\frac{120}{30}\right)(30)$$

$$= 120\ \text{rad/s}$$

Choose the larger value for $\min \omega_{AB}$. $\min \omega_{AB} = 120\ \text{rad/s}$ $\min \omega_{AB} = 1146\ \text{rpm}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



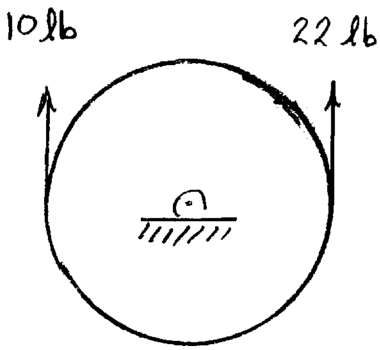
PROBLEM 17.51

The experimental setup shown is used to measure the power output of a small turbine. When the turbine is operating at 200 rpm, the readings of the two spring scales are 10 and 22 lb, respectively. Determine the power being developed by the turbine.

SOLUTION

Angular velocity. $\omega = 200 \text{ rpm} = 20.944 \text{ rad/s}$

Moments about the fixed axle.



$$M = (22 \text{ lb} - 10 \text{ lb})\left(\frac{9}{12}\text{ft}\right) = 9 \text{ lb}\cdot\text{ft}$$

$$\text{Power} = M\omega = (9)(20.994) = 188.5 \text{ lb}\cdot\text{ft/s}$$

$$\frac{188.5 \text{ lb}\cdot\text{ft/s}}{550 \text{ lb}\cdot\text{ft/s/hp}} = \qquad \text{Power} = 0.343 \text{ hp} \blacktriangleleft$$

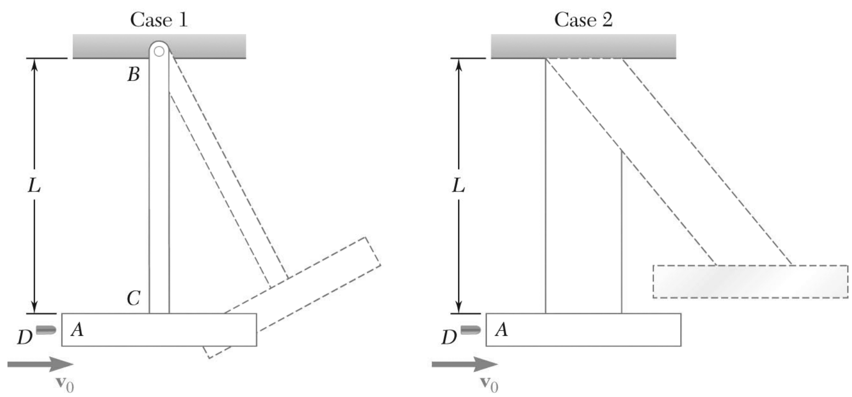
PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.CQ6

Slender bar A is rigidly connected to a massless rod BC in Case 1 and two massless cords in Case 2 as shown. The vertical thickness of bar A is negligible compared to L. If bullet D strikes A with a speed v_0 and becomes embedded in it, how will the speeds of the center of gravity of A immediately after the impact compare for the two cases?

(*a*) Case 1 will be larger.
(*b*) Case 2 will be larger.
(*c*) The speeds will be the same.

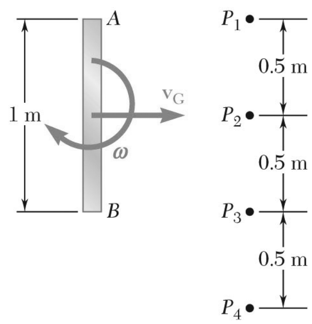


SOLUTION

Answer: (*b*)

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.CQ7

A 1-m long uniform slender bar *AB* has an angular velocity of 12 rad/s and its center of gravity has a velocity of 2 m/s as shown. About which point is the angular momentum of *A* smallest at this instant?

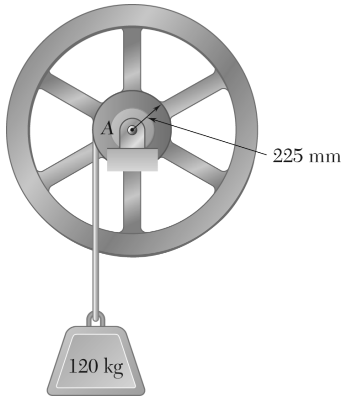
(*a*) P_1
(*b*) P_2
(*c*) P_3
(*d*) P_4
(*e*) It is the same about all the points.

SOLUTION

Answer: (*a*)

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

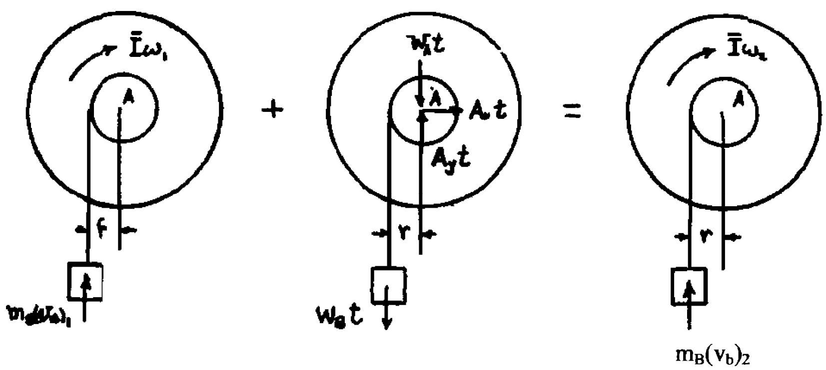


PROBLEM 17.F1

The 350-kg flywheel of a small hoisting engine has a radius of gyration of 600 mm. If the power is cut off when the angular velocity of the flywheel is 100 rpm clockwise, draw an impulse-momentum diagram that can be used to determine the time required for the system to come to rest.

SOLUTION

Answer:

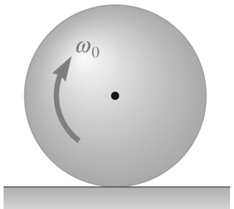


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\rightarrow 2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

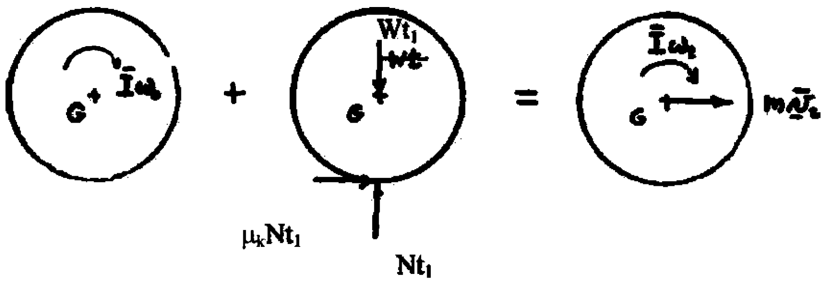
PROBLEM 17.F2



A sphere of radius r and mass m is placed on a horizontal floor with no linear velocity but with a clockwise angular velocity $\boldsymbol{\omega}_0$. Denoting by μ_k the coefficient of kinetic friction between the sphere and the floor, draw the impulse-momentum diagram that can be used to determine the time t_1 at which the sphere will start rolling without sliding.

SOLUTION

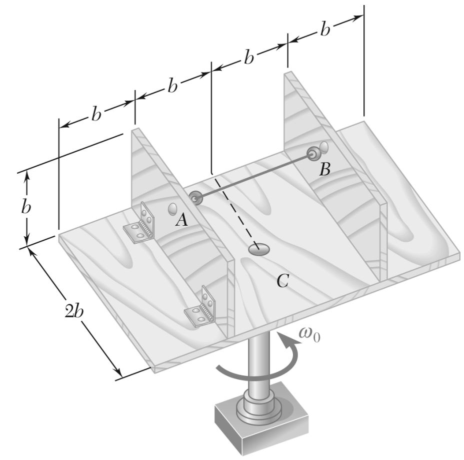
Answer:



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

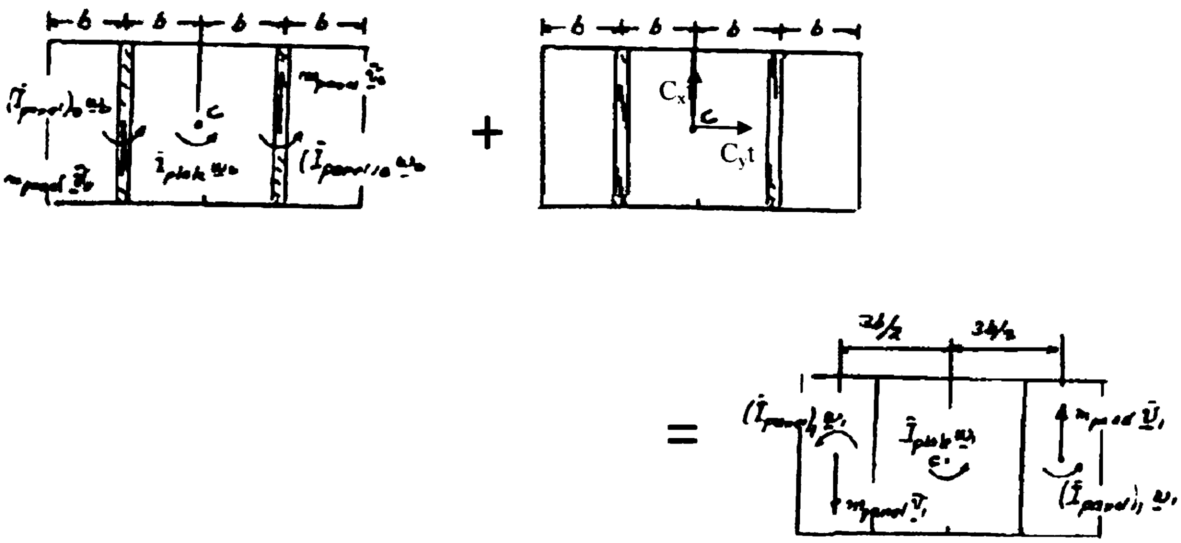


PROBLEM 17.F3

Two panels A and B are attached with hinges to a rectangular plate and held by a wire as shown. The plate and the panels are made of the same material and have the same thickness. The entire assembly is rotating with an angular velocity ω_0 when the wire breaks. Draw the impulse-momentum diagram that is needed to determine the angular velocity of the assembly after the panels have come to rest against the plate.

SOLUTION

Answer:



PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.52

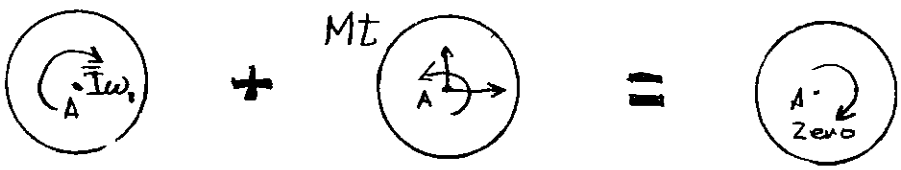
The rotor of an electric motor has a mass of 25 kg, and it is observed that 4.2 min is required for the rotor to coast to rest from an angular velocity of 3600 rpm. Knowing that kinetic friction produces a couple of magnitude 1.2 N·m, determine the centroidal radius of gyration for the rotor.

SOLUTION

Coasting time: $t = 4.2 \text{ min} = 252 \text{ s}$

Initial angular velocity: $\omega_1 = 3600 \text{ rpm} = 120\pi \text{ rad/s}$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **ΣSyst. Impulses.$_{1\to 2}$** = **Syst. Momenta$_2$**

↺ Moments about axle *A*: $\bar{I}\omega_1 - Mt = 0$

$$\bar{I} = \frac{Mt}{\omega_1}$$

$$= \frac{(1.2 \text{ N}\cdot\text{m})(252 \text{ s})}{120\pi \text{ rad/s}}$$

$$= 0.80214 \text{ kg}\cdot\text{m}^2$$

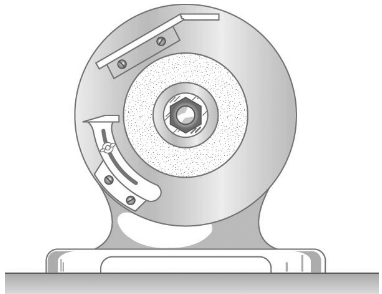
$$\bar{I} = m\bar{k}^2$$

Radius of gyration: $\bar{k} = \sqrt{\frac{\bar{I}}{m}} = \sqrt{\frac{0.80214 \text{ kg}\cdot\text{m}^2}{25 \text{ kg}}} = 0.1791 \text{ m}$

$\bar{k} = 179.1 \text{ mm}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.53

A small grinding wheel is attached to the shaft of an electric motor which has a rated speed of 3600 rpm. When the power is turned off, the unit coasts to rest in 70 s. The grinding wheel and rotor have a combined weight of 6 lb and a combined radius of gyration of 2 in. Determine the average magnitude of the couple due to kinetic friction in the bearings of the motor.

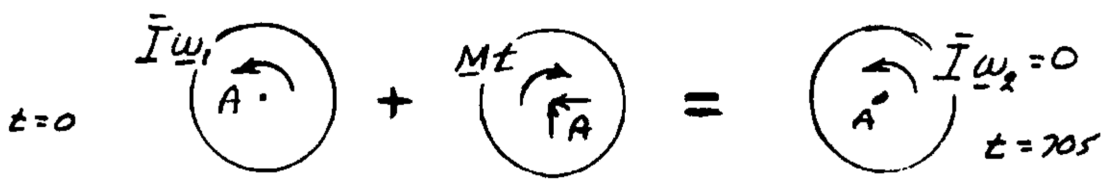
SOLUTION

Use the principle of impulse and momentum applied to the grinding wheel and rotor with

$$t_1 = 0 \qquad t_2 = 70 \text{ s}$$

$$\omega_1 = 3600 \text{ rpm} = 120\pi \text{ rad/s} \qquad \omega_2 = 0$$

Moment of inertia. $$\bar{I} = m\bar{k}^2 = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2}\left(\frac{2}{12}\text{ ft}\right)^2 = 0.00518 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

+) Moments about A: $$\bar{I}\omega_1 - Mt = 0$$

$$(0.00518)(120\pi) - M(70 \text{ s}) = 0$$

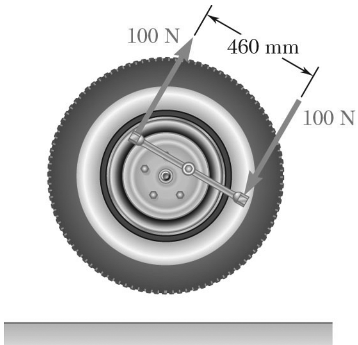
$$M = 0.02788 \text{ lb} \cdot \text{ft}$$

$$M = 0.33451 \text{ lb} \cdot \text{in.}$$

$M = 0.335 \text{ lb} \cdot \text{in.}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



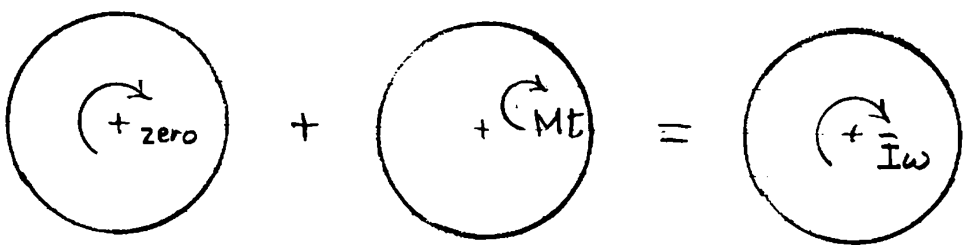
PROBLEM 17.54

A bolt located 50 mm from the center of an automobile wheel is tightened by applying the couple shown for 0.10 s. Assuming that the wheel is free to rotate and is initially at rest, determine the resulting angular velocity of the wheel. The wheel has a mass of 19 kg and has a radius of gyration of 250 mm.

SOLUTION

Moment of inertia. $\bar{I} = m\bar{k}^2 = (19 \text{ kg})(0.25 \text{ m})^2 = 1.1875 \text{ kg-m}^2$

Applied couple. $M = (100 \text{ N})(0.460 \text{ m}) = 46 \text{ N-m}$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

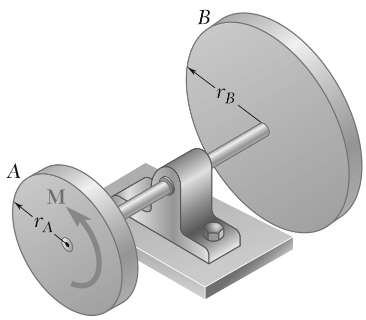
Moments about axle: $0 + Mt = \bar{I}\omega$

$$0 + (46 \text{ N-m})(0.10 \text{ s}) = (1.1875 \text{ kg-m}^2)\omega$$

$\omega = 3.87 \text{ rad/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.55

Two disks of the same thickness and same material are attached to a shaft as shown. The 8-lb disk A has a radius $r_A = 3$ in., and disk B has a radius $r_B = 4.5$ in. Knowing that a couple $\mathbf{M}$ of magnitude 20 lb · in. is applied to disk A when the system is at rest, determine the time required for the angular velocity of the system to reach 960 rpm.

SOLUTION

Weight of disk B.

$$W_B = \left(\frac{r_B}{r_A}\right)^2 W_B$$

$$= \left(\frac{4.5 \text{ in.}}{3 \text{ in.}}\right)^2 (8 \text{ lb})$$

$$= 18 \text{ lb}$$

Moment of inertia.

$$\bar{I} = \bar{I}_A + \bar{I}_B$$

$$= \frac{1}{2}\frac{8 \text{ lb}}{32.2}\left(\frac{3}{12}\text{ft}\right)^2 + \frac{1}{2}\frac{18 \text{ lb}}{32.2}\left(\frac{4.5}{12}\text{ft}\right)^2$$

$$= 0.04707 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Angular velocity. $\omega_2 = 960 \text{ rpm} = 100.53 \text{ rad/s}$

Moment. $M = 20 \text{ lb} \cdot \text{in.} = 1.667 \text{ lb} \cdot \text{ft}$

Principle of impulse and momentum.

Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about C: $0 + Mt = \bar{I}\omega_2$

Required time.

$$t = \frac{\bar{I}\omega_2}{M}$$

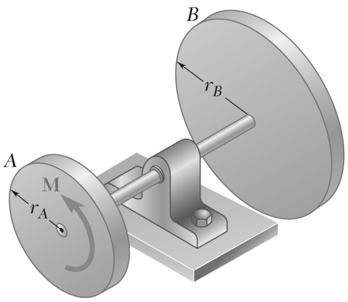
$$= \frac{(0.04707 \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(100.53 \text{ rad/s})}{1.667 \text{ lb} \cdot \text{ft}}$$

$$t = 2.839 \text{ s}$$

$t = 2.84$ s ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.56

Two disks of the same thickness and same material are attached to a shaft as shown. The 3-kg disk A has a radius $r_A = 100$ mm, and disk B has a radius $r_B = 125$ mm. Knowing that the angular velocity of the system is to be increased from 200 rpm to 800 rpm during a 3-s interval, determine the magnitude of the couple **M** that must be applied to disk A.

SOLUTION

Mass of disk B.

$$m_B = \left(\frac{r_B}{r_A}\right)^2 m_A$$

$$= \left(\frac{125\text{ mm}}{100\text{ mm}}\right)^2 3\text{ kg}$$

$$= 4.6875\text{ kg}$$

Moment of inertia.

$$\bar{I} = \bar{I}_A + \bar{I}_B$$

$$= \frac{1}{2}(3\text{ kg})(0.1\text{ m})^2 + \frac{1}{2}(4.6875\text{ kg})(0.125\text{ m})^2$$

$$= 0.05162\text{ kg}\cdot\text{m}^2$$

Angular velocities.

$$\omega_1 = 200\text{ rpm} = 20.944\text{ rad/s}$$

$$\omega_2 = 800\text{ rpm} = 83.776\text{ rad/s}$$

Principle of impulse and momentum.

Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about B: $\bar{I}\omega_1 + Mt = \bar{I}\omega_2$

Couple M.

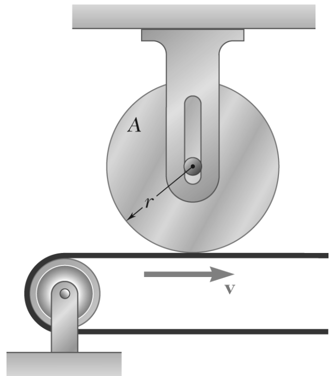
$$M = \frac{\bar{I}}{t}(\omega_2 - \omega_1)$$

$$= \frac{0.05162\text{ kg}\cdot\text{m}^2}{3\text{ s}}(83.776\text{ rad/s} - 20.944\text{ rad/s}) \qquad M = 1.081\text{ N}\cdot\text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.57

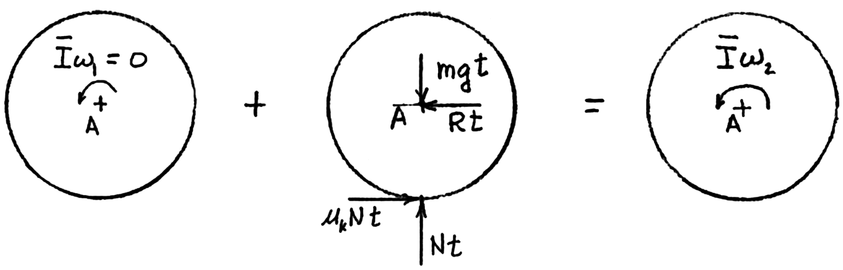


A disk of constant thickness, initially at rest, is placed in contact with a belt that moves with a constant velocity **v**. Denoting by μ_k the coefficient of kinetic friction between the disk and the belt, derive an expression for the time required for the disk to reach a constant angular velocity.

SOLUTION

Moment of inertia. $$\bar{I} = \frac{1}{2}mr^2$$

Final state of constant angular velocity. $$\omega_2 = \frac{v}{r}$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$+\uparrow$ y components: $\quad 0 + Nt - mgt = 0 \quad N = mg$

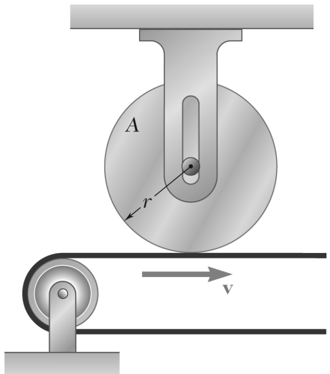
$+\circlearrowleft$ Moments about A: $\quad 0 + \mu_k Ntr = \bar{I}\omega_2$

$$t = \frac{\bar{I}\omega_2}{\mu_k mgr} = \frac{\frac{1}{2}mr^2\,\frac{v}{r}}{\mu_k mgr} = \frac{v}{2\mu_k g}$$

$$t = \frac{v}{2\mu_k g} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



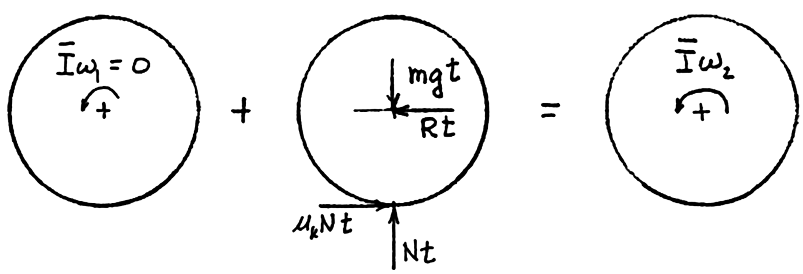
PROBLEM 17.58

Disk A, of weight 5 lb and radius $r = 3$ in., is at rest when it is placed in contact with a belt which moves at a constant speed $v = 50$ ft/s. Knowing that $\mu_k = 0.20$ between the disk and the belt, determine the time required for the disk to reach a constant angular velocity.

SOLUTION

Moment of inertia. $\qquad \bar{I} = \frac{1}{2}mr^2$

Final state of constant angular velocity. $\qquad \omega_2 = \frac{v}{r}$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$+\uparrow$ y components: $\qquad 0 + Nt - mgt = 0 \qquad N = mg$

$+\circlearrowright$ Moments about A: $\qquad 0 + \mu_k Ntr = \bar{I}\omega_2$

$$t = \frac{\bar{I}\omega_2}{\mu_k mgr} = \frac{\frac{1}{2}mr^2 \frac{v}{r}}{\mu_k mgr} = \frac{v}{2\mu_k g}$$

$$t = \frac{v}{2\mu_k g}$$

Data: $\qquad v = 50$ ft/s

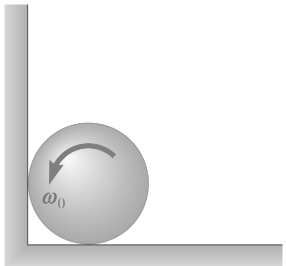
$\mu_k = 0.20$

$$t = \frac{50}{(2)(0.20)(32.2)}$$

$t = 3.88$ s ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



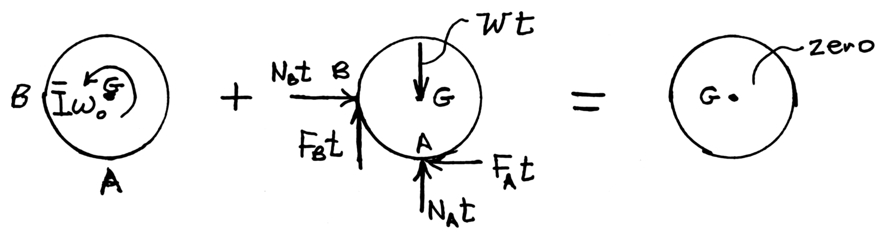
PROBLEM 17.59

A cylinder of radius r and weight W with an initial counterclockwise angular velocity ω_0 is placed in the corner formed by the floor and a vertical wall. Denoting by μ_k the coefficient of kinetic friction between the cylinder and the wall and the floor derive an expression for the time required for the cylinder to come to rest.

SOLUTION

For the cylinder $\qquad \bar{I} = \frac{1}{2}mr^2, \quad W = mg$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Linear momentum $\xrightarrow{+}$: $\qquad 0 + N_Bt - F_At = 0$

$$N_B = F_A$$

Linear momentum $+\uparrow$: $\qquad 0 + N_At + F_Bt - Wt = 0$

$$N_A + F_B = N_A + \mu_k N_B$$

$$= N_A + \mu_k F_A + N_A + \mu_k^2 N_A = W$$

$$N_A = \frac{W}{1+\mu_k^2}$$

$$F_A = \mu_k N_A = \frac{\mu_k W}{1+\mu_k^2}$$

$$N_B = \frac{\mu_k W}{1+\mu_k^2}$$

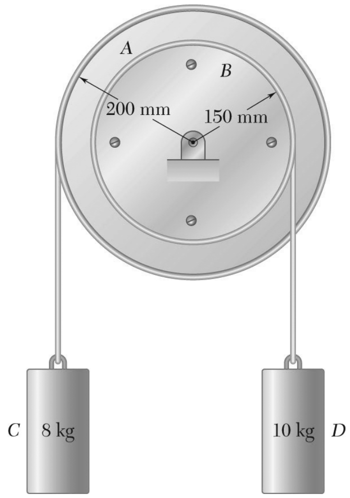
$$F_B = \frac{\mu_k^2 W}{1+\mu_k^2}$$

$+\circlearrowleft$ Moments about G: $\qquad \bar{I}\omega_0 - F_Art - F_Brt = 0$

$$t = \frac{\bar{I}\omega_0}{(F_A + F_B)r} = \frac{(1+\mu_k^2)\bar{I}\omega_0}{\mu_k(1+\mu_k)Wr} \qquad t = \frac{1+\mu_k^2}{2\mu_k(1+\mu_k)}\frac{r\omega_0}{g} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.60

Two uniform disks and two cylinders are assembled as indicated. Disk A has a mass of 10 kg and disk B has a mass of 6 kg. Knowing that the system is released from rest, determine the time required for cylinder C to have a speed of 0.5 m/s.

Disks A and B are bolted together and the cylinders are attached to separate cords wrapped on the disks.

SOLUTION

Moments of inertia.

Disk A: $$I_A = \frac{1}{2}m_A r_A^2 = \frac{1}{2}(10\text{ kg})(0.200\text{ m})^2 = 0.2\text{ kg}\cdot\text{m}^2$$

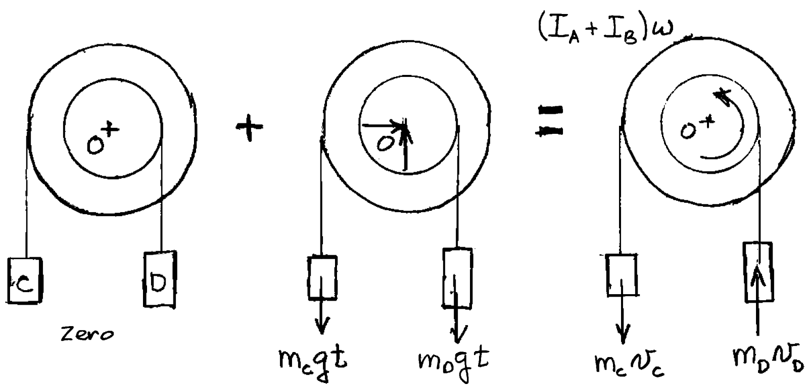
Disk B: $$I_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}(6\text{ kg})(0.150\text{ m})^2 = 0.0675\text{ kg}\cdot\text{m}^2$$

Kinematics: $$\mathbf{v}_C = v_C \downarrow \qquad \boldsymbol{\omega}_A = \omega_A \circlearrowleft = \frac{v_C}{r_A} \circlearrowleft$$

$$\boldsymbol{\omega}_B = \boldsymbol{\omega}_A = \boldsymbol{\omega}$$

$$\mathbf{v}_D = v_D \uparrow = \omega r_B \uparrow = \frac{r_B}{r_A} v_C \uparrow$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

$+\circlearrowleft$ Moments about axle 0.

$$0 + m_0 gtr_A - m_D gtr_B = m_C v_C r_A + m_D v_D r_B + (I_A + I_B)\omega \qquad (1)$$

Data: $v_C = 0.5 \text{ m/s} \qquad t = ?$

$$m_C gr_A = (8 \text{ kg})(9.81 \text{ m/s}^2)(0.200 \text{ m}) = 15.696 \text{ N}\cdot\text{m}$$

$$m_D gr_B = (10 \text{ kg})(9.81 \text{ m/s}^2)(0.150 \text{ m}) = 14.715 \text{ N}\cdot\text{m}$$

$$m_C gr_A - m_D gr_B = 0.981 \text{ N}\cdot\text{m}$$

$$m_C v_C r_A = (8 \text{ kg})(0.5 \text{ m/s})(0.200 \text{ m}) = 0.8 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$m_D v_D r_B = (10 \text{ kg})\left(\frac{0.150 \text{ m}}{0.200 \text{ m}}\right)(0.5 \text{ m/s})(0.150 \text{ m}) = 0.5625 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(I_A + I_B)\omega = (0.2675 \text{ kg}\cdot\text{m}^2)\left(\frac{0.5 \text{ m/s}}{0.200 \text{ m}}\right) = 0.66875 \text{ kg}\cdot\text{m}^2/\text{s}$$

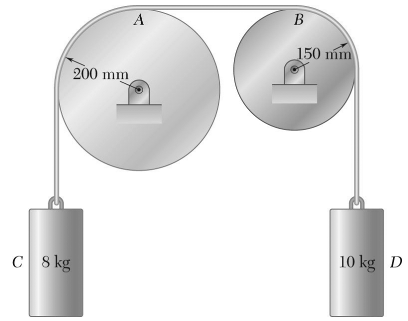
$$m_C v_C r_A + m_D v_D r_B + (I_A + I_B)\omega = 2.03125 \text{ kg}\cdot\text{m}^2/\text{s}$$

Solving Eq. (1) for t, $$t = \frac{2.03125 \text{ kg}\cdot\text{m}^2/\text{s}}{0.981 \text{ N}\cdot\text{m}}$$

$t = 2.07 \text{ s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.61

Two uniform disks and two cylinders are assembled as indicated. Disk A has a mass of 10 kg and disk B has a mass of 6 kg. Knowing that the system is released from rest, determine the time required for cylinder C to have a speed of 0.5 m/s.

The cylinders are attached to a single cord that passes over the disks. Assume that no slipping occurs between the cord and the disks.

SOLUTION

Moments of inertia.

Disk A: $$I_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2}(10 \text{ kg})(0.200 \text{ m})^2 = 0.2 \text{ kg} \cdot \text{m}^2$$

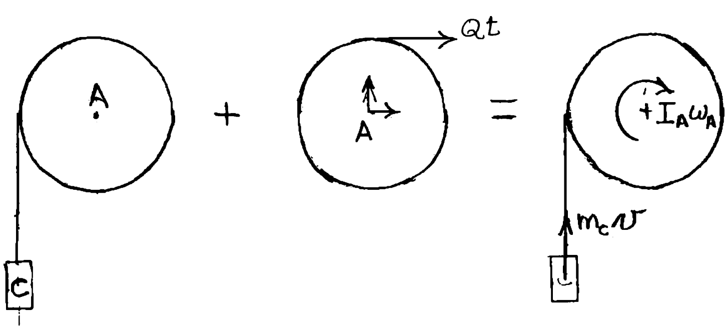
Disk B: $$I_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2}(6 \text{ kg})(0.150 \text{ m})^2 = 0.0675 \text{ kg} \cdot \text{m}^2$$

Kinematics: $$\mathbf{v}_C = v \uparrow \qquad \mathbf{v}_D = v \downarrow$$

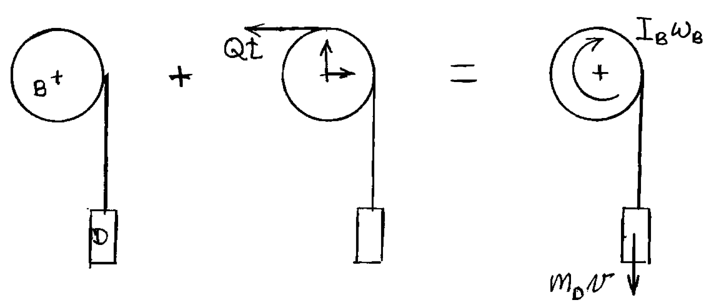
$$\boldsymbol{\omega}_A = \frac{v}{r_A} \circlearrowright \qquad \boldsymbol{\omega}_B = \frac{v}{r_B} \circlearrowright$$

Principle of impulse and momentum.

Disk A and cylinder C



Disk B and cylinder D



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.61 (Continued)

Disk A and cylinder C. $\circlearrowleft +$ Moments about A:

$$Qtr_A - m_C gtr_A = m_A vr_A + I_A \omega_A \tag{1}$$

Disk B and cylinder D. $\circlearrowleft +$ Moments about B:

$$-Qtr_B - m_D gtr_B = m_D r_B v + I_B \omega_B \tag{2}$$

To eliminate Qt divide Equation (1) by r_A and Equation (2) by r_B, and then add the resulting equations.

$$(m_D - m_C)gt = \left(m_A + \frac{I_A}{r_A^2} + m_B + \frac{I_B}{r_B^2} \right) v \tag{3}$$

Data: $v = 0.5 \text{ m/s} \qquad t = ?$

$$(m_D - m_C)g = (2 \text{ kg})(9.81 \text{ m/s}^2) = 19.62 \text{ N}$$

$$m_C + \frac{I_A}{r_A^2} + m_D + \frac{I_B}{r_B^2} = 8 \text{ kg} + \frac{0.2 \text{ kg} \cdot \text{m}^2}{(0.200 \text{ m})^2} + 10 \text{ kg} + \frac{0.0675 \text{ kg} \cdot \text{m}^2}{(0.150 \text{ m})^2} = 26 \text{ kg}$$

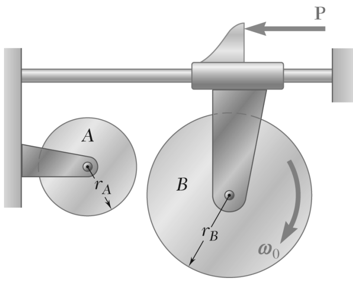
Equation (3) becomes $\qquad (19.62 \text{ N})t = (26 \text{ kg})(0.5 \text{ m})$

$$t = 0.66259 \text{ s}$$

$t = 0.663 \text{ s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.62

Disk B has an initial angular velocity ω_0 when it is brought into contact with disk A which is at rest. Show that the final angular velocity of disk B depends only on ω_0 and the ratio of the masses m_A and m_B of the two disks.

SOLUTION

Let Points A and B be the centers of the two disks and Point C be the contact point between the two disks.

Let ω_A and ω_B be the final angular velocities of disks A and B, respectively, and let v_C be the final velocity at C common to both disks.

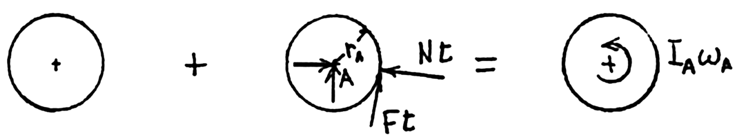
Kinematics: No slipping $\qquad v_C = r_A\omega_A = r_B\omega_B$

Moments of inertia. Assume that both disks are uniform cylinders.

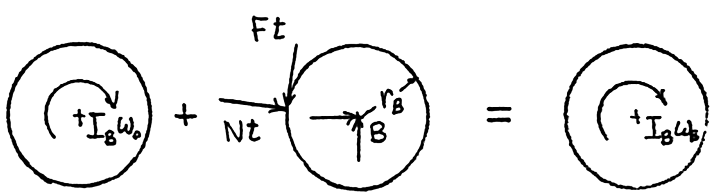
$$I_A = \frac{1}{2}m_A r_A^2 \qquad I_B = \frac{1}{2}m_B r_B^2$$

Principle of impulse and momentum.

Disk A



Disk B



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Disk A: Moments about A: $\qquad 0 + r_A Ft = I_A\omega_A$

$$Ft = \frac{I_A\omega_A}{r_A} = \frac{1}{2}\frac{m_A r_A^2 v_C}{r_A^2}$$

$$= \frac{1}{2}m_A v_C$$

$$= \frac{1}{2}m_A r_B\omega_B$$

Disk B: Moments about B: $\qquad I_B\omega_0 - r_B Ft = I_B\omega_B$

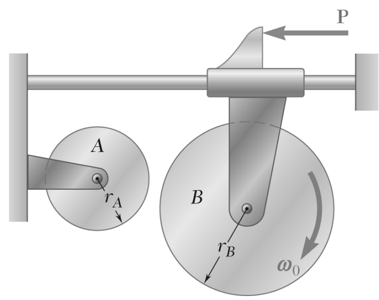
$$\frac{1}{2}m_B r_B^2\omega_0 - r_B\left(\frac{1}{2}m_A r_B\omega_B\right) = \frac{1}{2}mr_B^2\omega_B$$

$$\omega_B = \frac{\omega_0}{1 + \frac{m_A}{m_B}} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.63



The 7.5-lb disk A has a radius $r_A = 6$ in. and is initially at rest. The 10-lb disk B has a radius $r_B = 8$ in. and an angular velocity ω_0 of 900 rpm when it is brought into contact with disk A. Neglecting friction in the bearings, determine (*a*) the final angular velocity of each disk, (*b*) the total impulse of the friction force exerted on disk A.

SOLUTION

Let Points A and B be the centers of the two disks and Point C be the contact point between the two disks.

Let ω_A and ω_B be the final angular velocities of disks A and B, respectively, and let v_C be the final velocity at C common to both disks.

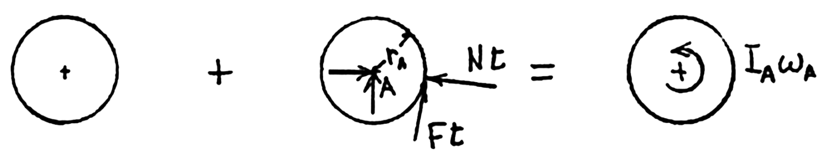
Kinematics: No slipping $\qquad v_C = r_A\omega_A = r_B\omega_B$

Moments of inertia. Assume that both disks are uniform cylinders.

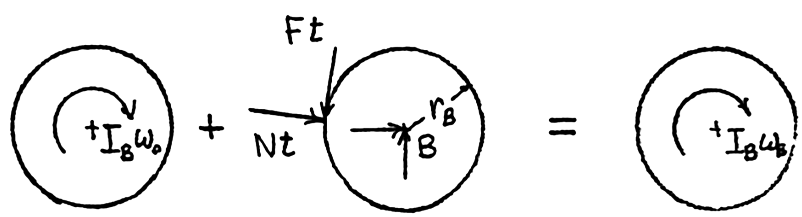
$$I_A = \frac{1}{2}m_A r_A^2 \qquad I_B = \frac{1}{2}m_B r_B^2$$

Principle of impulse and momentum.

Disk A



Disk B



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Disk A: Moments about A: $\qquad 0 + r_A Ft = I_A\omega_A$

$$Ft = \frac{I_A\omega_A}{r_A}$$

$$= \frac{1}{2}\frac{m_A r_A^2 v_C}{r_A^2} = \frac{1}{2}m_A v_C$$

$$= \frac{1}{2}m_A r_B\omega_B$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Disk B: Moments about B:

$$I_B\omega_0 - r_B Ft = I_B\omega_B$$

$$\frac{1}{2}m_B r_B^2\omega_0 - r_B\left(\frac{1}{2}m_A r_B\omega_B\right) = \frac{1}{2}m r_B^2\omega_B$$

$$\omega_B = \frac{\omega_0}{1+\frac{m_A}{m_B}}$$

Data:

$$m_A = \frac{7.5}{32.2} = 0.23292 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\frac{m_A}{m_B} = \frac{W_A}{W_B} = \frac{7.5}{10} = 0.75$$

$$r_B = \frac{8}{12} \text{ ft}$$

$$\frac{r_B}{r_A} = \frac{8}{6}$$

$$\omega_0 = 900 \text{ rpm} = 30\pi \text{ rad/s}$$

(*a*)

$$\omega_B = \frac{\omega_0}{1+0.75} = \frac{30\pi}{1.75} = 53.856 \text{ rad/s}$$

$$\omega_A = \frac{r_B}{r_A}\omega_B = \left(\frac{8}{6}\right)(53.856) = 71.808 \text{ rad/s}$$

$\boldsymbol{\omega}_A = 686 \text{ rpm}$ ◀

$\boldsymbol{\omega}_B = 514 \text{ rpm}$ ◀

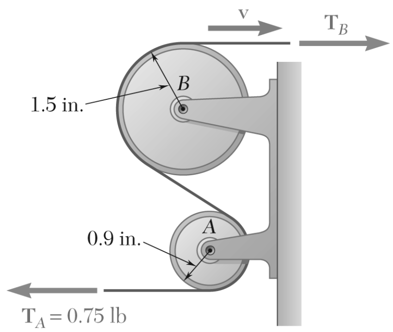
(*b*)

$$Ft = \frac{1}{2}\frac{(0.23292)\left(\frac{8}{12}\right)(30\pi)}{1+0.75}$$

$\mathbf{F}t = 4.18 \text{ lb}\cdot\text{s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.64

A tape moves over the two drums shown. Drum A weighs 1.4 lb and has a radius of gyration of 0.75 in., while drum B weighs 3.5 lb and has a radius of gyration of 1.25 in. In the lower portion of the tape the tension is constant and equal to $T_A = 0.75$ lb. Knowing that the tape is initially at rest, determine (*a*) the required constant tension T_B if the velocity of the tape is to be $v = 10$ ft/s after 0.24 s, (*b*) the corresponding tension in the portion of tape between the drums.

SOLUTION

Kinematics. Drums A and B rotate about fixed axes. Let v be the tape velocity in ft/s.

$$v = r_A \omega_A = \frac{0.9}{12}\omega_A \qquad \omega_A = 13.3333v$$

$$v = r_B \omega_B = \frac{1.5}{12}\omega_B \qquad \omega_B = 8v$$

Moments of inertia.

$$\bar{I}_A = m_A \bar{k}_A^2 = \left(\frac{1.4}{32.2}\right)\left(\frac{0.75}{12}\right)^2 = 169.837 \times 10^{-6} \text{lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\bar{I}_B = m_B \bar{k}_B^2 = \left(\frac{3.5}{32.2}\right)\left(\frac{1.25}{12}\right)^2 = 1.17942 \times 10^{-3} \text{lb} \cdot \text{s}^2 \cdot \text{ft}$$

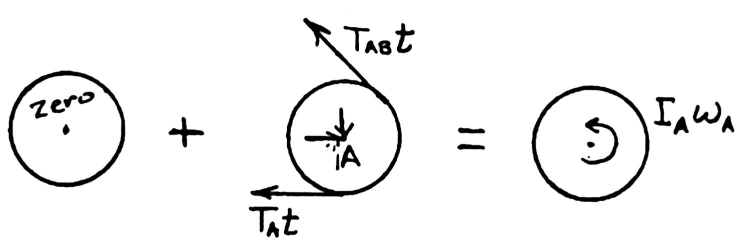
State 1. $t = 0 \qquad v = 0 \qquad \omega_A = \omega_B = 0$

State 2. $t = 0.24$ s, $\qquad v = 10$ ft/s

$$\omega_A = (13.3333)(10) = 133.333 \text{ rad/s}$$

$$\omega_B = (8)(10) = 80 \text{ rad/s}$$

Drum A.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

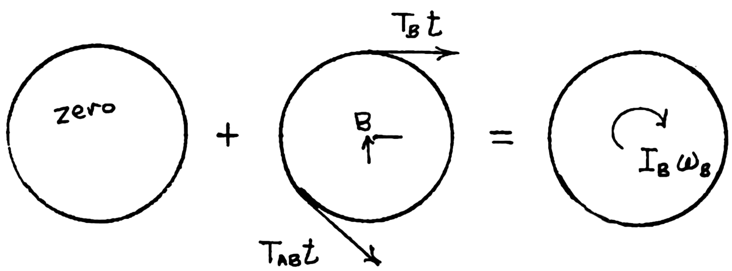
PROBLEM 17.64 (Continued)

$+\circlearrowright$ Moments about A: $\quad 0 + r_A T_{AB} t - r_A T_A t = I_A \omega_A$

$$0 + \left(\frac{0.9}{12}\right)(T_{AB} t) - \left(\frac{0.9}{12}\right)(0.75)(0.24) = (169.837 \times 10^{-6})(133.333)$$

$$T_{AB} t = 0.48193 \text{ lb} \cdot \text{s}$$

Drum B.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$+\circlearrowleft$ Moments about B: $\quad 0 + r_B T_B t - r_B T_{AB} t = I_B \omega_B$

$$0 + \frac{1.5}{12}(T_B t) - \left(\frac{1.5}{12}\right)(0.48193) = (1.17942 \times 10^{-3})(80)$$

$$T_B t = 1.23676 \text{ lb} \cdot \text{s}$$

(*a*) $$T_B = \frac{T_B t}{t} = \frac{1.23676}{0.24}$$ $T_B = 5.15 \text{ lb}$ ◀

(*b*) $$T_{AB} = \frac{T_{AB} t}{t} = \frac{0.48193}{0.24}$$ $T_{AB} = 2.01 \text{ lb}$ ◀

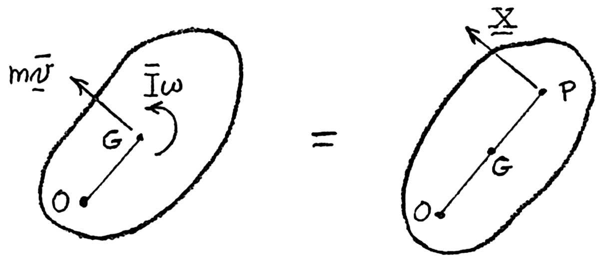
PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.65

Show that the system of momenta for a rigid slab in plane motion reduces to a single vector, and express the distance from the mass center G to the line of action of this vector in terms of the centroidal radius of gyration $\bar{k}$ of the slab, the magnitude $\bar{v}$ of the velocity of G, and the angular velocity ω.

SOLUTION



Syst. Momenta = **Single Vector**

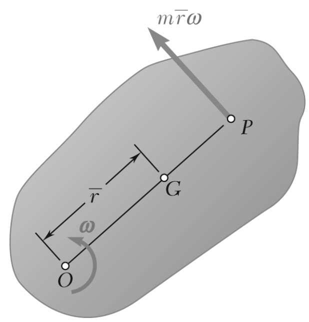
Components parallel to $m\bar{\mathbf{v}}$: $\quad m\bar{\mathbf{v}} = \mathbf{X} \qquad \mathbf{X} = m\bar{\mathbf{v}}$ ◀

Moments about G: $\quad \bar{I}\omega = (m\bar{v})d$

$$d = \frac{\bar{I}\omega}{m\bar{v}} = \frac{m\bar{k}^2\omega}{m\bar{v}} \qquad d = \frac{\bar{k}^2\omega}{\bar{v}} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

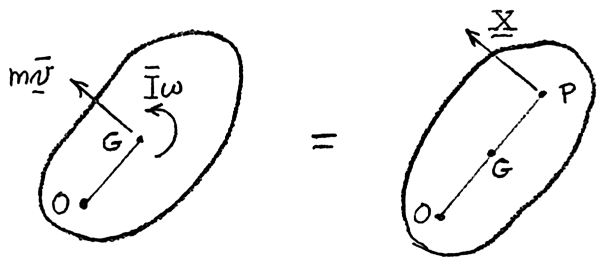
STUDENTS-HUB.com



PROBLEM 17.66

Show that, when a rigid slab rotates about a fixed axis through O perpendicular to the slab, the system of the momenta of its particles is equivalent to a single vector of magnitude $m\bar{r}\omega$, perpendicular to the line OG, and applied to a Point P on this line, called the *center of percussion*, at a distance $GP = \bar{k}^2/\bar{r}$ from the mass center of the slab.

SOLUTION



Kinematics. Point O is fixed. $\qquad \bar{v} = \bar{r}\omega$

System momenta.

Components parallel to $m\bar{\mathbf{v}}$: $\qquad X = m\bar{v} = m\bar{r}\omega \qquad X = m\bar{r}\omega$ ◀

Moments about G: $\qquad (GP)X = \bar{I}\omega$

$$(GP)m\,r\,\omega = m\bar{k}^2\omega \qquad (GP) = \frac{\bar{k}^2}{\bar{r}}$$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

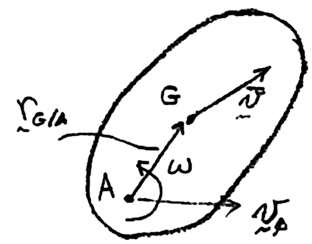
STUDENTS-HUB.com

PROBLEM 17.67

Show that the sum $\mathbf{H}_A$ of the moments about a Point A of the momenta of the particles of a rigid slab in plane motion is equal to $I_A\omega$, where ω is the angular velocity of the slab at the instant considered and I_A the moment of inertia of the slab about A, if and only if one of the following conditions is satisfied: (*a*) A is the mass center of the slab, (*b*) A is the instantaneous center of rotation, (*c*) the velocity of A is directed along a line joining Point A and the mass center G.

SOLUTION

Kinematics.



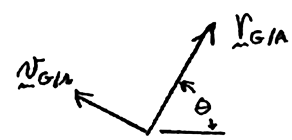
Let $\boldsymbol{\omega} = \omega\mathbf{k}$

and $\mathbf{r}_{G/A} = r_{G/A} \measuredangle\, \theta$

Then, $\mathbf{v}_{G/A} = \boldsymbol{\omega} \times \mathbf{r}_{G/A} = (\omega r_{G/A}) \measuredangle\, \beta$

where $\beta = \theta + 90°$

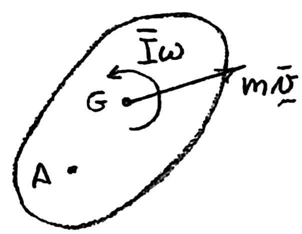
Also $\overline{\mathbf{v}} = \mathbf{v}_A + \mathbf{v}_{G/A}$



Define $\mathbf{h} = \mathbf{r}_{G/A} \times \mathbf{v}_{G/A}$

$$\mathbf{h} = (r_{G/A})(v_{G/A})\mathbf{k} = (r_{G/A})^2\omega\mathbf{k} = (r_{G/A})^2\boldsymbol{\omega}$$

System momenta. Moments about A:



$$\begin{aligned}
\mathbf{H}_A &= \mathbf{r}_{G/A} \times m\overline{\mathbf{v}} + \overline{I}\boldsymbol{\omega} \\
&= \mathbf{r}_{G/A} \times m(\mathbf{v}_A + \mathbf{v}_{G/A}) + \overline{I}\,\omega \\
&= \mathbf{r}_{G/A} \times m\mathbf{v}_A + m\mathbf{r}_{G/A} \times \mathbf{v}_{G/A} + \overline{I}\boldsymbol{\omega} \\
&= \mathbf{r}_{G/A} \times m\mathbf{v}_A + m\mathbf{h} + \overline{I}\boldsymbol{\omega} \\
&= \mathbf{r}_{G/A} \times m\mathbf{v}_A + mr_{G/A}^2\boldsymbol{\omega} + \overline{I}\boldsymbol{\omega} \\
&= \mathbf{r}_{G/A} \times m\mathbf{v}_A + (m\,r_{G/A}^2 + \overline{I}\,)\boldsymbol{\omega}
\end{aligned}$$

The first term on the right hand side is equal to zero if

(*a*) $\mathbf{r}_{G/A} = 0$ (A is the mass center)

or (*b*) $\mathbf{v}_A = 0$ (A is the instantaneous center of rotation)

or (*c*) $\mathbf{r}_{G/A}$ is perpendicular to $\mathbf{v}_A$.

In the second term, $mr_{G/A}^2 + \overline{I} = I_A$ by the parallel axis theorem.

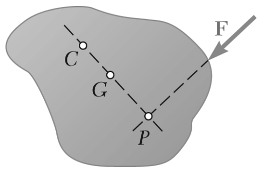
Thus, $\mathbf{H}_A = I_A\boldsymbol{\omega}$

when one or more of the conditions (*a*), (*b*) or (*c*) is satisfied.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

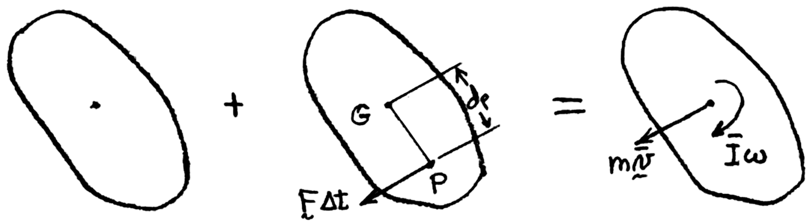
PROBLEM 17.68



Consider a rigid slab initially at rest and subjected to an impulsive force **F** contained in the plane of the slab. We define the *center of percussion* P as the point of intersection of the line of action of **F** with the perpendicular drawn from G. (a) Show that the instantaneous center of rotation C of the slab is located on line GP at a distance $GC = \bar{k}^2/GP$ on the opposite side of G. (b) Show that if the center of percussion were located at C the instantaneous center of rotation would be located at P.

SOLUTION

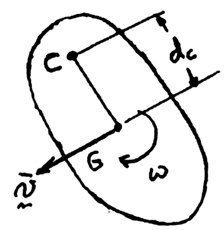
(a) Locate the instantaneous center C corresponding to center of percussion P. Let $d_P = GP$.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Components parallel to $\mathbf{F}\Delta t$: $\quad 0 + F\Delta t = m\bar{v}$

Moments about G: $\quad 0 + d_P(F\Delta t) = \bar{I}\omega$



Eliminate $F\Delta t$ to obtain

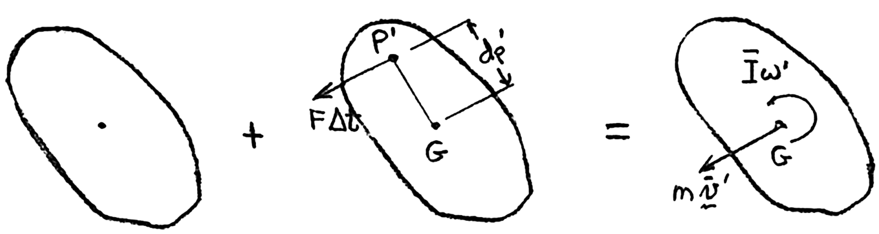
$$\frac{\bar{v}}{\omega} = \frac{\bar{I}}{m d_P} = \frac{\bar{k}^2}{d_P}$$

Kinematics. Locate Point C. $\quad GC = d_C = \dfrac{\bar{v}}{\omega} = \dfrac{\bar{k}^2}{d_P}$

$$GC = \frac{\bar{k}^2}{GP} \blacktriangleleft$$

(b) Place the center of percussion at $P' = C$. Locate the corresponding instantaneous center C'. Let

$$d_{P'} = GP' = GC = d_C.$$

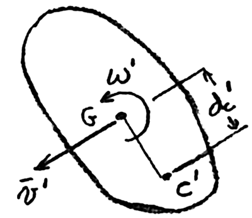


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.68 (Continued)



Components parallel to $\mathbf{F}\Delta t$: $\quad 0 + F\Delta t = m\overline{v}'$

Moments about G: $\quad 0 + d_{P'}(F\Delta t) = \overline{I}\,\omega'$

Eliminate $F\Delta t$ to obtain $\quad \dfrac{\overline{v}'}{\omega'} = \dfrac{\overline{I}}{md_{P'}} = \dfrac{\overline{k}^2}{d_{P'}}$

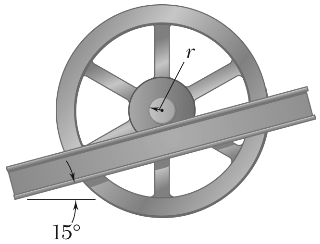
Kinematics. Locate Point C'. $\quad GC' = d_{C'} = \dfrac{\overline{v}'}{\omega'} = \dfrac{\overline{k}^2}{d_{P'}} = \dfrac{\overline{k}^2}{d_C}$

Using $d_C = d_{P'} = \dfrac{\overline{k}^2}{d_P}$ gives $\quad d_{C'} = d_P$ or $GC' = GP$ ◀

Thus Point C' coincides with Point P.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

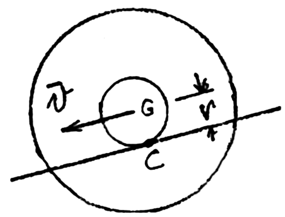
STUDENTS-HUB.com



PROBLEM 17.69

A flywheel is rigidly attached to a 1.5-in.-radius shaft that rolls without sliding along parallel rails. Knowing that after being released from rest the system attains a speed of 6 in./s in 30 s, determine the centroidal radius of gyration of the system.

SOLUTION

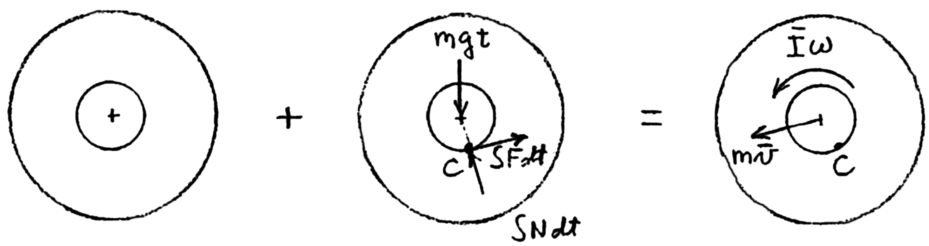


Kinematics. Rolling motion. Instantaneous center at C.

$$\bar{v} = v_G = r\omega$$

Moment of inertia. $\quad \bar{I} = m\bar{k}^2$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about C: $\quad 0 + (mgt)r\sin\beta = m\bar{v}r + \bar{I}\,\omega$

$$mgtr\sin\beta = m\left(r + \frac{\bar{k}^2}{r}\right)\bar{v}$$

Solving for $\bar{k}^2$,

$$\bar{k}^2 = r^2\left(\frac{gt\sin\beta}{\bar{v}} - 1\right)$$

Data:

$$r = 1.5 \text{ in.} = 0.125 \text{ ft}$$
$$g = 32.2 \text{ ft/s}$$
$$t = 30 \text{ s}$$
$$\bar{v} = 6 \text{ in./s} = 0.5 \text{ ft/s}$$

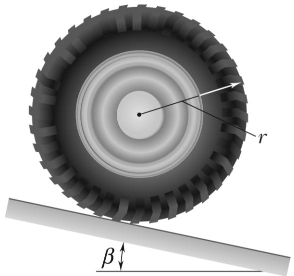
$$\bar{k}^2 = (0.125)^2\left[\frac{(32.2)(30)\sin 15°}{0.5} - 1\right]$$
$$= 7.7974 \text{ ft}^2$$

$\bar{k} = 2.79$ ft ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

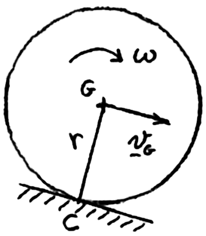
PROBLEM 17.70



A wheel of radius r and centroidal radius of gyration $\bar{k}$ is released from rest on the incline shown at time $t = 0$. Assuming that the wheel rolls without sliding, determine (*a*) the velocity of its center at time t, (*b*) the coefficient of static friction required to prevent slipping.

SOLUTION

Kinematics. Rolling motion. Instantaneous center at C.

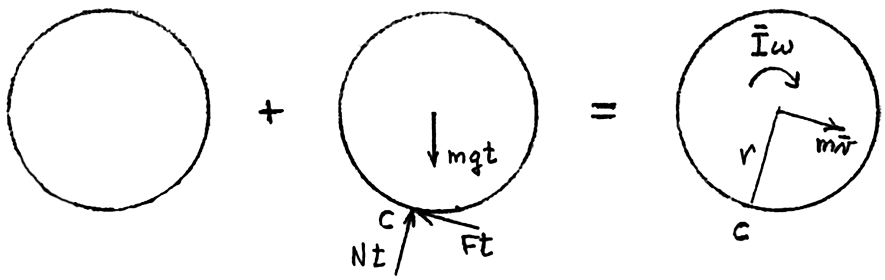


$$\bar{v} = v_G = r\omega$$

$$\omega = \frac{\bar{v}}{r}$$

Moment of inertia. $\bar{I} = m\bar{k}^2$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

moments about C: $0 + (mgt)r\sin\beta = m\bar{v}r + \bar{I}\omega$

$$(mgr\sin\beta)t = mr\bar{v} + \frac{m\bar{k}^2\bar{v}}{r}$$

(*a*) *Velocity of Point G.* $\bar{\mathbf{v}} = \dfrac{r^2 gt\sin\beta}{r^2 + \bar{k}^2} \searrow \beta$ ◀

$+\searrow$ components parallel to incline:

$$0 + mgt\sin\beta - Ft = m\bar{v}$$

$$Ft = mgt\sin\beta - \frac{mr^2 gt\sin\beta}{r^2 + k^2}$$

$$= \frac{\bar{k}^2 mgt\sin\beta}{r^2 + k^2}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.70 (Continued)

$+\nearrow$ components normal to incline:

$$0 + Nt - mgt\cos\beta = 0$$

$$Nt = mgt\cos\beta$$

(*b*) *Required coefficient of static friction.*

$$\mu_s \geq \frac{F}{N}$$

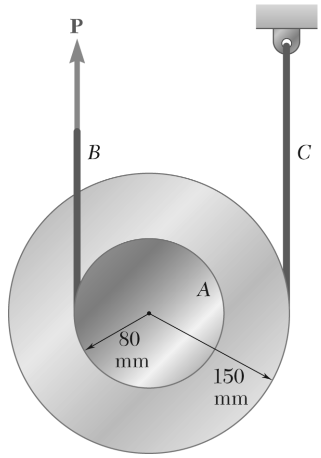
$$= \frac{Ft}{Nt}$$

$$= \frac{\bar{k}^2 mgt\sin\beta}{(r^2 + \bar{k}^2)mgt\cos\beta}$$

$$\mu_s \geq \frac{\bar{k}^2 \tan\beta}{r^2 + \bar{k}^2} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.71

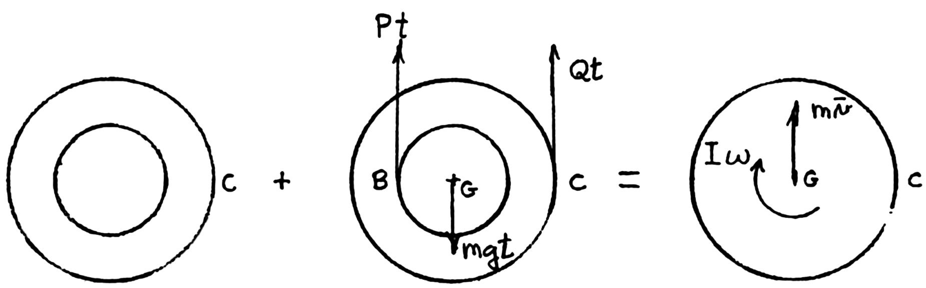
The double pulley shown has a mass of 3 kg and a radius of gyration of 100 mm. Knowing that when the pulley is at rest, a force **P** of magnitude 24 N is applied to cord B, determine (a) the velocity of the center of the pulley after 1.5 s, (b) the tension in cord C.

SOLUTION

For the double pulley,

$$r_C = 0.150 \text{ m}$$
$$r_B = 0.080 \text{ m}$$
$$k = 0.100 \text{ m}$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics. Point C is the instantaneous center. $\bar{v} = r_C \omega$

Moments about C: $0 + Pt(r_C + r_B) - mgtr_C = \bar{I}\omega + m\bar{v}r_C$

$$= mk^2\omega + m(r_C\omega)r_C$$

$$\omega = \frac{Pt(r_C + r_B) - mgtr_C}{m\left(k^2 + r_C^2\right)}$$

$$= \frac{(24)(1.5)(0.230) - (3)(9.81)(1.5)(0.150)}{3(0.100^2 + 0.150^2)}$$

$$= 17.0077 \text{ rad/s}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

(*a*) $\bar{v} = (0.150)(17.0077) = 2.55115 \text{ m/s}$ $\qquad \bar{\mathbf{v}} = 2.55 \text{ m/s} \uparrow$ ◀

$+\uparrow$ Linear components: $\quad 0 + Pt - mgt + Qt = m\bar{v}$

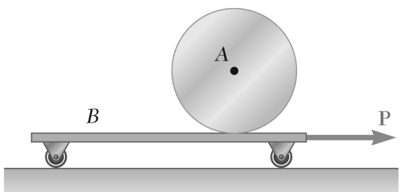
$$Q = \frac{m\bar{v}}{t} + mg - P$$

$$= \frac{(3)(2.55115)}{1.5} + (3)(9.81) - 24$$

(*b*) *Tension in cord C.* $\qquad Q = 10.53 \text{ N}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.72

A 9-in.-radius cylinder of weight 18 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 2.5 lb is applied as shown for 1.2 s. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine the resulting velocity of (*a*) the carriage, (*b*) the center of the cylinder.

SOLUTION

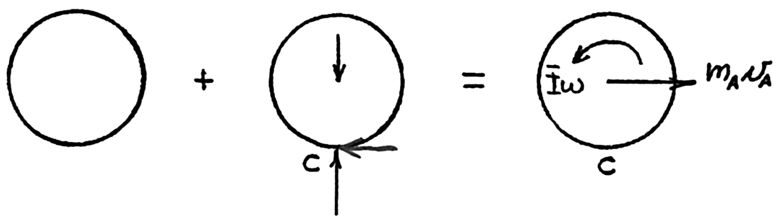
Moment of inertia.

$$\bar{I} = \frac{1}{2} m_A r^2$$

$$= \frac{1}{2}\left(\frac{18 \text{ lb}}{32.2}\right)\left(\frac{9 \text{ in.}}{12}\right)^2$$

$$= 0.15722 \text{ slug} \cdot \text{ft}^2$$

Cylinder alone:

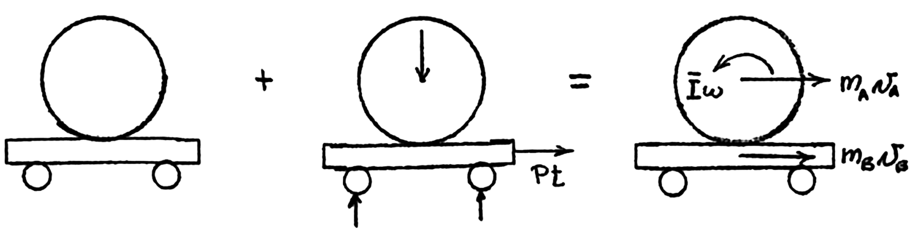


$$\textbf{Syst. Momenta}_1 + \textbf{Syst. Ext. Imp.}_{1\to 2} = \textbf{Syst. Momenta}_2$$

Moments about C: $$0 + 0 = \bar{I}\omega - m_A v_A r$$

or $$0 = 0.15722\omega - \left(\frac{18}{32.2}\right)\left(\frac{9}{12}\right) v_A \qquad (1)$$

Cylinder and carriage:



$$\textbf{Syst. Momenta}_1 + \textbf{Syst. Ext. Imp.}_{1\to 2} = \textbf{Syst. Momenta}_2$$

$\xrightarrow{+}$ Horizontal components: $$0 + Pt = m_A v_A + m_B v_B$$

or $$0 + (2.5)(1.2) = \left(\frac{18}{32.2}\right) v_A + \left(\frac{6}{32.2}\right) v_B \qquad (2)$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.72 (Continued)

Kinematics. $v_A = v_B - r\omega$

$$v_A = v_B - \left(\frac{9}{12}\right)\omega \qquad (3)$$

Solving Equations (1), (2) and (3) simultaneously gives $\boldsymbol{\omega} = 7.16 \text{ rad/s}$ ⟳

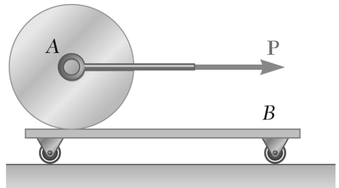
(*a*) Velocity of the carriage. $\mathbf{v}_B = 8.05 \text{ ft/s} \longrightarrow$ ◀

(*b*) Velocity of the center of the cylinder. $\mathbf{v}_A = 2.68 \text{ ft/s} \longrightarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.73



A 9-in.-radius cylinder of weight 18 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 2.5 lb is applied as shown for 1.2 s. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine the resulting velocity of (*a*) the carriage, (*b*) the center of the cylinder.

SOLUTION

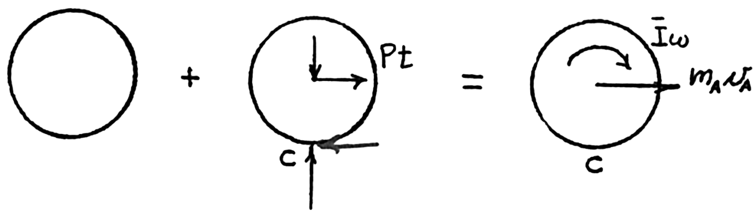
Moment of inertia.

$$\bar{I} = \frac{1}{2} m_A r^2$$

$$= \frac{1}{2}\left(\frac{18 \text{ lb}}{32.2}\right)\left(\frac{9 \text{ in.}}{12}\right)^2$$

$$= 0.15722 \text{ slug} \cdot \text{ft}^2$$

Cylinder alone:



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

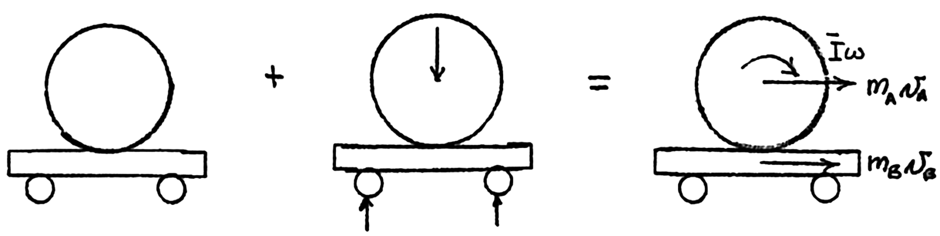
Moments about C:

$$0 + Ptr = \bar{I}\omega + m_A v_A r$$

or

$$0 + (2.5)(1.2)\left(\frac{9}{12}\right) = 0.15722\omega + \left(\frac{18}{32.2}\right)\left(\frac{9}{12}\right)v_A \tag{1}$$

Cylinder and carriage:



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

$\xrightarrow{+}$ Horizontal components:

$$0 + Pt = m_A v_A + m_B v_B$$

or

$$0 + (2.5)(1.2) = \left(\frac{18}{32.2}\right)v_A + \left(\frac{6}{32.2}\right)v_B \tag{2}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Kinematics. $$v_A = v_B + r\omega$$

$$v_A = v_B + \left(\frac{9}{12}\right)\omega \qquad (3)$$

Solving Equations (1), (2) and (3) simultaneously gives $\boldsymbol{\omega} = 2.39 \text{ rad/s}$ ↻

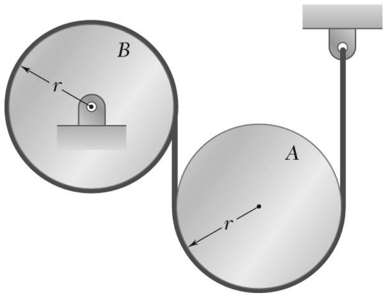
(*a*) Velocity of the carriage. $\mathbf{v}_B = 2.68 \text{ ft/s} \longrightarrow$ ◀

(*b*) Velocity of the center of the cylinder. $\mathbf{v}_A = 4.47 \text{ ft/s} \longrightarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

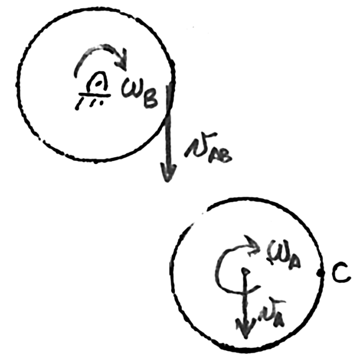
STUDENTS-HUB.com

PROBLEM 17.74



Two uniform cylinders, each of mass $m = 6$ kg and radius $r = 125$ mm, are connected by a belt as shown. If the system is released from rest when $t = 0$, determine (*a*) the velocity of the center of cylinder B at $t = 3$ s, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics. $v_{AB} = r\omega_B$

Point C is the instantaneous center of cylinder A.

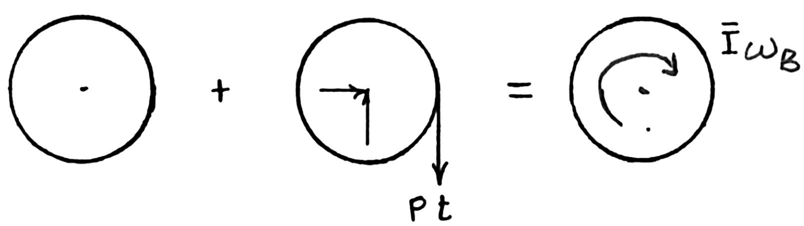
$$\omega_A = \frac{v_{AB}}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

Moment of inertia. $\bar{I} = \frac{1}{2}mr^2$

(*a*) Velocity of the center of *A*.

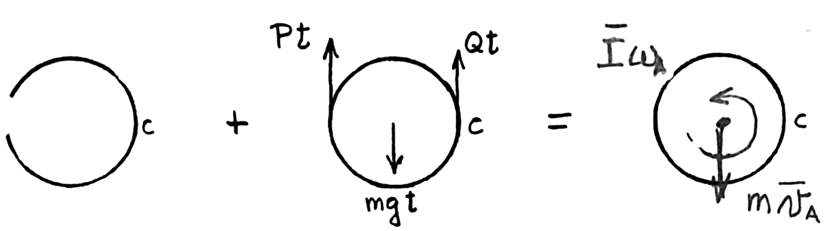
Cylinder B:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$+\circlearrowright$ Moments about B: $\quad 0 + Ptr = \bar{I}\omega_B \qquad (1)$

Cylinder. A:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.74 (Continued)

Moments about C: $$0 - 2Ptr + mgtr = m\bar{v}_A r + \bar{I}\omega_A$$

$$0 - 2\bar{I}\,\omega_B + mgtr = m\left(\frac{1}{2}r\omega_B\right)r + \bar{I}\left(\frac{1}{2}\omega_B\right)$$

$$\left(\frac{5}{2}\bar{I} + \frac{1}{2}mr^2\right)\omega_B = mgrt$$

$$\left(\frac{5}{2}\frac{mr^2}{2} + \frac{1}{2}mr^2\right)\omega_B = mgrt$$

$$\frac{7}{4}r\omega_B = gt$$

$$\omega_B = \frac{4}{7}\frac{gt}{r} \qquad (2)$$

$$\bar{v}_A = \frac{1}{2}r\omega_B = \frac{2}{7}gt = \frac{2}{7}(9.81)(3) \qquad \bar{\mathbf{v}}_A = 8.41 \text{ m/s}\downarrow \blacktriangleleft$$

(*b*) Tension in the belt.

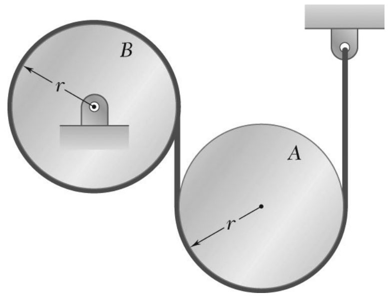
From Eqs. (1) and (2), $$Ptr = \bar{I}\left(\frac{4}{7}\frac{gt}{r}\right)$$

$$P = \frac{1}{tr}\left(\frac{1}{2}mr^2\right)\left(\frac{4}{7}\frac{gt}{r}\right) = \frac{2}{7}mg = \frac{2}{7}(6)(9.81) = 16.817 \text{ N} \qquad P = 16.82 \text{ N} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

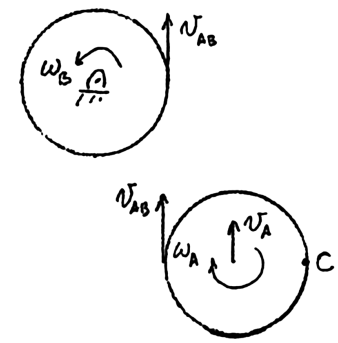
STUDENTS-HUB.com

PROBLEM 17.75



Two uniform cylinders, each of mass $m = 6$ kg and radius $r = 125$ mm, are connected by a belt as shown. Knowing that at the instant shown the angular velocity of cylinder A is 30 rad/s counterclockwise, determine (a) the time required for the angular velocity of cylinder A to be reduced to 5 rad/s, (b) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics. $$v_{AB} = r\omega_B$$

Point C is the instantaneous center of cylinder A.

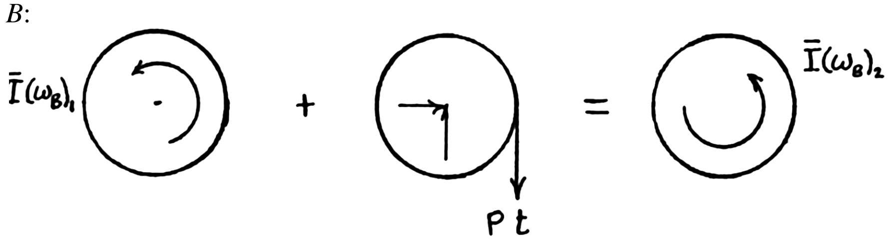
$$\omega_A = \frac{v_{AB}}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

Moment of inertia. $$\bar{I} = \frac{1}{2}\frac{W}{g}r^2$$

(a) *Required time*.

Cylinder B:



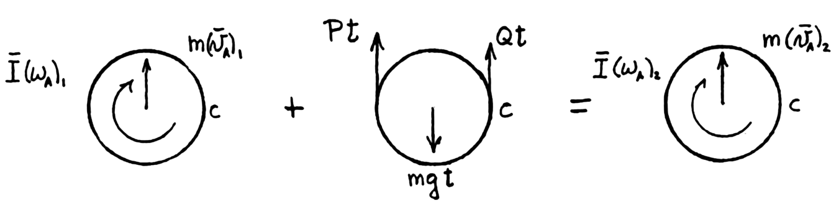
Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

Moments about B: $$\bar{I}(\omega_B)_1 - Ptr = \bar{I}(\omega_B)_2$$

$$Ptr = \bar{I}[(\omega_B)_1 - (\omega_B)_2]$$

$$= \frac{1}{2}mr^2[(\omega_B)_1 - (\omega_B)_2] \tag{1}$$

Cylinder A:



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.75 (Continued)

Moments about C: $\bar{I}(\omega_A)_1 + m(v_A)_1 r + 2Ptr - mgtr = \bar{I}(\omega_A)_2 + m(v_A)_2 r$

$$\frac{1}{2}mr^2[(\omega_A)_1 - (\omega_A)_2 + mr[(\omega_A)_1 - (\omega_A)_2]r + 2Ptr - mgtr = 0$$

$$\frac{3}{2}mr^2\left[\left(\frac{1}{2}\omega_B\right)_1 - \frac{1}{2}(\omega_B)_2\right] + 2\left\{\frac{1}{2}mr^2[(\omega_B)_1 - (\omega_B)_2]\right\} - mgtr = 0$$

$$\frac{7}{4}mr^2[(\omega_B)_1 - (\omega_B)_2] - mgtr = 0$$

$$t = \frac{7r[(\omega_B)_1 - (\omega_B)_2]}{4g} \qquad (2)$$

Data: $m = 6\text{ kg}$

$r = 125\text{ mm} = 0.125\text{ m}$

From Equation (2), $t = \dfrac{(7)(0.125)(30-5)}{(4)(9.81)} = 0.55747$ $\quad t = 0.557\text{ s}$ ◀

(*b*) *Tension in belt between cylinders.*

From Equation (1), $Ptr = \dfrac{1}{2}(6)(0.125)^2(30-5)$

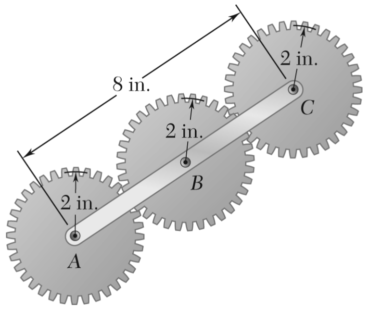
$= 1.172\text{ N}\cdot\text{m}\cdot\text{s}$

$$P = \frac{Ptr}{tr} = \frac{1.172}{(0.55747)(0.125)} = 16.817$$ $\quad P = 16.82\text{ N}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

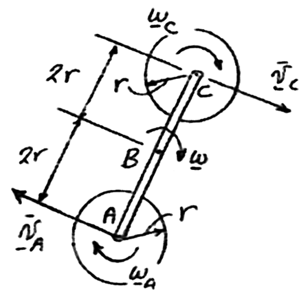
STUDENTS-HUB.com

PROBLEM 17.76



In the gear arrangement shown, gears A and C are attached to rod ABC, which is free to rotate about B, while the inner gear B is fixed. Knowing that the system is at rest, determine the magnitude of the couple **M** which must be applied to rod ABC, if 2.5 s later the angular velocity of the rod is to be 240 rpm clockwise. Gears A and C weigh 2.5 lb each and may be considered as disks of radius 2 in.; rod ABC weighs 4 lb.

SOLUTION



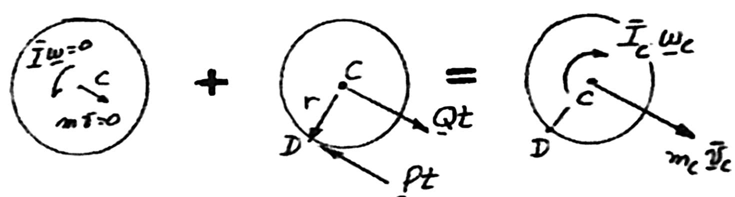
Kinematics of motion

Let $\omega_{ABC} = \omega \qquad \overline{v}_A = \overline{v}_C = (BC)\omega = 2r\omega$

Since gears A and C roll on the fixed gear B,

$$\omega_A = \omega_C = \frac{v_C}{r} = \frac{2r\omega}{r} = 2\omega$$

Principle of impulse and momentum.

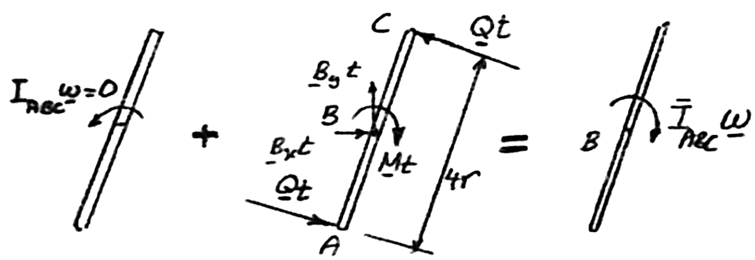


Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

+) Moments about D: $\qquad 0 + (Qt)r = m_C\overline{v}_C r + \overline{I}_C w_C$

$$(Qt)r = m_C(2r\omega)r + \frac{1}{2}m_C r^2(2\omega)$$

$$Qt = 3m_C r\omega \qquad (1)$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.76 (Continued)

$+\circlearrowright$ Moments about B: $$Mt - Qt(4r) = \bar{I}_{ABC}\omega$$

$$Mt - 4(Qt)r = \frac{1}{12}m_{ABC}(4r)^2\omega$$

$$Mt - 4(Qt)r = \frac{4}{3}m_{ABC}r^2\omega \qquad (2)$$

Substitute for (Qt) from (1) into (2):

$$Mt - 4(3m_C r\omega)r = \frac{4}{3}m_{ABC}r^2\omega$$

$$Mt = \frac{4}{3}r^2\omega(m_{ABC} + 9m_C) \qquad (3)$$

Couple M.

Data: $$t = 2.5\text{ s}$$

$$r = \frac{2}{12}\text{ ft}$$

$$m_{ABC} = \frac{4\text{ lb}}{32.2\text{ ft/s}}$$

$$m_C = \frac{2.5\text{ lb}}{32.2\text{ ft/s}^2}$$

$$\omega = 240\text{ rpm}$$
$$= 8\pi\text{ rad/s}$$

Eq. (3): $$M(2.5\text{ s}) = \frac{4}{3}\left(\frac{2}{12}\text{ ft}\right)^2(8\pi\text{ rad/s})\left[\frac{4}{32.2} + 9\left(\frac{2.5}{32.2}\right)\right]$$

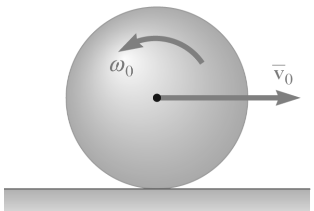
$$2.5\text{ M} = 0.76607$$
$$M = 0.3064\text{ lb}\cdot\text{ft}$$

$\mathbf{M} = 0.306\text{ lb}\cdot\text{ft} \circlearrowright$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

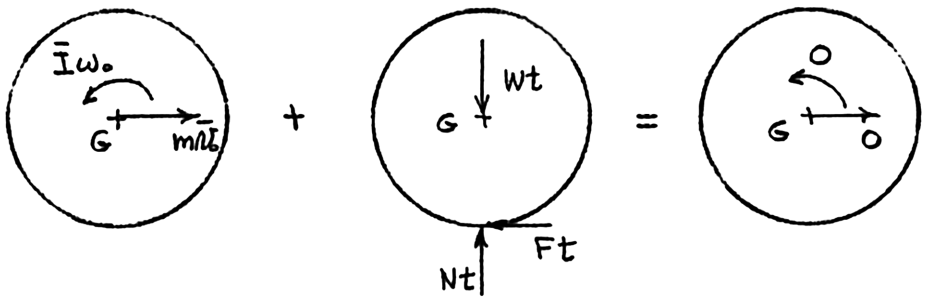
PROBLEM 17.77



A sphere of radius r and mass m is projected along a rough horizontal surface with the initial velocities shown. If the final velocity of the sphere is to be zero, express (*a*) the required magnitude of $\boldsymbol{\omega}_0$ in terms of v_0 and r, (*b*) the time required for the sphere to come to rest in terms of v_0 and coefficient of kinetic friction μ_k.

SOLUTION

Moment of inertia. Solid sphere. $\bar{I} = \frac{2}{5}mr^2$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$+\uparrow$ y components: $Nt - Wt = 0 \quad N = W = mg \quad (1)$

$\xrightarrow{+}$ x components: $m\bar{v}_0 - Ft = 0 \quad Ft = m\bar{v}_0 \quad (2)$

$+\circlearrowright$ Moments about G: $\bar{I}\omega_0 - Ftr = 0 \quad (3)$

$$\frac{2}{5}mr^2\omega_0 - m\bar{v}_0 r = 0$$

(*a*) *Solving for* ω_0, $\omega_0 = \frac{5}{2}\frac{\bar{v}_0}{r}$ ◀

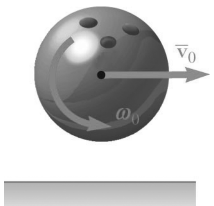
(*b*) *Time to come to rest.*

From Equation (2), $t = \frac{mv_0}{F} = \frac{m\bar{v}_0}{\mu_k mg}$ $\quad t = \frac{\bar{v}_0}{\mu_k g}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.78



A bowler projects an 8.5-in.-diameter ball weighing 16 lb along an alley with a forward velocity $\mathbf{v}_0$ of 25 ft/s and a backspin ω_0 of 9 rad/s. Knowing that the coefficient of kinetic friction between the ball and the alley is 0.10, determine (*a*) the time t_1 at which the ball will start rolling without sliding, (*b*) the speed of the ball at time t_1.

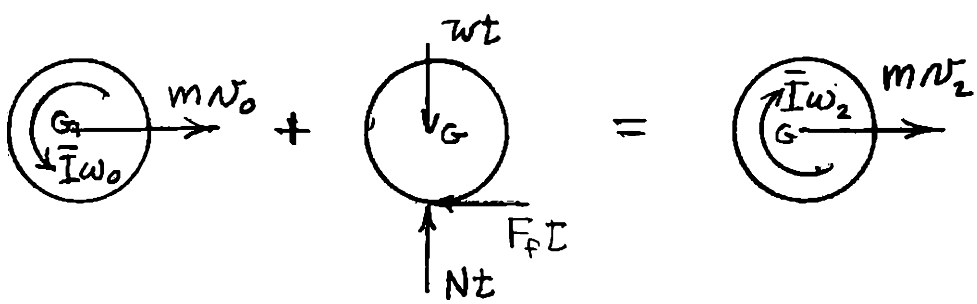
SOLUTION

Radius: $r = \frac{1}{2}d = \frac{1}{2}(8.5 \text{ in.}) = 4.25 \text{ in.} = 0.35417 \text{ ft}$

Mass: $m = \frac{W}{g} = \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.49689 \text{ lb} \cdot \text{s}^2/\text{ft}$

Moment of inertia: $\bar{I} = \frac{2}{5}mr^2 = \left(\frac{2}{5}\right)(0.49689)(0.35417)^2 = 0.02493 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$

Use the principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

$+\uparrow$: $Nt - Wt = 0 \qquad N = W = 16 \text{ lb}$

Friction force: $F_F = \mu_k N = (0.10)(16) = 1.6 \text{ lb.}$

$\xrightarrow{+}$: $mv_0 - F_F t = mv_2$

$$v_2 = v_0 - \frac{F_F t}{m} = 25 \text{ ft/s} - \frac{1.6t}{0.49689}$$
$$= 25 - 3.22t$$

$+\circlearrowright$ Moments about G: $\bar{I}\omega_0 - F_F tr = -\bar{I}\omega_2$

$$\omega_2 = \frac{F_F tr}{\bar{I}} - \omega_0 = \frac{(1.6t)(0.35417)}{0.02493} - 9$$
$$= 22.731t - 9$$

Slipping stops when $v_2 = r\omega_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.78 (Continued)

(a) Time t when slipping stops.

$$(25 - 3.22t) = (0.35417 \text{ ft})(22.731t - 9)$$

$$(25 + 3.1875) = (3.22 + 8.0506)t$$

$$t = 2.501 \text{ s}$$

$t = 2.50$ s ◀

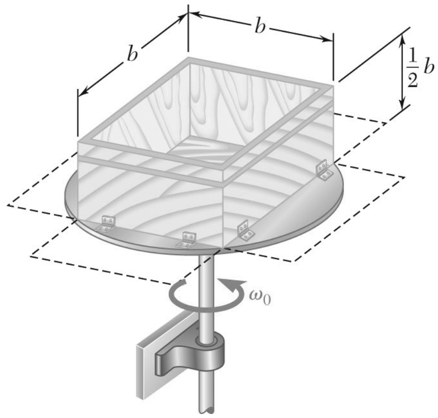
(b) Corresponding velocity.

$$v_2 = 25 - 3.22t$$

$v_2 = 16.95$ ft/s ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.79

Four rectangular panels, each of length b and height $\frac{1}{2}b$, are attached with hinges to a circular plate of diameter $\sqrt{2}b$ and held by a wire loop in the position shown. The plate and the panels are made of the same material and have the same thickness. The entire assembly is rotating with an angular velocity ω_0 when the wire breaks. Determine the angular velocity of the assembly after the panels have come to rest in a horizontal position.

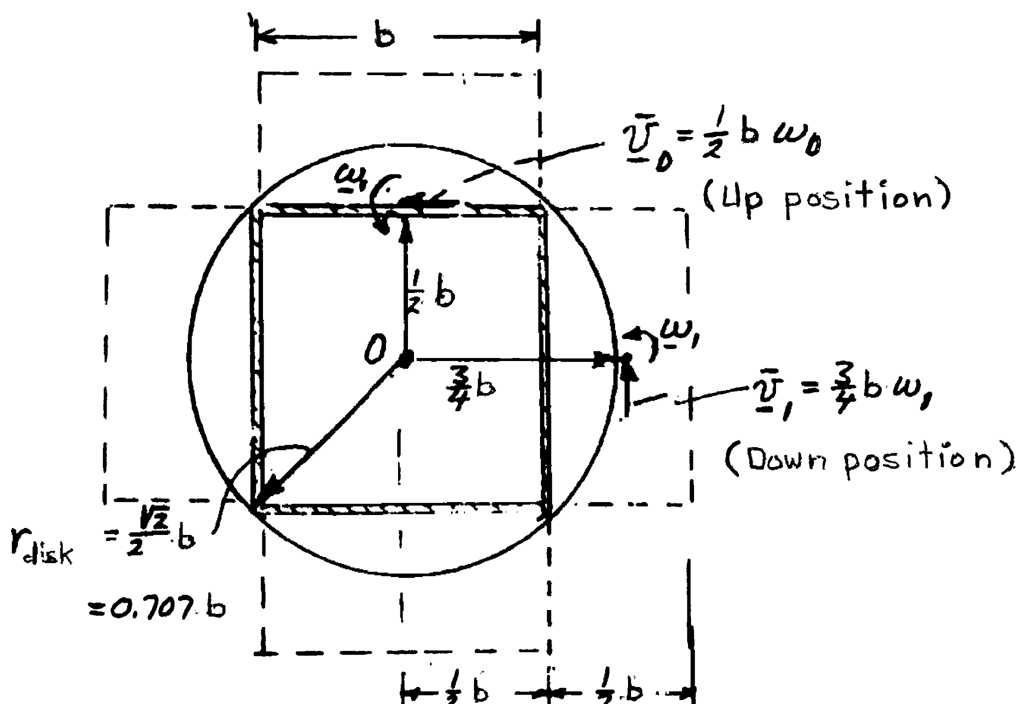
SOLUTION

Kinematics: When the panels are in the up position, the speed of the mass center of each panel is

$$v_0 = \frac{1}{2}b\omega_0$$

When the panels are in the down position the speed of the mass center of each panel is

$$v_1 = \frac{3}{4}b\omega_1$$



PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Let $\rho =$ mass density of plate and of panels

$t =$ thickness of plate and of panels

Disk: $$m = \rho V = \rho \pi t(0.707b)^2 = \rho t \pi b^2 (0.500)$$

$$I_{\text{disk}} = I_D = \frac{1}{2} m(r_{\text{disk}})^2 = \frac{1}{2} \rho t \pi b^2 (0.500)(0.707b)^2$$

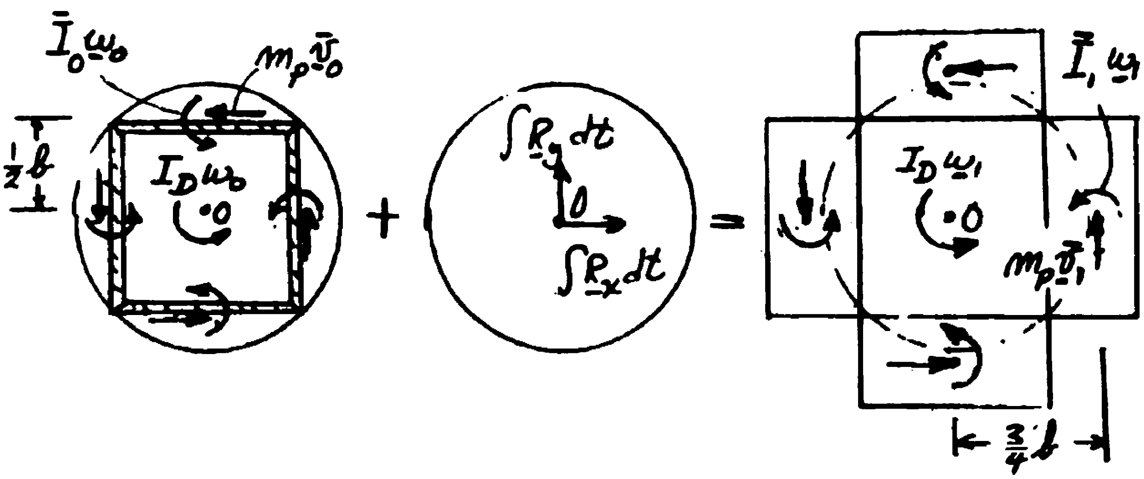
$$I_D = \frac{1}{8} \rho t \pi b^4$$

Each panel: $$m_\rho = b\left(\frac{1}{2}b\right)\rho t = \frac{1}{2}\rho t b^2$$

In up position $$\bar{I}_0 = \frac{1}{12} m_\rho b^2 = \frac{1}{12}\left(\frac{1}{2}\rho t b^2\right) b^2 = \frac{1}{24}\rho t b^4$$

In down position $$\bar{I}_1 = \frac{1}{12} m_\rho \left(b^2 + \left(\frac{1}{2}b\right)^2\right) = \frac{1}{12}\left(\frac{1}{2}\rho t b^2\right)\frac{5}{4}b^2 = \frac{5}{96}\rho t b^4$$

Principle of impulse and momentum.



Syst. Momenta$_0$ + **Syst. Ext. Imp.** $_{0\to1}$ = **Syst. Momenta**$_1$

In the up position, the angular momentum of one panel about the vertical axle is

$$m_\rho v_0 \left(\frac{1}{2}b\right) + \bar{I}_0 \omega_0$$

In the down position it is

$$m_\rho \bar{v}_1 \frac{3}{4}b + \bar{I}_1 \omega_1$$

Conservation of angular momentum.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.79 (Continued)

$$\bar{I}_{\text{disk}}\omega_0 + 4\left[m_\rho v_D\left(\frac{1}{2}b\right) + \bar{I}_0\omega_0\right] = \bar{I}_{\text{disk}}\omega_1 + 4\left[m_\rho \bar{v}_1\left(\frac{3}{4}b\right) + \bar{I}_1\omega_1\right]$$

$$\bar{I}_{\text{disk}}\omega_0 + 4\left[m_\rho\left(\frac{1}{2}b\right)^2 + \bar{I}_0\right]\omega_0 = \bar{I}_{\text{disk}}\omega_1 + 4\left[m_\rho\left(\frac{3}{4}b\right)^2 + \bar{I}_1\right]\omega_1$$

$$\left\{\frac{1}{8}\rho t\pi b^4 + 4\left[\frac{1}{2}\rho t b^2\left(\frac{1}{2}b\right)^2 + \frac{1}{24}\rho t b^4\right]\right\}\omega_0$$

$$= \left\{\frac{1}{8}\rho t\pi b^4 + 4\left[\frac{1}{2}\rho t b^2\left(\frac{3}{4}b\right)^2 + \frac{5}{96}\rho t b^4\right]\right\}\omega_1$$

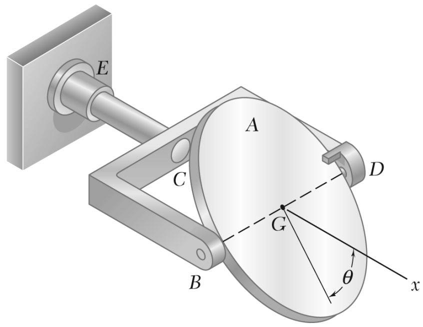
$$\left\{\frac{\pi}{8} + \frac{1}{2} + \frac{1}{6}\right\}\omega_0 = \left\{\frac{\pi}{8} + \frac{9}{8} + \frac{5}{24}\right\}\omega_1$$

$$\{1.059\}\omega_0 = \{1.726\}\omega_1$$

$$\boldsymbol{\omega}_1 = 0.614\omega_0 \circlearrowleft \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.80

A 2.5-lb disk of radius 4 in. is attached to the yoke *BCD* by means of short shafts fitted in bearings at *B* and *D*. The 1.5-lb yoke has a radius of gyration of 3 in. about the *x* axis. Initially the assembly is rotating at 120 rpm with the disk in the plane of the yoke $(\theta = 0)$. If the disk is slightly disturbed and rotates with respect to the yoke until $\theta = 90°$, where it is stopped by a small bar at *D*, determine the final angular velocity of the assembly.

SOLUTION

Moment of inertia of yoke: $$I_C = mk_C^2 = \left(\frac{1.5}{32.2}\right)\left(\frac{3}{12}\right)^2 = 2.9115\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Moment of inertia of disk:

$$\theta = 0:\quad I_A = \frac{1}{4}mr^2$$
$$= \frac{1}{4}\left(\frac{2.5}{32.2}\right)\left(\frac{4}{12}\right)^2$$
$$= 2.15666\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\theta = 90°:\quad I_A = \frac{1}{2}mr^2$$
$$= \frac{1}{2}\left(\frac{2.5}{32.2}\right)\left(\frac{4}{12}\right)^2$$
$$= 4.3133\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Total moment of inertia about the *x* axis:

$$\theta = 0:\quad (I_x)_1 = I_C + I_A$$
$$= 5.0682\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\theta = 90°:\quad (I_x)_2 = I_C + I_A$$
$$= 7.2248\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Angular momentum about the *x* axis:

$$\theta = 0:\quad H_1 = (I_x)_1\omega_1$$
$$= 5.0682\times10^{-3}\omega_1$$

$$\theta = 90°:\quad H_2 = (I_x)_2\omega_2$$
$$= 7.2248\times10^{-3}\omega_2$$

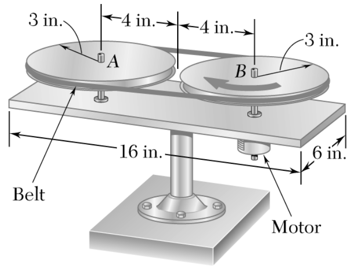
Conservation of angular momentum.

$$H_1 = H_2:\quad 5.0682\times10^{-3}\,\omega_1 = 7.2248\times10^{-3}\,\omega_2$$
$$\omega_2 = 0.7015\omega_1 = (0.7015)(120\text{ rpm})$$

$\omega_2 = 84.2$ rpm ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.81

Two 10-lb disks and a small motor are mounted on a 15-lb rectangular platform which is free to rotate about a central vertical spindle. The normal operating speed of the motor is 180 rpm. If the motor is started when the system is at rest, determine the angular velocity of all elements of the system after the motor has attained its normal operating speed. Neglect the mass of the motor and of the belt.

SOLUTION

Kinematics. Motor speed: $\omega_M = 180 \text{ rpm} = 6\pi \text{ rad/s}$

Let ω_A, ω_B, and ω_P be the angular velocities, respectfully, of disk A, disk B and the platform. Since the motor speed is the angular velocity of disk B relative to the platform,

$$\omega_B = \omega_P + \omega_M = \omega_P + 6\pi \tag{1}$$

Since, the disks have the same outer radius, $\omega_B = \omega_A$ (2)

Velocity of the center of disk A $\quad v_A = \frac{4}{12}\omega_P$ (3)

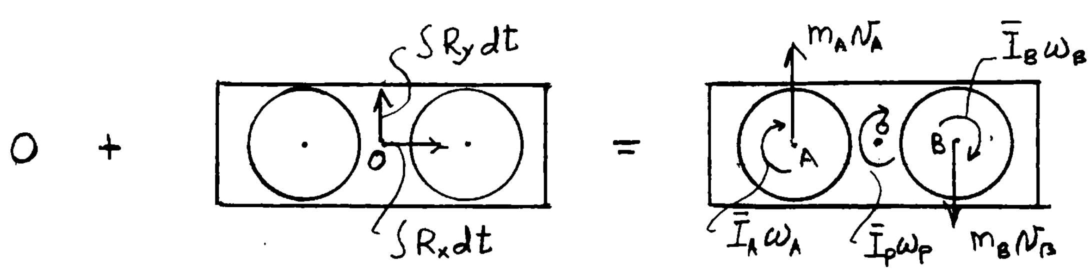
Velocity of the center of disk B $\quad v_B = \frac{4}{12}\omega_P$ (4)

Moments of inertia.

Disks A and B: $\quad \bar{I}_A = \bar{I}_B = \frac{1}{2}\frac{W}{g}r^2 = \frac{1}{2}\left(\frac{10}{32.2}\right)\left(\frac{3}{12}\right)^2 = 9.705 \times 10^{-3} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$

Platform: $\quad \bar{I}_P = \frac{1}{12}\frac{W}{g}(a^2 + b^2) = \frac{1}{12}\left(\frac{15}{32.2}\right)\left[\left(\frac{16}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right] = 78.718 \times 10^{-3} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$

Principle of impulse and momentum for system.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.81 (Continued)

+↻ Moments about O:

$$0 + 0 = \bar{I}_P\omega_P + m_A v_A l_{OA} + \bar{I}_A\omega_A + m_B v_B l_{OB} + \bar{I}_B\omega_B$$

$$= (78.718\times 10^{-3})\omega_P + \left(\frac{10}{32.2}\right)\left(\frac{4}{12}\omega_P\right)\left(\frac{4}{12}\right) + (9.705\times 10^{-3})(\omega_P + 6\pi) + \left(\frac{10}{32.2}\right)\left(\frac{4}{12}\omega_P\right)\left(\frac{4}{12}\right)$$

$$+ (9.705\times 10^{-3})(\omega_P + 6\pi)$$

$$= 167.141\times 10^{-3}\omega_P + 365.87\times 10^{-3}$$

$$\omega_P = -2.189 \text{ rad/s}$$

$$\omega_A = \omega_B = -2.189 + 6\pi = 16.66 \text{ rad/s}$$

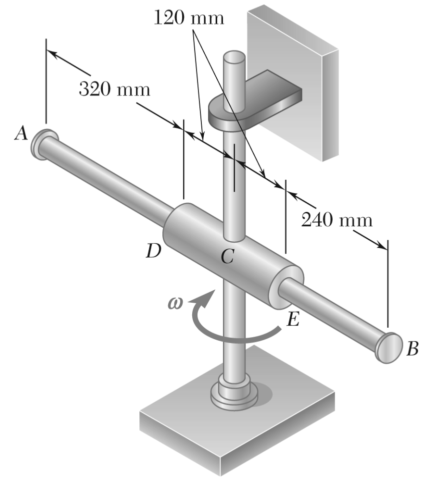
Angular velocities. $\omega_A = 159.1$ rpm ↻ ◀

$\omega_B = 159.1$ rpm ↻ ◀

$\omega_P = 20.9$ rpm ↺ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

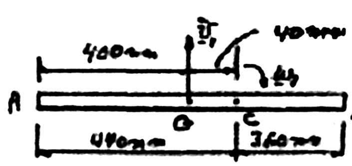


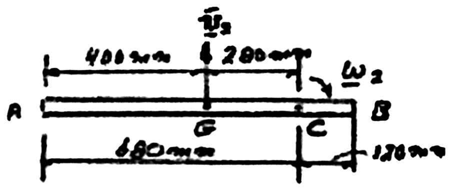
PROBLEM 17.82

A 3-kg rod of length 800 mm can slide freely in the 240-mm cylinder DE, which in turn can rotate freely in a horizontal plane. In the position shown the assembly is rotating with an angular velocity of magnitude $\omega = 40$ rad/s and end B of the rod is moving toward the cylinder at a speed of 75 mm/s relative to the cylinder. Knowing that the centroidal mass moment of inertia of the cylinder about a vertical axis is $0.025\ \text{kg}\cdot\text{m}^2$ and neglecting the effect of friction, determine the angular velocity of the assembly as end B of the rod strikes end E of the cylinder.

SOLUTION

Kinematics and geometry.





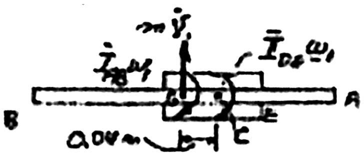
$\bar{v}_1 = (0.04\ \text{m})\omega_1 = (0.4\ \text{m})(40\ \text{rad/s})$

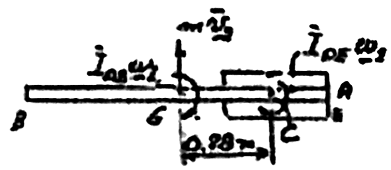
$\bar{v}_1 = 1.6\ \text{m/s}$

$\bar{v}_2 = (0.28\ \text{m})\omega_2$

Initial position — Final position

Conservation of angular momentum about C.





$+\circlearrowleft$ Moments about C: $\quad \bar{I}_{AB} = \dfrac{1}{12}(3\ \text{kg})(0.8\ \text{m})^2 = 0.16\ \text{kg}\cdot\text{m}^2$

$$\bar{I}_{AB}\omega_1 + m\bar{v}_1(0.04\ \text{m}) + \bar{I}_{DE}\omega_1 = \bar{I}_{AB}\omega_2 + m\bar{v}_2(0.028\ \text{m}) + \bar{I}_{DE}\omega_2$$

$$(0.16\ \text{kg}\cdot\text{m}^2)(40\ \text{rad/s}) + (3\ \text{kg})(1.6\ \text{m/s})(0.04\ \text{m}) + (0.025\ \text{kg}\cdot\text{m}^2)(40\ \text{rad/s})$$

$$= (0.16\ \text{kg}\cdot\text{m}^2)\omega_2 + (3\ \text{kg})(0.28\omega_2)(0.28) + (0.025\ \text{kg}\cdot\text{m}^2)\omega_2$$

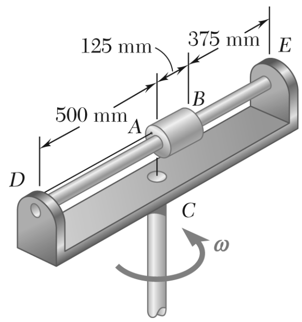
$$(6.4 + 0.192 + 1.00) = (0.16 + 0.2352 + 0.025)\omega_2$$

$$7.592 = 0.4202\omega_2; \quad \omega_2 = 18.068\ \text{rad/s}$$

Angular velocity. $\qquad \omega_2 = 18.07\ \text{rad/s}$ ◄

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.83

A 1.6-kg tube *AB* can slide freely on rod *DE*, which in turn can rotate freely in a horizontal plane. Initially the assembly is rotating with an angular velocity $\omega = 5$ rad/s and the tube is held in position by a cord. The moment of inertia of the rod and bracket about the vertical axis of rotation is $0.30\ \text{kg} \cdot \text{m}^2$ and the centroidal moment of inertia of the tube about a vertical axis is $0.0025\ \text{kg} \cdot \text{m}^2$. If the cord suddenly breaks, determine (*a*) the angular velocity of the assembly after the tube has moved to end *E*, (*b*) the energy lost during the plastic impact at *E*.

SOLUTION

Let Point *C* be the intersection of axle *C* and rod *DE*. Let Point *G* be the mass center of tube *AB*.

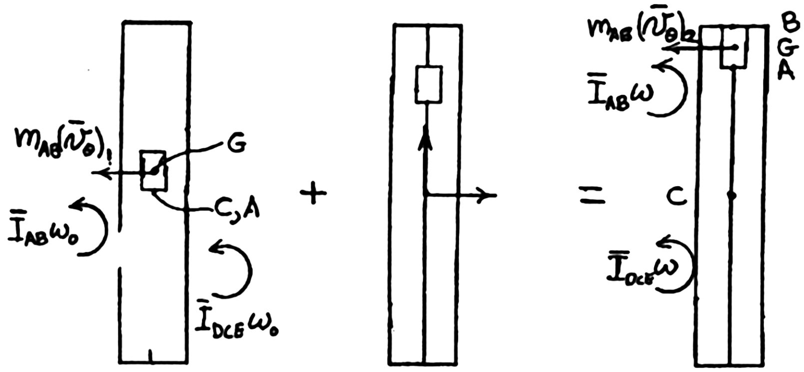
Masses and moments of inertia about vertical axes.

$$m_{AB} = 1.6\ \text{kg}$$
$$\bar{I}_{AB} = 0.0025\ \text{kg} \cdot \text{m}^2$$
$$\bar{I}_{DCE} = 0.30\ \text{kg} \cdot \text{m}^2$$

State 1.
$$(r_{G/A})_1 = \frac{1}{2}(125)$$
$$= 62.5\ \text{mm}$$
$$\omega_1 = 5\ \text{rad/s}$$

State 2.
$$(r_{G/A})_2 = 500 - 62.5$$
$$= 437.5\ \text{mm}$$
$$\omega = \omega_2$$

Kinematics.
$$(v_G)_\theta = \bar{v}_\theta = r_{G/C}\,\omega$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Moments about C:

$$\bar{I}_{AB}\omega_1 + \bar{I}_{DCE}\omega_1 + m_{AB}(\bar{v}_\theta)_1(r_{G/C})_1 + 0 = \bar{I}_{AB}\omega_2 + \bar{I}_{DCE}\omega_2 + m_{AB}(\bar{v}_\theta)_2(r_{G/C})_2$$

$$\left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/G})_1^2\right]\omega_1 = \left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_2^2\right]\omega_2$$

$$[0.0025 + 0.30 + (1.6)(0.0625)^2](5) = [0.0025 + 0.30 + (1.6)(0.4375)^2]\omega_2$$

$$(0.30875)(5) = 0.60875\omega_2$$

$$\omega_2 = 2.5359 \text{ rad/s}$$

(*a*) *Angular velocity after the plastic impact.* 2.54 rad/s ◀

Kinetic energy. $$T = \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\bar{v}^2$$

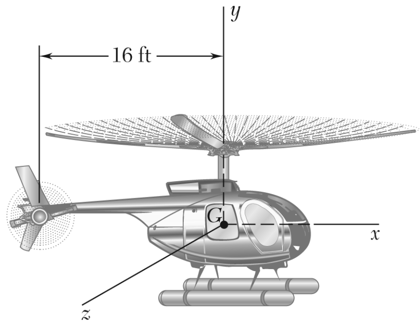
$$T_1 = \frac{1}{2}(0.0025)(5)^2 + \frac{1}{2}(0.30)(5)^2 + \frac{1}{2}(1.6)(0.0625)^2(5)^2$$
$$= 3.859375 \text{ J}$$

$$T_2 = \frac{1}{2}(0.0025)(2.5359)^2 + \frac{1}{2}(0.30)(2.5359)^2 + \frac{1}{2}(1.6)(0.4375)^2(2.5359)^2$$
$$= 1.9573 \text{ J}$$

(*b*) *Energy lost.* $T_1 - T_2 = 1.902 \text{ J}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.84

In the helicopter shown, a vertical tail propeller is used to prevent rotation of the cab as the speed of the main blades is changed. Assuming that the tail propeller is not operating, determine the final angular velocity of the cab after the speed of the main blades has been changed from 180 to 240 rpm. (The speed of the main blades is measured relative to the cab, and the cab has a centroidal moment of inertia of $650\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$. Each of the four main blades is assumed to be a slender 14-ft rod weighting 55 lb.)

SOLUTION

Let $\boldsymbol{\Omega}$ be the angular velocity of the cab and ω be the angular velocity of the blades relative to the cab. The absolute angular velocity of the blades is $\Omega + \omega$.

$$\omega_1 = 180\ \text{rpm} = 6\pi\ \text{rad/s}$$

$$\omega_2 = 240\ \text{rpm} = 8\pi\ \text{rad/s}$$

Moments of inertia.

Cab: $$I_C = 650\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Blades: $$I_B = 4\left(\frac{1}{3}mL^2\right)$$

$$= (4)\left(\frac{1}{3}\right)\left(\frac{55}{32.2}\right)(14)^2$$

$$= 446.38\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Assume $\Omega_1 = 0$.

Conservation of angular momentum about shaft.

$$I_B(\omega_1 + \Omega_1) + I_C\Omega_1 = I_B(\omega_2 + \Omega_2) + I_C\Omega_2$$

$$\Omega_2 = -\frac{I_B(\omega_2 - \omega_1)}{I_C + I_B}$$

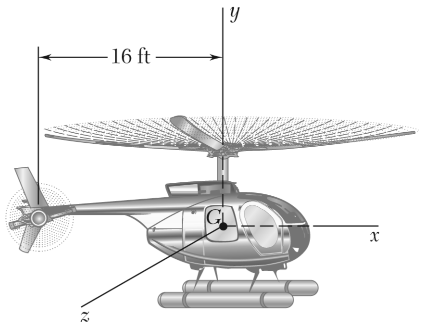
$$= -\frac{(446.38)(8\pi - 6\pi)}{446.38 + 650}$$

$$= -2.5581\ \text{rad/s}$$

$\Omega_2 = -24.4\ \text{rpm}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.85

Assuming that the tail propeller in Problem 17.84 is operating and that the angular velocity of the cab remains zero, determine the final horizontal velocity of the cab when the speed of the main blades is changed from 180 to 240 rpm. The cab weighs 1250 lb and is initially at rest. Also determine the force exerted by the tail propeller if the change in speed takes place uniformly in 12 s.

SOLUTION

Let $\mathbf{\Omega}$ be the angular velocity of the cab and ω be the angular velocity of the blades relative to the cab. The absolute angular velocity of the blades is $\Omega + \omega$.

$$\omega_1 = 180 \text{ rpm} = 6\pi \text{ rad/s}$$

$$\omega_2 = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

Moments of inertia.

Cab: $I_C = 650 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$

Blades: $$I_B = 4\left(\frac{1}{3}mL^2\right) = (4)\left(\frac{1}{3}\right)\left(\frac{55}{32.2}\right)(14)^2$$

$$= 446.38 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

The cab does not rotate. $\Omega_1 = \Omega_2 = 0$

$$\textbf{Syst. Momenta}_1 + \textbf{Syst. Ext. Imp.}_{1\to 2} = \textbf{Syst. Momenta}_2$$

Moments about shaft: $I_B(\omega_1 + \Omega_1) + I_C\Omega_1 + Frt = I_B(\omega_2 + \Omega_2) + I_C\Omega_2$

$$Frt = I_B(\omega_2 - \omega_1)$$

$$= (446.38)(8\pi - 6\pi)$$

$$= 2804.7 \text{ lb} \cdot \text{ft} \cdot \text{s}$$

$$Ft = \frac{Frt}{r} = \frac{2804.7}{16} = 175.29 \text{ lb} \cdot \text{s}$$

Linear components: $mv_1 + Ft = mv_2$

$$v_2 - v_1 = \frac{Ft}{m} = \frac{175.29}{\frac{1250}{32.2} + (4)\left(\frac{55}{32.2}\right)}$$

$$= 3.8398 \text{ ft/s}$$

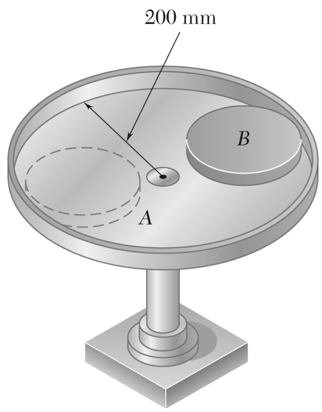
(*a*) Assume $v_1 = 0$. $v_2 = 3.84 \text{ ft/s}$ ◀

(*b*) Force. $F = \dfrac{Ft}{t} = \dfrac{175.29}{12}$ $F = 14.61 \text{ lb}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

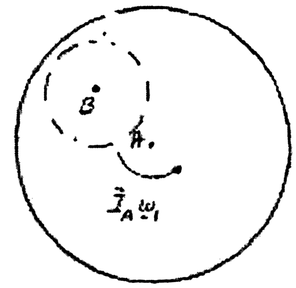
STUDENTS-HUB.com

PROBLEM 17.86



The circular platform A is fitted with a rim of 200-mm inner radius and can rotate freely about the vertical shaft. It is known that the platform-rim unit has a mass of 5 kg and a radius of gyration of 175 mm with respect to the shaft. At a time when the platform is rotating with an angular velocity of 50 rpm, a 3-kg disk B of radius 80 mm is placed on the platform with no velocity. Knowing that disk B then slides until it comes to rest relative to the platform against the rim, determine the final angular velocity of the platform.

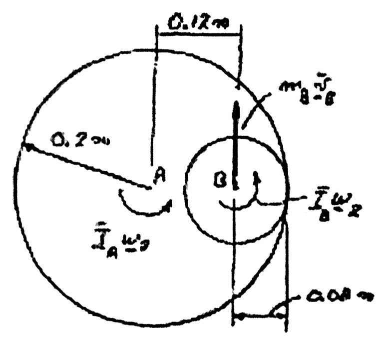
SOLUTION



Syst. Momenta$_1$

Moments of inertia.

$$\bar{I}_A = m_A k^2$$
$$= (5 \text{ kg})(0.175 \text{ m})^2$$
$$= 0.153125 \text{ kg}\cdot\text{m}^2$$
$$\bar{I}_B = \frac{1}{2} m_B r_B^2$$
$$= \frac{1}{2}(3 \text{ kg})(0.08 \text{ m})^2$$
$$= 9.6\times 10^{-3} \text{ kg}\cdot\text{m}^2$$



Syst. Momenta$_2$

<u>State 1</u> Disk B is at rest.

<u>State 2</u> Disk B moves with platform A.

Kinematics. In <u>State 2</u>, $\bar{v}_B = (0.12 \text{ m})\omega_2$

Principle of conservation of angular momentum.

$+\circlearrowleft$ Moments about D: $\bar{I}_A\omega_1 = \bar{I}_A\omega_2 + \bar{I}_B\omega_2 + m_B\bar{v}_B(0.12 \text{ m})$

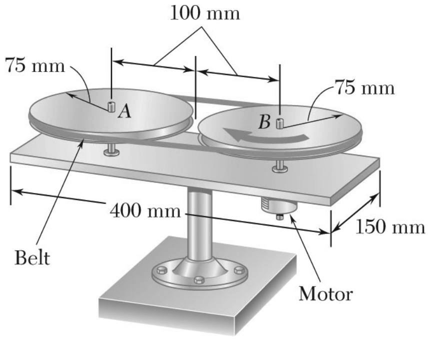
$$(0.153125 \text{ kg}\cdot\text{m}^2)\omega_1 = (0.153125 \text{ kg}\cdot\text{m}^2)\omega_2 + (9.6\times 10^{-3} \text{ kg}\cdot\text{m}^2)\omega_2 + (3 \text{ kg})(0.12 \text{ m})^2\omega_2$$

$$0.153125\omega_2 = 0.20593\omega_1$$
$$\omega_2 = 0.7436\omega_1$$
$$= 0.7436(50 \text{ rpm})$$

Final angular velocity $\omega_2 = 37.2 \text{ rpm}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.87

Two 4-kg disks and a small motor are mounted on a 6-kg rectangular platform which is free to rotate about a central vertical spindle. The normal operating speed of the motor is 240 rpm. If the motor is started when the system is at rest, determine the angular velocity of all elements of the system after the motor has attained its normal operating speed. Neglect the mass of the motor and of the belt.

SOLUTION

Moments of inertia.

Disks: $$\bar{I}_A = \bar{I}_B = \frac{1}{2}mr^2 = \frac{1}{2}(4\text{ kg})(0.075\text{ m})^2 = 0.01125\text{ kg}\cdot\text{m}$$

Platform: $$\bar{I}_P = \frac{1}{12}m(a^2+b^2) = \frac{1}{12}(6\text{ kg})[(0.15\text{ m})^2 + (0.4\text{ m})^2] = 0.09125\text{ kg}\cdot\text{m}^2$$

Kinematics:

$$\omega_M = \left(\frac{240\text{ rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = 8\pi\text{ rad/s}$$

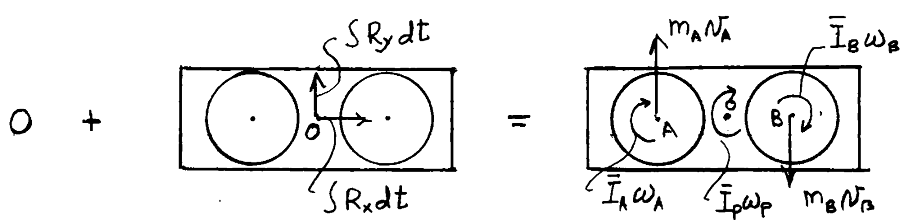
Let ω_A, ω_B and ω_P be the angular velocities of A, B, and the platform. The motor speed is the angular velocity of B relative to the platform.

$$\omega_B = \omega_P + \omega_M = \omega_P + 8\pi$$
$$\omega_A = \omega_B$$

Velocity of center of disk A. $\quad v_A = \omega_P r_{A/O} = 0.1\,\omega_A$

Velocity of center of disk B. $\quad v_B = \omega_P r_{B/O} = 0.1\,\omega_A$

Principle of impulse and momentum for system.



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.87 (Continued)

$+\circlearrowright$ Moments about O:

$$0 + 0 = \bar{I}_P\omega_P + m_A v_A r_{A/O} + \bar{I}_A\omega_A + m_B v_B r_{B/O} + \bar{I}_B\omega_B$$

$$0 = \bar{I}_P\omega_P + m_A(\omega_P r_{A/O}^2) + \bar{I}_A(\omega_P + 8\pi) + m_B(\omega_P r_{B/O}^2) + \bar{I}_B(\omega_P + 8\pi)$$

$$\omega_P = -\frac{8\pi(\bar{I}_A + \bar{I}_B)}{\bar{I}_P + m_A r_{A/O} + \bar{I}_A + m_B r_{B/O}^2 + \bar{I}_B}$$

$$\omega_P = -\frac{8\pi(0.01125 + 0.01125)}{0.09125 + (4)(0.1)^2 + 0.01125 + 4(0.1)^2 + 0.01125}$$

$$= -2.9186 \text{ rad/s} = -27.87 \text{ rpm}$$

$$\omega_B = \omega_A = -2.9186 + 8\pi = 22.214 \text{ rad/s}$$

$$= 22.214 \text{ rad/s}\left(\frac{60\text{ s}}{\text{min}}\right)\left(\frac{1\text{ rev}}{2\pi}\right) = 212.13 \text{ rpm}$$

Angular velocities.

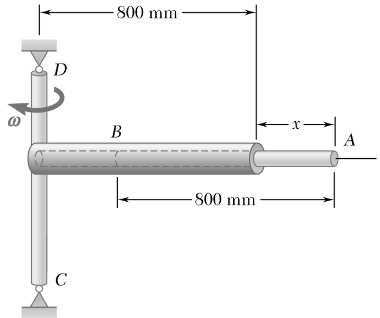
$\omega_A = 212 \text{ rpm} \circlearrowright$ ◀

$\omega_B = 212 \text{ rpm} \circlearrowright$ ◀

$\omega_P = 27.9 \text{ rpm} \circlearrowleft$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.88

The 4-kg rod AB can slide freely inside the 6-kg tube CD. The rod was entirely within the tube ($x = 0$) and released with no initial velocity relative to the tube when the angular velocity of the assembly was 5 rad/s. Neglecting the effect of friction, determine the speed of the rod relative to the tube when $x = 400$ mm.

SOLUTION

Let l be the length of the tube and the rod and Point O be the point of intersection of the tube and the axle.

Moments of inertia. $\quad \bar{I}_T = \frac{1}{12}m_T l^2, \qquad \bar{I}_R = \frac{1}{12}m_R l^2$

Kinematics. $\quad (\bar{v}_\theta)_T = \bar{r}_T\omega = \frac{l}{2}\omega$

$$(\bar{v}_\theta)_R = \bar{r}_R\omega = \left(\frac{l}{2} + x\right)\omega, \qquad (\bar{v}_r)_T = v_r$$

Angular momentum about Point O.

$$H_O = m_T\bar{r}_T(\bar{v}_\theta) + \bar{I}_T\omega + m_R\bar{r}_R(\bar{v}_\theta)_R + \bar{I}_R\omega$$

$$= m_T\frac{l}{2}\left(\frac{l}{2}\omega\right) + \frac{1}{12}m_T l^2\omega + m_R\left(\frac{l}{2} + x\right)\left(\frac{l}{2} + x\right)\omega + \frac{1}{12}m_R l^2\omega$$

$$= \left[\frac{1}{3}m_T l^2 + m_R\left(\frac{1}{3}l^2 + lx + x^2\right)\right]\omega$$

Kinetic energy.

$$T = \frac{1}{2}m_T(\bar{v}_\theta)_T^2 + \frac{1}{2}\bar{I}_T\omega^2 + \frac{1}{2}m_R(\bar{v}_\theta{}^2)_R + \frac{1}{2}m_R v_r^2 + \frac{1}{2}\bar{I}_R\omega^2$$

$$= \frac{1}{2}\left[m_T\left(\frac{l}{2}\omega\right)^2 + \frac{1}{12}m_T l^2\omega^2 + \frac{1}{2}m_R\left(\frac{l}{2} + x\right)^2\omega^2 + \frac{1}{2}m_R v_r^2 + \frac{1}{12}m_R l^2\omega^2\right]$$

$$= \frac{1}{2}\left[\frac{1}{3}m_T l^2 + m_R\left(\frac{1}{3}l^2 + lx + x^2\right)\right]\omega^2 + \frac{1}{2}m_R v_r^2 = \frac{1}{2}H_O\omega + \frac{1}{2}m_R v_r^2$$

Potential energy. $\quad$ All motion is horizontal. $\quad V = 0$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.88 (Continued)

State 1. $x = 0, \qquad \omega = \omega_1 = 5 \text{ rad/s}, \qquad v_r = 0$

$$(H_O)_1 = \frac{1}{3}(m_T + m_R)l^2\omega = \frac{1}{3}(6+4)(0.800)^2(5) = 10.6667 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$T_1 = \frac{1}{2}\left[\frac{1}{3}(m_T + m_R)l^2\omega^2\right] + 0 = \frac{1}{2}(H_O)_1\omega = \frac{1}{2}(10.6667)(5) = 26.667 \text{ J}$$

$$V_1 = 0$$

State 2. $x = \frac{l}{2} = 0.400 \text{ m}, \qquad \omega = \omega_2 = ?, \qquad v_r = ?$

$$(H_O)_2 = \left[\frac{1}{3}(6)(0.800)^2 + (4)\left\{\frac{1}{3}(0.800)^2 + (0.800)(0.400) + (0.400)^2\right\}\right]\omega_2$$

$$= 4.05333\omega_2$$

$$T_2 = \frac{1}{2}(4.05333\omega_2)\omega_2 + \frac{1}{2}(4)v_r^2 = 2.026667\omega_2^2 + 2v_r^2$$

$$V_2 = 0$$

Conservation of angular momentum: $(H_O)_1 = (H_O)_2$

$$10.6667 = 4.05333\,\omega_2 \qquad \omega_2 = 2.6316 \text{ rad/s}$$

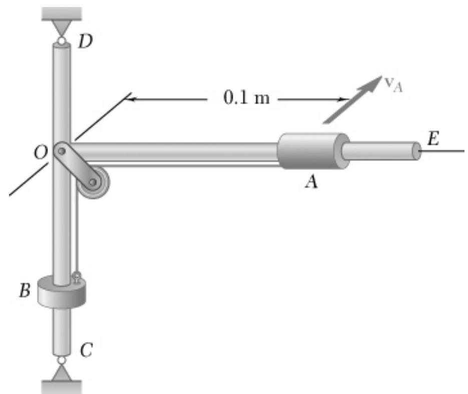
Conservation of energy. $T_1 + V_1 = T_2 + V_2$

$$26.667 + 0 = (2.02667)(2.6316)^2 + 2v_r^2 + 0$$

$$v_r^2 = 6.3158 \text{ m}^2/\text{s}^2 \qquad v_r = 2.51 \text{ m/s} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.89

A 1.8-kg collar A and a 0.7-kg collar B can slide without friction on a frame, consisting of the horizontal rod OE and the vertical rod CD, which is free to rotate about its vertical axis of symmetry. The two collars are connected by a cord running over a pulley that is attached to the frame at O. At the instant shown, the velocity $\mathbf{v}_A$ of collar A has a magnitude of 2.1 m/s and a stop prevents collar B from moving. The stop is suddenly removed and collar A moves toward E. As it reaches a distance of 0.12 m from O, the magnitude of its velocity is observed to be 2.5 m/s. Determine at that instant the magnitude of the angular velocity of the frame and the moment of inertia of the frame and pulley system about CD.

SOLUTION

Components of velocity of collar A. $$v_A^2 = (v_A)_r^2 + (v_A)_\theta^2 \qquad (1)$$

Constraint of rod OE. $$(v_A)_\theta = r_A\omega \qquad (2)$$

Constraint of cable AB. $$\Delta r_A = \Delta y_B, \quad (v_A)_r = v_B \qquad (3)$$

Position 1. $$(\Delta r_A) = 0.1\text{ m}, \quad [(v_A)_r]_1 = 0, \quad (v_A)_1 = 2.1\text{ m/s}$$

From Equation (1), $$(2.1)^2 = 0 + [(v_A)_\theta]_1^2 \qquad [(v_A)_\theta]_1 = 2.1\text{ m/s}$$

From Equation (2), $$(2.1) = 0.1\omega_1 \qquad \omega_1 = 21\text{ rad/s}$$

From Equation (3), $$v_B = 0$$

Potential energy. Take position 1 as datum. $$V_1 = 0 \qquad (4)$$

Angular momentum. $$(H_O)_1 = I\omega_1 + m_A[(v_A)_r]_1(r_A)_1:$$

$$(H_O)_1 = I(21) + (1.8)(2.1)(0.1) \qquad (H_O)_1 = 21I + 0.378 \qquad (5)$$

Kinetic energy. $$T_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2:$$

$$T_1 = \frac{1}{2}I(21)^2 + \frac{1}{2}(1.8)(2.1)^2 \qquad T_1 = 220.5I + 3.969 \qquad (6)$$

Position 2. $$(r_A)_2 = 0.12\text{ m}, \quad (v_A)_2 = 2.5\text{ m/s} \qquad \omega = \omega_2$$

From Equation (2), $$[(v_A)_\theta]_2 = 0.12\omega_2$$

From Equation (1), $$[(v_A)_r]_2^2 = (v_A)_2^2 - [(v_A)_\theta]_2^2 = (2.5)^2 - (0.12)^2\omega_2^2$$
$$= 6.25 - 0.0144\omega_2^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

From Equation (3), $$v_B^2 = 6.25 - 0.0144\omega_2^2$$

Change in radial position. $$\Delta r_A = (r_A)_2 - (r_A)_1 = 0.02 \text{ m}$$

From Equation (3), $$\Delta y_B = 0.02 \text{ m}$$

Potential energy. $$V_2 = m_B g(\Delta y_B) = (0.7)(9.81)(0.02)$$

$$V_2 = 0.13734 \text{ J} \quad (7)$$

Angular momentum. $$(H_O)_2 = I\omega_2 + m_A[(v_A)_\theta]_2(r_A)_2:$$

$$(H_O)_2 = I\omega_2 + (1.8)(0.12\omega_2)(0.12) \qquad (H_O)_2 = (I + 0.02592)\omega_2 \quad (8)$$

Kinetic energy. $$T_2 = \frac{1}{2}I\omega_2^2 + \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2:$$

$$T_2 = \frac{1}{2}I\omega_2^2 + \frac{1}{2}(1.8)(2.5)^2 + \frac{1}{2}(0.7)(6.25 - 0.0144\omega_2^2)$$

$$T_2 = (0.5I - 0.00504)\omega_2^2 + 7.8125 \quad (9)$$

Conservation of angular momentum. $$(H_O)_1 = (H_O)_2:$$

$$21I + 0.378 = (I + 0.02592)\omega_2$$

Solving for ω_2, $$\omega_2 = \frac{21I + 0.378}{I + 0.02592} = \frac{N}{D} \quad (10)$$

Conservation of energy. $$T_1 + V_1 = T_2 + V_2:$$

$$220.5I + 3.969 = (0.5I - 0.00504)\omega_2^2 + 7.8125 + 0.13734$$

$$220.5I - (0.5I - 0.00504)\frac{N^2}{D^2} - 3.98084 = 0$$

$$220.5ID^2 - 0.5IN^2 + 0.00504N^2 - 3.98084D^2 = 0$$

$$220.5I(I^2 + 0.05184I + 0.0006718464) - 0.5I(441I^2 + 15.876I + 0.142884)$$

$$+ 0.00504(441I^2 + 15.876I + 0.142884) - (3.98084)(I^2 + 0.05184I + 0.0006718464) = 0$$

$$0I^3 + 1.73452I^2 - 0.04965167I - 0.001954378 = 0$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.89 (Continued)

Solving the quadratic equation for I,

$$I = \frac{0.04965167 \pm 0.126590}{3.46904} = 0.050804 \qquad \text{and} \qquad -0.022179$$

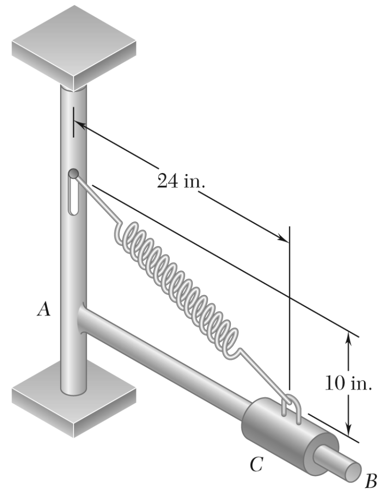
Reject the negative root.

From Equation (10), $$\omega_2 = \frac{(21)(0.050804) + 0.378}{0.050804 + 0.02592}$$ $\omega = 18.83 \text{ rad/s}$ ◀

$I = 0.0508 \text{ kg}\cdot\text{m}^2$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



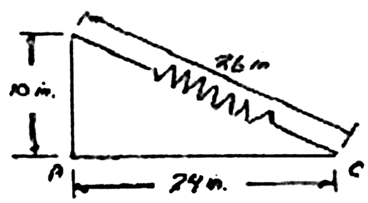
PROBLEM 17.90

A 6-lb collar C is attached to a spring and can slide on rod AB, which in turn can rotate in a horizontal plane. The mass moment of inertia of rod AB with respect to end A is $0.35\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$. The spring has a constant $k = 15\ \text{lb/in.}$ and an undeformed length of 10 in. At the instant shown the velocity of the collar relative to the rod is zero, and the assembly is rotating with an angular velocity of 12 rad/s. Neglecting the effect of friction, determine (a) the angular velocity of the assembly as the collar passes through a point located 7.5 in. from end A of the rod, (b) the corresponding velocity of the collar relative to the rod.

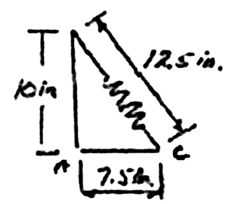
SOLUTION

Potential energy of spring: undeformed length = 10 in.

Position 1:



Position 2:



$$\Delta = 26\ \text{in.} - 10\ \text{in.} = 16\ \text{in.}$$

$$V_1 = \frac{1}{2}k\Delta^2 = \frac{1}{2}(15\ \text{lb/in.})(16\ \text{in.})^2$$

$$= 1920\ \text{in.} \cdot \text{lb}$$

$$V_1 = 160\ \text{ft} \cdot \text{lb}$$

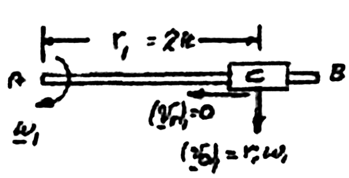
$$\Delta = 12.5\ \text{in.} - 10\ \text{in.} = 2.5\ \text{in.}$$

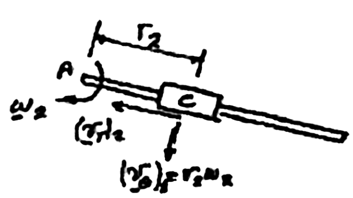
$$V_2 = \frac{1}{2}k\Delta^2 = \frac{1}{2}(15\ \text{lb/in.})(2.5\ \text{in.})^2$$

$$= 46.875\ \text{in.} \cdot \text{lb}$$

$$V_2 = 3.91\ \text{ft} \cdot \text{lb}$$

Kinematics:





Kinetics: Since moments of all forces about shaft at A are zero, $(\mathbf{H}_A)_1 = (\mathbf{H}_A)_2$

$$I_R\omega_1 + m_C(v_0)_1 r_1 = I_R\omega_C + m_C(v_0)_2 r_2$$

$$\left(I_R + m_C r_1^2\right)\omega_1 = \left(I_R + m_C r_2^3\right)\omega_2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Data: $$I_R = 0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2, \quad m_C = \frac{6 \text{ lb}}{32.2}$$

$$r_1 = 2 \text{ ft}, \quad r_2 = \frac{7.5}{12}\text{ft}, \quad \omega_1 = 12 \text{ rad/s}$$

$$\left[0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2 + \frac{6 \text{ lb}}{32.2}(2 \text{ ft})^2\right](12 \text{ rad/s}) = \left[0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2 + \frac{6 \text{ lb}}{32.2}\left(\frac{7.5}{12}\text{ft}\right)^2\right]\omega_2$$

$$13.1441 = 0.42279\omega_2; \quad \omega_2 = 31.089 \text{ rad/s}$$

(*a*) Angular velocity. $\omega_2 = 31.1 \text{ rad/s}$ ◀

Kinetic energy. $$T_1 = \frac{1}{2}I_A\omega_1^2 + \frac{1}{2}m_C(v_D)_1^2 + \frac{1}{2}m_C(v_r)_1^2$$

$$= \frac{1}{2}(0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)(12 \text{ rad/s})^2 + \frac{1}{2}\left(\frac{6 \text{ lb}}{32.2}\right)(2 \text{ ft})^2(12 \text{ rad/s})^2 + 0$$

$$T_1 = 78.865 \text{ ft}\cdot\text{lb}$$

$$T_2 = \frac{1}{2}I_R\omega_2^2 + \frac{1}{2}m(v_B)_2^2 + \frac{1}{2}m_2(v_r)_2^2$$

$$= \frac{1}{2}(0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)(31.089 \text{ rad/s})^2$$

$$+\frac{1}{2}\left(\frac{6 \text{ lb}}{32.2}\right)\left(\frac{7.5}{12}\text{ft}\right)^2(31.089 \text{ rad/s})^2 + \frac{1}{2}\left(\frac{6 \text{ lb}}{32.2}\right)(v_r)_2^2$$

$$T_2 = 204.32 + 0.09317(v_r)_2^2$$

Principle of conservation of energy: $T_1 + V_1 = T_2 + V_2$

Recall: $V_1 = 160 \text{ ft}\cdot\text{lb}$ and $V_2 = 3.91 \text{ ft}\cdot\text{lb}$

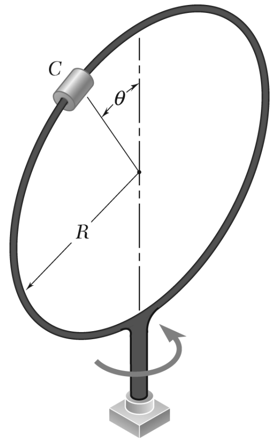
$$78.865 + 160 = 204.32 + 0.09317(v_r)_2^2 + 3.91$$

$$30.638 = 0.09317(v_r)_2^2$$

(*b*) Velocity of collar relative to rod. $(v_r)_2 = 18.13 \text{ ft/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

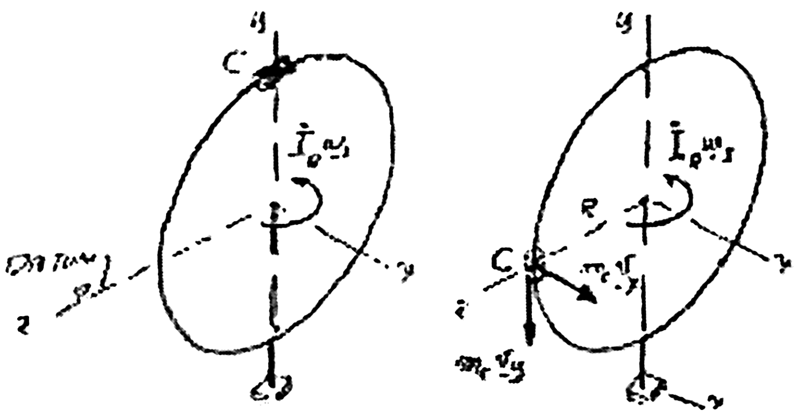


PROBLEM 17.91

A small 4-lb collar C can slide freely on a thin ring of weight 6 lb and radius 10 in. The ring is welded to a short vertical shaft, which can rotate freely in a fixed bearing. Initially the ring has an angular velocity of 35 rad/s and the collar is at the top of the ring $(\theta = 0)$ when it is given a slight nudge. Neglecting the effect of friction, determine (*a*) the angular velocity of the ring as the collar passes through the position $\theta = 90°$, (*b*) the corresponding velocity of the collar relative to the ring.

SOLUTION

Moment of inertia of ring. $\qquad \bar{I}_R = \dfrac{1}{2} m_R R^2$



Position 1 $\qquad\qquad$ Position 2

Position 1. $\qquad \theta = 0$

$$v_C = 0$$

Position 2. $\qquad \theta = 90°$

$$(v_C)_y = v_y = R\omega_2$$

Conservation of angular momentum about y axis for system.

$$\bar{I}_R \omega_1 = \bar{I}_R \omega_2 + m_C v_y R$$

$$\frac{1}{2} m_R R^2 \omega_1 = \frac{1}{2} m_R R^2 \omega_2 + m_C R^2 \omega_2$$

$$m_R R^2 \omega_1 = (m_R + 2m_C) R^2 \omega_2$$

$$\omega_2 = \frac{m_R}{m_R + 2m_C} \omega_1 \qquad (1)$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Potential energy. Datum is the center of the ring.

$$V_1 = m_C g R \qquad V_2 = 0$$

Kinetic energy:

$$T_1 = \frac{1}{2}\bar{I}_R \omega_1^2 = \frac{1}{2}\left(\frac{1}{2}m_R R^2\right)\omega_1^2$$

$$= \frac{1}{4}m_R R^2 \omega_1^2$$

$$T_2 = \frac{1}{2}\bar{I}_R \omega_2^2 + \frac{1}{2}m_C\left(v_x^2 + v_y^2\right)$$

$$= \frac{1}{4}m_R R^2 \omega_2^2 + \frac{1}{2}m_C R^2 \omega_2^2 + \frac{1}{2}m_C v_y^2$$

Principle of conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{4}m_R R^2 \omega_1^2 + m_C g R = \left(\frac{1}{4}m_R + \frac{1}{2}m_C\right)R^2\omega_2^2 + \frac{1}{2}m_C v_y^2 \tag{2}$$

Data:

$$W_C = 4 \text{ lb}$$

$$W_R = 6 \text{ lb}$$

$$R = 10 \text{ in.} = 0.83333 \text{ ft}$$

$$\omega_1 = 35 \text{ rad/s}$$

(*a*) Angular velocity.

From Eq. (1),

$$\omega_2 = \frac{\frac{6 \text{ lb}}{g}}{\frac{6 \text{ lb}}{g} + 2\left(\frac{4 \text{ lb}}{g}\right)}(35 \text{ rad/s}) \qquad \omega_2 = 15.00 \text{ rad/s} \blacktriangleleft$$

(*b*) Velocity of collar relative to ring.

From Eq. (2),

$$\frac{1}{4}\left(\frac{6 \text{ lb}}{32.2}\right)\left(\frac{10}{12}\text{ft}\right)^2(35 \text{ rad/s})^2 + (4 \text{ lb})\left(\frac{10}{12}\text{ft}\right)$$

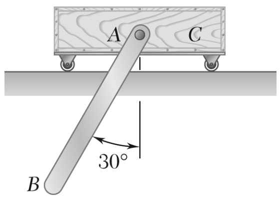
$$= \left[\frac{1}{4}\left(\frac{6 \text{ lb}}{32.2}\right) + \frac{1}{2}\left(\frac{4 \text{ lb}}{32.2}\right)\right]\left(\frac{10}{12}\text{ft}\right)^2(15 \text{ rad/s})^2 + \frac{1}{2}\left(\frac{4 \text{ lb}}{32.2}\right)v_y^2$$

$$39.629 + 3.3333 = 16.984 + 0.062112 v_y^2$$

$$v_y^2 = 418.25 \qquad v_y = 20.5 \text{ ft/s} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

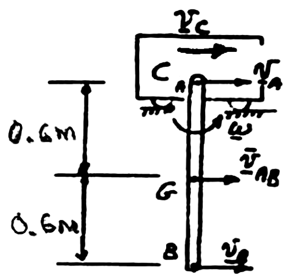


PROBLEM 17.92

A uniform rod AB, of mass 7 kg and length 1.2 m, is attached to the 11-kg cart C. Knowing that the system is released from rest in the position shown and neglecting friction, determine (*a*) the velocity of Point B as rod AB passes through a vertical position (*b*) the corresponding velocity of cart C.

SOLUTION

Kinematics $\quad \mathbf{v}_C = \mathbf{v}_A$



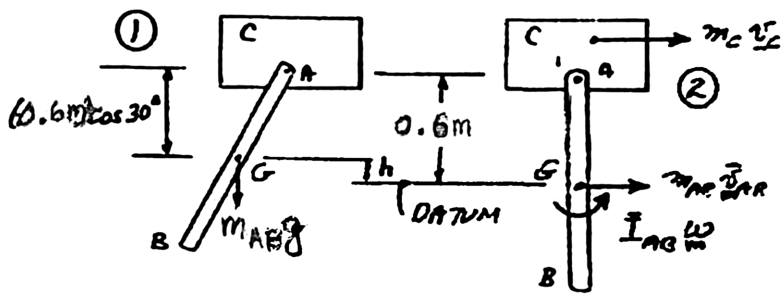
$$\bar{v}_{AB} \longrightarrow = v_C \longrightarrow + (0.6 \text{ m})\omega \longrightarrow$$

$$v_C = \bar{v}_{AB} - 0.6\omega \tag{1}$$

$$AB = 1.2 \text{ m}$$

Weights.

Kinetics



Linear momentum

$$\xrightarrow{+}: \quad 0 = m_C v_C + m_{AB}\bar{v}_{AB}$$

$$\bar{v}_{AB} = -\frac{m_C}{m_{AB}} v_C = -\frac{(11 \text{ kg})}{(7 \text{ kg})} v_C, \quad \bar{v}_{AB} = -\frac{11}{7} v_C \tag{2}$$

Substitute into Eq. (1): $\quad v_C = -\frac{11}{7} v_C - 0.6\omega$

$$\frac{18}{7} v_C = -0.6\omega \qquad v_C = -0.23333\omega \tag{3}$$

Substitute into Eq. (2): $\quad \bar{v}_{AB} = -\frac{11}{7}(-0.23333\omega)$

$$\bar{v}_{AB} = 0.36667\omega \tag{4}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Kinetic and potential energies.

$$T_1 = 0$$

$$V_1 = m_{AB}gb = (7\text{ kg})(9.81)(0.6\text{ m})(1-\cos 30°)$$
$$= 5.520\text{ N}\cdot\text{m}$$

$$V_2 = 0$$

$$T_2 = \frac{1}{2}m_C v_C^2 + \frac{1}{2}m_{AB}\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\omega^2$$
$$= \frac{1}{2}(11)(-0.23333\omega)^2 + \frac{1}{2}(7)(0.36667\omega)^2 + \frac{1}{2}\left(\frac{1}{12}(7)(1.2)^2\right)\omega^2$$
$$= (0.29944 + 0.47056 + 0.4200)\omega^2$$
$$= 1.190\omega^2$$

Conservation of energy: $\quad T_1 + V_1 = T_2 + V_2$

$$0 + 5.52 = 1.190\omega^2$$

$$\omega^2 = 4.6387 \qquad \omega = 2.1538\text{ rad/s}$$

(*b*) Velocity of *C*: Eq. (3) $\quad v_C = -0.23333(2.1538) \qquad \mathbf{v}_C = 0.503\text{ m/s} \longleftarrow$ ◀

(*a*) Velocity of *B*: $\quad \mathbf{v}_B = \mathbf{v}_C + [(1.2)\omega \longrightarrow] = [0.50254\text{ m/s} \longleftarrow] + [1.2(2.1538) \longrightarrow]$

$$\mathbf{v}_B = [0.50254 \longleftarrow] + [2.5845 \longrightarrow] \qquad \mathbf{v}_B = 2.08\text{ m/s} \longrightarrow$$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

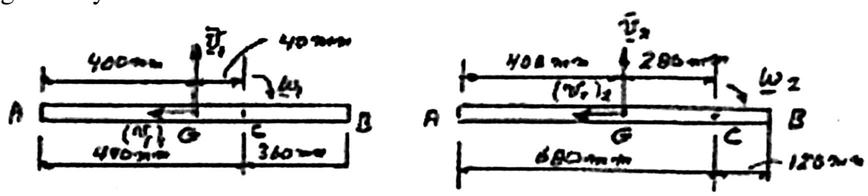
STUDENTS-HUB.com

PROBLEM 17.93

In Problem 17.82, determine the velocity of rod *AB* relative to cylinder *DE* as end *B* of the rod strikes end *E* of the cylinder.

SOLUTION

Kinematics and geometry.



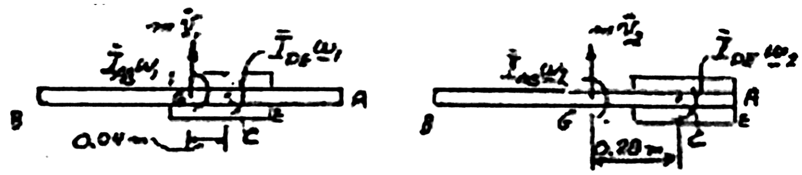
$\bar{v}_1 = (0.04 \text{ m})\omega_1 = (0.4 \text{ m})(40 \text{ rad/s})$

$\bar{v}_1 = 1.6 \text{ m/s}$

$\bar{v}_2 = (0.28 \text{ m})\omega_2$

Initial position　　　　Final position

Conservation of angular momentum about *C*.



$+\circlearrowright$ Moments about *C*: $\quad \bar{I}_{AB} = \frac{1}{12}(3 \text{ kg})(0.8 \text{ m})^2 = 0.16 \text{ kg}\cdot\text{m}^2$

$$\bar{I}_{AB}\omega_1 + m\bar{v}_1(0.04 \text{ m}) + I_{DE}\omega_1 = I_{AB}\omega_2 + m\bar{v}_2(0.28 \text{ m}) + I_{DE}\omega_2$$

$$(0.16 \text{ kg}\cdot\text{m}^2)(40 \text{ rad/s}) + (3 \text{ kg})(1.6 \text{ m/s})(0.04 \text{ m}) + (0.025 \text{ kg}\cdot\text{m}^2)(40 \text{ rad/s})$$

$$= (0.16 \text{ kg}\cdot\text{m}^2)\omega_2 + (3 \text{ kg})(0.28\omega_2)(0.28) + (0.025 \text{ kg}\cdot\text{m}^2)\omega_2$$

$$(6.4 + 0.192 + 1.00) = (0.16 + 0.2352 + 0.025)\omega_2$$

$$7.592 = 0.4202\omega_2; \quad \omega_2 = 18.068 \text{ rad/s}: \quad \omega_2 = 18.07 \text{ rad/s}$$

Conservation of energy $\quad (v_r) = 0.075 \text{ m/s}$

$$V_1 = V_2 = 0$$

$$T_1 = \frac{1}{2}\bar{I}_{DE}\omega_1^2 + \frac{1}{2}\bar{I}_{AB}\omega_1^2 + \frac{1}{2}m_{AB}\bar{v}_1^2 + \frac{1}{2}m_{AB}(v_r)_1^2$$

$$= \frac{1}{2}(0.025 \text{ kg}\cdot\text{m}^2)(40 \text{ rad/s})^2 + \frac{1}{2}(0.16 \text{ kg}\cdot\text{m}^2)(40 \text{ rad/s})^2$$

$$+ \frac{1}{2}(3 \text{ kg})(1.6 \text{ m/s})^2 + \frac{1}{2}(3 \text{ kg})(0.075 \text{ m/s})^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.93 (Continued)

$$T_1 = 20 \text{ J} + 128 \text{ J} + 3.84 \text{ J} + 0.008 \text{ J} = 151.85 \text{ J}$$

$$\bar{v}_2 = (0.28 \text{ m})\omega_2 = (0.28 \text{ m})(18.068 \text{ rad/s}) = 5.059 \text{ m/s}$$

$$T_2 = \frac{1}{2}\bar{I}_{DE}\omega_2^2 + \frac{1}{2}\bar{I}_{AB}\omega_2^2 + \frac{1}{2}m_{AB}v_2^2 + \frac{1}{2}m_{AB}(v_r)_2^2$$

$$= \frac{1}{2}(0.025 \text{ kg}\cdot\text{m}^2)(18.068 \text{ rad/s})^2$$

$$+ \frac{1}{2}(0.16 \text{ kg}\cdot\text{m}^2)(18.068 \text{ rad/s})^2$$

$$= \frac{1}{2}(3 \text{ kg})(5.059 \text{ m/s})^2 + \frac{1}{2}(3 \text{ kg})(v_r)_2^2$$

$$T_2 = 4.081 \text{ J} + 26.116 \text{ J} + 38.391 \text{ J} + 1.5(v_r)_2^2$$

$$T_2 = 68.587 \text{ J} + 1.5(v_r)_2^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 151.85 \text{ J} + 0 = 68.587 \text{ J} + 1.5(v_r)_2^2$$

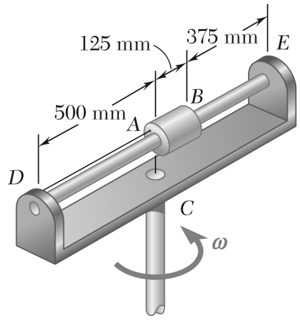
$$83.263 = 1.5(v_r)_2^2$$

Velocity of rod relative to cylinder. $(v_r)_2 = 7.45 \text{ m/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.94



In Problem 17.83 determine the velocity of the tube relative to the rod as the tube strikes end *E* of the assembly.

PROBLEM 17.83 A 1.6-kg tube *AB* can slide freely on rod *DE* which in turn can rotate freely in a horizontal plane. Initially the assembly is rotating with an angular velocity $\omega = 5$ rad/s and the tube is held in position by a cord. The moment of inertia of the rod and bracket about the vertical axis of rotation is $0.30\ \text{kg}\cdot\text{m}^2$ and the centroidal moment of inertia of the tube about a vertical axis is $0.0025\ \text{kg}\cdot\text{m}^2$. If the cord suddenly breaks, determine (*a*) the angular velocity of the assembly after the tube has moved to end *E*, (*b*) the energy lost during the plastic impact at *E*.

SOLUTION

Let Point *C* be the intersection of axle *C* and rod *DE*. Let Point *G* be the mass center of tube *AB*.

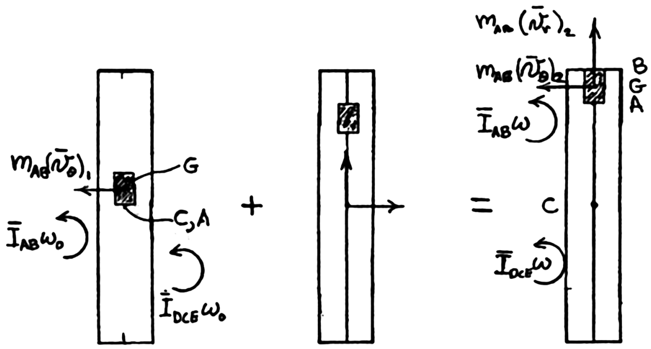
Masses and moments of inertia about vertical axes.

$$m_{AB} = 1.6\ \text{kg}, \qquad \bar{I}_{AB} = 0.0025\ \text{kg}\cdot\text{m}^2, \qquad \bar{I}_{DCE} = 0.30\ \text{kg}\cdot\text{m}^2$$

State 1. $\quad (r_{G/A})_1 = \frac{1}{2}(125) = 62.5\ \text{mm}, \qquad \omega_1 = 5\ \text{rad/s}, \quad (v_r)_1 = 0$

State 2. $\quad (r_{G/A})_2 = 500 - 62.5 = 437.5\ \text{mm}, \qquad \omega = \omega_2, \qquad v_r = (v_r)_2 = 0$

Kinematics. $\quad (v_G)_\theta = \bar{v}_\theta = r_{G/C}\,\omega$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about *C*:

$$\bar{I}_{AB}\omega_1 + \bar{I}_{DCE}\omega_1 + m_{AB}(\bar{v}_\theta)_1(r_{G/C})_1 + 0 = \bar{I}_{AB}\omega_2 + \bar{I}_{DCE}\omega_2 + m_{AB}(\bar{v}_\theta)_2(r_{G/C})_2$$

$$\left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_1^2\right]\omega_1 = \left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_2^2\right]\omega_2$$

$$[0.0025 + 0.30 + (1.6)(0.0625)^2](5) = [0.0025 + 0.30 + (1.6)(0.4375)^2]\omega_2$$

$$(0.30875)(5) = 0.60875\omega_2 \qquad \omega_2 = 2.5359\ \text{rad/s}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Kinetic energy.
$$T = \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\bar{v}^2$$
$$= \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\left(r_{G/C}^2\omega^2 + \bar{v}_r^2\right)$$
$$T_1 = \frac{1}{2}(0.0025)(5)^2 + \frac{1}{2}(0.3)(5)^2 + \frac{1}{2}(1.6)(0.0625)^2 + 0 = 3.859375 \text{ J}$$
$$T_2 = \frac{1}{2}(0.0025)(2.5359)^2 + \frac{1}{2}(0.30)(2.5359)^2$$
$$+\frac{1}{2}(1.6)(0.4375)^2(2.5359)^2 + \frac{1}{2}(1.6)(\bar{v}_r)_2^2$$
$$= 1.95737 + 0.8(\bar{v}_r)_2^2$$

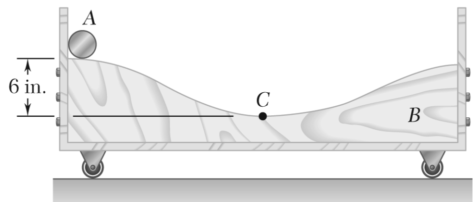
Work. The work of the bearing reactions at C is zero. Since the sliding contact between the rod and the tube is frictionless, the work of the contact force is zero.
$$U_{1\to 2} = 0$$

Principle of work and energy.
$$T_1 + U_{1\to 2} = T_2$$
$$3.859375 + 0 = 1.95737 + 0.8(\bar{v}_r)_2^2$$

Velocity of the tube relative to the rod. $(\bar{v}_r)_2 = 1.542 \text{ m/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

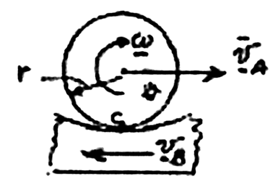
STUDENTS-HUB.com



PROBLEM 17.95

The 6-lb steel cylinder A and the 10-lb wooden cart B are at rest in the position shown when the cylinder is given a slight nudge, causing it to roll without sliding along the top surface of the cart. Neglecting friction between the cart and the ground, determine the velocity of the cart as the cylinder passes through the lowest point of the surface at C.

SOLUTION

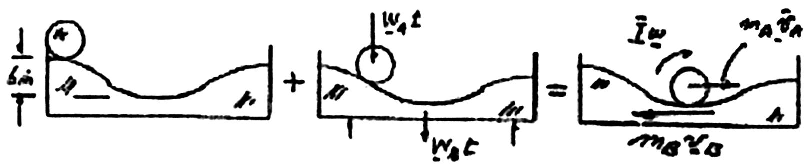


Kinematics (when cylinder is passing C)

$$\overset{+}{\longleftarrow} \quad v_B = v_C = r\omega - \bar{v}_A$$

$$\omega = \frac{\bar{v}_A + v_B}{r}$$

Principle of impulse and momentum.



Syst. of Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\overset{+}{\longrightarrow}$ x components: $\quad m_A\bar{v}_A - m_B v_B = 0$

$$\frac{6\text{ lb}}{g}\bar{v}_A = \frac{10\text{ lb}}{g}v_B; \quad v_B = 0.6\,\bar{v}_A$$

Work: $\quad U_{1\to 2} = W_A(6\text{ in.}) = (6\text{ lb})\left(\frac{6}{12}\text{ ft}\right) = 3\text{ ft}\cdot\text{lb}; \quad T_1 = 0$

Kinetic energy: $\quad T_2 = \frac{1}{2}m_A\bar{v}_A^2 + \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m_B v_3^2$

$$v_B = 0.6\bar{v}_A; \quad \omega = \frac{\bar{v}_A + v_0}{r} = \frac{v_A + 0.6v_A}{r} = \frac{1.6v_A}{r}$$

$$T_2 = \frac{1}{2}\left(\frac{6\text{ lb}}{g}\right)\bar{v}_A^2 + \frac{1}{2}\left[\frac{1}{2}\frac{6\text{ lb}}{g}r^2\right]\left(\frac{1.6v_A}{r}\right)^2 + \frac{1}{2}\frac{10\text{ lb}}{g}(0.6v_A)^2$$

$$= \frac{3}{g}\bar{v}_A^2 + \frac{3.84}{g}\bar{v}_A^2 + \frac{1.8}{g}\bar{v}_A^2 = \frac{8.64}{g}\bar{v}_A^2$$

Principle of work and energy: $\quad T_1 + U_{1\to 2} = T_2$

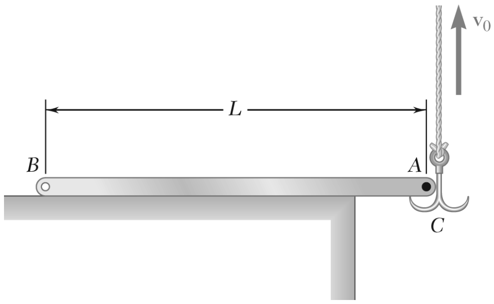
$$0 + 3\text{ ft}\cdot\text{lb} = \frac{8.64}{32.2}v_A^2$$

$$\bar{v}_A^2 = 11.181 \quad \bar{\mathbf{v}}_A = 3.344\text{ ft/s} \longrightarrow$$

$$v_B = 0.6\bar{v}_A = 0.6(3.344) \qquad \mathbf{v}_B = 2.01\text{ ft/s} \longleftarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

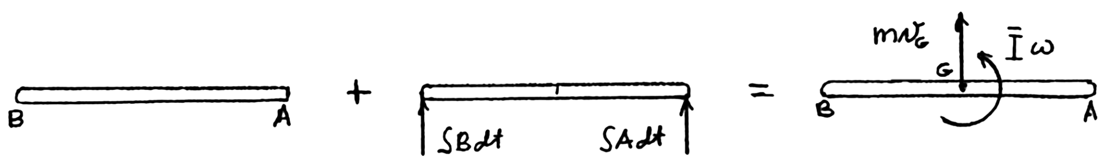


PROBLEM 17.F4

A uniform slender rod AB of mass m is at rest on a frictionless horizontal surface when hook C engages a small pin at A. Knowing that the hook is pulled upward with a constant velocity $\mathbf{v}_0$, draw the impulse-momentum diagram that is needed to determine the impulse exerted on the rod at A and B. Assume that the velocity of the hook is unchanged and that the impact is perfectly plastic.

SOLUTION

Answer:

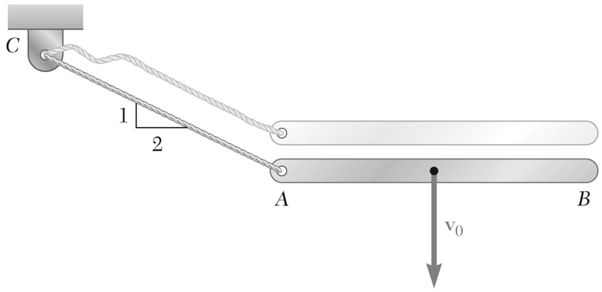


Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to2}$** = **Syst. Momenta$_2$**

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.F5

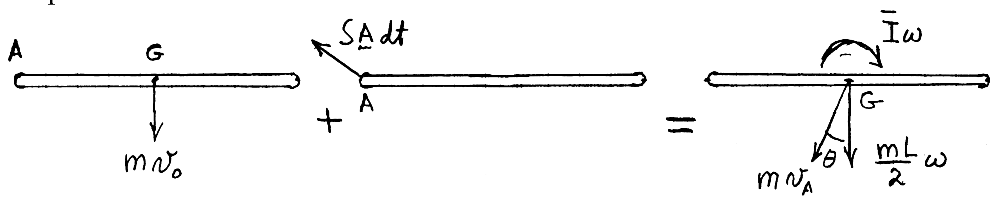


A uniform slender rod AB of length L is falling freely with a velocity $\mathbf{v_0}$ when cord AC suddenly becomes taut. Assuming that the impact is perfectly plastic, draw the impulse-momentum diagram that is needed to determine the angular velocity of the rod and the velocity of its mass center immediately after the cord becomes taut.

SOLUTION

Answer:

Principle of impulse and momentum.



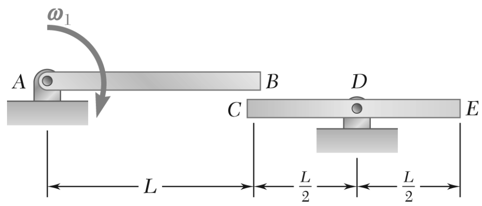
Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Note: For the momentum after the impact a general a_{Gx} and a_{Gy} can be used. These can be related to ω and $\mathbf{v}_A$ using kinematics.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.F6

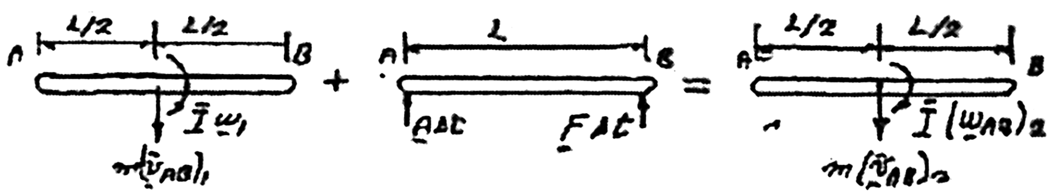


A slender rod *CDE* of length *L* and mass *m* is attached to a pin support at its midpoint *D*. A second and identical rod *AB* is rotating about a pin support at *A* with an angular velocity $\boldsymbol{\omega}_1$ when its end *B* strikes end *C* of rod *CDE*. The coefficient of restitution between the rods is *e*. Draw the impulse-momentum diagrams that are needed to determine the angular velocity of each rod immediately after the impact.

SOLUTION

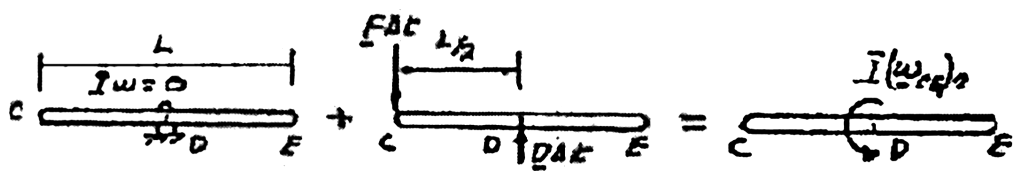
Answer:

Rod *AB*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Rod *CE*.

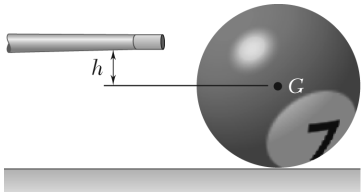


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.96

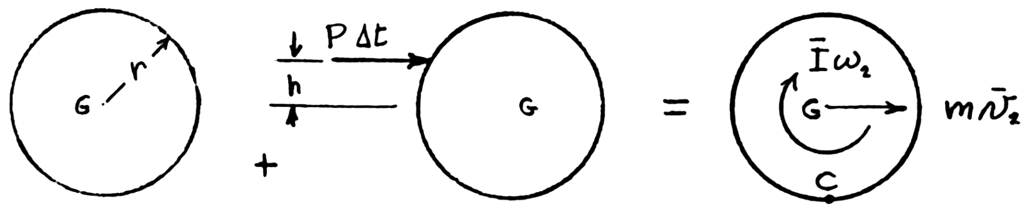


At what height h above its center G should a billiard ball of radius r be struck horizontally by a cue if the ball is to start rolling without sliding?

SOLUTION

Moment of inertia. $\bar{I} = \frac{2}{5}mr^2$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to2}$ = **Syst. Momenta**$_2$

Kinematics. Rolling without sliding. Point C is the instantaneous center of rotation.

$\xrightarrow{+}$ Linear components:

$$0 + P\Delta t = m\bar{v}_2$$
$$= mr\omega_2$$

↺ Moments about G:

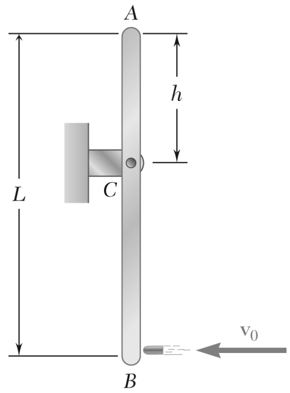
$$0 + hP\Delta t = \bar{I}\omega_2$$
$$0 + h(mr\omega_2) = \left(\frac{2}{5}mr^2\right)\omega_2$$

$h = \frac{2}{5}r$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.97



A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the lower end of a slender 15-lb bar of length $L = 30$ in. Knowing that $h = 12$ in. and that the bar is initially at rest, determine (*a*) the angular velocity of the bar immediately after the bullet becomes embedded, (*b*) the impulsive reaction at *C*, assuming that the bullet becomes embedded in 0.001 s.

SOLUTION

Bar: $L = 30 \text{ in.} = 2.5 \text{ ft} \quad m = \dfrac{15}{32.2} = 0.46584 \text{ lb}\cdot\text{s}^2/\text{ft}$

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(0.46584)(2.5)^2 = 0.24262 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

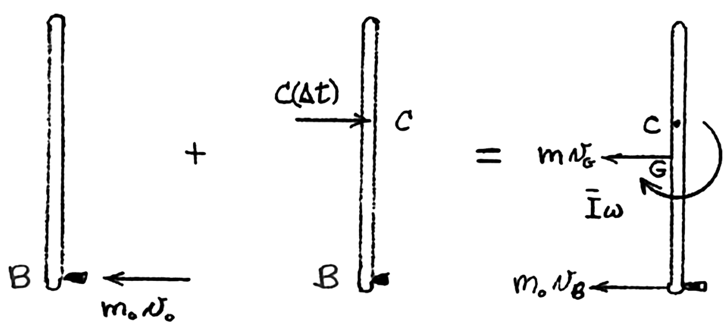
Bullet: $m_0 = \dfrac{0.08}{32.2} = 0.0024845 \text{ lb}\cdot\text{s}^2/\text{ft}$

Support location: $h = 12 \text{ in.} = 1.0 \text{ ft}$

Kinematics. $v_B = (L-h)\omega = (2.5 - 1.0)\omega = 1.5\omega$

$$v_G = \left(\frac{L}{2} - h\right)\omega = (1.25 - 1.0)\omega = 0.25\omega$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about *C*: $m_0 v_0 (L-h) = m_0 v_B (L-h) + m v_0 \left(\dfrac{L}{2} - h\right) + \bar{I}\omega$

$$(0.0024845)(1800)(1.5) = (0.0024845)(1.5\omega) + (0.46584)(0.25\omega)(0.25) + (0.24262\omega)$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.97 (Continued)

(*a*) $$6.7082 = 0.27546\omega \qquad \text{or} \qquad \omega = 24.353$$ $\boldsymbol{\omega} = 24.4\ \text{rad/s}$ ◀

$$v_B = (1.5)(24.353) = 36.53\ \text{ft/s}$$
$$v_G = (0.25)(24.353) = 6.0881\ \text{ft/s}$$

$\xrightarrow{+}$ Horizontal components:

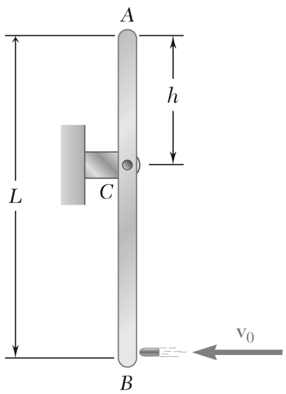
$$-m_0 v_0 + C(\Delta t) = -m_0 v_B - m v_G: \quad C(\Delta t) = m_0(v_0 - v_B) - m v_0$$

$$C(\Delta t) = (0.0024845)(1800 - 36.53) - (0.46584)(6.0881)$$
$$= 1.545\ \text{lb} \cdot \text{s}$$

(*b*) $$C = \frac{C\Delta t}{\Delta t} = \frac{1.545}{0.001}$$ $\mathbf{C} = 1545\ \text{lb} \longrightarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.98

In Problem 17.97, determine (*a*) the required distance h if the impulsive reaction at C is to be zero, (*b*) the corresponding angular velocity of the bar immediately after the bullet becomes embedded.

PROBLEM 17.97 A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the lower end of a slender 15-lb bar of length $L = 30$ in. Knowing that $h = 12$ in. and that the bar is initially at rest, determine (*a*) the angular velocity of the bar immediately after the bullet becomes embedded, (*b*) the impulsive reaction at C, assuming that the bullet becomes embedded in 0.001 s.

SOLUTION

Bar: $$L = 30\text{ in.} = 2.5\text{ ft} \quad m = \frac{15}{32.2} = 0.46584\text{ lb}\cdot\text{s}^2/\text{ft}$$

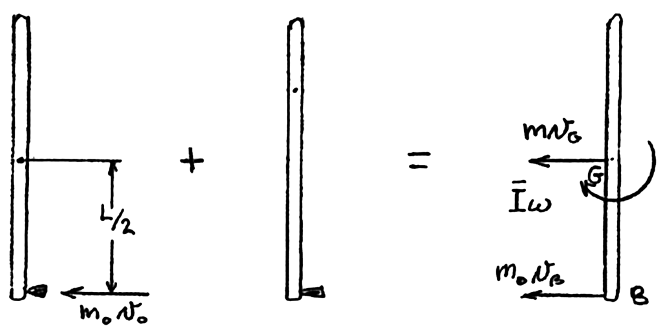
$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(0.46584)(2.5)^2 = 2.24262\text{ lb}\cdot\text{s}^2/\text{ft}$$

Bullet: $$m_0 = \frac{0.08}{32.2} = 0.0024845\text{ lb}\cdot\text{s}^2/\text{ft}$$

Kinematics. $$v_B = (L-h)\omega = (2.5-h)\omega$$

$$v_G = \left(\frac{L}{2} - h\right)\omega = (1.25-h)\omega$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

moment about B: $$0 + 0 = \bar{I}\omega - mv_G\left(\frac{L}{2}\right)$$

$$0 + 0 = 0.24262\omega - (0.46584)(1.25-h)\omega(1.25)$$

Divide by ω $$0 = 0.24262 - 0.5823(1.25-h)$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.98 (Continued)

(*a*) $h = 0.8333$ ft $\qquad h = 10.00$ in. ◀

$$v_B = (2.5 - 0.8333)\omega = 1.6667\omega$$
$$v_G = (1.25 - 0.8333)\omega = 0.4167\omega$$

$\xleftarrow{+}$ Horizontal components: $m_0 v_0 + 0 = m v_G + m_0 v_B$

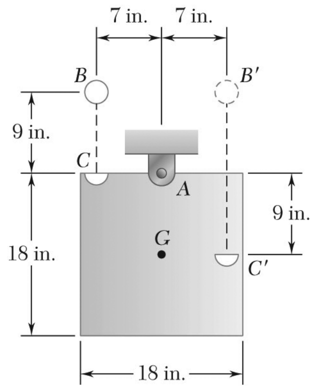
$$(0.0024845)(1800) + 0 = (0.46584)(0.4167\omega) + (0.0024845)(1.6667\omega)$$

(*b*) $\omega = 22.56$ $\qquad \boldsymbol{\omega} = 22.6$ rad/s ↻ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.99



A 16-lb wooden panel is suspended from a pin support at *A* and is initially at rest. A 4-lb metal sphere is released from rest at *B* and falls into a hemispherical cup *C* attached to the panel at a point located on its top edge. Assuming that the impact is perfectly plastic, determine the velocity of the mass center *G* of the panel immediately after the impact.

SOLUTION

Mass and moment of inertia

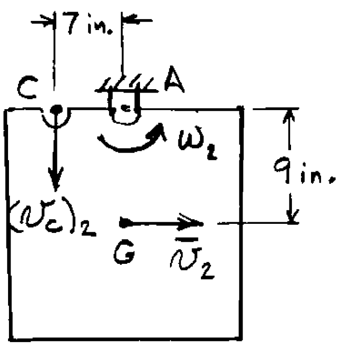
$$W_s = 4 \text{ lb} \quad W_P = 16 \text{ lb}$$

$$\bar{I} = \frac{1}{6} m_P (L)^2 = \frac{1}{6}\left(\frac{16}{32.2}\right)\left(\frac{18}{12}\right)^2 = 0.18634 \text{ slug} \cdot \text{ft}^2$$

Velocity of sphere at *C*. $\quad (v_C)_1 = \sqrt{2gy} = \sqrt{2(32.2 \text{ ft/s}^2)\left(\frac{9}{12} \text{ft}\right)} = 6.9498 \text{ ft/s}$

Impact analysis.

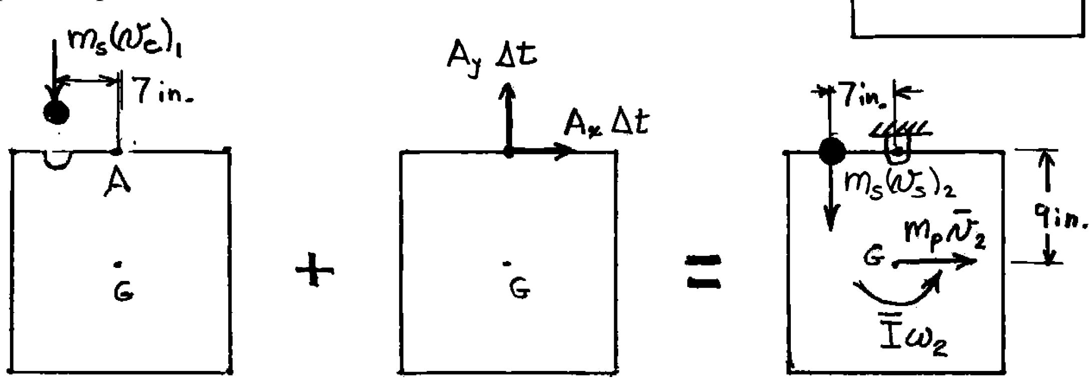
Kinematics: Immediately after impact in terms of ω_2



$$\bar{v}_2 = \frac{9}{12}\omega_2$$

$$(v_C)_2 = \frac{7}{12}\omega_2$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

$+\circlearrowleft$ Moments about A:

$$m_s(v_C)_1\left(\frac{7}{12}\text{ft}\right)+0=m_s(v_C)_2\left(\frac{7}{12}\text{ft}\right)+\bar{I}\omega_2+m_P\bar{v}_2\left(\frac{9}{12}\text{ft}\right)$$

$$\left(\frac{4\text{ lb}}{32.2}\right)(6.9498\text{ ft/s})\left(\frac{7}{12}\text{ft}\right)=\left(\frac{4\text{ lb}}{32.2}\right)\left(\frac{7}{12}\omega_2\right)\left(\frac{7}{12}\text{ft}\right)+0.18634\omega_2+\left(\frac{16\text{ lb}}{32.2}\right)\left(\frac{9}{12}\omega_2\right)\left(\frac{9}{12}\text{ft}\right)$$

$$0.50361=(0.042271+0.18634+0.2795)\omega_2$$

$$\omega_2=0.99115\text{ rad/s}\qquad \boldsymbol{\omega}_2=0.99115\text{ rad/s}\circlearrowleft$$

Velocity of the mass center

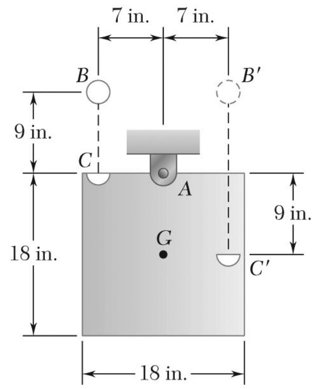
$$\bar{v}_2=\left(\frac{9}{12}\text{ft}\right)\omega_2=\left(\frac{9}{12}\text{ft}\right)(0.99115\text{ rad/s})$$

$$\bar{v}_2=0.74336\text{ ft/s}$$

$$\bar{\mathbf{v}}_2=8.92\text{ in./s}\longrightarrow\blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.100

An 16-lb wooden panel is suspended from a pin support at A and is initially at rest. A 4-lb metal sphere is released from rest at B' and falls into a hemispherical cup C' attached to the panel at the same level as the mass center G. Assuming that the impact is perfectly plastic, determine the velocity of the mass center G of the panel immediately after the impact.

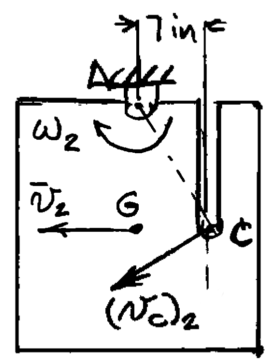
SOLUTION

Mass and moment of inertia.

$$W_s = 4 \text{ lb}$$
$$W_P = 16 \text{ lb}$$
$$\bar{I} = \frac{1}{6} m_P (0.5 \text{ m})^2$$
$$= \frac{1}{6}\left(\frac{16}{32.2}\right)\left(\frac{18}{12}\right)^2$$
$$= 0.18634 \text{ slug} \cdot \text{ft}^2$$

Velocity of sphere at C'.

$$(v_{C'})_1 = \sqrt{2gy}$$
$$= \sqrt{2(32.2 \text{ ft/s}^2)\left(\tfrac{18}{12} \text{ ft}\right)}$$
$$= 9.8285 \text{ ft/s}$$



Impact analysis.

Kinematics: Immediately after impact in terms of ω_2.

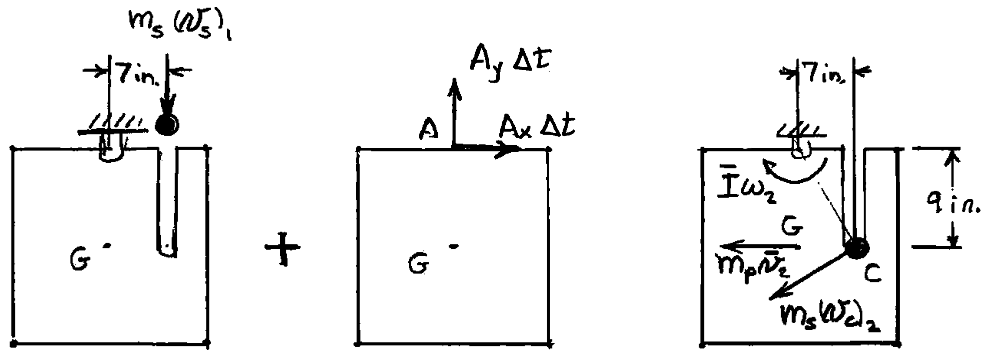
$$AC' = \sqrt{\left(\frac{7}{12}\right)^2 + \left(\frac{9}{12}\right)^2} = 0.95015 \text{ ft}$$

$$(\mathbf{v}_{C'})_2 = AC'\omega_2$$
$$= 0.95015\omega_2 \;\swarrow\; \theta \quad \text{(perpendicular to } AC.\text{)}$$
$$\bar{v}_2 = \frac{9}{12}\omega_2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1-2}$ = **Syst. Momenta**$_2$

+↻ Moments about A:

$$m_s(v_C)_1\left(\frac{7}{12}\text{ ft}\right)+0=m_s(v_C)_2(0.95015\text{ ft})+\bar{I}\omega_2+m_P\bar{v}_2\left(\frac{9}{12}\text{ft}\right)$$

$$\left(\frac{4\text{ lb}}{32.2}\right)(9.8285\text{ ft/s})\left(\frac{7}{12}\text{ft}\right)=\left(\frac{4\text{ lb}}{32.2}\right)(0.95015\omega_2)(0.95015\text{ ft})+0.18634\omega_2$$

$$+\left(\frac{16\text{ lb}}{32.2}\right)\left(\frac{9}{12}\omega_2\right)\left(\frac{9}{12}\text{ft}\right)$$

$$0.71221=(0.11215+0.18634+0.2795)\omega_2$$

$$\omega_2=1.2322$$

Velocity of the mass center.

$$\bar{v}_2=\left(\frac{9\text{ ft}}{12}\right)\omega_2$$

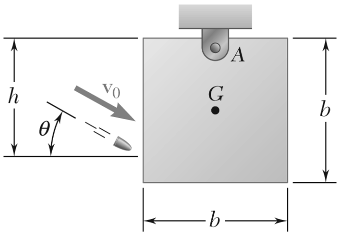
$$=\left(\frac{9\text{ ft}}{12}\right)(1.2322\text{ rad/s})$$

$$=0.92418\text{ ft/s}$$

$\bar{\mathbf{v}}_2=11.09\text{ in./s}\longleftarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.101

A 45-g bullet is fired with a velocity of 400 m/s at $\theta = 30°$ into a 9-kg square panel of side $b = 200$ mm. Knowing that $h = 150$ mm and that the panel is initially at rest, determine (*a*) the velocity of the center of the panel immediately after the bullet becomes embedded, (*b*) the impulsive reaction at *A*, assuming that the bullet becomes embedded in 2 ms.

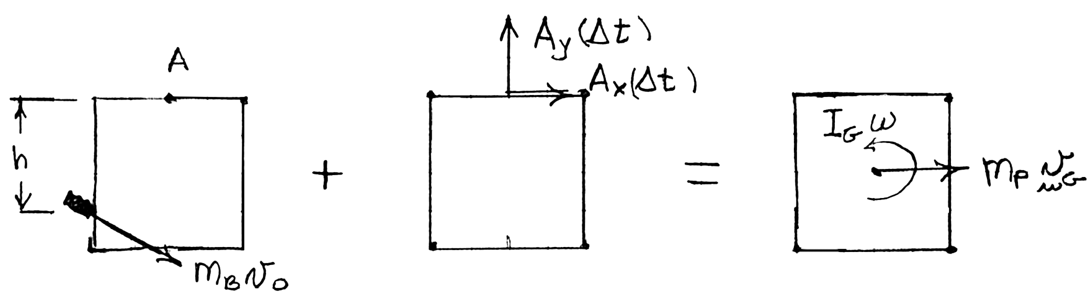
SOLUTION

$$m_B = 0.045 \text{ kg} \quad m_P = 9 \text{ kg} \quad I_G = \frac{1}{6}m_P b^2 = \frac{1}{6}(9)(0.200)^2 = 0.06 \text{ kg}\cdot\text{m}^2$$

Kinematics. After impact, the plate is rotating about the fixed Point *A* with angular velocity $\boldsymbol{\omega} = \omega \circlearrowright$.

$$\mathbf{v}_G = \frac{b}{2}\omega \longrightarrow$$

Principle of impulse and momentum. To simplify the analysis, neglect the mass of the bullet after impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

(*a*) $+\circlearrowright$ Moments about *A*:

$$(m_B v_0 \cos 30°)h + m_B v_0 \sin 30°\left(\frac{b}{2}\right) + 0 = I_G\omega + m_P v_G \frac{b}{2}$$

$$m_B v_0\left(h\cos 30° + \frac{b}{2}\sin 30°\right) = \left(I_G + \frac{1}{4}m_P b^2\right)\omega$$

$$(0.045)(400)(0.150\cos 30° + 0.100\sin 30°)$$

$$= \left[0.06 + \frac{1}{4}(9)(0.2)^2\right]\omega = 0.15\omega$$

$$\omega = 21.588 \text{ rad/s}$$

$$v_B = (0.100)(21.588) = 2.1588 \text{ m/s} \qquad \mathbf{v}_G = 2.16 \text{ m/s} \longrightarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.101 (Continued)

(*b*) $\xrightarrow{+}$ Linear momentum:

$$m_B v_0 \cos 30° + A_x(\Delta t) = m_P v_G$$

$$(0.045)(400\cos 30°) + A_x(0.002) = (9)(2.1588)$$

$$A_x = 1920 \text{ N} \qquad \mathbf{A}_x = 1920 \text{ N} \longrightarrow$$

$+\uparrow$ Linear momentum:

$$-m_B v_0 \sin 30° + A_y(\Delta t) = 0$$

$$-(0.045)(400)\sin 30° + A_y(0.002) = 0$$

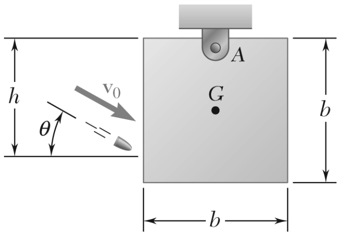
$$A_y = 4500 \text{ N} \qquad \mathbf{A}_y = 4500 \text{ N} \uparrow$$

$$A = 4892 \text{ N} = 4.892 \text{ kN} \qquad \tan\beta = \frac{4500}{1920} \qquad \beta = 66.9°$$

$$\mathbf{A} = 4.87 \text{ kN} \measuredangle 66.9° \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.102

A 45-g bullet is fired with a velocity of 400 m/s at $\theta = 5°$ into a 9-kg square panel of side $b = 200$ mm. Knowing that the panel is initially at rest, determine (*a*) the required distance *h* if the horizontal component of the impulsive reaction at *A* is to be zero, (*b*) the corresponding velocity of the center of the panel immediately after the bullet becomes embedded.

SOLUTION

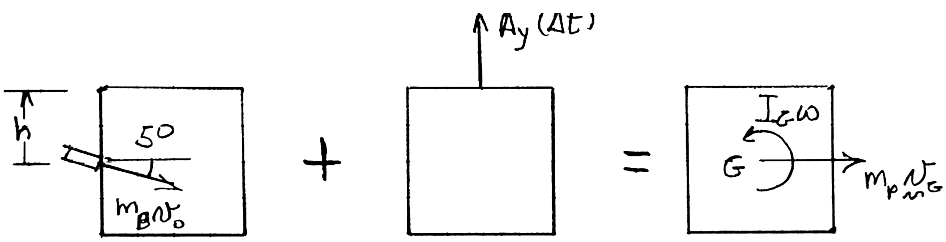
$$m_B = 0.045 \text{ kg} \quad m_P = 9 \text{ kg} \quad I_G = \frac{1}{6}m_P b^2 = \frac{1}{6}(9)(0.200)^2 = 0.06 \text{ kg}\cdot\text{m}^2$$

Kinematics. After impact, the plate is rotating about the fixed Point *A* with angular velocity $\boldsymbol{\omega} = \omega \circlearrowright$.

$$\mathbf{v}_G = \frac{b}{2}\omega \longrightarrow.$$

Principle of impulse and momentum. To simplify the analysis, neglect the mass of the bullet after impact.

Also $\qquad A_X(\Delta t) = 0.$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Linear momentum: $\qquad m_B v_0 \cos 5° + 0 = m_P v_G = m_P\left(\frac{b}{2}\omega\right)$

$$(0.045)(400\cos 5°) = (9)(0.100)\omega \quad \omega = 19.9239 \text{ rad/s}$$

$$v_G = (0.100)(19.9239) = 1.99239 \text{ m/s} \qquad (1)$$

$+\circlearrowright$ Moments about *A*: $\qquad (m_B v_0 \cos 5°)h + (m_B v_0 \sin 5°)\frac{b}{2} = I_G\omega + m_P v_G \frac{b}{2}$

$$m_B v_0\left(h\cos 5° + \frac{b}{2}\sin 5°\right) = \left(I_G + \frac{1}{4}m_P b^2\right)\omega$$

$$(0.045)(400)(h\cos 5° + 0.100\sin 5°) = \left[0.06 + \frac{1}{4}(9)(0.200)^2\right](19.9239)$$

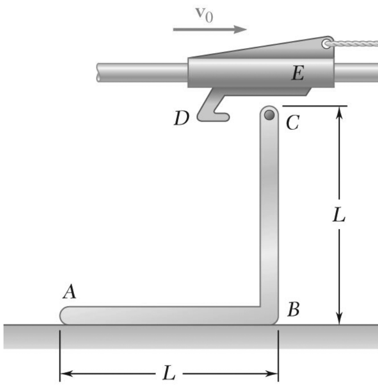
$$17.9315h + 0.1569 = 2.9886$$

(*a*) $\qquad h = 0.15792$ m $\qquad h = 158 \text{ mm}$ ◀

(*b*) From Eq. (1), $\qquad \mathbf{v}_G = 1.992 \text{ m/s} \longrightarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.103

The uniform rods, each of mass m, form the L-shaped rigid body ABC which is initially at rest on the frictionless horizontal surface when hook D of the carriage E engages a small pin at C. Knowing that the carriage is pulled to the right with a constant velocity $\mathbf{v}_0$, determine immediately after the impact (*a*) the angular velocity of the body, (*b*) the velocity of corner B. Assume that the velocity of the carriage is unchanged and that the impact is perfectly plastic.

SOLUTION

Kinematics: $\mathbf{v}_B = v_B \longrightarrow, \quad \boldsymbol{\omega} = \omega \circlearrowright$

$$\bar{\mathbf{v}}_{BC} = [v_0 \longrightarrow] + \left[\frac{L}{2}\omega \longleftarrow\right]$$

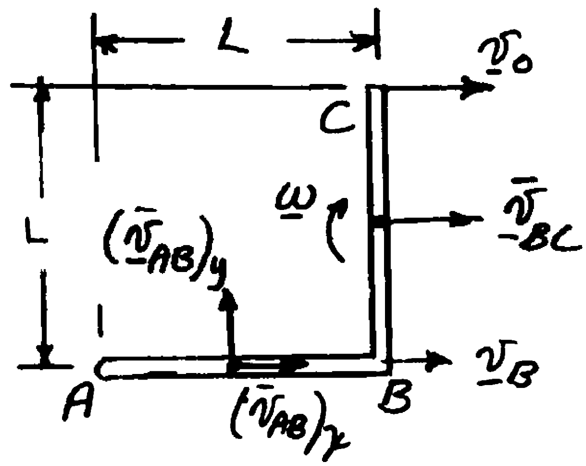
$$\bar{\mathbf{v}}_{BC} = \left[v_0 - \frac{L}{2}\omega\right] \longrightarrow$$

$$\mathbf{v}_B = [v_0 \longrightarrow] + [L\omega \longleftarrow]$$

$$\mathbf{v}_B = [v_0 - L\omega] \longrightarrow$$

$$(\bar{v}_{AB})_x = v_B = [v_0 - L\omega] \longrightarrow$$

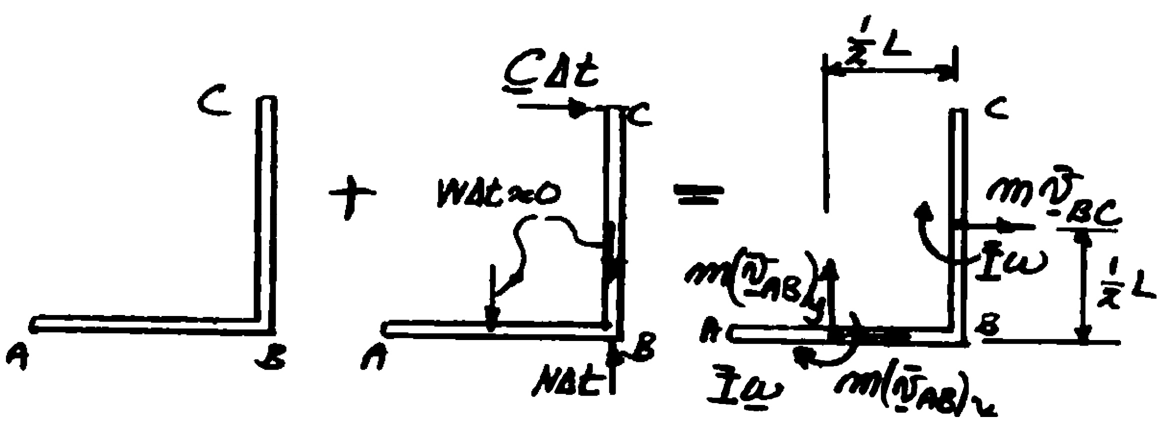
$$(\bar{v}_{AB})_y = (v_B)_y + \frac{L}{2}\omega \uparrow = \frac{L}{2}\omega \uparrow$$



Let m be the mass of each rod.

Moment of inertia of each rod. $\bar{I} = \frac{1}{12} mL^2$

(a) *Principle of impulse and momentum.*



Syst. Momenta$_0$ + **Syst. Ext. Imp.$_{0-1}$** = **Syst. Momenta$_1$**

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.103 (Continued)

$+\circlearrowright$ Moments about C:

$$0+0=m(v_{AB})_y\left(\frac{1}{2}L\right)-m(v_{AB})_x L+\bar{I}\omega-m(\bar{v}_{BC})\left(\frac{1}{2}L\right)+\bar{I}\omega$$

$$0=m\left(\frac{L}{2}\omega\right)\left(\frac{1}{2}L\right)-m(v_0-L\omega)L+\frac{1}{12}mL^2\omega-m\left(v_0-\frac{L}{2}\omega\right)\left(\frac{1}{2}L\right)+\frac{1}{12}mL^2\omega$$

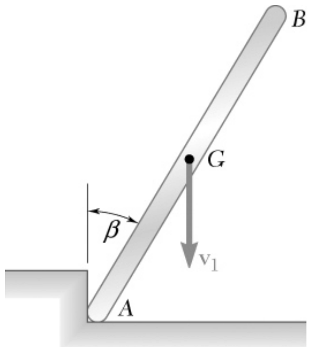
$$0=-\frac{3}{2}mLv_0+\frac{5}{3}mL^2\omega \qquad \omega=\frac{9}{10}\frac{v_0}{L}$$

(*a*) *Angular velocity* $\boldsymbol{\omega}=0.900\ v_0/L \circlearrowright$ ◀

(*b*) *Velocity of B.* $v_B=v_0-L\omega=\frac{1}{10}v_0$ $\mathbf{v}_B=0.100\ v_0 \longrightarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



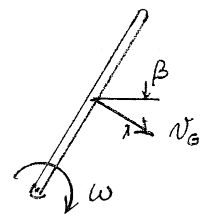
PROBLEM 17.104

The uniform slender rod AB of weight 5 lb and length 30 in. forms an angle $\beta = 30°$ with the vertical as it strikes the smooth corner shown with a vertical velocity $\mathbf{v}_1$ of magnitude 8 ft/s and no angular velocity. Assuming that the impact is perfectly plastic, determine the angular velocity of the rod immediately after the impact.

SOLUTION

Moment of inertia.

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}\left(\frac{5}{32.2}\right)\left(\frac{30}{12}\right)^2 = 80.875\times 10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

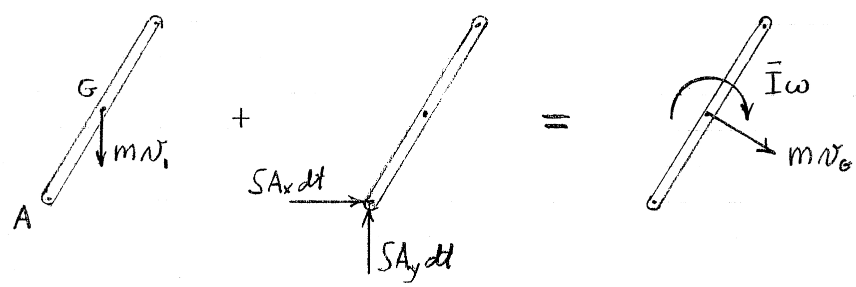


Kinematics. (Rotation about A)

$$\beta = 30°$$

$$v_G = \frac{L}{2}\omega = \frac{15}{12}\omega$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

moments about A:

$$mv_1\frac{L}{2}\sin\beta + 0 = \bar{I}\omega + mv_G\frac{L}{2}$$

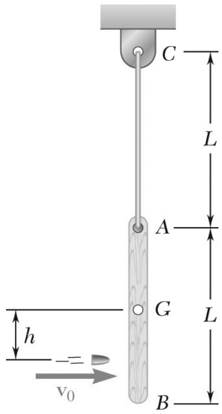
$$\left(\frac{5}{32.2}\right)(8)\left(\frac{15}{12}\right)\sin 30° + 0 = 80.875\times 10^{-3}\omega + \left(\frac{5}{32.2}\right)\left(\frac{15}{12}\omega\right)\left(\frac{15}{12}\right)$$

$$\omega = 2.4\ \text{rad/s}$$

$\boldsymbol{\omega} = 2.40\ \text{rad/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.105

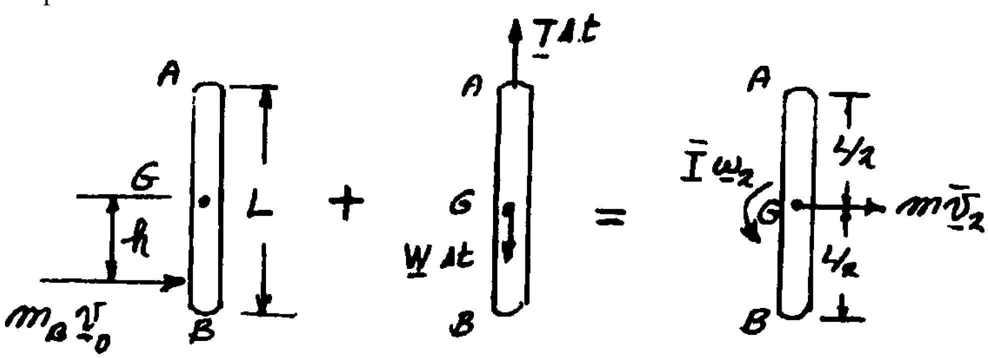
A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the 15-lb wooden rod AB of length $L = 30$ in. The rod, which is initially at rest, is suspended by a cord of length $L = 30$ in. Determine the distance h for which, immediately after the bullet becomes embedded, the instantaneous center of rotation of the rod is Point C.

SOLUTION

Let m_B be the mass of the bullet and m the mass of the rod. The moment of inertia I of the rod is

$$\bar{I} = \frac{1}{12} mL^2$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

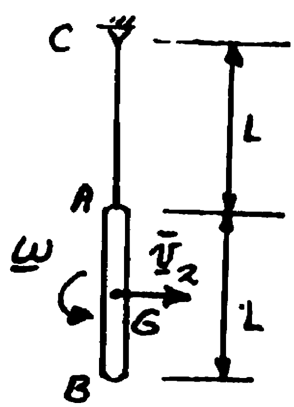
$+\circlearrowright$ Moments about G: $\quad m_B v_0 h = \bar{I}\omega_2 \quad$ (1)

$\xrightarrow{+}$ x components: $\quad m_B v_0 = m\bar{v}_2 \quad$ (2)

From Eq. (2). $\quad \bar{v}_2 = \frac{m_B}{m} v_0 = \frac{W_B}{W} v_0 \quad$ (3)

From Eq. (3). $\quad w_2 = \frac{m_B v_0 h}{\bar{I}} = \frac{\frac{W_B}{g} v_0 h}{\frac{1}{12}\frac{W}{g}L^2} = 12\frac{W_B}{W}\frac{v_0 h}{L^2} \quad$ (4)

For the instantaneous center to lie at Point C, $\quad \bar{v}_2 = \frac{3}{2} L\omega_2$



PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Substitute for $\bar{v}_2$ and ω_2 from Equations (3) and (4).

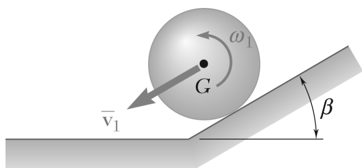
$$\frac{W_B}{W} v_0 = \frac{3}{2} L \left[12 \frac{W_B}{W} \cdot \frac{v_0 h}{L^2} \right]$$

$$h = \frac{1}{18} L = \frac{30 \text{ in.}}{18}$$

$h = 1.667$ in. ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.106

A uniform sphere of radius r rolls down the incline shown without slipping. It hits a horizontal surface and, after slipping for a while, it starts rolling again. Assuming that the sphere does not bounce as it hits the horizontal surface, determine its angular velocity and the velocity of its mass center after it has resumed rolling.

SOLUTION

Moment of inertia. Solid sphere. $$\bar{I} = \frac{2}{5}mr^2$$

Kinematics.

Before After

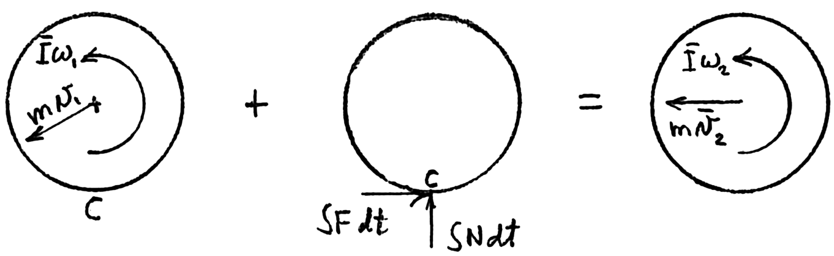
Before impact (rolling).

$$v_1 = r\omega_1$$

After slipping has stopped.

$$\bar{v}_2 = r\omega_2$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about C:
$$\bar{I}\omega_1 + mv_1 r\cos\beta + 0 = \bar{I}\omega_2 + m\bar{v}_2 r$$
$$\bar{I}\omega_1 + mr^2\omega_1\cos\beta = \bar{I}\omega_2 + mr^2\omega_2$$

$$\omega_2 = \frac{\bar{I} + mr^2\cos\beta}{\bar{I} + mr^2}\omega_1 = \frac{\frac{2}{5}mr^2 + mr^2\cos\beta}{\frac{2}{5}mr^2 + mr^2}\omega_1$$

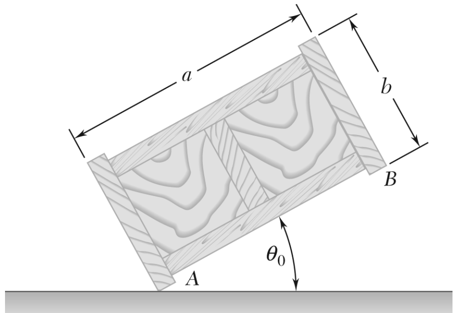
$$\boldsymbol{\omega}_2 = \frac{1}{7}(2 + 5\cos\beta)\bar{v}_1/r \ \circlearrowleft \blacktriangleleft$$

$$\bar{v}_2 = r\omega_2 = \frac{2 + 5\cos\beta}{7}r\omega_1$$

$$\bar{\mathbf{v}}_2 = \frac{1}{7}(2 + 5\cos\beta)\bar{v}_1 \longleftarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

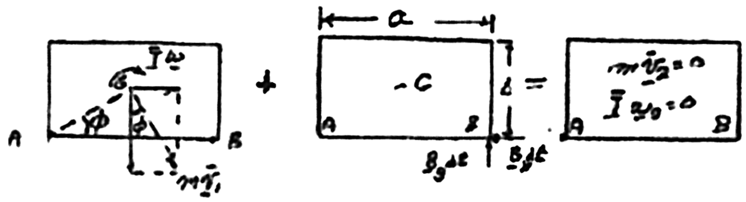


PROBLEM 17.107

A uniformly loaded rectangular crate is released from rest in the position shown. Assuming that the floor is sufficiently rough to prevent slipping and that the impact at B is perfectly plastic, determine the smallest value of the ratio a/b for which corner A will remain in contact with the floor.

SOLUTION

We consider the limiting case when the crate is just ready to rotate about B. At that instant the velocities must be zero and the reaction at corner A must be zero. Use the principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about B:

$$\bar{I}\omega_1 + (m\bar{v}_1)_2\frac{b}{2} - (m\bar{v}_1)_y\frac{a}{2} + 0 = 0 \tag{1}$$

Note: $$\sin\phi = \frac{b}{\sqrt{a^2+b^2}}, \quad \cos\phi = \frac{a}{\sqrt{a^2+b^2}}$$

$$\bar{v}_1 = (AG)v_1 = \frac{1}{2}\sqrt{a^2+b^2}\,\omega_1$$

Thus: $$(m\bar{v}_1)_x = (m\bar{v}_1)\sin\phi = \frac{m}{2}\sqrt{a^2+b^2}\,\omega_1\frac{b}{\sqrt{a^2+b^2}} = \frac{1}{2}mb\omega_1$$

Also, $$(m\bar{v}_1)_y = (m\bar{v}_1)\cos\phi = \frac{1}{2}ma\omega$$

$$\bar{I} = \frac{1}{12}m(a^2+b^2)$$

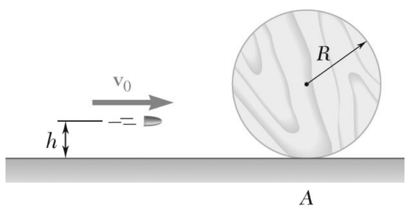
From Eq. (1) $$\frac{1}{12}m(a^2+b^2)\omega_1 + \frac{1}{2}(mb\omega_1)\frac{b}{2} - \frac{1}{2}(ma\omega_1)\frac{a}{2} = 0$$

$$\frac{1}{3}mb^2\omega_1 - \frac{1}{6}ma^2v_1 = 0 \qquad \frac{a^2}{b^2} = 2 \quad \frac{a}{b} = \sqrt{2} \quad \frac{a}{b} = 1.414 \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.108

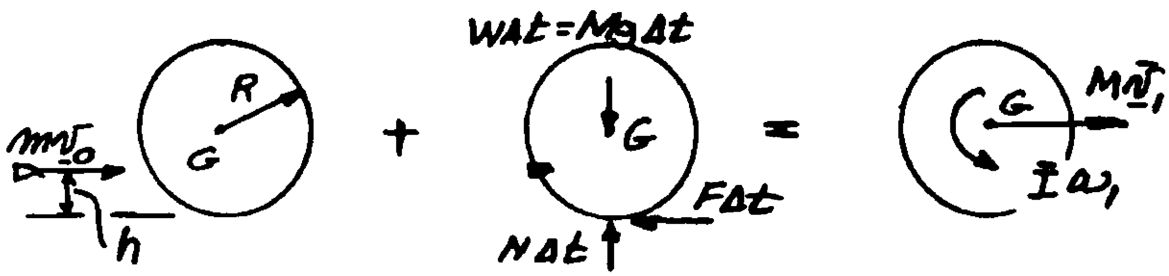


A bullet of mass m is fired with a horizontal velocity v_0 and at a height $h = \frac{1}{2}R$ into a wooden disk of much larger mass M and radius R. The disk rests on a horizontal plane and the coefficient of friction between the disk and the plane is finite. (*a*) Determine the linear velocity $\overline{\mathbf{v}}_1$ and the angular velocity $\boldsymbol{\omega}_1$ of the disk immediately after the bullet has penetrated the disk. (*b*) Describe the ensuing motion of the disk and determine its linear velocity after the motion has become uniform.

SOLUTION

(*a*) Conditions immediately after the bullet has penetrated the disk.

Principle of impulse and momentum:



Syst. Momenta$_0$ + **Syst. Ext. Imp.**$_{0\to 1}$ = **Syst. Momenta**$_1$

$+\uparrow$ y components: $\quad 0 + N\Delta t - W\Delta t = 0 \quad N = W$

$\xrightarrow{+}$ x components:

$$mv_0 - F\Delta t = M\,\bar{v}_1$$

$$mv_0 - \mu W\Delta t = M\,\bar{v}_1$$

Since $\Delta t \approx 0$, $\quad mv_0 = m\bar{v}_1 \qquad \overline{\mathbf{v}}_1 = \dfrac{mv_0}{M} \longrightarrow \ (1)$ ◀

$+\circlearrowright$ Moments about G: $\quad mv_0 = (R-h) - R(\mu W\Delta t) = \bar{I}\,\omega_1$

Since $\Delta t \approx 0$, $\quad mv_0 = (R-h) = \dfrac{1}{2}MR^2\omega_1$

$$\omega_1 = 2\frac{m}{M}\frac{R-h}{R^2}v_0 \circlearrowright \ (2)$$

But $\quad h = \dfrac{1}{2}R$

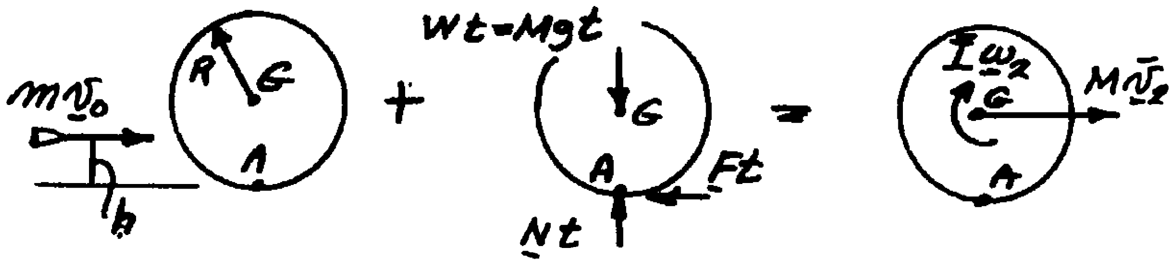
$$\omega_1 = 2\frac{m}{M}\frac{R-\frac{1}{2}R}{R^2} \qquad \omega_1 = \frac{mv_0}{MR} \circlearrowright$$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

(*b*) After the motion becomes uniform, the disk rolls without slipping.

Kinematics. $\bar{v}_2 = R\omega_2$



Syst. Momenta$_0$ + **Syst. Ext. Imp.**$_{0\to 2}$ = **Syst. Momenta**$_2$

+) Moments about *A*: $mv_0 h + 0 = M\,\bar{v}_2 R + \bar{I}\,\omega_2$

Since $h = \frac{1}{2}R$ is given:

$$mv_0\left(\frac{1}{2}R\right) = (MR\omega_2)R + \frac{1}{2}MR^2\omega_2$$

$$\frac{1}{2}mv_0 = \frac{3}{2}MR\omega_2$$

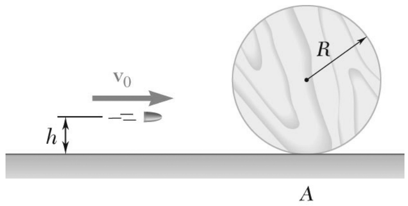
$$\omega_2 = \frac{mv_0}{3MR}$$

$$\bar{v}_2 = R\omega_2 \qquad \bar{\mathbf{v}}_2 = \frac{mv_0}{3M} \longrightarrow \blacktriangleleft$$

At first the disk slides $\longrightarrow$ and rotates), it latter rolls with a constant linear velocity $\bar{\mathbf{v}}_2$ and a constant angular velocity $\boldsymbol{\omega}_2$.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

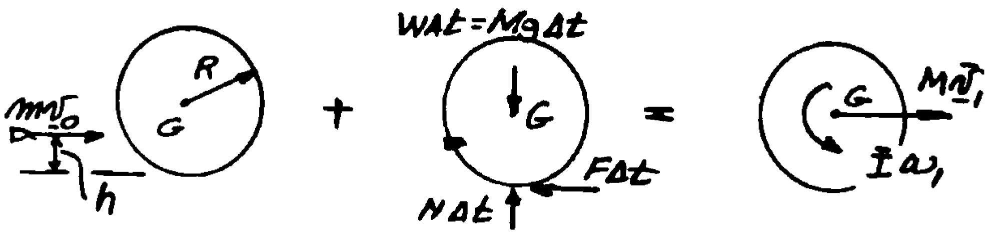


PROBLEM 17.109

Determine the height h at which the bullet of Problem 17.108 should be fired (*a*) if the disk is to roll without sliding immediately after impact, (*b*) if the disk is to slide without rolling immediately after impact.

SOLUTION

Principle of impulse and momentum:



Syst. Momenta$_0$ + **Syst. Ext. Imp.** = **Syst. Momenta$_1$**

$+\uparrow$ y components: $\quad 0 + N\Delta t - W\Delta t = 0 \qquad N = W$

$\xrightarrow{+}$ x components: $\quad mv_0 - F\Delta t = M\,\bar{v}_1$

$$mv_0 - \mu W\Delta t = M\,\bar{v}_1$$

Since $\Delta t \approx 0$; $\qquad mv_0 = m\bar{v}_1 \qquad\qquad \bar{\mathbf{v}}_1 = \dfrac{mv_0}{M} \longrightarrow \quad (1)$

$+\circlearrowright$ Moments about G: $\quad mv_0 = (R-h) - R(\mu W\Delta t) = \bar{I}\,\omega_1$

Since $\Delta t \approx 0$; $\qquad mv_0 = (R-h) = \dfrac{1}{2}MR^2\omega_1$

$$\omega_1 = 2\frac{m}{M}\frac{R-h}{R^2}v_0 \circlearrowright \qquad (2)$$

(*a*) If disk is to roll without sliding immediately after impact, we must have

$\omega_1 \circlearrowright$ and $\bar{v}_1 = R\omega_1 \longrightarrow$

$$\frac{mv_0}{M} = -R\left[\frac{2m}{M}\cdot\frac{R-h}{R^2}v_0\right]$$

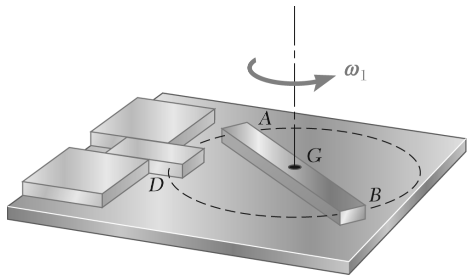
$$1 = -2\frac{R-h}{R} \qquad\qquad h = \frac{3}{2}R \blacktriangleleft$$

(*b*) If disk is to slide without rotating,

$$\omega_1 = \frac{2m}{M}\cdot\frac{R-h}{R^2}v_0 = 0 \qquad\qquad h = R \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.110

A uniform slender bar of length $L = 200$ mm and mass $m = 0.5$ kg is supported by a frictionless horizontal table. Initially the bar is spinning about its mass center G with a constant angular velocity $\boldsymbol{\omega}_1 = 6$ rad/s. Suddenly latch D is moved to the right and is struck by end A of the bar. Knowing that the coefficient of restitution between A and D is $e = 0.6$, determine the angular velocity of the bar and the velocity of its mass center immediately after the impact.

SOLUTION

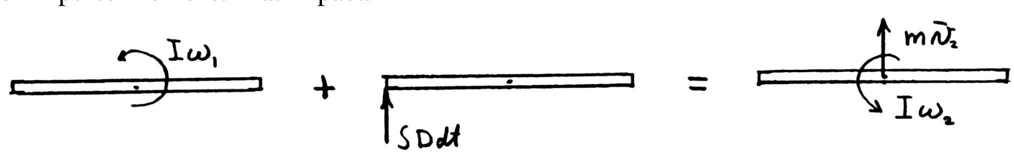
Moment of inertia. $\quad \bar{I} = \frac{1}{12}mL^2$

Before impact. $\quad (\mathbf{v}_A)_1 = \frac{L}{2}\omega_1 \downarrow$

Impact condition. $\quad (\mathbf{v}_A)_2 = -e(\mathbf{v}_A)_1 = \frac{1}{2}eL\omega_1 \uparrow$

Kinematics after impact. $\quad \bar{v}_2 = (v_A)_2 + \frac{L}{2}\omega_2 = \frac{1}{2}eL\omega_1 + \frac{1}{2}L\omega_2$

Principle of impulse-momentum at impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about D:

$$I\omega_1 + 0 = I\omega_2 + m\bar{v}_2\frac{L}{2}$$

$$I\omega_1 = I\omega_2 + m\left(\frac{1}{2}eL\omega_1 + \frac{1}{2}L\omega_2\right)\frac{L}{2}$$

$$\frac{1}{12}mL^2\omega_1 = \frac{1}{12}mL^2\omega_2 + \frac{1}{4}mL^2e\omega_1 + \frac{1}{4}mL^2\omega_2$$

$$\omega_2 = \frac{1}{4}(1-3e)\omega_1$$

$$\bar{v}_2 = \frac{1}{2}Le\omega_1 + \frac{1}{2}L\left(\frac{1}{4}\right)(1-3e)\omega_1 = \frac{1}{8}(1+e)\omega_1 L$$

Coefficient of restitution, $\quad e = 0.6$

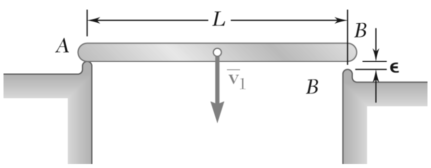
$$\omega_2 = \frac{1}{4}(1-(3)(0.6))(6 \text{ rad/s}) = -1.200 \qquad \boldsymbol{\omega}_2 = 1.200 \text{ rad/s} \circlearrowright \blacktriangleleft$$

$$v_2 = \frac{1}{8}(1+0.6)(6 \text{ rad/s})(0.2 \text{ m}) = 0.240 \text{ m/s} \qquad \mathbf{v}_2 = 0.240 \text{ m/s} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.111

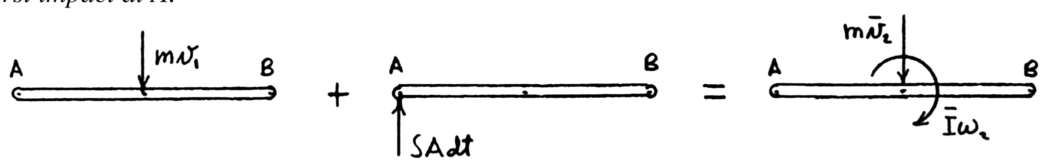


A uniform slender rod of length L is dropped onto rigid supports at A and B. Since support B is slightly lower than support A, the rod strikes A with a velocity $\bar{\mathbf{v}}_1$ before it strikes B. Assuming perfectly elastic impact at both A and B, determine the angular velocity of the rod and the velocity of its mass center immediately after the rod (*a*) strikes support A, (*b*) strikes support B, (*c*) again strikes support A.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{12}mL^2$

(*a*) *First impact at A.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Condition of impact: $e = 1: \quad (\mathbf{v}_A)_2 = v_1 \uparrow$

Kinematics: $\bar{v}_2 = \frac{L}{2}\omega - (v_A)_2 = \frac{L}{2}\omega - v_1$

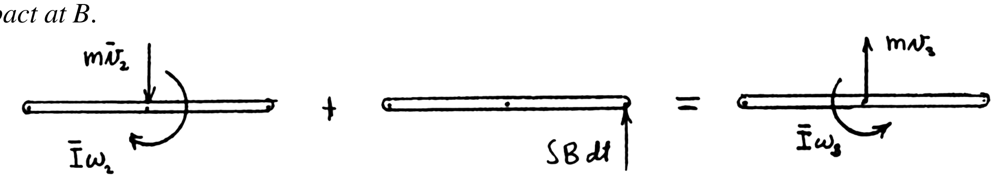
↻ Moments about A: $mv_1\frac{L}{2} + 0 = m\bar{v}_2\frac{L}{2} + \bar{I}\omega_2$

$$= m\left(\frac{L}{2}\omega - v_1\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\omega_2 \qquad \boldsymbol{\omega}_2 = \frac{3v_1}{L} \circlearrowright \blacktriangleleft$$

$$\bar{v}_2 = \frac{L}{2}\left(\frac{3v_1}{L}\right) - v_1 = \frac{1}{2}v_1 \qquad \bar{\mathbf{v}}_2 = \frac{1}{2}v_1 \downarrow \blacktriangleleft$$

$$(v_B)_2 = L\omega - (v_A)_2 = 3v_1 - v_1 = 2v_1 \downarrow$$

(*b*) *Impact at B.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_3$

Condition of impact. $e = 1: \quad (\mathbf{v}_B)_3 = 2v_1 \uparrow$

Kinematics: $\bar{v}_3 = (v_B)_2 - \frac{L}{2}\omega = 2v_1 - \frac{L}{2}\omega$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

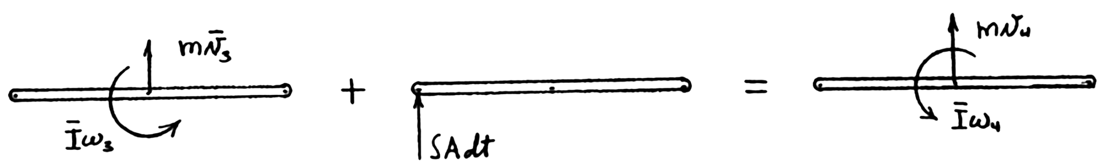
Moments about B: $$-m\bar{v}_2\frac{L}{2}+\bar{I}\omega_2+0=m\bar{v}_3\frac{L}{2}-\bar{I}\omega_3$$

$$-m\left(\frac{1}{2}v_1\right)\frac{L}{2}+\left(\frac{1}{12}mL^2\right)\left(\frac{3v_1}{L}\right)+0=m\left(2v_1-\frac{L}{2}\omega_3\right)\frac{L}{2}-\left(\frac{1}{12}mL^2\right)\omega_3 \qquad \omega_3=\frac{3v_1}{L} ◀$$

$$\bar{v}_3=2v_1-\frac{L}{2}\left(\frac{3v_1}{L}\right)=\frac{1}{2}v_1 \qquad \bar{\mathbf{v}}_3=\frac{1}{2}v_1 \uparrow ◀$$

$$(v_A)_3=L\omega-(v_B)_3=3v_1-2v_1=v_1 \downarrow$$

(*c*) *Second impact at A.*



Syst. Momenta$_3$ + **Syst. Ext. Imp.**$_{3\to4}$ = **Syst. Momenta**$_4$

Condition of impact. $e=1$: $(\mathbf{v}_A)_4=v_1 \uparrow$

Kinematics: $$\bar{v}_4=(v_A)_4+\frac{L}{2}\omega_4=v_1+\frac{L}{2}\omega_4$$

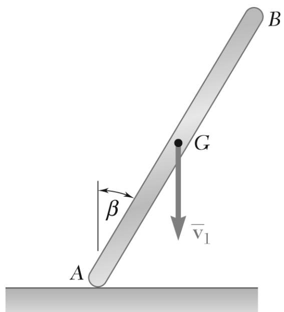
Moments about A: $$m\bar{v}_3\frac{L}{2}+\bar{I}\omega_3+0=m\bar{v}_4\frac{L}{2}+\bar{I}\omega_4$$

$$m\left(\frac{1}{2}v_1\right)\frac{L}{2}+\left(\frac{1}{12}mL^2\right)\left(\frac{3v_1}{L}\right)+0=m\left(v_1+\frac{L}{2}\omega_4\right)\frac{L}{2}+\left(\frac{1}{12}mL^2\right)\omega_4 \qquad \omega_4=0 ◀$$

$$\bar{v}_4=v_1+0 \qquad \bar{\mathbf{v}}_4=v_1 \uparrow ◀$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.112

The slender rod AB of length L forms an angle β with the vertical as it strikes the frictionless surface shown with a vertical velocity $\overline{\mathbf{v}}_1$ and no angular velocity. Assuming that the impact is perfectly plastic, derive an expression for the angular velocity of the rod immediately after the impact.

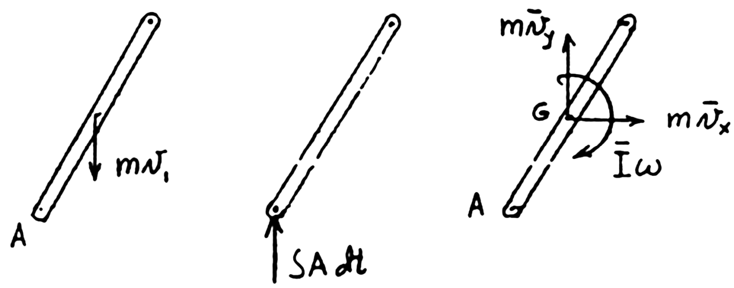
SOLUTION

Moment of inertia. $\quad \overline{I} = \frac{1}{12}mL^2$

Perfectly plastic impact. $\quad e = 0 \qquad [(v_A)_y]_2 = -e(v_A)_{y1} = 0$

$$\mathbf{v}_A = (v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = (v_A)_x\mathbf{i}$$

Kinetics.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

$\xrightarrow{+}$ horizontal components: $\quad 0 + 0 = m\overline{v}_x \qquad m\overline{v}_x = 0$

Kinematics. $\quad \mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A} \qquad [\overline{v}_y \uparrow] = [(v_A)_x \longrightarrow] + \left[\frac{L}{2}\omega \nwarrow \beta\right]$

Velocity components $\uparrow$: $\quad v_y = -\frac{L}{2}\omega\sin\beta$

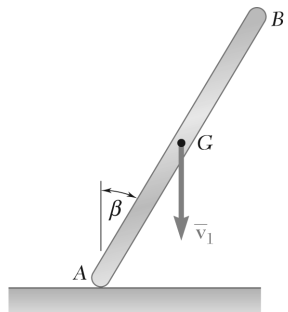
$\curvearrowright$ Moments about A: $\quad mv_1\frac{L}{2}\sin\beta + 0 = -m\overline{v}_y\frac{L}{2}\sin\beta + \overline{I}\omega$

$$mv_1\frac{L}{2}\sin\beta = m\left(\frac{L}{2}\omega\sin\beta\right)\frac{L}{2}\sin\beta + \frac{1}{12}mL^2\omega$$

$$\left(\frac{1}{12}mL^2 + \frac{1}{4}mL^2\sin^2\beta\right)\omega = \frac{1}{2}mv_1L\sin\beta \qquad \omega = \frac{6\sin\beta}{1+3\sin^2\beta}\frac{v_1}{L} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.113

The slender rod AB of length $L = 1$ m forms an angle $\beta = 30°$ with the vertical as it strikes the frictionless surface shown with a vertical velocity $\bar{\mathbf{v}}_1 = 2$ m/s and no angular velocity. Knowing that the coefficient of restitution between the rod and the ground is $e = 0.8$, determine the angular velocity of the rod immediately after the impact.

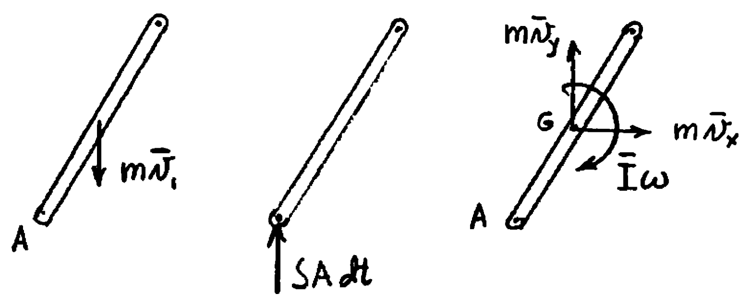
SOLUTION

Moment of inertia. $$\bar{I} = \frac{1}{12}mL^2$$

Apply coefficient of restitution. $$[(v_A)_y]_2 = -e[(v_A)_y]_1 = e\bar{v}_1 \uparrow$$

$$\mathbf{v}_A = (v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = (v_A)_x\mathbf{i} + e\bar{v}_1\mathbf{j}$$

Kinetics.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

$\xrightarrow{+}$ horizontal components: $$0 + 0 = m\bar{v}_x \qquad m\bar{v}_x = 0$$

Kinematics. $$\mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A} \qquad [\bar{v}_y \uparrow] = [e\bar{v}_1 \uparrow] + [(v_A)_x \longrightarrow] + \left[\frac{L}{2}\omega \nwarrow \beta\right]$$

Velocity components $\uparrow$: $$v_y = e\bar{v}_1 - \frac{L}{2}\omega \sin\beta$$

$\circlearrowright$ Moments about A: $$mv_1\frac{L}{2}\sin\beta + 0 = -m\bar{v}_y\frac{L}{2}\sin\beta + \bar{I}\omega$$

$$mv_1\frac{L}{2}\sin\beta = m\left(\frac{L}{2}\omega\sin\beta - e\bar{v}_1\right)\frac{L}{2}\sin\beta + \frac{1}{12}mL^2\omega$$

$$\left(\frac{1}{12}mL^2 + \frac{1}{4}mL^2\sin^2\beta\right)\omega = \frac{1+e}{2}mv_1L\sin\beta$$

$$\omega = \frac{6(1+e)\sin\beta}{1+3\sin^2\beta}\frac{\bar{v}_1}{1L}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.113 (Continued)

Data: $L = 1\text{ m}, \quad \beta = 30°, \quad \bar{v}_1 = 2\text{ m/s}, \quad e = 0.8$

$$\omega = \frac{(6)(1.8)\sin 30°}{1 + 3\sin^2 30°} \cdot \frac{2\text{ m/s}}{1\text{ m}}$$

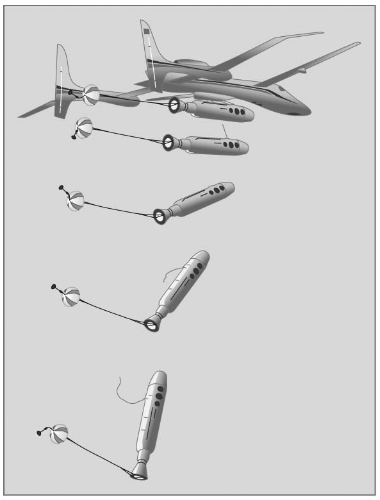
$\boldsymbol{\omega} = 6.17\text{ rad/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

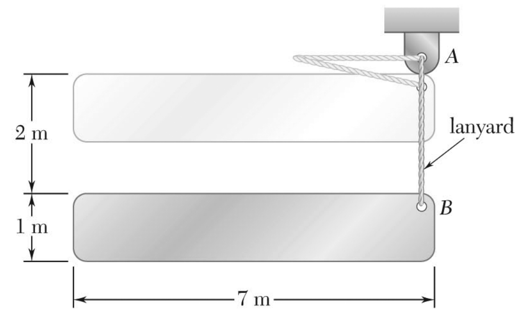
STUDENTS-HUB.com

PROBLEM 17.114

The trapeze/lanyard air drop (t/LAD) launch is a proposed innovative method for airborne launch of a payload-carrying rocket. The release sequence involves several steps as shown in (1) where the payload rocket is shown at various instances during the launch. To investigate the first step of this process where the rocket body drops freely from the carrier aircraft until the 2-m lanyard stops the vertical motion of B, a trial rocket is tested as shown in (2). The rocket can be considered a uniform 1-m by 7-m rectangle with a mass of 4000 kg. Knowing that the rocket is released from rest and falls vertically 2 m before the lanyard becomes taut, determine the angular velocity of the rocket immediately after the lanyard is taut.



(1)



(2)

SOLUTION

While the lanyard is slack, the rocket falls freely without rotation. Considering its motion relative to the airplane (a Newtonian frame of reference), its vertical velocity is

$$v_1^2 = v_0^2 + 2gy = 2gy$$

$$v_1 = \sqrt{2gy} = \sqrt{(2)(9.81\text{ m/s})(2\text{ m})} \qquad \mathbf{v}_1 = 6.2642\text{ m/s}\downarrow \qquad (1)$$

Moment of inertia:

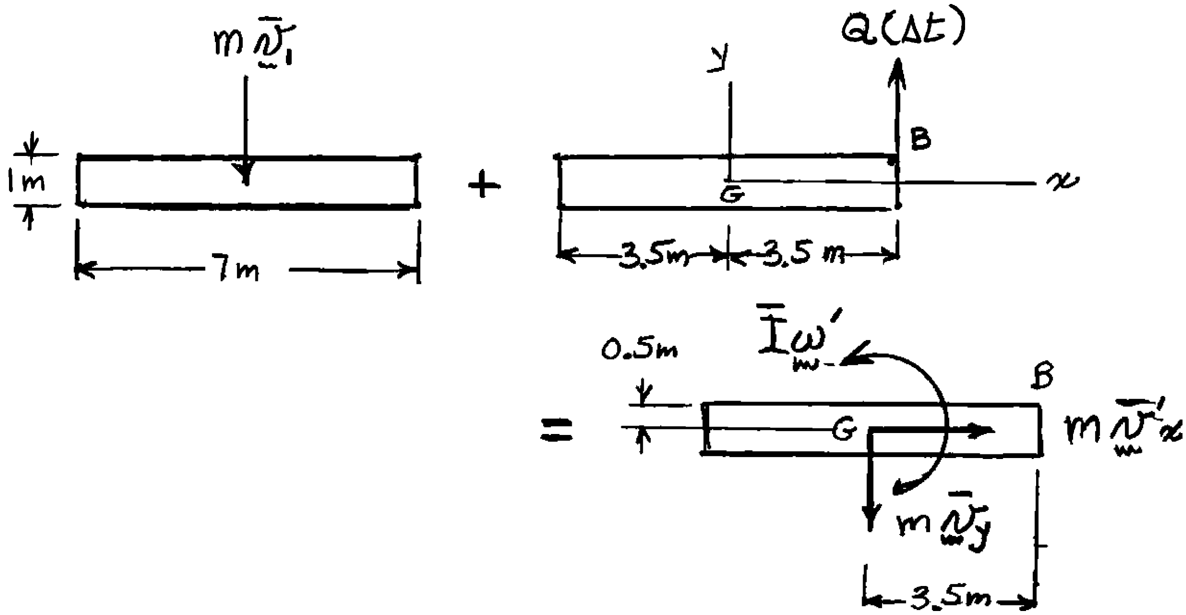
$$\bar{I} = \frac{1}{12}m(a^2+b^2)$$

$$= \frac{1}{12}(4000\text{ kg})[(7\text{ m})^2 + (1\text{ m})^2] = 16.667\text{ kg}\cdot\text{m}^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

For impact use the principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to2}$** = **Syst. Momenta$_2$**

x-components $\longrightarrow$: $\quad 0 + 0 = m\bar{v}'_x \qquad \bar{v}'_x = 0$

$+\circlearrowright$ Moments about B: $\quad mv_1(3.5) + 0 = \bar{I}\omega' + m\bar{v}'_y(3.5)$ (2)

Kinematics. $\quad \mathbf{v}_B = v_B \longrightarrow$

$$\bar{v} = [\bar{v}_x \longrightarrow] + [\bar{v}_y \downarrow] = [v_B \longrightarrow] + [3.5\omega' \downarrow] + [0.5\omega \longrightarrow]$$

y-components $\downarrow$: $\quad \bar{v}_y = 3.5\omega'$ (3)

Substitute from Eqs. (1) and (3) into Eq. (2).

$$mv_1(3.5) = [\bar{I} + m(3.5)^2]\omega'$$

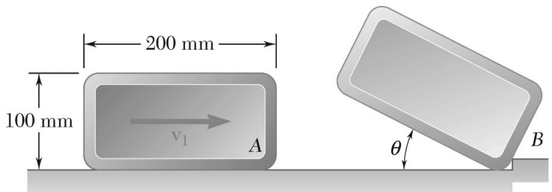
$$(4000\text{ kg})(6.2642\text{ m/s})(3.5\text{ m}) = [16667\text{ kg}\cdot\text{m}^2 + (4000\text{ kg})(3.5\text{ m})^2]\omega'$$

Angular velocity. $\qquad \boldsymbol{\omega}' = 1.336\text{ rad/s} \circlearrowright$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.115



The uniform rectangular block shown is moving along a frictionless surface with a velocity $\overline{\mathbf{v}}_1$ when it strikes a small obstruction at B. Assuming that the impact between corner A and obstruction B is perfectly plastic, determine the magnitude of the velocity $\overline{\mathbf{v}}_1$ for which the maximum angle θ through which the block will rotate is 30°.

SOLUTION

Let m be the mass of the block.

Dimensions: $a = 0.200$ m

$b = 0.100$ m

Moment of inertia about the mass center.

$$\bar{I} = \frac{1}{12}m(a^2 + b^2)$$

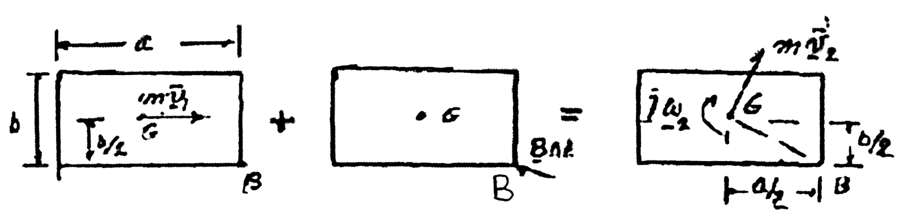
Let d be one half the diagonal. $d = \frac{1}{2}\sqrt{a^2 + b^2} = 0.1118$ m

Kinematics. Before impact $\overline{\mathbf{v}}_1 = v_1 \longrightarrow, \quad \boldsymbol{\omega}_1 = 0$

After impact, the block is rotating about corner at B.

$$\boldsymbol{\omega}_2 = \omega_2 \circlearrowright \qquad \mathbf{v}_2 = d\omega_2 \nearrow$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

$\circlearrowleft +$ Moments about B:

$$\frac{mv_1b}{2} + 0 = \bar{I}\omega_2 + mdv_2$$

$$\frac{1}{2}mv_1b = \frac{1}{12}m(a^2 + b^2)\omega_2 + md^2\omega_2$$

$$= \frac{1}{3}m(a^2 + b^2)\omega_2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Angular velocity after impact $\qquad \boldsymbol{\omega}_2 = \dfrac{3v_1 b}{2(a^2+b^2)}$ ↻ $\qquad$ (1)

The motion after impact is a rotation about corner B.

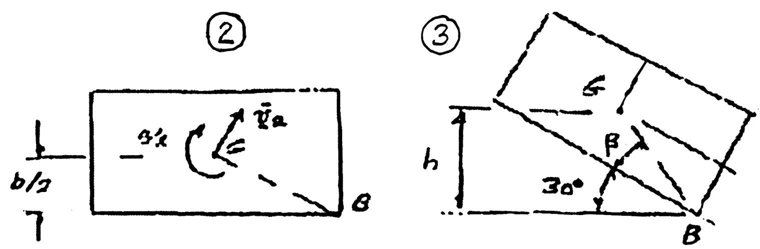
Position 2 (immediately after impact). $\qquad \bar{v}_2 = d\omega_2$

Position 3 $\quad (\theta = 30°)$.

$$\beta = \tan^{-1}\frac{b}{a} = \tan^{-1}0.5 = 26.565°$$

$$h = d\sin(\beta + 30°) = 0.11180\sin 56.565° = 0.093301\text{ m}$$

$$\omega_3 = 0 \qquad \bar{v}_3 = 0$$



Potential energy: $\qquad V_2 = \dfrac{mgb}{2} \qquad V_3 = mgh$

Kinetic energy:

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_2^2 = \frac{1}{2}(\bar{I} + md^2)\omega_2^2$$

$$= \frac{1}{6}m(a^2+b^2)\omega_2^2 \qquad T_3 = 0$$

Principle of conservation of energy:

$$T_2 + V_3 = T_3 + V_3$$

$$\frac{1}{6}m(a^2+b^2)\omega_2^2 + \frac{mgb}{2} = 0 + mgh$$

$$\omega_2^2 = \frac{3g(2h-b)}{(a^2+b^2)} = \frac{(3)(9.81)(0.18660-0.100)}{(0.200)^2+(0.100)^2}$$

$$= 50.974\ (\text{rad/s})^2 \qquad \boldsymbol{\omega}_2 = 7.1396\text{ rad/s} ↻$$

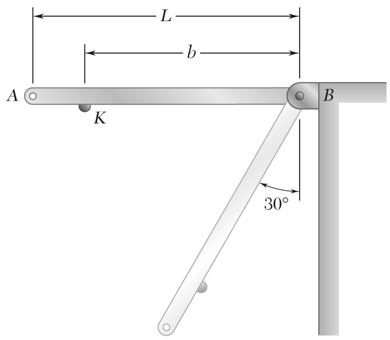
Magnitude of initial velocity.

Solving Eq. (1) for v_1 $\qquad v_1 = \dfrac{2(a^2+b^2)\omega_2}{3b}$

$$v_1 = \frac{(2)[(0.200)^2+(0.100)^2](7.1396)}{(3)(0.100)} \qquad v_1 = 2.38\text{ m/s} ◀$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



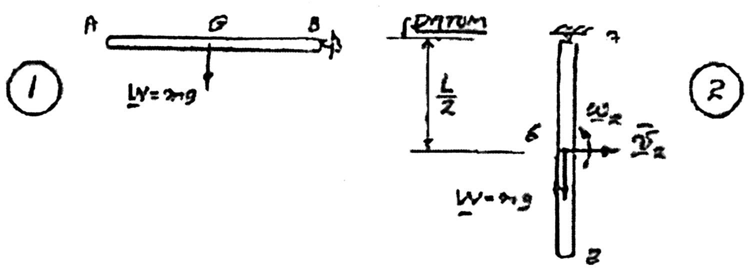
PROBLEM 17.116

A slender rod of length L and mass m is released from rest in the position shown. It is observed that after the rod strikes the vertical surface it rebounds to form an angle of 30° with the vertical. (*a*) Determine the coefficient of restitution between knob K and the surface. (*b*) Show that the same rebound can be expected for any position of knob K.

SOLUTION

For analysis of the downward swing of the rod before impact and for the upward swing after impact use the principle of conservation of energy.

Before impact.



$$V_1 = 0$$

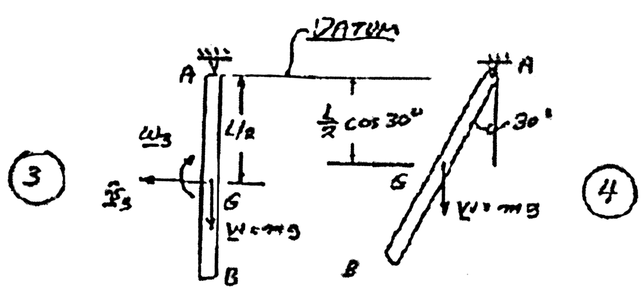
$$V_2 = -W\frac{L}{2} = -mg\frac{L}{2}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_2^2 = \frac{1}{2}\left(\frac{1}{12}mL^2\right)v_2^2 + \frac{1}{2}m\left(\frac{1}{2}\omega_2\right)^2 = \frac{1}{6}mL^2\omega_2^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 = \frac{1}{6}mL^2\omega_2^2 = -mg\frac{L}{2}; \quad \omega_2^2 = 3\frac{g}{L} \qquad \boldsymbol{\omega}_2 = 1.73205\sqrt{\frac{g}{L}}$$

After impact.



$$V_3 = -W\frac{L}{2} = -mg\frac{L}{2}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.116 (Continued)

$$V_4 = -W\frac{L}{2}\cos 30°$$

$$T_3 = \frac{1}{2}\bar{I}\omega_3^2 + \frac{1}{2}m\bar{v}_3^2$$

$$= \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_3^2 + \frac{1}{2}m\left(\frac{1}{2}\omega_3\right)^2$$

$$= \frac{1}{6}mL^2\omega_3^2$$

$$T_4 = 0$$

$$T_3 + V_3 = T_4 + V_4: \quad \frac{1}{6}mL^2\omega_3^2 - mg\frac{L}{2} = 0 - mg\frac{L}{2}\cos 30°$$

$$\omega_3^2 = 3(1-\cos 30°)\frac{g}{L} \qquad \boldsymbol{\omega}_3 = 0.63397\sqrt{\frac{g}{L}} \circlearrowright$$

Analysis of impact.

Let r be the distance BK.

Before impact, $\quad (\mathbf{v}_k)_3 = r\omega_3 \longrightarrow = 1.73205r\sqrt{\frac{g}{L}} \longrightarrow$

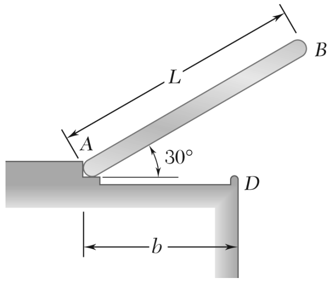
After impact, $\quad (\mathbf{v}_k)_4 = r\omega_4 \longleftarrow = 0.63397r\sqrt{\frac{g}{L}} \longleftarrow$

Coefficient of restitution. $\quad e = \frac{|(v_k)_{4n}|}{|(v_k)_{3n}|}$

$$e = \frac{0.63397}{1.73205} \qquad e = 0.366 \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

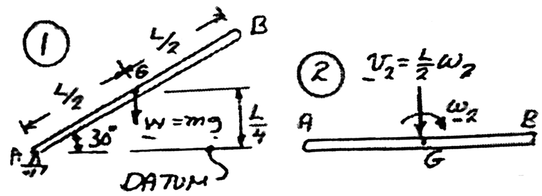


PROBLEM 17.117

A slender rod of mass m and length L is released from rest in the position shown and hits edge D. Assuming perfectly plastic impact at D, determine for $b = 0.6L$, (*a*) the angular velocity of the rod immediately after the impact, (*b*) the maximum angle through which the rod will rotate after the impact.

SOLUTION

For analysis of the falling motion before impact use the principle of conservation of energy.



Position 1: $\quad T_1 = 0, \quad V_1 = mg\dfrac{L}{4}$

Position 2: $\quad V_2 = 0$

$$T_2 = \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$

$$T_2 = \frac{1}{6}mL^2\omega_2^2$$

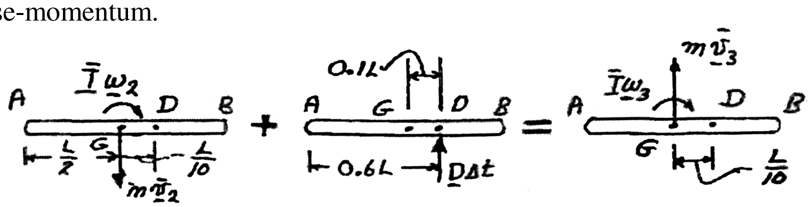
$$T_1 + V_1 = T_2 + V_2: \quad 0 + mg\frac{L}{4} = \frac{1}{6}mL^2\omega_2^2 \qquad \omega_2 = \sqrt{\frac{3g}{2L}}$$

Analysis of impact. Kinematics

Before impact, rotation is about Point A. $\qquad \bar{v}_2 = \dfrac{L}{2}\omega_2$

After impact, rotation is about Point D. $\qquad \bar{v}_3 = \dfrac{L}{10}\omega_3$

Principle of impulse-momentum.



Syst. Momenta$_2$ $\quad+\quad$ **Syst. Ext. Imp.**$_{2\to3}$ $\quad=\quad$ **Syst. Momenta**$_3$

+) Moments about D:

$$\bar{I}\omega_2 - m\bar{v}_2\left(\frac{L}{10}\right) = \bar{I}\omega_3 + m\bar{v}_3\left(\frac{L}{10}\right) \qquad (1)$$

$$\frac{1}{12}mL^2\omega_2 - m\left(\frac{L}{2}\omega_2\right)\frac{L}{10} = \frac{1}{12}mL^2\omega_3 + m\left(\frac{L}{10}\omega_3\right)\frac{L}{10}$$

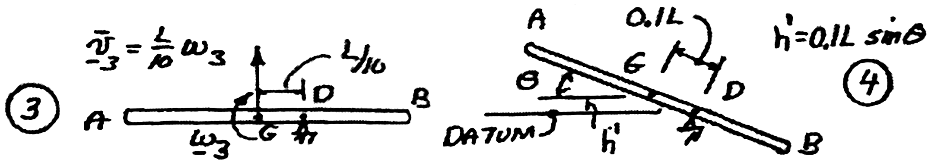
$$\left(\frac{1}{12} - \frac{1}{20}\right)\omega_2 = \left(\frac{1}{12} + \frac{1}{100}\right)\omega_3$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

(*a*) Angular velocity. $\omega_3 = \dfrac{5}{14}\omega_2 = \dfrac{5}{14}\sqrt{\dfrac{3g}{2L}}$ $\qquad \omega_3 = 0.437\sqrt{\dfrac{g}{L}}$ ◀

For analysis of the rotation about Point *D* after the impact use the principle of conservation of energy.



Position 3 (just after impact)

$$\bar{v}_3 = \frac{L}{10}\omega_3 \quad V_3 = 0$$

$$T_3 = \frac{1}{2}m\left(\frac{L}{10}\omega_3\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_3^2 = \frac{14}{300}mL^2\omega_3^2$$

$$= \frac{14}{300}mL^2\left(\frac{5}{14}\sqrt{\frac{3g}{2L}}\right)^2 = \frac{mgL}{112}$$

Position 4. $\theta =$ maximum rotation angle.

$$h' = \frac{L}{10}\sin\theta$$

$$V_4 = mgh' = \frac{mgL}{10}\sin\theta$$

$$\bar{v}_4 = 0, \quad \omega_4 = 0, \quad T_4 = 0$$

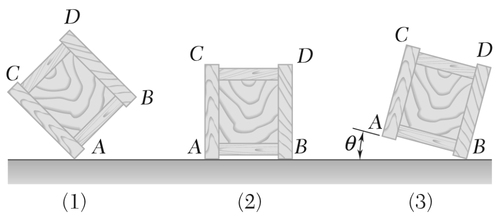
$$T_3 + V_3 = T_4 + V_4; \quad \frac{mgL}{112} + 0 = 0 + \frac{mgL}{10}\sin\theta$$

(*b*) Maximum rotation angle. $\sin\theta = \dfrac{10}{112}$ $\qquad \theta = 5.12°$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.118

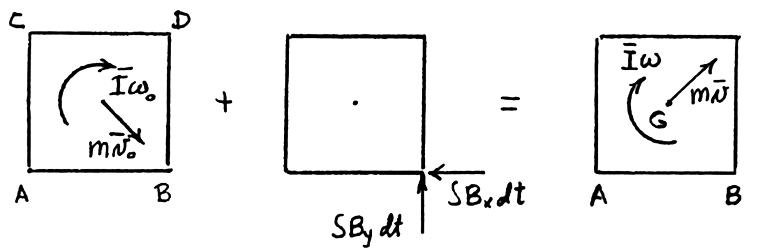


A uniformly loaded square crate is released from rest with its corner D directly above A; it rotates about A until its corner B strikes the floor, and then rotates about B. The floor is sufficiently rough to prevent slipping and the impact at B is perfectly plastic. Denoting by $\boldsymbol{\omega}_0$ the angular velocity of the crate immediately before B strikes the floor, determine (a) the angular velocity of the crate immediately after B strikes the floor, (b) the fraction of the kinetic energy of the crate lost during the impact, (c) the angle θ through which the crate will rotate after B strikes the floor.

SOLUTION

Let m be the mass of the crate and c be the length of an edge.

Moment of inertia $$\bar{I} = \frac{1}{12}m(c^2 + c^2) = \frac{1}{6}mc^2$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

Kinematics: $$\bar{v}_0 = r_{G/A}\omega_0 = \frac{1}{2}\sqrt{2}c\omega_0$$

$$\bar{v} = r_{G/B}\omega = \frac{1}{2}\sqrt{2}c\omega$$

Moments about B: $$\bar{I}\omega_0 + 0 = \bar{I}\omega + r_{G/B}m\bar{v}$$

$$\frac{1}{6}mc^2\omega_0 + 0 = \frac{1}{6}mc^2\omega + \left(\frac{1}{2}\sqrt{2}c\right)m\left(\frac{1}{2}\sqrt{2}c\omega\right) = \frac{2}{3}mc^2\omega$$

(a) Solving for ω, $$\boldsymbol{\omega} = \frac{1}{4}\omega_0 \circlearrowright \blacktriangleleft$$

Kinetic Energy.

Before impact: $$T_1 = \frac{1}{2}\bar{I}\omega_0^2 + \frac{1}{2}m\bar{v}_0^2$$

$$= \frac{1}{2}\left(\frac{1}{6}mc^2\right)\omega_0^2 + \frac{1}{2}m\left(\frac{1}{2}\sqrt{2}c\omega_0\right)^2$$

$$= \frac{1}{3}mc^2\omega_0^2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.118 (Continued)

After impact:
$$T_2 = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}\left(\frac{1}{6}mc^2\right)\omega^2 + \frac{1}{2}m\left(\frac{1}{2}\sqrt{2}c\omega\right)^2$$
$$= \frac{1}{3}mc^2\omega^2 = \frac{1}{3}mc^2\left(\frac{1}{4}\omega_0\right)^2 = \frac{1}{48}mc^2\omega_0$$

(*b*) Fraction of energy lost:
$$\frac{T_1 - T_2}{T_1} = \frac{\frac{1}{3} - \frac{1}{48}}{\frac{1}{3}} = 1 - \frac{1}{16}$$
$$\frac{15}{16} \blacktriangleleft$$

Conservation of energy during falling. $T_0 + V_0 = T_1 + V_1$ (1)

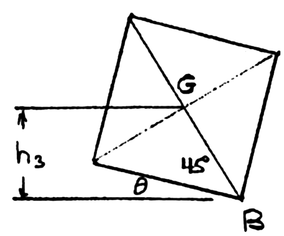
Conservation of energy during rising. $T_3 + V_3 = T_2 + V_2$ (2)

Conditions:
$$T_0 = 0, \quad T_3 = 0 \quad T_2 = \frac{1}{16}T_1$$
$$V_0 = mg\left(\frac{1}{2}\sqrt{2}c\right) \quad V_1 = V_2 = mg\left(\frac{1}{2}c\right) \quad V_3 = mgh_3$$

From Equation (1),
$$T_1 = V_0 - V_1 = \frac{1}{2}(\sqrt{2} - 1)mgc$$

From Equation (2),
$$T_2 = V_3 - V_2 = mgh_3 - \frac{1}{2}mgc$$
$$\frac{h_3 - \frac{1}{2}c}{\frac{1}{2}\sqrt{2} - 1} = \frac{1}{16} \qquad h_3 = \left[\frac{1}{2} + \frac{1}{16}(\sqrt{2} - 1)\right]c$$

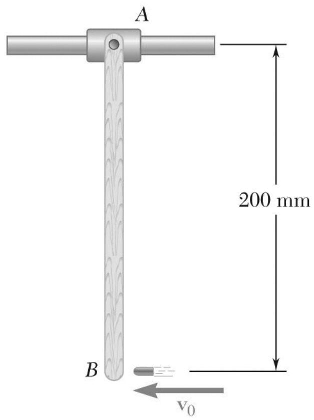
(*c*) From geometry,
$$h_3 = \frac{1}{2}\sqrt{2}c\sin(\theta + 45°)$$



Equating the two expressions for h_3,
$$\sin(45° + \theta) = \frac{\frac{1}{2} + \frac{1}{16}(\sqrt{2} - 1)}{\frac{1}{2}\sqrt{2}}$$
$$45° + \theta = 46.503° \qquad \theta = 1.50° \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.119

A 1-oz bullet is fired with a horizontal velocity of 750 mi/h into the 18-lb wooden beam *AB*. The beam is suspended from a collar of negligible mass that can slide along a horizontal rod. Neglecting friction between the collar and the rod, determine the maximum angle of rotation of the beam during its subsequent motion.

SOLUTION

Mass of bullet. $W' = 1 \text{ ounce} = 0.0625 \text{ lb}$

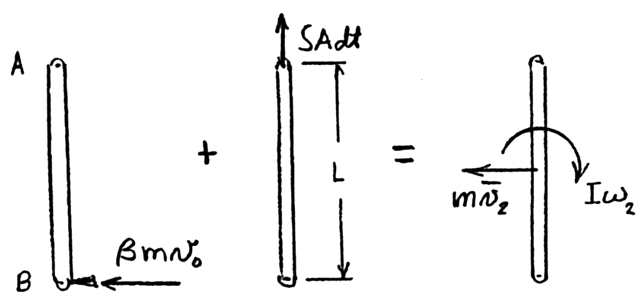
Mass of beam *AB*. $W = 18 \text{ lb}$

Mass ratio. $\beta = \dfrac{W'}{W} = 0.0034722 \qquad W' = \beta W \text{ and } m' = \beta m$

Since β is so small, the mass of the bullet will be neglected in comparison with that of the beam in determining the motion after the impact.

Moment of inertia. $\bar{I} = \dfrac{1}{12} mL^2$

Impact kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xleftarrow{+}$ linear components: $-\beta m v_0 + 0 = m\bar{v}_2 \qquad \bar{v}_2 = \beta v_0$

$\circlearrowright$ Moments about *B*: $0 + 0 = \bar{I}\omega - m\bar{v}_2 \dfrac{L}{2}$

$$\omega = \frac{m\bar{v}_2 L}{2\bar{I}} = \frac{12 m\beta v_0 L}{2mL^2}$$

$$\omega = \frac{6\beta v_0}{L}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Motion during rising. *Position 2*. Just after the impact.

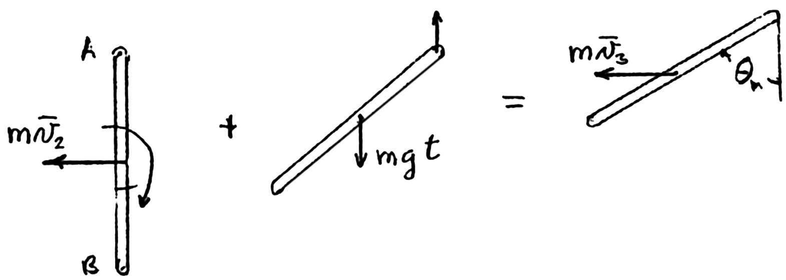
$$V_2 = -mg\frac{L}{2} \quad \text{(datum at level } A\text{)}$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m(\beta v_0)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\left(\frac{6\beta v_0}{L}\right)^2$$

$$= 2\beta^2 m v_0^2$$

Position 3. $\omega = 0, \quad \theta = \theta_m.$



Syst. Momenta$_2$ + **Syst. Ext. Imp.**$_{2\to3}$ = **Syst. Momenta**$_3$

$$V_3 = -mg\frac{L}{2}\cos\theta_m$$

$$T_3 = \frac{1}{2}m\bar{v}_3^2$$

$\xleftarrow{+}$ linear components: $m\bar{v}_2 + 0 = m\bar{v}_3 \qquad \bar{v}_3 = \bar{v}_2 = \beta v_0$ where $v_0 = 750 \text{ mi/h} = 1100 \text{ ft/s}$

Conservation of energy. $T_2 + V_2 = T_3 + V_3: \quad 2\beta^2 m v_0^2 - mg\frac{L}{2} = \frac{1}{2}m(\beta v_0)^2 - mg\frac{L}{2}\cos\theta_m$

$$\frac{3\beta^2 v_0^2}{gL} = 1 - \cos\theta_m$$

$$\cos\theta_m = 1 - \frac{3\beta^2 v_0^2}{gL}$$

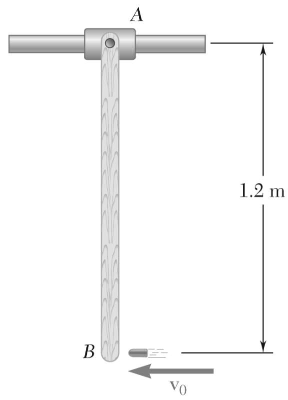
$$= 1 - \frac{(3)(0.0034722)^2(1100)^2}{(32.2)(4)}$$

$$= 0.66021$$

$\theta_m = 48.7° \blacktriangleleft$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.120

For the beam of Problem 17.119, determine the velocity of the 1-oz bullet for which the maximum angle of rotation of the beam will be 90°.

PROBLEM 17.119 A 1-oz bullet is fired with a horizontal velocity of 350 m/s into the 18-lb wooden beam *AB*. The beam is suspended from a collar of negligible weight that can slide along a horizontal rod. Neglecting friction between the collar and the rod, determine the maximum angle of rotation of the beam during its subsequent motion.

SOLUTION

Mass of bullet. $W' = 1 \text{ ounce} = 0.0625 \text{ lb}$

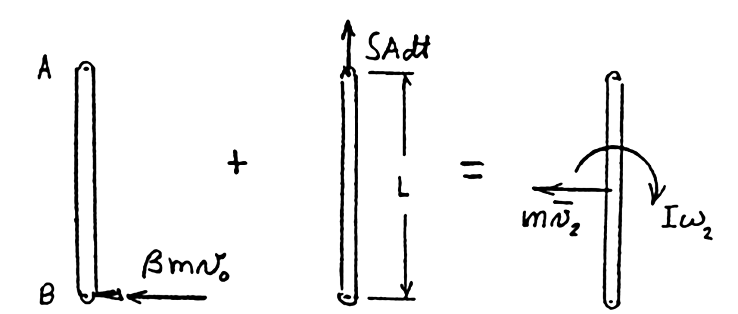
Mass of beam *AB*. $W = 18 \text{ lb}$

Mass ratio. $\beta = \dfrac{W'}{W} = 0.0034722 \qquad W' = \beta W \text{ and } m' = \beta m$

Since β is so small, the mass of the bullet will be neglected in comparison with that of the beam in determining the motion after the impact.

Moment of inertia. $\bar{I} = \dfrac{1}{12} mL^2$

Impact Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\overset{+}{\longleftarrow}$ linear components: $-\beta m v_0 + 0 = m\bar{v}_2 \qquad \bar{v}_2 = \beta v_0$

$\circlearrowright$ Moments about *B*: $0 + 0 = \bar{I}\omega - m\bar{v}_2 \dfrac{L}{2}$

$$\omega = \frac{m\bar{v}_2 L}{2\bar{I}} = \frac{12 m\beta v_0 L}{2mL^2}$$

$$\omega = \frac{6\beta v_0}{L}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Motion during rising. *Position 2*. Just after the impact.

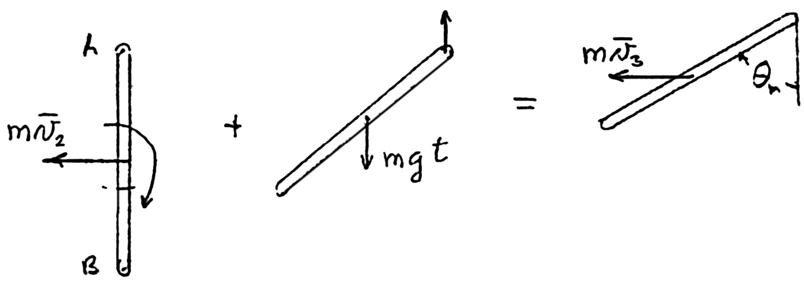
$$V_2 = -mg\frac{L}{2} \quad \text{(datum at level } A\text{)}$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m(\beta v_0)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\left(\frac{6\beta v_0}{L}\right)^2$$

$$= 2\beta^2 m v_0^2$$

Position 3. $\qquad \omega = 0, \quad \theta = \theta_m.$



Syst. Momenta$_2$ + **Syst. Ext. Imp.**$_{2\to3}$ = **Syst. Momenta**$_3$

$$V_3 = -mg\frac{L}{2}\cos\theta_m$$

$$T_3 = \frac{1}{2}m\bar{v}_3^2$$

$\xleftarrow{+}$ linear components: $\qquad m\bar{v}_2 + 0 = m\bar{v}_3 \qquad \bar{v}_3 = \bar{v}_2 = \beta v_0$

Conservation of energy.

$$T_2 + V_2 = T_3 + V_3: \quad 2\beta^2 m v_0^2 - mg\frac{L}{2} = \frac{1}{2}m(\beta v_0)^2 - mg\frac{L}{2}\cos\theta_m$$

$$\beta v_0 = \sqrt{\frac{1}{3}gL(1-\cos\theta_m)}$$

$$= \sqrt{\left(\frac{1}{3}\right)(32.2)(4)(1-\cos 90°)}$$

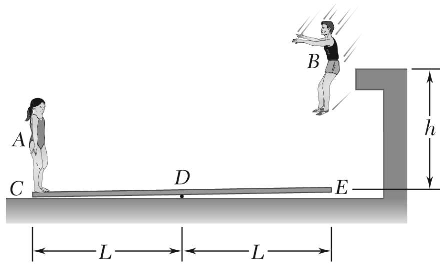
$$= 6.5524 \text{ ft/s}$$

$$v_0 = \frac{6.5524}{0.0034722}$$

$v_0 = 1887$ ft/s ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.121

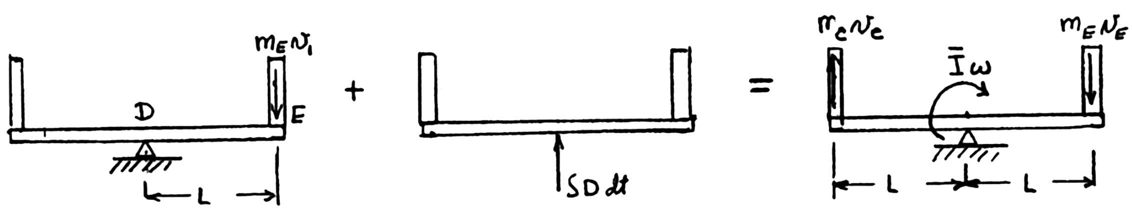
The plank *CDE* has a mass of 15 kg and rests on a small pivot at *D*. The 55-kg gymnast *A* is standing on the plank at *C* when the 70-kg gymnast *B* jumps from a height of 2.5 m and strikes the plank at *E*. Assuming perfectly plastic impact and that gymnast *A* is standing absolutely straight, determine the height to which gymnast *A* will rise.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{12} m_P (2L)^2 = \frac{1}{3} m_P L^2$

Velocity of jumper at *E*. $(v)_1 = \sqrt{2gh_1}$ (1)

Principle of impulse-momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

Kinematics: $v_C = L\omega \quad v_D = L\omega$

Moments about *D*: $m_E v_1 L + 0 = m_E v_E L + m_C v_C L + \bar{I}\omega$

$$= m_E L^2 \omega + m_C L^2 \omega + \frac{1}{3} m_P L^2 \omega$$

$$\omega = \frac{m_E}{m_E + m_C + \frac{1}{3} m_P} \frac{v_1}{L}$$

$$v_C = L\omega = \frac{m_E v_1}{m_E + m_C + \frac{1}{3} m_P} \qquad (2)$$

Gymnast (flier) rising. $h_C = \frac{v_C^2}{2g}$ (3)

Data:

$$m_E = m_B = 70 \text{ kg}$$
$$m_C = m_A = 55 \text{ kg}$$
$$m_P = 15 \text{ kg}$$
$$h_1 = 2.5 \text{ m}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.121 (Continued)

From Equation (1) $$v_1 = \sqrt{(2)(9.81)(2.5)}$$
$$= 7.0036 \text{ m/s}$$

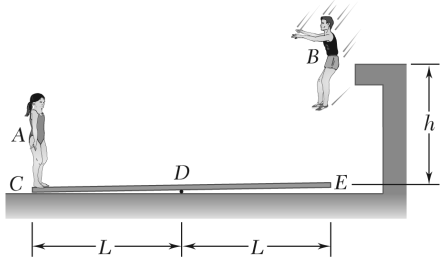
From Equation (2) $$v_C = \frac{(70)(7.0036)}{70+55+5}$$
$$= 3.7712 \text{ m/s}$$

From Equation (3) $$h_2 = \frac{(3.7712)^2}{(2)(9.81)}$$
$$= 0.725 \text{ m}$$

$h_2 = 725 \text{ mm}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.122

Solve Problem 17.121, assuming that the gymnasts change places so that gymnast A jumps onto the plank while gymnast B stands at C.

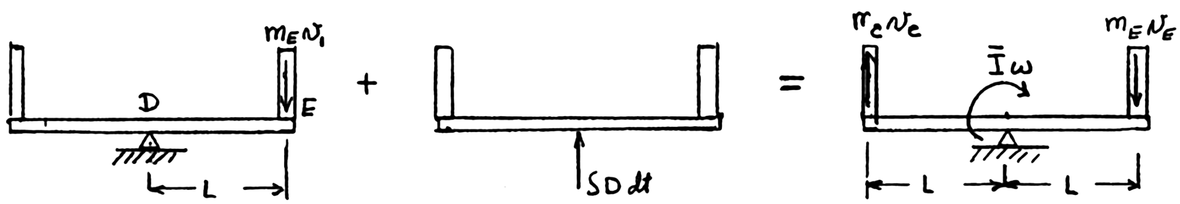
PROBLEM 17.121 The plank CDE has a mass of 15 kg and rests on a small pivot at D. The 55-kg gymnast A is standing on the plank at C when the 70-kg gymnast B jumps from a height of 2.5 m and strikes the plank at E. Assuming perfectly plastic impact and that gymnast A is standing absolutely straight, determine the height to which gymnast A will rise.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{12} m_P (2L)^2 = \frac{1}{3} m_P L^2$

Velocity of jumper at E. $(v)_1 = \sqrt{2gh_1}$ (1)

Principle of impulse-momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics: $v_C = L\omega \quad v_D = L\omega$

Moments about D: $m_E v_1 L + 0 = m_E v_E L + m_C v_C L + \bar{I}\omega$

$$= m_E L^2\omega + m_C L^2\omega + \frac{1}{3} m_P L^2 \omega$$

$$\omega = \frac{m_E}{m_E + m_C + \frac{1}{3}m_P}\frac{v_1}{L}$$

$$v_C = L\omega = \frac{m_E v_1}{m_E + m_C + \frac{1}{3}m_P} \quad (2)$$

Gymnast (flier) rising. $h_C = \frac{v_C^2}{2g}$ (3)

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Data:

$$m_E = m_A = 55 \text{ kg}$$
$$m_C = m_B = 70 \text{ kg}$$
$$m_P = 15 \text{ kg}$$
$$h_1 = 2.5 \text{ m}$$

From Equation (1)

$$v_1 = \sqrt{(2)(9.81)(2.5)}$$
$$= 7.0036 \text{ m/s}$$

From Equation (2)

$$v_C = \frac{(55)(7.0036)}{55 + 70 + 5}$$
$$= 2.9631 \text{ m/s}$$

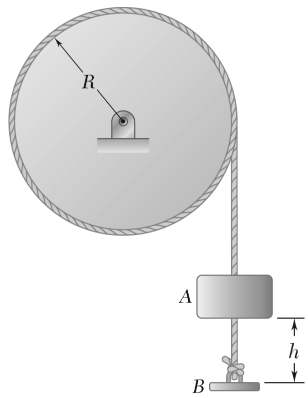
From Equation (3)

$$h_2 = \frac{(2.9631)^2}{(2)(9.81)}$$
$$= 0.447 \text{ m}$$

$h_2 = 447 \text{ mm}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.123

A small plate B is attached to a cord that is wrapped around a uniform 8-lb disk of radius $R = 9$ in. A 3-lb collar A is released from rest and falls through a distance $h = 15$ in. before hitting plate B. Assuming that the impact is perfectly plastic and neglecting the weight of the plate, determine immediately after the impact (*a*) the velocity of the collar, (*b*) the angular velocity of the disk.

SOLUTION

The collar A falls a distance h. From the principle of conservation of energy.

$$v_1 = \sqrt{2gh}$$

Impact analysis: $e = 0$

Kinematics. Collar A and plate B move together. The cord is inextensible.

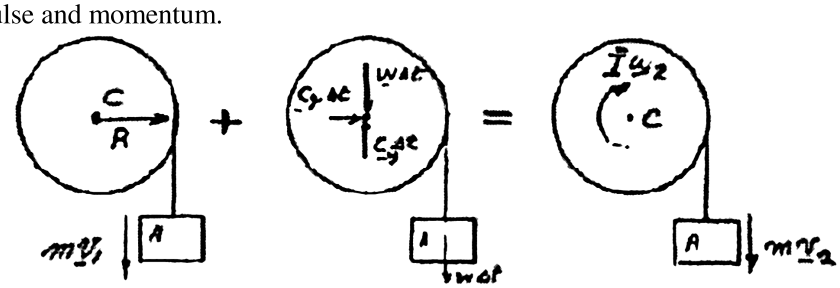
$$\bar{v}_2 = R\omega \quad \text{or} \quad \omega_2 = \frac{\bar{v}_2}{R}$$

Let m = mass of collar A and M = mass of disk.

Moment of inertia of disk: $\bar{I} = \frac{1}{2}MR^2$

Principle of impulse and momentum.

$\bar{I}\omega_1 = 0$



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

+) Moments about C: $$m v_1 R = \bar{I}\omega_2 + m v_2 R \qquad (1)$$

$$m v_1 R = \frac{1}{2}MR^2\left(\frac{v_2}{R}\right) + m v_2 R$$

$$m v_1 = \frac{1}{2}Mv_2 + m v_2$$

$$v_2 = \frac{2m}{2m + M}v_1$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.123 (Continued)

Data:

$$m = 3 \text{ lb/g}$$
$$M = 8 \text{ lb/g}$$
$$h = 15 \text{ in.} = 1.25 \text{ ft}$$
$$R = 9 \text{ in.} = 0.75 \text{ ft}$$

$$v_1 = \sqrt{(2)(32.2)(1.25)}$$
$$= 8.972 \text{ ft/s}$$

(*a*) Velocity of *A*.

$$v_2 = \frac{(2)(3)}{(2)(3)+8} v_1 = \frac{3}{7} v_1$$

$$v_2 = \frac{3}{7}(8.972) = 3.8452 \text{ ft/s}$$

$\mathbf{v}_2 = 3.85 \text{ ft/s} \downarrow$ ◀

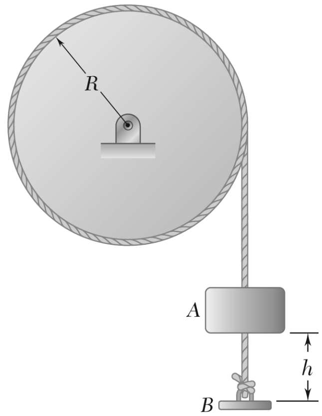
(*b*) Angular velocity.

$$\omega_2 = \frac{3.8452}{0.75}$$

$\boldsymbol{\omega}_2 = 5.13 \text{ rad/s}$ ↻ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.124

Solve Problem 17.123, assuming that the coefficient of restitution between A and B is 0.8.

PROBLEM 17.123 A small plate B is attached to a cord that is wrapped around a uniform 8-lb disk of radius $R = 9$ in. A 3-lb collar A is released from rest and falls through a distance $h = 15$ in. before hitting plate B. Assuming that the impact is perfectly plastic and neglecting the weight of the plate, determine immediately after the impact (*a*) the velocity of the collar, (*b*) the angular velocity of the disk.

SOLUTION

$$W_D = 8 \text{ lb}$$

$$m_D = \frac{W_D}{g} = \frac{8}{32.2} = 0.2484 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$R = 9 \text{ in.} = 0.75 \text{ ft}$$

$$I_D = \frac{1}{2} m_D R^2 = 0.06988 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$W_A = 3 \text{ lb}$$

$$m_A = \frac{W_A}{g} = \frac{3}{32.2} = 0.09317 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$h = 15 \text{ in.} = 1.25 \text{ ft}$$

Collar A falls through distance h. Use conservation of energy.

$$T_1 = 0$$

$$V_1 = W_A h$$

$$T_2 = \frac{1}{2} m_A v_A^2$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + W_A h = \frac{1}{2} m_A v_A^2 + 0$$

$$v_A^2 = \frac{2 m_A h}{W_A} = 2gh$$

$$= (2)(32.2)(1.25)$$

$$= 80.5 \text{ ft}^2/\text{s}^2$$

$$\mathbf{v}_A = 8.972 \text{ ft/s} \downarrow$$

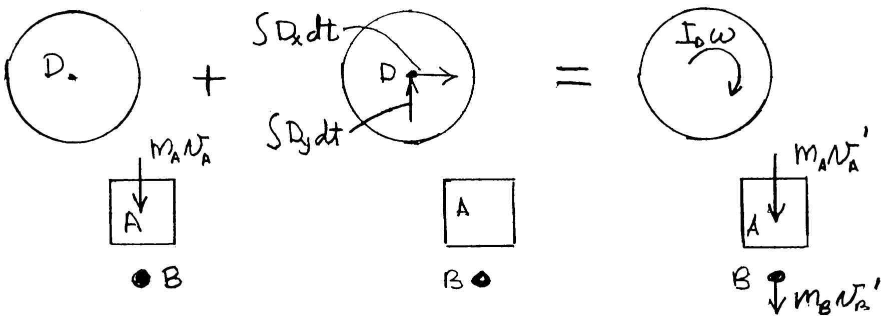
Impact. Neglect the mass of plate B. Neglect the effect of weight over the duration of the impact.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Kinematics. $\boldsymbol{\omega}' = \omega$ $\quad \mathbf{v}'_B = R\omega\downarrow = 0.75\omega'\downarrow$

Conservation of momentum.



$+\circlearrowright$ Moments about D: $\quad m_A v_A R + 0 = m_A v'_A R + I_D \omega' + m_B v'_B R$

$$(0.09317)(8.972)(0.75) = (0.09317)(0.75)v'_A + 0.06988\,\omega' \qquad (1)$$

Coefficient of restitution. $\quad v'_B - v'_A = e(v_A - v_B)$

$$0.75\omega' - v'_A = 0.8(8.972 - 0) \qquad (2)$$

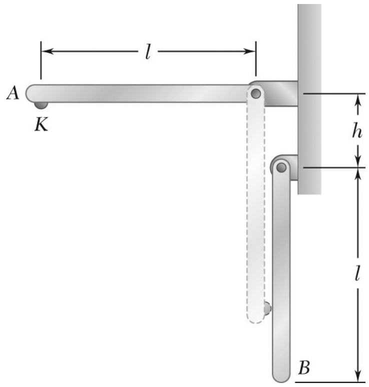
Solving Eqs. (1) and (2) simultaneously

(*a*) <u>Velocity of *A*</u>. $\quad v'_A = -0.25648$ ft/s $\quad \mathbf{v}'_A = 0.256$ ft/s $\uparrow$ ◀

(*b*) <u>Angular velocity</u>. $\quad \omega' = 9.228$ rad/s $\quad \boldsymbol{\omega}' = 9.23$ rad/s $\circlearrowright$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.125

Two identical slender rods may swing freely from the pivots shown. Rod A is released from rest in a horizontal position and swings to a vertical position, at which time the small knob K strikes rod B which was at rest. If $h = \frac{1}{2}l$ and $e = \frac{1}{2}$, determine (*a*) the angle through which rod B will swing, (*b*) the angle through which rod A will rebound.

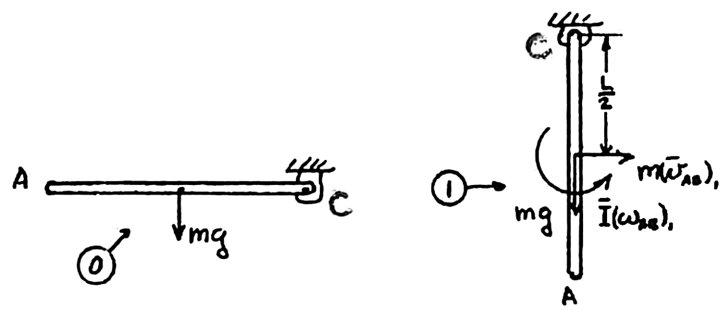
SOLUTION

Let $$m = m_{AC} = m_{BD}$$

Moment of inertia. $$\bar{I}_{AC} = \bar{I} = \frac{1}{12}mL^2$$

$$\bar{I}_{BD} = \frac{1}{12}mL^2$$

Rod AB falls to vertical position.



Position 0. $$V_1 = 0 \qquad T_1 = 0$$

Position 1. $$V_2 = -mg\frac{L}{2}$$

$$(\bar{v}_{AC})_1 = \frac{L}{2}(\omega_{AC})_1$$

$$T_1 = \frac{1}{2}m(\bar{v}_{AC})_1^2 + \frac{1}{2}I(\omega_{AC})_1^2$$

$$= \frac{1}{6}mL^2(\omega_{AC})_1^2$$

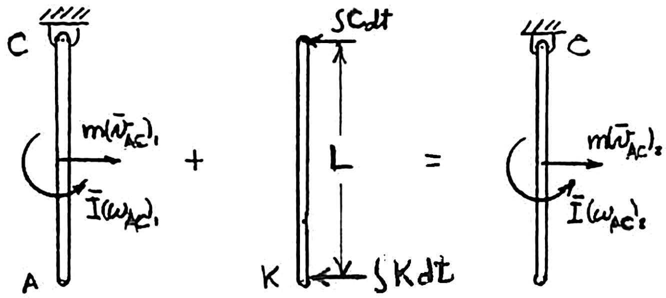
Conservation of energy. $$T_0 + V_0 = T_1 + V_1: \quad 0 + 0 = \frac{1}{6}mL^2(\omega_{AC})_1^2 - \frac{1}{2}mgL$$

$$(\omega_{AC})_1^2 = \frac{3g}{L} \qquad (1)$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Impact.

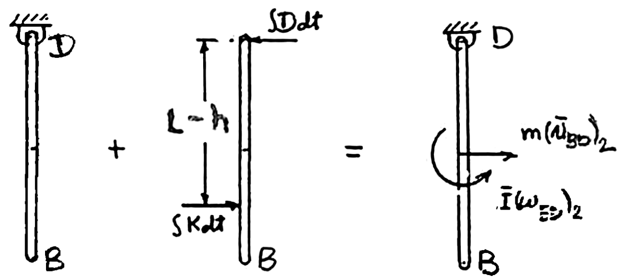


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics
$$(\bar{v}_{AC})_1 = \frac{L}{2}(\omega_{AC})_1$$
$$(\bar{v}_{AC})_2 = \frac{L}{2}(\omega_{AC})_2$$

Moments about C:
$$m(\bar{v}_{AC})_1\frac{L}{2} + \bar{I}(\omega_{AC})_1 - L\int K dt = m(\bar{v}_{AC})_2\left(\frac{L}{2}\right) + \bar{I}(\omega_{AC})_2$$

$$\frac{1}{3}mL^2(\omega_{AC})_1 - L\int K dt = \frac{1}{3}mL^2(\omega_{AC})_2 \qquad (2)$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics
$$(\bar{v}_{BD})_2 = \frac{L}{2}(\omega_{BD})_2$$

Moments about D:
$$0 + (L-h)\int K dt = m(\bar{v}_{BD})_2\frac{L}{2} + \bar{I}(\omega_{BD})_2$$

$$(L-h)\int K dt = \frac{1}{3}mL^2(\omega_{BD})_2 \qquad (3)$$

Multiply Eq. (2) by $(L-h)$ and Eq. (3) by L and then add to eliminate $\int Kdt$.

$$\frac{1}{3}mL^2(L-h)(\omega_{AC})_1 = \frac{1}{3}mL^2(L-h)(\omega_{AC})_2 + \frac{1}{3}mL^3(\omega_{BD})_2 \qquad (1)$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Condition of impact: $L(\omega_{AC})_2 - (L-h)(\omega_{BD})_2 = -eL(\omega_{AC})_1$ (2)

For $h = \frac{1}{2}L$ and $e = 0.5$ Eqs. (1) and (2) become

$$\frac{1}{6}mL^3(\omega_{AC})_1 = \frac{1}{6}mL^3(\omega_{AC})_2 + \frac{1}{3}mL^3(\omega_{BD})_2 \quad (3)$$

$$-\frac{1}{2} = 0.5L(\omega_{AB})_1 \quad (4)$$

Dividing Eq. (3) by $\frac{1}{6}mL^3$ and transposing terms gives

$$(\omega_{AC})_2 + 2(\omega_{BD})_2 = (\omega_{AC})_1 \quad (5)$$

Dividing Eq. (4) by $L/2$ and transposing terms gives

$$2(\omega_{AC})_2 - (\omega_{BD})_2 = -(\omega_{AC})_1 \quad (6)$$

Solving Eqs. (5) and (6) simultaneously for $(\omega_{AC})_2$ and $(\omega_{BD})_2$ gives

$$(\omega_{AC})_2 = -0.2(\omega_{AC})_1 \quad (7)$$

$$(\omega_{BD})_2 = 0.6(\omega_{AC})_1 \quad (8)$$

(*a*) *Angle of swing* θ_B *for rod B.*

Apply the principle of conservation of energy to rod *B*.

$$T_2 + V_2 = T_3 + V_3$$

Position (2): Just after impact.

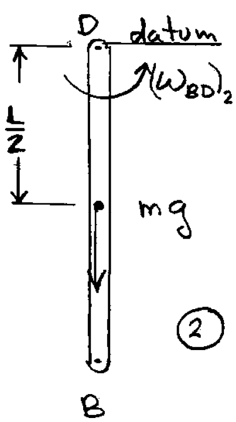
Position (3): At maximum angle of swing.

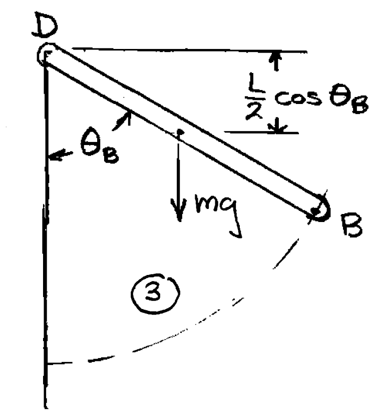
Potential energy.

Use the pivot point *D* as the datum.

$$V_2 = -mg\frac{L}{2}$$

$$V_3 = -mg\frac{L}{2}\cos\theta_B$$





PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Kinetic energy.

$$T_2 = \frac{1}{2} I_D (\omega_{BD})_2^2 = \frac{1}{6} mL^3 (\omega_{BD})_2^2$$

$$T_3 = 0$$

$$\frac{1}{6} mL^2 (\omega_{BD})_2^2 - mg\frac{L}{2} = 0 - mg\frac{L}{2}\cos\theta_B$$

$$1 - \cos\theta_B = \frac{1}{3}\frac{L}{g}(\omega_{BD})_2^2$$

$$= \frac{1}{3}\frac{L}{g}[0.6(\omega_{AB})_1]^2$$

$$= \left(\frac{1}{3}\right)(0.36)\left(\frac{L}{g}\right)\left(\frac{3g}{L}\right) = 0.36$$

$$\cos\theta_B = 0.64 \qquad\qquad \theta_B = 50.2° \blacktriangleleft$$

(*b*) *Angle of rebound θ_A for rod A.*

Apply the principle of conservation of energy to rod *A*.

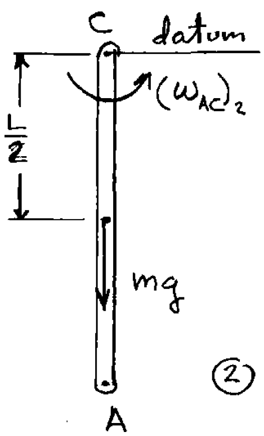
$$T_2 + V_2 = T_4 + V_4$$

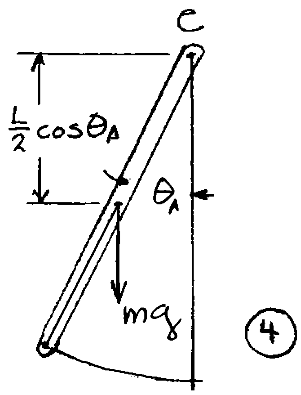
Position (2): Just after impact.

Position (4): At maximum angle of rebound.

Potential energy. Use the pivot Point *C* as the datum.

$$V_2 = -mg\frac{L}{2} \qquad V_4 = -mg\frac{L}{2}\cos\theta_A$$





Kinetic energy.

$$T_2 = \frac{1}{2} I_C (\omega_{AB})_2^2 = \frac{1}{6} mL^2 (\omega_{AC})_2^2 \qquad T_4 = 0$$

$$\frac{1}{6} mL^2 (\omega_{AC})_2^2 - mg\frac{L}{2} = 0 - mg\frac{L}{2}\cos\theta_A$$

$$1 - \cos\theta_A = \frac{1}{3}\frac{L}{g}(\omega_{AC})_2^2$$

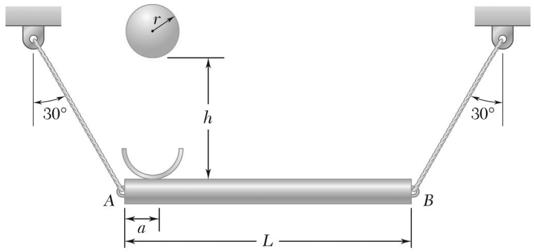
$$= \frac{1}{3}\frac{L}{g}[-0.2(\omega_{AB})_1]^2$$

$$= \left(\frac{1}{3}\right)(0.04)\left(\frac{L}{g}\right)\left(\frac{3g}{L}\right) = 0.04$$

$$\cos\theta_A = 0.96 \qquad\qquad \theta_A = 16.26° \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.126

A 2-kg solid sphere of radius $r = 40$ mm is dropped from a height $h = 200$ mm and lands on a uniform slender plank AB of mass 4 kg and length $L = 500$ mm which is held by two inextensible cords. Knowing that the impact is perfectly plastic and that the sphere remains attached to the plank at a distance $a = 40$ mm from the left end, determine the velocity of the sphere immediately after impact. Neglect the thickness of the plank.

SOLUTION

Masses and moments of inertia.

Sphere: $m_S = 2 \text{ kg}, \quad r = 40 \text{ mm} = 0.040 \text{ m}$

$$\bar{I}_S = \frac{2}{5} m_S r^2 = \left(\frac{2}{5}\right)(2 \text{ kg})(0.04 \text{ m})^2 = 1.28 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Plank AB: $m_{AB} = 4 \text{ kg}, \quad L = 500 \text{ mm} = 0.5 \text{ m}$

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L^2 = \left(\frac{1}{12}\right)(4 \text{ kg})(0.5 \text{ m})^2 = 83.333 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Velocity of sphere at impact.

$$v_S = \sqrt{2gh} = \sqrt{(2)(9.81 \text{ m/s})(0.200 \text{ m})} = 1.9809 \text{ m/s}$$

Before impact.

Linear momentum: $m_S \mathbf{v}_S = (4 \text{ kg})(1.9809 \text{ m/s})\downarrow = 7.9236 \text{ kg} \cdot \text{m/s}\downarrow$

with its line of action lying at distance $\frac{L}{2} - a$ from the midpoint of the plank.

$$\frac{L}{2} - a = 0.25 \text{ m} - 0.04 \text{ m} = 0.21 \text{ m}.$$

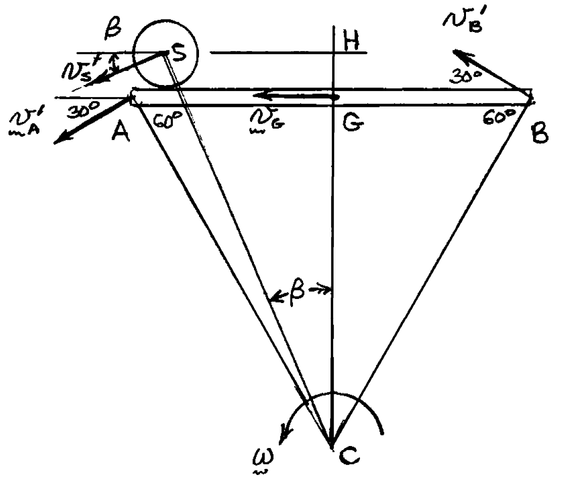
After impact. Assume that both cables are taut so $\mathbf{v}_A$ is perpendicular to the cable at A and $\mathbf{v}_B$ is perpendicular to the cable at B.

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Kinematics.

To locate the instantaneous center C draw line $\overline{AC}$ perpendicular to $\mathbf{v}_A$ and line $\overline{BC}$ perpendicular to $\mathbf{v}_B$. Let point G be the mass center of the plank AB and Point S be that of the sphere.



$$\overline{CH} = L\cos 30° + r$$
$$= (0.500\text{ m})\cos 30° + 0.040\text{ m}$$
$$= 0.47301\text{ m}$$

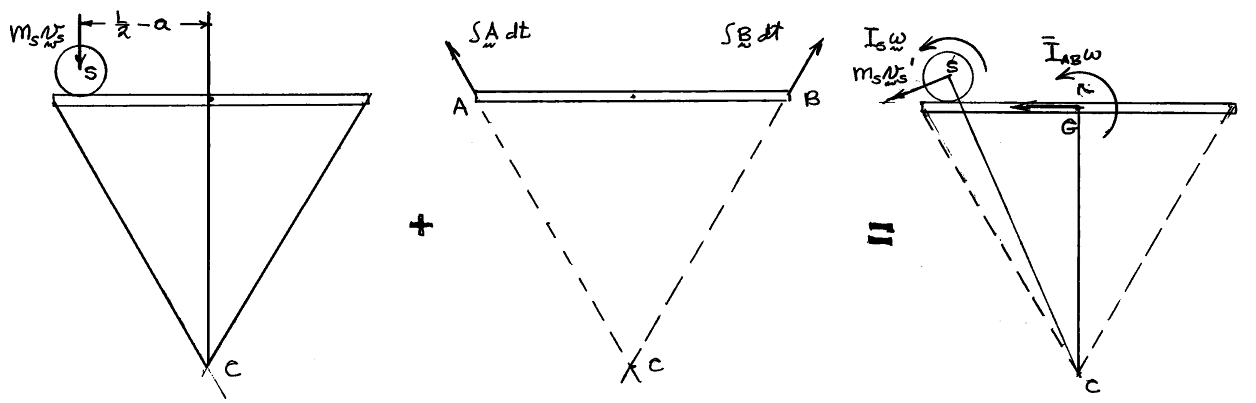
$$\overline{HS} = \frac{L}{2} - a = 0.21\text{ m}$$

$$\overline{CS} = \sqrt{\overline{CH}^2 + \overline{HS}^2} = 0.51753\text{ m}$$

$$\tan\beta = \frac{\overline{HS}}{\overline{CH}} = 0.44397 \qquad \beta = 23.94°$$

$$v_S = (\overline{CS})\omega = 0.51753\omega$$
$$v_G = (L\cos 30°)\omega = 0.43301\omega$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about C:

$$m_S v_S\left(\frac{L}{2} - a\right) + 0 = m_S v_S^1(\overline{CS}) + m_{AB}v_G^1(CH) + \bar{I}_S\omega + \bar{I}_{AB}\omega$$

$$m_S v_S\left(\frac{L}{2} - a\right) = [m_S(\overline{CS})^2 + m_{AB}\overline{CG}^2 + \bar{I}_S + \bar{I}_{AB}]\omega = I_C\omega$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.126 (Continued)

where $$m_S v_S\left(\frac{L}{2}-a\right)=(2\text{ kg})(1.9809\text{ m/s})(0.21\text{ m})=0.83198\text{ kg}\cdot\text{m}^2/\text{s}$$

and $$I_C=(2\text{ kg})(0.51753\text{ m})^2+(4\text{ kg})(0.43301\text{ m})^2+1.28\times10^{-3}\text{kg}\cdot\text{m}^2+83.333\times10^{-3}\text{ kg}\cdot\text{m}^2$$
$$=(0.53567+0.75+0.00128+0.08333)\text{ kg}\cdot\text{m}^2=1.37028\text{ kg}\cdot\text{m}^2$$

$$0.83198\text{ kg}\cdot\text{m}^2/\text{s}=(1.37028\text{ kg}\cdot\text{m}^2)\omega \qquad \boldsymbol{\omega}=0.60716\text{ rad/s} ◀$$

$$v_S^1=(0.51753\text{ m})(0.60716\text{ rad/s})=0.31422\text{ m/s}$$
$$v_G^1=(0.43301\text{ m})(0.60716\text{ rad/s})=0.26291\text{ m/s}$$

To check that neither cable becomes slack during the impact, we show that $\int A dt$ and $\int B\,dt$ are positive quantities.

$+\uparrow$ components: $$-m_S v_S+(\int Adt+\int Bdt)\cos 30°=-mv_S^1\sin\beta$$
$$\frac{\sqrt{3}}{2}(\int Adt+\int Bdt)=[m_S v_S-m_S v_1^1\sin\beta]/\cos 30°$$
$$=7.9236-(2)(0.31422)\sin 23.94°$$
$$=7.6686\text{ N}\cdot\text{s}$$

$\xleftarrow{+}$ components: $$0+(\int Adt-\int Bdt)\sin 30°=m_{AB}v_G^1+m_S v_S\cos\beta$$
$$\frac{1}{2}(\int Adt-\int Bdt)=(4)(0.26291)+(2)(0.31422)\cos 23.94°$$
$$=1.6260\text{ N}\cdot\text{s}$$

Solving the simultaneous equation gives

$$\int Adt=6.05\text{ N}\cdot\text{s} \qquad \int Bdt=2.80\text{ N}\cdot\text{s}$$

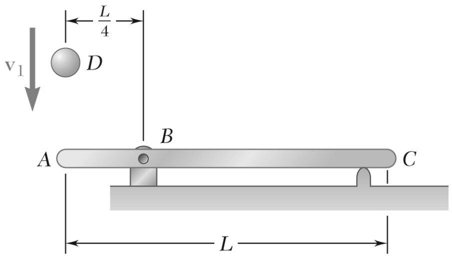
The cables remain taut as assumed.

Velocity of sphere: $\mathbf{v}_S^1=0.314\text{ m/s}$ ⦣ $23.9°$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.127



Member ABC has a mass of 2.4 kg and is attached to a pin support at B. An 800-g sphere D strikes the end of member ABC with a vertical velocity $\mathbf{v}_1$ of 3 m/s. Knowing that $L = 750$ mm and that the coefficient of restitution between the sphere and member ABC is 0.5, determine immediately after the impact (*a*) the angular velocity of member ABC, (*b*) the velocity of the sphere.

SOLUTION

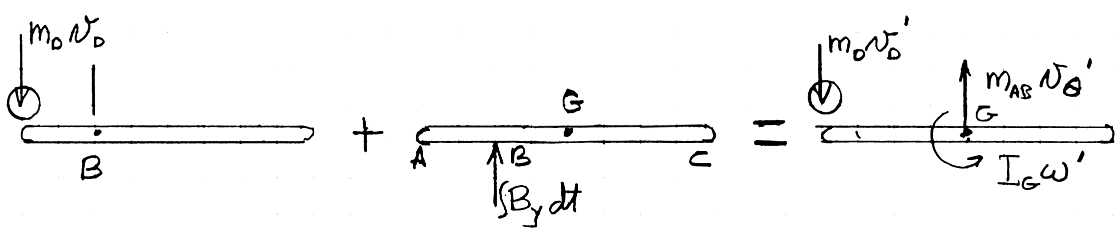
$$m_D = 0.800 \text{ kg}$$
$$L = 0.750 \text{ m}$$
$$\frac{1}{4}L = 0.1875 \text{ m}$$
$$m_{AC} = 2.4 \text{ kg}$$

Let Point G be the mass center of member ABC.

$$I_G = \frac{1}{12} m_{AC} L^2$$
$$= \frac{1}{12}(2.4)(0.750)^2$$
$$= 0.1125 \text{ kg} \cdot \text{m}^2$$

Kinematics after impact. $\quad \boldsymbol{\omega}' = \omega' \circlearrowright, \qquad \mathbf{v}'_G = \frac{L}{4}\omega' \uparrow, \qquad \mathbf{v}'_A = \frac{L}{4}\omega' \downarrow$

Conservation of momentum.



$+\circlearrowright$ Moments about B:

$$m_D v_D \frac{L}{2} + 0 = m_D v'_D \frac{L}{2} + I_G \omega' + m_{AC} v'_G \frac{L}{4}$$

$$m_D v_D \frac{L}{4} = m_D v'_D \frac{L}{4} + \left[I_G + m_{AD} \left(\frac{L}{4} \right)^2 \right] \omega'$$

$$(0.800)(3)(0.1875) = (0.800)(0.1875) v'_D + [0.1125 + (2.4)(0.1875)^2] \omega'$$

$$0.45 = 0.15 v'_D + 0.196875\, \omega' \qquad (1)$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Coefficient of restitution.
$$v'_D - v'_A = v'_D - \frac{L}{4}\omega'$$
$$= -e(v_D - v_A)$$
$$v'_D - 0.1875\omega' = -(0.5)(3-0) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously.

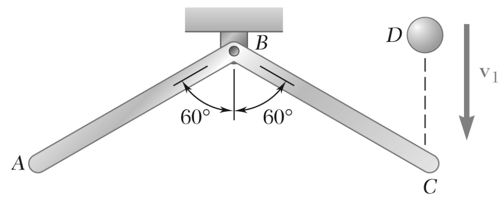
(a) Angular velocity. $\omega' = 3$ $\qquad \boldsymbol{\omega}' = 3.00 \text{ rad/s} \circlearrowright$ ◀

(b) Velocity of D. $v'_D = -0.9375$ $\qquad v'_D = 0.938 \text{ m/s} \uparrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.128



Member *ABC* has a mass of 2.4 kg and is attached to a pin support at *B*. An 800-g sphere *D* strikes the end of member *ABC* with a vertical velocity $\mathbf{v}_1$ of 3 m/s. Knowing that $L = 750\,\text{mm}$ and that the coefficient of restitution between the sphere and member *ABC* is 0.5, determine immediately after the impact (*a*) the angular velocity of member *ABC*, (*b*) the velocity of the sphere.

SOLUTION

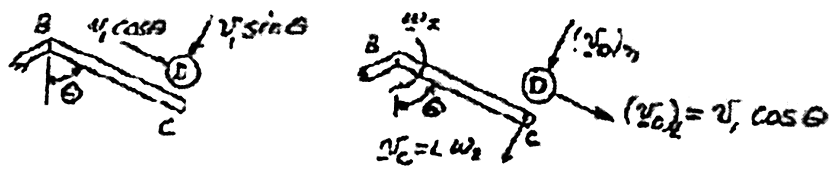
Let M be the mass of member *ABC* and $\bar{I}$ its moment of inertia about *B*.

$$M = 2.4\ \text{kg} \qquad \bar{I} = \frac{1}{12}M(2L)^2$$

where $\qquad L = 750\ \text{mm} = 0.75\ \text{m}$

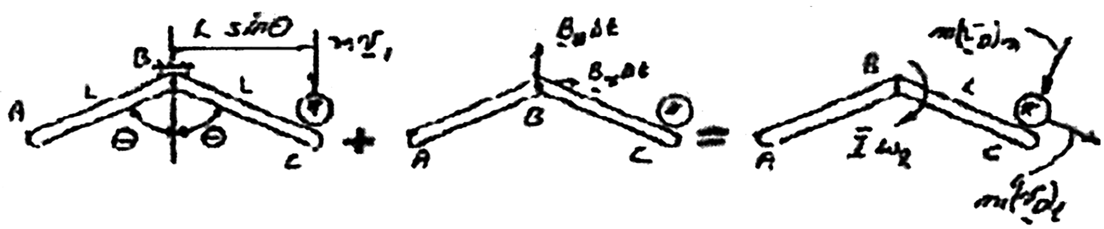
Let m be the mass of sphere *D*. $\qquad m = 800\ \text{g} = 0.8\ \text{kg}$

Impact kinematics and coefficient of restitution.



$$(v_1 \sin\theta)e = L\omega_2 - (v_D)_n: \quad (v_D)_n = L\omega_2 - (v_1 \sin\theta)e \tag{1}$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$+\circlearrowright$ Moments about *B*: $\qquad mv_1 L\sin\theta = \bar{I}\omega_2 + m(v_D)_n L$

$$mv_1 L\sin\theta = \frac{1}{12}M(2L)^2\omega_2 + m[L\omega_2 - (v_1 \sin\theta)e]L$$

$$mv_1 \sin\theta = \frac{1}{3}ML\omega_2 - mL\omega_2 - m(v_1 \sin\theta)e$$

$$m(1+e)\frac{v_1}{L}\sin\theta = \left(\frac{1}{3}M + m\right)\omega_2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.128 (Continued)

(*a*) Angular velocity.

$$\omega_2 = \frac{(3)(1+e)mv_1 \sin\theta}{(M+3m)L}$$

$$\omega_2 = \frac{(3)(1.5)(0.8)(3)\sin 60^\circ}{(2.4+2.4)(0.75)}$$

$$= 2.5981$$

$\boldsymbol{\omega}_2 = 2.60$ rad/s ◀

(*b*) Velocity of *D*.

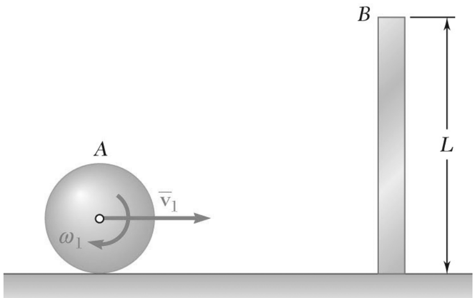
From Eq. (1),

$$(v_D)_n = (0.75)(2.5981) - (3\sin 60^\circ)(0.5)$$
$$= 0.64976 \text{ m/s}$$
$$(v_D)_t = v_1 \cos 60^\circ$$
$$= 3\cos 60^\circ$$
$$= 1.5 \text{ m/s}$$
$$(\mathbf{v}_D)_n = 0.64976 \text{ m/s} \angle 30^\circ$$
$$(\mathbf{v}_D)_t = 1.5 \text{ m/s} \angle 30^\circ$$
$$v_D = \sqrt{(0.64976)^2 + (1.5)^2}$$
$$= 1.63468 \text{ m/s}$$
$$\tan\theta = \frac{0.64976}{1.5}$$
$$\theta = 23.4^\circ$$
$$\theta + 30^\circ = 53.4^\circ$$

$\mathbf{v}_D = 1.635$ m/s $\angle$ 53.4° ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.129

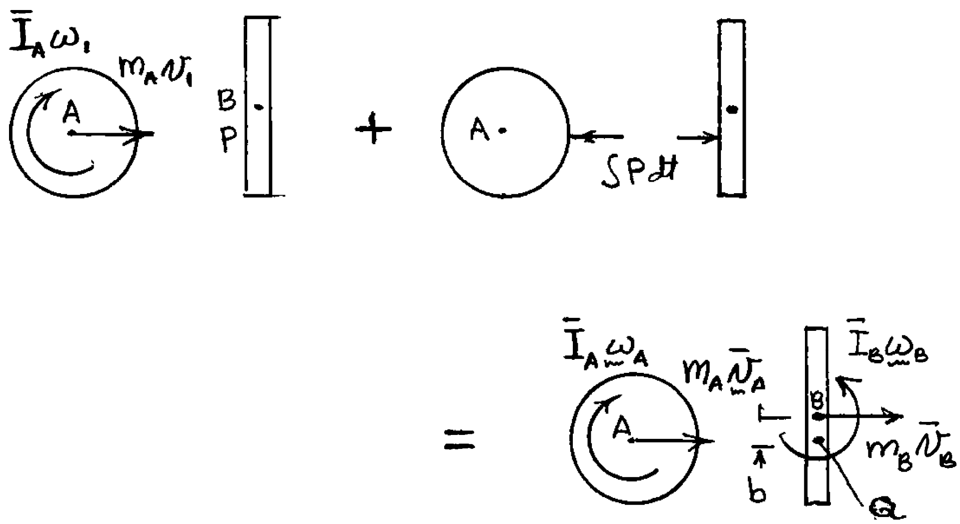
Sphere A of mass $m_A = 2$ kg and radius $r = 40$ mm rolls without slipping with a velocity $\overline{\mathbf{v}}_1 = 2$ m/s on a horizontal surface when it hits squarely a uniform slender bar B of mass $m_B = 0.5$ kg and length $L = 100$ mm that is standing on end and at rest. Denoting by μ_k the coefficient of kinetic friction between the sphere and the horizontal surface, neglecting friction between the sphere and the bar, and knowing the coefficient of restitution between A and B is 0.1, determine the angular velocities of the sphere and the bar immediately after the impact.

SOLUTION

Before impact sphere A rolls without slipping so that its instantaneous center of rotation is its contact point with the floor.

$$\omega_1 = \frac{v_1}{r} = \frac{2 \text{ m/s}}{0.040 \text{ m}} = 50 \text{ rad/s} \qquad \boldsymbol{\omega}_1 = 50 \text{ rad/s} \circlearrowright \blacktriangleleft$$

Analysis of impact. Use the principle of impulse and momentum. Let point A be the center of sphere A, point B be the mass center of bar B, and Points P and Q the contact point between the sphere and the bar, Point P being on sphere A and Point Q being on the bar B.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\boldsymbol{\omega}_A = \omega_A \circlearrowright$ $\qquad$ $\boldsymbol{\omega}_B = \omega_B \circlearrowright$

$\overline{\mathbf{v}}_A = \overline{v}_A \longrightarrow$ $\qquad$ $\overline{\mathbf{v}}_B = \overline{v}_B \longrightarrow,$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Sphere A alone.

$+$ Moments about A: $\quad \bar{I}_A \omega_1 + 0 = \bar{I}_A \omega_A \qquad \omega_A = \omega_1$

$$\boldsymbol{\omega}_A = 50.0 \text{ rad/s} \circlearrowright \blacktriangleleft$$

Kinematics: $$b = \frac{L}{2} - r = 50 \text{ mm} - 40 \text{ mm} = 10 \text{ mm} = 0.010 \text{ m}$$

$$\mathbf{v}_Q = (\bar{v}_B + b\omega_B) \longrightarrow$$

Condition of impact. $\quad \bar{\mathbf{v}}_A - \mathbf{v}_Q = -e\bar{\mathbf{v}}_1$

$$v_A - v_B - b\omega_B = -e\bar{v}_1 \qquad (1)$$

Bar B alone:

$+$ Moments about Q: $$0 + 0 = \bar{I}_B \omega_B - b m_B v_B$$

$$\frac{1}{12} m_B L^2 \omega_B - b m_B v_B = 0$$

$$b v_B - \frac{1}{12} L^2 \omega_B = 0 \qquad (2)$$

Sphere A and bar B together.

$\xrightarrow{+}$ components:

$$m_A \bar{v}_1 + 0 = m_A v_A + m_B v_B$$

$$m_A \bar{v}_A + m_B \bar{v}_B = m_A \bar{v}_1 \qquad (3)$$

Data: $\quad m_A = 2 \text{ kg}, \quad m_B = 0.5 \text{ kg}, \quad e = 0.1$

$\bar{v}_1 = 2 \text{ m/s}, \quad L = 0.100 \text{ m}, \quad b = 0.010 \text{ m}$

$$v_A - v_B - (0.010 \text{ m})\omega_B = -(0.1)(2 \text{ m/s}) \qquad (1)'$$

$$(0.010 \text{ m})v_B - \frac{1}{12}(0.100 \text{ m})^2 \omega_B = 0 \qquad (2)'$$

$$(2 \text{ kg})v_A + (0.5 \text{ kg})v_B = (2 \text{ kg})(2 \text{ m/s}) \qquad (3)'$$

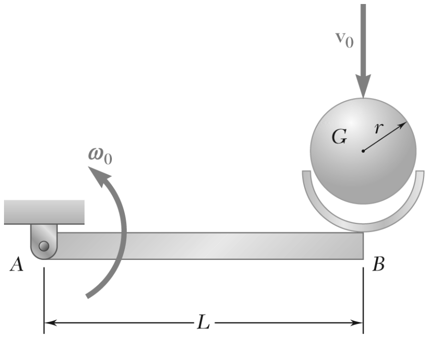
Solving Eqs. (1)′, (2)′, and (3)′ simultaneously,

$$v_A = 1.599 \text{ m/s} \qquad v_B = 1.606 \text{ m/s} \qquad \omega_B = 19.27 \text{ rad/s}$$

$$\boldsymbol{\omega}_B = 19.27 \text{ rad/s} \circlearrowright \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.130

A large 3-lb sphere with a radius $r = 3$ in. is thrown into a light basket at the end of a thin, uniform rod weighing 2 lb and length $L = 10$ in. as shown. Immediately before the impact the angular velocity of the rod is 3 rad/s counterclockwise and the velocity of the sphere is 2 ft/s down. Assume the sphere sticks in the basket. Determine after the impact (*a*) the angular velocity of the bar and sphere, (*b*) the components of the reactions at *A*.

SOLUTION

Let Point *G* be the mass center of the sphere and Point *C* be that of the rod *AB*.

Rod *AB*: $W_{AB} = 2\text{ lb.}\quad m_{AB} = \dfrac{2}{32.2} = 0.06211\text{ lb}\cdot\text{s}^2/\text{ft}$

$$I_{AB} = \frac{1}{12}m_P L^2 = \frac{1}{12}(0.06211)\left(\frac{10}{12}\right)^2 = 0.003594\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

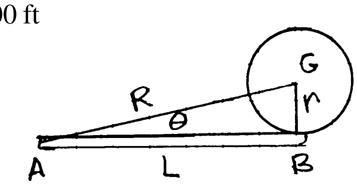
Sphere: $W_S = 3\text{ lb}\quad m_S = \dfrac{3}{32.2} = 0.09317\text{ lb}\cdot\text{s}^2/\text{ft}$

$$I_G = \frac{2}{5}m_S r^2 = \frac{2}{5}(0.09317)\left(\frac{3}{12}\right)^2 = 0.002329\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Impact. Before impact, bar *AB* is rotating about *A* with angular velocity $\boldsymbol{\omega}_0 = \omega_0 \circlearrowleft$ ($\omega_0 = 3$ rad/s) and the sphere is falling with velocity $\mathbf{v}_0 = v_0\downarrow$ ($v_0 = 2$ ft/s). After impact, the rod and the sphere move together, rotating about *A* with angular velocity $\boldsymbol{\omega} = \omega \circlearrowright$.

Geometry. $R = \sqrt{L^2 + r^2} = \sqrt{10^2 + 3^2} = 10.44\text{ in.} = 0.8700\text{ ft}$

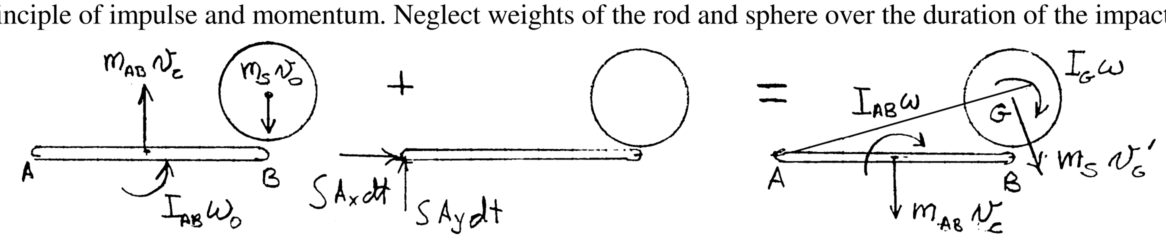
$$\tan\theta = \frac{r}{L} = \frac{3}{10}\qquad \theta = 16.7°$$



Kinematics: Before impact, $v_C = \dfrac{L}{2}\omega_0 = \left(\dfrac{5}{12}\right)(3) = 1.25\text{ ft/s}\uparrow$

After impact, $\mathbf{v}_C = \dfrac{L}{2}\omega'\downarrow,\quad \mathbf{v}_G = R\omega' \searrow \theta$

Principle of impulse and momentum. Neglect weights of the rod and sphere over the duration of the impact.



PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

(*a*) + Moments about *A*:

$$m_S v_0 L - I_{AB}\omega_0 - m_{AB} v_C \frac{L}{2} + 0 = I_G \omega' + m_S v_G' R + I_{AB}\omega' + m_{AB} v_C \frac{L}{2}$$

or

$$m_S v_0 L - I_{AB}\omega_0^2 - m_{AB} v_C \frac{L}{2} = \left(I_G + m_S R^2 + I_{AB} + \frac{1}{4} m_{AB} L^2\right)\omega' \qquad (1)$$

$$(0.09317)(2)\left(\frac{10}{12}\right) - (0.003594)(3) - (0.06211)(1.25)\left(\frac{5}{12}\right)$$

$$= \left[0.002329 + (0.09317)(0.87)^2 + 0.003594 + \frac{1}{4}(0.06211)\left(\frac{10}{12}\right)^2\right]\omega'$$

$$0.112152 = 0.087226\omega' \quad \omega' = 1.2858 \qquad \boldsymbol{\omega}' = 1.286 \text{ rad/s} ◀$$

Normal accelerations at *C* and *G*.

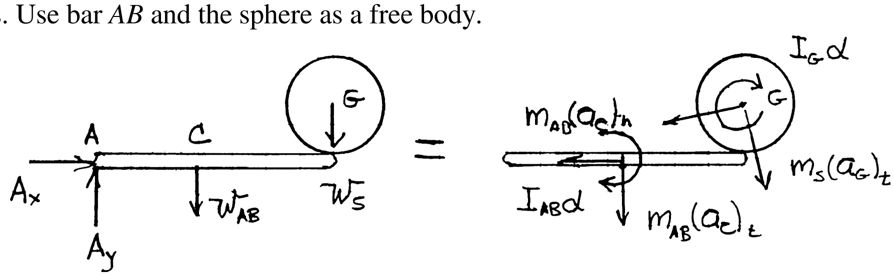
$$(\mathbf{a}_C)_n = \frac{L}{2}(\omega')^2 = \left(\frac{5}{12}\right)(1.2858)^2 = 0.6889 \text{ ft/s}^2 \longleftarrow$$

$$(a_G)_n = R(\omega')^2 = (0.87)(1.2858)^2 = 1.4384 \text{ ft/s}^2 \; 16.7°$$

Tangential accelerations at *C* and *G*. $\boldsymbol{\alpha} = \alpha$

$$(a_C)_t = \frac{L}{2}\alpha = \frac{5}{12}\alpha \downarrow \quad (a_G)_t = R\alpha = 0.87\alpha \; 16.7°$$

(*b*) Kinetics. Use bar *AB* and the sphere as a free body.



$$+\;\Sigma M_A = \Sigma(M_A)_{\text{eff}}:$$

$$W_{AB}\frac{L}{2} + W_S L = I_{AB}\alpha + \frac{L}{2} m_{AB}(a_C)_t + I_G\alpha + m_S(a_G)_t R$$

$$= \left(I_{AB} + \frac{1}{4} m_{AB} L^2 + I_G + m_S R^2\right)\alpha$$

$$(2)\left(\frac{5}{12}\right) + (3)\left(\frac{10}{12}\right) = \left[0.003594 + \frac{1}{4}(0.06211)\left(\frac{10}{12}\right)^2 + 0.002329 + (0.09317)(0.87)^2\right]\alpha$$

$$3.3333 = 0.087226\alpha \qquad \boldsymbol{\alpha} = 38.214 \text{ rad/s}^2$$

$$(\mathbf{a}_C)_t = \left(\frac{5}{12}\right)(38.214) = 15.923 \text{ ft/s}^2 \downarrow, \quad (\mathbf{a}_G)_t = (0.87)(38.214) = 33.247 \text{ ft/s}^2 \; 16.7°$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.130 (Continued)

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}$:

$$A_x = -m_{AB}(a_C)_n - m_S(a_G)_n \cos 16.7° + m_S(a_G)_t \sin 16.70$$

$$A_x = -(0.06211)(0.6889) - (0.09317)(1.4384)\cos 16.7° + (0.09317)(33.247)\sin 16.7°$$

$$A_x = 0.719 \text{ lb} \qquad \mathbf{A}_x = 0.719 \text{ lb} \longrightarrow \blacktriangleleft$$

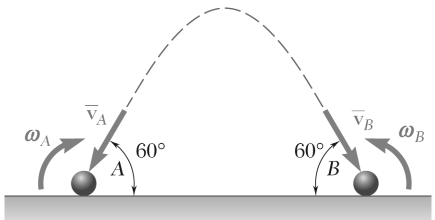
$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}$: $\quad A_y - W_{AB} - W_S = -m_{AB}(a_C)_t - m_S(a_G)_t \cos 16.7° - m_S(a_G)_n \sin 16.7°$

$$A_y - 2 - 3 = -(0.06211)(15.923) - (0.09317)(33.247)\cos 16.7° - (0.09317)(1.4384)\sin 16.7°$$

$$A_y = 1.006 \text{ lb} \qquad \mathbf{A}_y = 1.006 \text{ lb} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



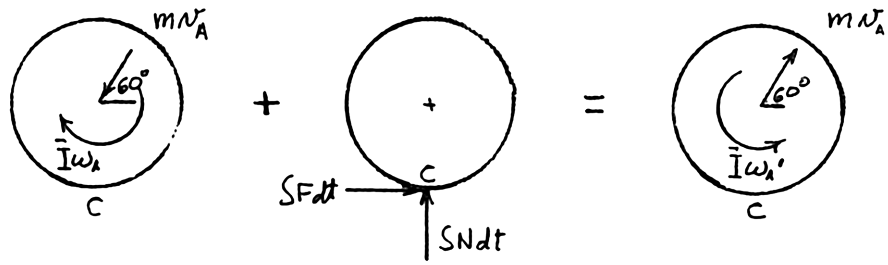
PROBLEM 17.131

A small rubber ball of radius r is thrown against a rough floor with a velocity $\overline{\mathbf{v}}_A$ of magnitude v_0 and a backspin $\boldsymbol{\omega}_A$ of magnitude ω_0. It is observed that the ball bounces from A to B, then from B to A, then from A to B, etc. Assuming perfectly elastic impact, determine the required magnitude ω_0 of the backspin in terms of $\overline{v}_0$ and r.

SOLUTION

Moment of inertia. $\quad \overline{I} = \frac{2}{5}mr^2$ Ball is assumed to be a solid sphere.

Impact at A.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

For the velocity of the ball to be reversed on each impact,

$$v_A' = v_A = v_0$$
$$\omega_A' = \omega_A = \omega_0$$

This is consistent with the assumption of perfectly elastic impact.

Moments about C: $\quad mv_A r\cos 60° - \overline{I}\,\omega_A + 0 = \overline{I}\,\omega_A' - mv_A' r\cos 60°$

$$mv_0 r\cos 60° - \frac{2}{5}mr^2\omega_0 + 0 = \frac{2}{5}mr^2\omega_0 - mv_0 r\cos 60°$$

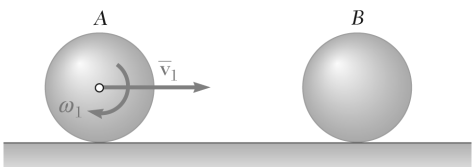
$$\frac{2}{5}r\omega_0 = v_0\cos 60°$$

$$\omega_0 = \frac{5}{4}\frac{v_0}{r} \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.132

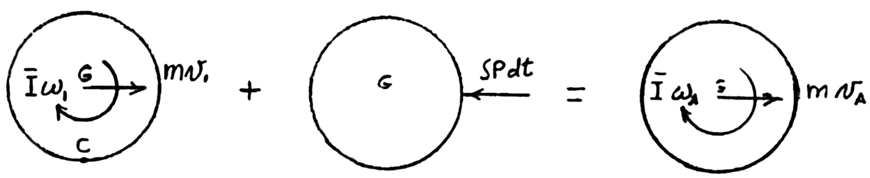


Sphere A of mass m and radius r rolls without slipping with a velocity $\bar{\mathbf{v}}_1$ on a horizontal surface when it hits squarely an identical sphere B that is at rest. Denoting by μ_k the coefficient of kinetic friction between the spheres and the surface, neglecting friction between the spheres, and assuming perfectly elastic impact, determine (a) the linear and angular velocities of each sphere immediately after the impact, (b) the velocity of each sphere after it has started rolling uniformly.

SOLUTION

Moment of inertia. $\bar{I} = \dfrac{2}{5}mr^2$

Analysis of impact. Sphere A.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

Kinematics: Rolling without slipping in Position 1.

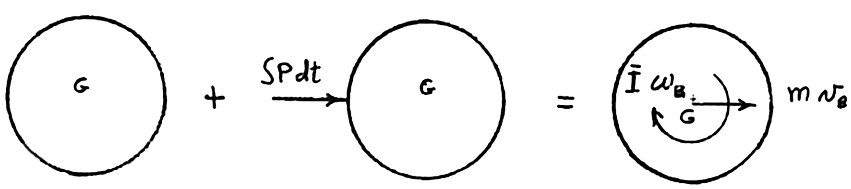
$$\omega_A = \frac{v_1}{r}$$

Moments about G: $\bar{I}\omega_1 + 0 = \bar{I}\omega_A$

$$\omega_A = \omega_1 = \frac{v_1}{r}$$

$\xrightarrow{+}$ Linear components: $mv_1 - \int Pdt = mv_A$ (1)

Analysis of impact. *Sphere B*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Linear components: $0 + \int Pdt = mv_B$ (2)

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Add Equations (1) and (2) to eliminate $\int P dt$.

$$mv_1 = mv_A + mv_B \quad \text{or} \quad v_B + v_A = v_1 \tag{3}$$

Condition of impact. $e = 1$. $\quad v_B - v_A = ev_1 = v_1 \quad (4)$

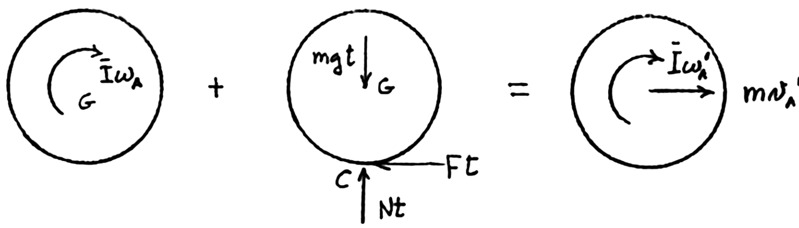
Solving Equations (3) and (4) simultaneously,

$$v_A = 0, \qquad v_B = v_1$$

Moments about G: $\quad 0 + 0 = \bar{I}\omega_B \quad \omega_B = 0$

(a) *Velocities after impact.* $\quad \mathbf{v}_A = 0; \quad \boldsymbol{\omega}_A = \dfrac{v_1}{r}$ ↻; $\quad \mathbf{v}_B = v_1 \longrightarrow; \quad \boldsymbol{\omega}_B = 0$ ◀

Motion after Impact. *Sphere A*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

Condition of rolling without slipping: $\quad v_A' = \omega_A' r$

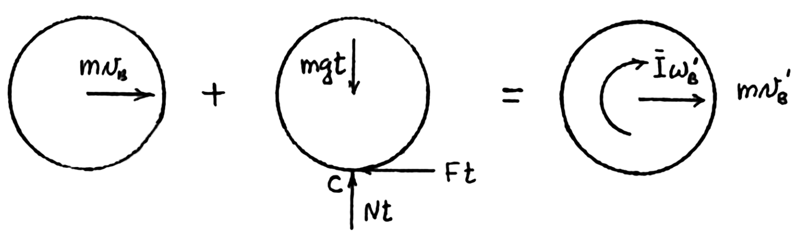
Moments about C: $\quad \bar{I}\omega_A + 0 + \bar{I}\omega_A' + mv_A' r$

$$\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right) + 0 = \left(\frac{2}{5}mr^2\right)\omega_A' + m(r\omega_A')r$$

$$\omega_A' = \frac{2}{7}\frac{v_1}{r}$$

$$v_A' = \frac{2}{7}v_1$$

Motion after impact. *Sphere B.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.132 (Continued)

Condition of rolling without slipping: $v'_B = r\omega'_B$

Moments about C: $mv_B r + 0 = \bar{I}\omega'_B + mv'_B r$

$$mv_1 r + 0 = \left(\frac{2}{5}mr^2\right)\omega'_B + m(r\omega'_B)r$$

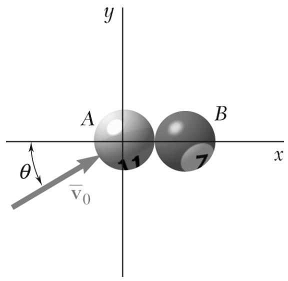
$$\omega'_B = \frac{5}{7}\frac{v_1}{r}$$

$$v'_B = \frac{5}{7}v_1$$

(*b*) *Final Rolling Velocities.* $\mathbf{v}'_A = \frac{2}{7}v_1 \longrightarrow;\quad \mathbf{v}'_B = \frac{5}{7}v_1 \longrightarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.133

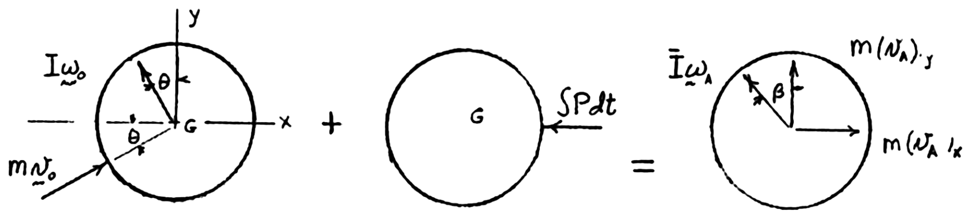
In a game of pool, ball A is rolling without slipping with a velocity $\overline{\mathbf{v}}_0$ as it hits obliquely ball B, which is at rest. Denoting by r the radius of each ball and by μ_k the coefficient of kinetic friction between a ball and the table and assuming perfectly elastic impact, determine (a) the linear and angular velocity of each ball immediately after the impact, (b) the velocity of ball B after it has started rolling uniformly.

SOLUTION

Moment of inertia. $\qquad \bar{I} = \dfrac{2}{5}mr^2$

(a) Impact analysis.

Ball A:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

Kinematics of rolling: $\qquad \omega_0 = \dfrac{v_0}{r}$

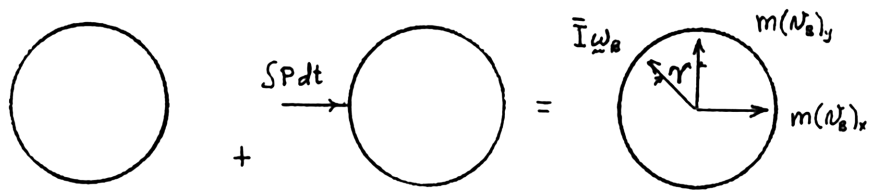
$\xrightarrow{+}$ Linear components: $\quad mv_0\cos\theta - \int Pdt = m(v_A)_x \qquad (1)$

$+\uparrow$ Linear components: $\quad mv_0\sin\theta + 0 = m(v_A)_y \qquad (2)$

Moments about y axis: $\quad \bar{I}\omega_0\cos\theta + 0 = \bar{I}\omega_A\cos\beta \qquad (3)$

Moments about x axis: $\quad -\bar{I}\omega_0\sin\theta + 0 = -\bar{I}\omega_A\sin\beta \qquad (4)$

Ball B:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Linear components: $\quad 0 + \int Pdt = m(v_B)_x \qquad (5)$

$+\uparrow$ Linear components: $\quad 0 + 0 = m(v_B)_y \qquad (6)$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Moments about y axis: $$0+0=\bar{I}\omega_B\cos\gamma \tag{7}$$

Moments about x axis: $$0+0=\bar{I}\omega_B\sin\gamma \tag{8}$$

Adding Equations (1) and (5) to eliminate $\int P dt$,

$$mv_0\cos\theta+0=m(v_A)_x+m(v_B)_x$$

or $$(v_B)_x+(v_A)_x=v_0\cos\theta \tag{9}$$

Condition of impact. $e=1$: $$(v_B)_x-(v_A)_x=ev_0\cos\theta=v_0\cos\theta \tag{10}$$

Solving Equations (9) and (10) simultaneously,

$$(v_A)_x=0,\quad (v_B)_x=v_0\cos\theta$$

From Equations (2) and (6), $(v_A)_y=v_0\sin\theta,\quad (v_B)_y=0$ $\qquad v_A=(v_0\sin\theta)\mathbf{j}$ ◀

$v_B=(v_0\cos\theta)\mathbf{i}$ ◀

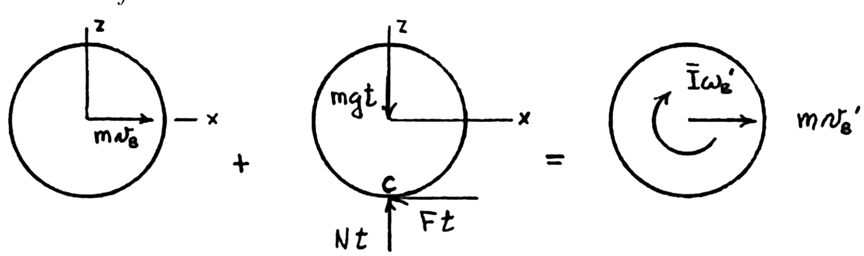
From Equations (3) and (4) simultaneously,

$$\beta=\theta,\quad \omega_A=\omega_0=\frac{v_0}{r}$$ $\boldsymbol{\omega}_A=\dfrac{v_0}{r}(-\sin\theta\mathbf{i}+\cos\theta\mathbf{j})$ ◀

From Equations (7) and (8) simultaneously,

$$\omega_B=0$$ $\boldsymbol{\omega}_B=0$ ◀

(*b*) *Subsequent motion of ball B.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics of rolling without slipping. $v_B'=r\omega_B'$

↻ Moments about C: $$mv_Br+0=\bar{I}\omega_B'+mv_B'r$$

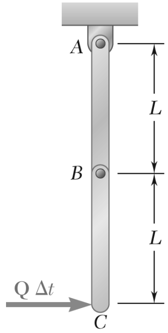
$$=\frac{2}{5}mr^2\omega_B'+m(r\omega_B')r$$

$$\omega_B'=\frac{5}{7}\frac{v_B'}{r}=\frac{5}{7}\frac{v_1\cos\theta}{r}$$

$$v_B'=\frac{5}{7}v_1\cos\theta$$ $\mathbf{v}_B'=\dfrac{5}{7}(v_0\cos\theta)\mathbf{i}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



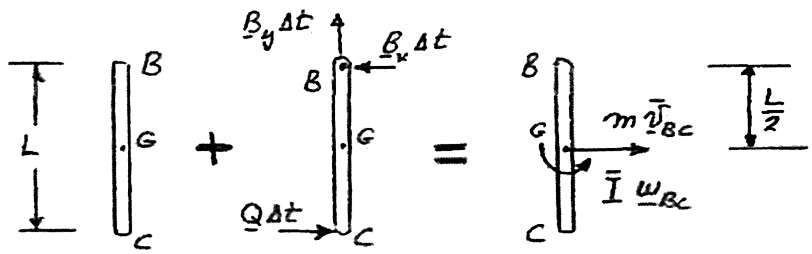
PROBLEM 17.134

Each of the bars AB and BC is of length $L = 400$ mm and mass $m = 1.2$ kg. Determine the angular velocity of each bar immediately after the impulse $Q\Delta t = (1.5\ \text{N}\cdot\text{s})\mathbf{i}$ is applied at C.

SOLUTION

Principle of impulse and momentum.

Bar BC:



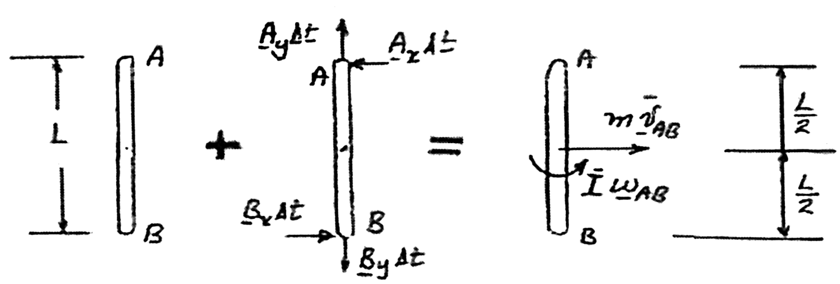
$$\textbf{Syst. Momenta}_1 + \textbf{Syst. Ext. Imp.}_{1\to2} = \textbf{Syst. Momenta}_2$$

Kinematics
$$\bar{v}_{BC} = v_B + \frac{L}{2}\omega_{BC} = L\omega_{AB} + \frac{L}{2}\omega_{BC}$$

$+\circlearrowleft$ Moments about B:
$$0 + (Q\Delta t)L = \bar{I}\,\omega_{BC} + m\bar{v}_{BC}\frac{L}{2}$$
$$(Q\Delta t)L = \frac{1}{12}mL^2\omega_{BC} + m\left(L\omega_{AB} + \frac{L}{2}\omega_{BC}\right)\frac{L}{2}$$
$$Q\Delta t = \frac{1}{2}mL\omega_{AB} + \frac{1}{3}mL\omega_{BC} \tag{1}$$

$\xrightarrow{+}$ x components:
$$Q\Delta t - B_x\Delta t = m\left(L\omega_{AB} + \frac{L}{2}\omega_{BC}\right) \tag{2}$$

Bar AB:



$$\textbf{Syst. Momenta}_1 + \textbf{Syst. Ext. Imp.}_{1\to2} = \textbf{Syst. Momenta}_2$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

+ Moments about A: $$0+(B_x\Delta t)L=\bar{I}\,\omega_{AB}+m\bar{v}_{AB}\frac{L}{2}$$

$$(B_x\Delta t)L=\frac{1}{12}mL^2\omega_{AB}+m\left(\frac{L}{2}\omega_{AB}\right)\frac{L}{2}$$

$$B_x\Delta t=\frac{1}{3}mL\omega_{AB} \qquad (3)$$

We now have 3 unknowns ($B_x\Delta t_1$, ω_{AB}, and ω_{BC}) and 3 equations.

Add Eqs. (2) and (3): $$Q\Delta t=\frac{4}{3}mL\omega_{AB}+\frac{1}{2}mL\omega_{BC} \qquad (4)$$

Subtract Eq. (1) from Eq. (4): $$0=\frac{5}{6}mL\omega_{AB}+\frac{1}{6}mL\omega_{BC}$$

$$\omega_{BC}=-5\omega_{AB} \qquad (5)$$

Substitute for ω_{BC} in Eq. (1): $$Q\Delta t=\frac{1}{2}mL\omega_{AB}+\frac{1}{3}mL(-5\omega_{AB})$$

$$=-\frac{7}{6}mL\omega_{AB}$$

$$\omega_{AB}=-\frac{6}{7}\frac{Q\Delta t}{mL} \qquad (6)$$

Substituting into Eq. (5): $$\omega_{BC}=-5\left(-\frac{6}{7}\frac{Q\Delta t}{mL}\right)$$

$$\omega_{BC}=\frac{30}{7}\frac{Q\Delta t}{mL} \qquad (7)$$

Given data: $$L=0.400\text{ m}$$
$$Q\Delta t=1.5\text{ N}\cdot\text{s}$$
$$m=1.2\text{ kg}$$

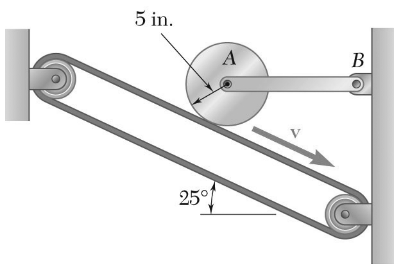
Angular velocity of bar AB. $$\omega_{AB}=-\frac{6}{7}\frac{Q\Delta t}{mL}=-\frac{(6)(1.5)}{(7)(1.2)(0.4)}$$ $\boldsymbol{\omega}_{AB}=2.68\text{ rad/s}$ ◀

Angular velocity of bar BC. $$\omega_{BC}=\frac{30}{7}\frac{Q\Delta t}{mL}=\frac{(30)(1.5)}{(7)(1.2)(0.4)}$$ $\boldsymbol{\omega}_{BC}=13.39\text{ rad/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

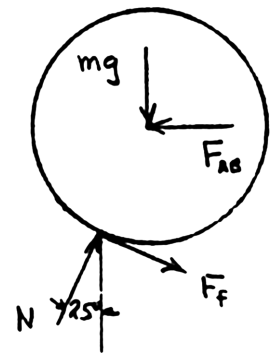
STUDENTS-HUB.com

PROBLEM 17.135



A uniform disk of constant thickness and initially at rest is placed in contact with the belt shown, which moves at a constant speed $v = 80$ ft/s. Knowing that the coefficient of kinetic friction between the disk and the belt is 0.15, determine (*a*) the number of revolutions executed by the disk before it reaches a constant angular velocity, (*b*) the time required for the disk to reach that constant angular velocity.

SOLUTION



Kinetic friction. $$F_f = \mu_k N = 0.15\text{ N}$$

$$+\uparrow \Sigma F_y = N\cos 25° - F_f \sin 25° - mg = 0$$

$$(\cos 25° - \mu_k \sin 25°)N = mg$$

$$N = \frac{mg}{\cos 25° - 0.15\sin 25°}$$

$$= 1.18636\, mg$$

$$F_f = (0.15)(1.18636)mg$$

$$= 0.177954\, mg$$

Final angular velocity. $$\omega_2 = \frac{v}{r}$$

Moment of inertia. $$\bar{I} = \frac{1}{2}mr^2$$

(*a*) *Principle of work and energy.*

$$T_1 + W_{1\to 2} = T_2: \quad T_1 = 0$$

$$W_{1\to 2} = F_f r\theta = 0.177954 mgr\theta$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2$$

$$0 + 0.177954 mgr\theta = \frac{1}{4}mv^2$$

$$\theta = 1.40486\frac{v^2}{gr}$$

$$= \frac{(1.40486)(80)^2}{(32.2)\left(\frac{5}{12}\right)}$$

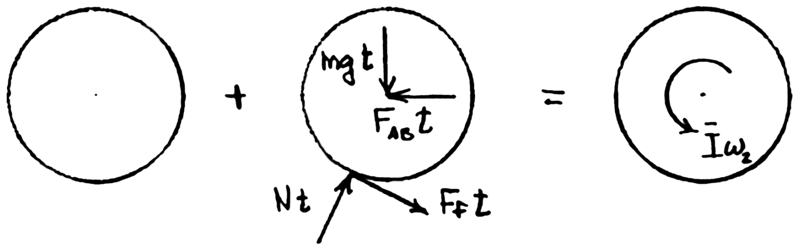
$$= 670.14 \text{ radians}$$

$\theta = 106.7$ rev ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

(*b*) *Principle of impulse-momentum.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about A: $\quad 0 + F_f tr = \bar{I}\omega_2$

$$t = \frac{\bar{I}\omega_2}{F_f r}$$

$$= \frac{\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)}{0.177954 mgr}$$

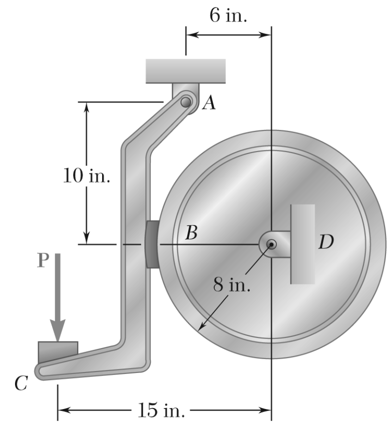
$$= 2.8097\frac{v}{g}$$

$$= \frac{(2.8097)(80)}{32.2}$$

$t = 6.98$ s ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

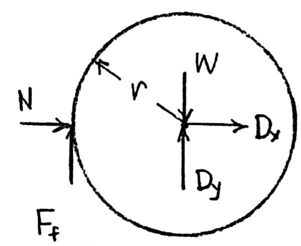
STUDENTS-HUB.com



PROBLEM 17.136

The 8-in.-radius brake drum is attached to a larger flywheel that is not shown. The total mass moment of inertia of the flywheel and drum is $14\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.35. Knowing that the initial angular velocity of the flywheel is 360 rpm counterclockwise, determine the vertical force **P** that must be applied to the pedal *C* if the system is to stop in 100 revolutions.

SOLUTION



Kinetic energy.

$$\omega_1 = 360\ \text{rpm} = 12\pi\ \text{rad/s}$$

$$\omega_2 = 0$$

$$T_1 = \frac{1}{2}I\omega_1^2$$

$$= \frac{1}{2}(14)(12\pi)^2$$

$$= 9.9486\times 10^3\ \text{ft}\cdot\text{lb}$$

$$T_2 = 0$$

Work.

$$\theta = (100)(2\pi) = 628.32\ \text{rad}$$

$$M_D = F_f r = F_f\left(\frac{8}{12}\right)$$

$$U_{1\to 2} = -M_D\theta = -F_f\left(\frac{8}{12}\right)(628.32)$$

$$= -418.88F_f$$

Principle of work and energy.

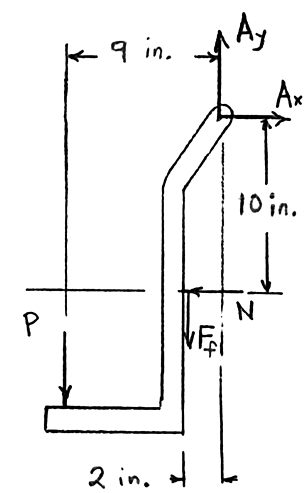
$$T_1 + U_{1\to 2} = T_2:\quad 9.9486\times 10^3 - 418.88F_f = 0$$

$$F_f = 23.75\ \text{lb}$$

Kinetic friction force.

$$F_f = \mu_k N$$

$$N = \frac{F_f}{\mu_k} = \frac{23.75}{0.35} = 67.859\ \text{lb}$$



Statics. $\quad +\circlearrowleft\ \Sigma M_A = 0:\quad (9\ \text{in.})P + (2\ \text{in.})F_f - (10\ \text{in.})N = 0$

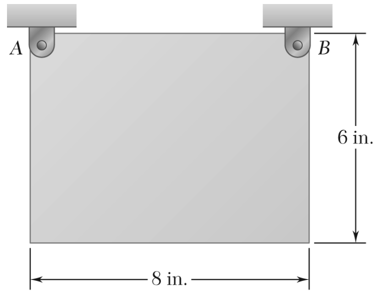
$$9P + (2)(23.75) - (10)(67.859) = 0$$

$$P = 70.12$$

$\mathbf{P} = 70.1\ \text{lb}\downarrow$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.137

A $6 \times 8-$in. rectangular plate is suspended by pins at A and B. The pin at B is removed and the plate swings freely about pin A. Determine (a) the angular velocity of the plate after it has rotated through 90°, (b) the maximum angular velocity attained by the plate as it swings freely.

SOLUTION

Let m be the mass of the plate.

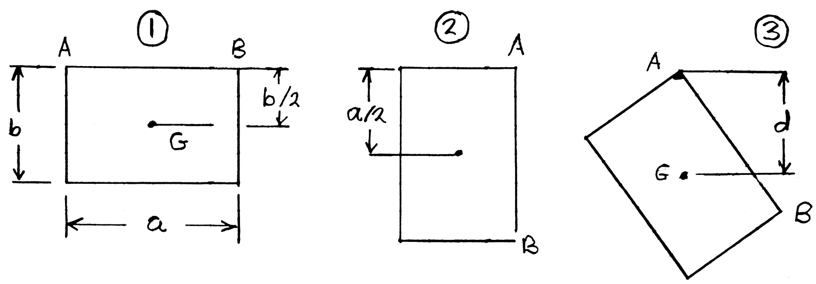
Dimensions: $a = 8 \text{ in.} = 0.66667 \text{ ft} \quad b = 6 \text{ in.} = 0.5 \text{ ft}$

Moment of inertia about A $\quad I_A = \frac{1}{3}m(a^2 + b^2)$

Position 1. Initial position. $\quad \omega_1 = 0$

Position 2. Plate has rotated about A through 90°.

Position 3. Mass center is directly below pivot A.



Potential energy. Use level A as datum.

$$V_1 = -\frac{mab}{2} \qquad V_2 = -\frac{mga}{2} \qquad V_3 = -mgd$$

Where $\quad d = \frac{1}{2}\sqrt{a^2 + b^2} = 0.41667 \text{ ft}$

Kinetic energy. $\quad T_1 = 0 \quad T_2 = \frac{1}{2}I_A\omega_2^2 \quad T_3 = \frac{1}{2}I_A\omega_3^2$

(a) 90° rotation. Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 - \frac{mgb}{2} = \frac{1}{2}\cdot\frac{1}{3}m(a^2 + b^2)\omega_2^2 + \frac{mga}{2}$$

$$\omega_2^2 = \frac{3g(a-b)}{a^2 + b^2} = \frac{(3)(32.2)(0.66667 - 0.5)}{(0.66667)^2 + (0.5)^2} = 23.184(\text{rad/s})^2$$

$$\boldsymbol{\omega}_2 = 4.81 \text{ rad/s} \circlearrowright \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

(*b*) ω is maximum. Conservation of energy.

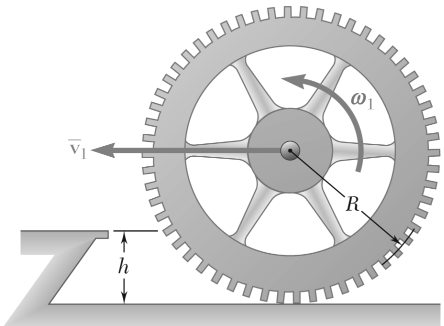
$$T_1 + V_1 = T_3 + V_3: \quad 0 - \frac{mgb}{2} = \frac{1}{2}\cdot\frac{1}{3}m(a^2 + b^2)\omega_3^2 - mgd$$

$$\omega_3^2 = \frac{g(6d - 3b)}{a^2 + b^2} = \frac{(32.2)(2.5 - 1.5)}{(0.66667)^2 + (0.5)^2} = 46.386\ (\text{rad/s})^2$$

$$\boldsymbol{\omega}_3 = 6.81\ \text{rad/s} \circlearrowright \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.138

The gear shown has a radius R = 150 mm and a radius of gyration $\bar{k} = 125$ mm. The gear is rolling without sliding with a velocity $\bar{\mathbf{v}}_1$ of magnitude 3 m/s when it strikes a step of height h = 75 mm. Because the edge of the step engages the gear teeth, no slipping occurs between the gear and the step. Assuming perfectly plastic impact, determine (a) the angular velocity of the gear immediately after the impact, (*b*) the angular velocity of the gear after it has rotated to the top of the step.

SOLUTION

Part (*a*) Conditions just after impact.

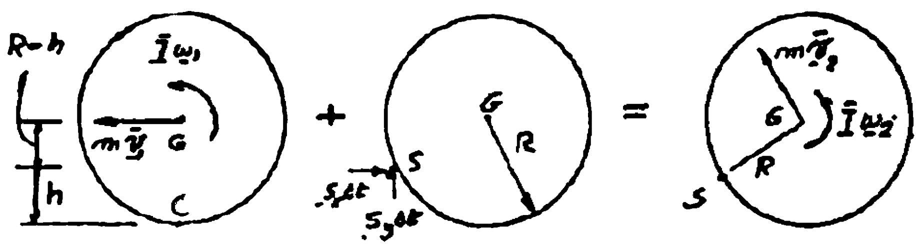
Kinematics. Just before impact, the contact Point C with the floor the instantaneous center of rotation of the gear.

$$\bar{\mathbf{v}}_1 = R\omega_1 \longleftarrow$$

Just after impact, Point S is the instantaneous center of rotation.

$$\mathbf{v}_2 = R\omega_2 \searrow \theta \quad \text{(perpendicular to } GS\text{)}$$

Principle of impulse and momentum.



+↺ Moments about S: $\quad m\bar{v}_2(R-h) + \bar{I}\omega_1 = m\bar{v}_2 R + \bar{I}\omega_2$

$$m(R\omega_1)(R-h) + m\bar{k}^2\omega_1 = m(R\omega_2)R + m\bar{k}^2\omega_2$$

$$[R(R-h)+\bar{k}^2]\omega_1 = (R^2+\bar{k}^2)\omega_2$$

$$\omega_2 = \frac{R^2+\bar{k}^2-Rh}{R^2+k^2}\omega_1 \qquad \omega_2 = \left[1-\frac{Rh}{R^2+k^2}\right]\omega_1 \tag{1}$$

Data: $\quad R = 150 \text{ mm}, \quad \bar{k} = 125 \text{ mm}, \quad v_1 = 3 \text{ m/s}, \quad h = 75 \text{ mm}$

$$\omega_1 = \frac{v_1}{R} = \frac{3 \text{ m/s}}{0.150 \text{ m}} = 20 \text{ rad/s}$$

Angular velocity.

From (1), $\quad \omega_2 = \left[1-\frac{(150)(75)}{(150^2+125^2)}\right](20 \text{ rad/s}) = 0.7049(20) \qquad \omega_2 = 14.10 \text{ rad/s} \circlearrowleft$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

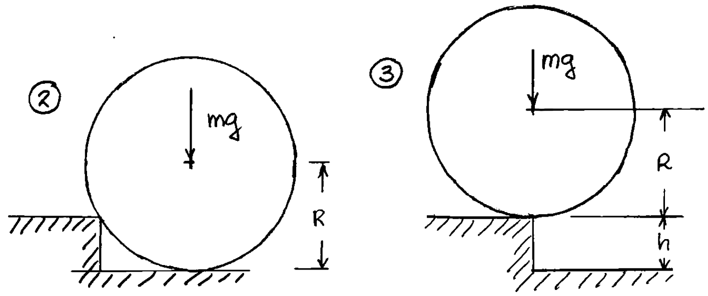
STUDENTS-HUB.com

Part (*b*) Conditions at the top of the step.

The gear pivots about the edge of the step. Use the principle of conservation of energy.

Position (2): The gear has just broken contact with the floor.

Position (3): The center of the gear is above the edge of the step.



Kinematics: (Rotation about *S*) $\bar{v} = \omega R$

Kinetic energy:
$$T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2$$
$$= \frac{1}{2}m\bar{k}^2\omega^2 + \frac{1}{2}mR^2\omega^2$$
$$= \frac{1}{2}m(\bar{k}^2 + R^2)\omega^2$$

Position (2):
$$T_2 = \frac{1}{2}m(\bar{k}^2 + R^2)\omega_2^2$$
$$V_2 = mgR$$

Position (3):
$$T_3 = \frac{1}{2}m(\bar{k}^2 + R^2)\omega_3^2$$
$$V_3 = mg(R+h)$$

Principle of conservation of energy: $T_2 + V_2 = T_3 + V_3$

$$\frac{1}{2}m(\bar{k}^2 + R^2)\omega_2^2 + mgR = \frac{1}{2}m(\bar{k}^2 + R^2)\omega_3^2 + mg(R+h)$$

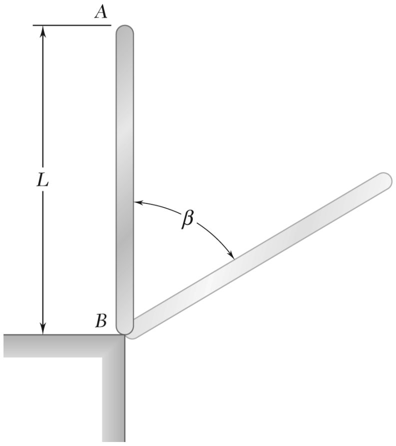
Angular velocity:

$$\omega_3^2 = \omega_3^2 - \frac{2gh}{\bar{k}^2 + R^2}$$
$$= (14.10 \text{ rad/s})^2 - \frac{(2)(9.81 \text{ m/s}^2)(0.075 \text{ m})}{(0.125 \text{ m})^2 + (0.150 \text{ m})^2}$$
$$= 160.21 \text{ rad}^2/\text{s}^2$$

$\omega_3 = 12.66 \text{ rad/s}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

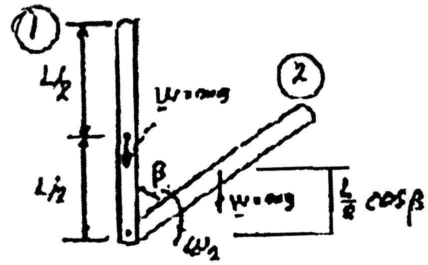
STUDENTS-HUB.com



PROBLEM 17.139

A uniform slender rod is placed at corner B and is given a slight clockwise motion. Assuming that the corner is sharp and becomes slightly embedded in the end of the rod, so that the coefficient of static friction at B is very large, determine (a) the angle β through which the rod will have rotated when it loses contact with the corner, (b) the corresponding velocity of end A.

SOLUTION



Position 1 $\quad T_1 = 0$

$$V_1 = mgh_1 = \frac{mgL}{2}$$

Position 2 $\quad V_2 = mgh_2 = \dfrac{mgL\cos\beta}{2}$

$$T_2 = \frac{1}{2}I\omega_2^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2$$

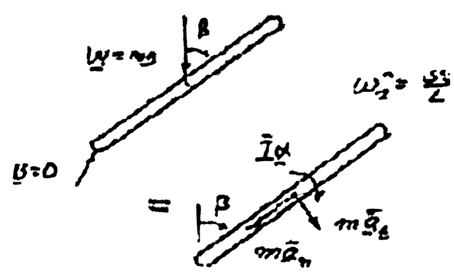
Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{mgL}{2} = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2 + \frac{mgL\cos\beta}{2}$$

$$\omega_2^2 = \frac{3g}{L}(1-\cos\beta) \qquad (1)$$

Normal acceleration of mass center.



$$a_n = \frac{L}{2}\omega_2^2 = \frac{3}{2}g(1-\cos\beta)$$

$$+\swarrow \Sigma F = +\Sigma F_{\text{eff}} = ma_n$$

$$mg\cos\beta = \frac{3}{2}mg(1-\cos\beta)$$

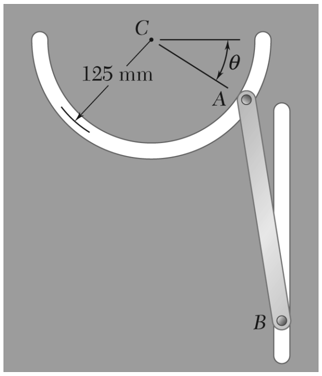
(a) <u>Angle β.</u> $\quad \dfrac{5}{2}\cos\beta = \dfrac{3}{2} \qquad \cos\beta = 0.6 \qquad \beta = 53.1° \blacktriangleleft$

From (1) $\quad \omega_2^2 = \dfrac{3g}{L}(1-0.6) = 1.2\dfrac{g}{L} \qquad \omega_2 = 1.09545\sqrt{\dfrac{g}{L}}$

(b) <u>Velocity of end A</u> $\quad v_A = L\omega_2 \qquad \mathbf{v}_A = 1.095\sqrt{gL}\ \nwarrow 53.1° \blacktriangleleft$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.140

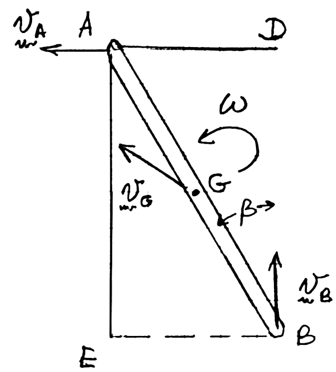
The motion of the slender 250-mm rod AB is guided by pins at A and B that slide freely in slots cut in a vertical plate as shown. Knowing that the rod has a mass of 2 kg and is released from rest when $\theta = 0$, determine the reactions at A and B when $\theta = 90°$.

SOLUTION

Let Point G be the mass center of rod AB.

$$m = 2 \text{ kg}$$
$$L = 0.25 \text{ m}$$
$$I_G = \frac{1}{12} mL^2 = 0.0104667 \text{ kg} \cdot \text{m}^2$$

Kinematics. $\theta = 90°$



$$\overline{AD} = R = 0.125 \text{ m}$$
$$AB = L = 0.25 \text{ m}$$
$$\sin\beta = \frac{R}{L} = \frac{1}{2} \qquad \beta = 30°$$
$$\overline{AG} = \frac{L}{2} = 0.125 \text{ m}$$
$$\overline{BG} = 0.125 \text{ m}$$

Point E is the instantaneous center of rotation of bar AB.

$$v_G = \frac{L}{2}\omega = 0.125\ \omega$$
$$v_A = (L\cos 30°)\omega = 0.21651\omega$$
$$v_B = (L\sin 30°)\omega = 0.125\omega$$

Use principle of conservation of energy to obtain the velocities when $\theta = 90°$:

Use level A as the datum for potential energy.

Position 1 $\theta = 0$

$$T_1 = 0$$
$$V_1 = -mg\frac{L}{2} = -(2)(9.81)(0.125) = -2.4525 \text{ J}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Position 2 $\quad\theta = 90°$

$$T_2 = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv_G^2$$
$$= \frac{1}{2}(0.0104667)\omega^2 + \frac{1}{2}(2)(0.125\omega)^2$$
$$= 0.0208583\omega^2$$
$$V_2 = -mg\left(R + \frac{L}{2}\cos\beta\right)$$
$$= -(2)(9.81)(0.125 + 0.125\cos 30°)$$
$$= -4.5764 \text{ J}$$

$T_1 + V_1 = T_2 + V_2:\quad 0 - 2.4525 = 0.0208583\omega^2 - 4.5764$

$$\omega^2 = 101.826 \text{ rad}^2/\text{s}^2$$
$$\omega = 10.091 \text{ rad/s}$$
$$v_A = (0.21651)(10.091)$$
$$= 2.1848 \text{ m/s}$$
$$v_G = (0.125)(10.091)$$
$$= 1.2614 \text{ m/s}$$

More kinematics: For Point A moving in the curved slot,

$$\mathbf{a}_A = (a_C)_x\mathbf{i} + \frac{v_A^2}{R}\mathbf{j}$$
$$= (a_C)_x\mathbf{i} + \frac{(2.1847)^2}{0.125}\mathbf{j}$$
$$= (a_C)_x\mathbf{i} + 38.1833\mathbf{j}$$

For the rod AB, $\quad \boldsymbol{\alpha} = \alpha\mathbf{k}, \quad \mathbf{v}_B = v_B\mathbf{j}$

$$\mathbf{r}_{A/B} = -L\sin 30°\mathbf{i} + L\cos 30°\mathbf{j}$$
$$= -0.125\mathbf{i} + 0.21651\mathbf{j}$$
$$\mathbf{r}_{G/B} = \frac{1}{2}\mathbf{r}_{A/B}$$
$$= -0.0625\mathbf{i} + 0.108253\mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} + \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$$
$$= a_B\mathbf{j} + \alpha\mathbf{k} \times (-0.125\mathbf{i} + 0.21651\mathbf{j})$$
$$-(10.091)^2(-0.125\mathbf{i} + 0.21651\mathbf{j})$$
$$= a_B\mathbf{j} - 0.125\alpha\mathbf{j} - 0.21651\alpha\mathbf{i} + 12.7285\mathbf{i} - 22.0468\mathbf{j}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Matching vertical components of $\mathbf{a}_A$

$$38.1833 = a_B - 0.125\alpha - 22.0468$$

$$a_B = 0.125\alpha + 60.2301$$

$$\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$

$$= \mathbf{a}_B + \alpha\mathbf{k}\times\mathbf{r}_{G/B} - \omega^2 r_{G/B}$$

$$= (0.125\alpha + 60.2324)\mathbf{j} + \alpha\mathbf{k}\times(-0.0625\mathbf{i} + 0.108253\mathbf{j})$$

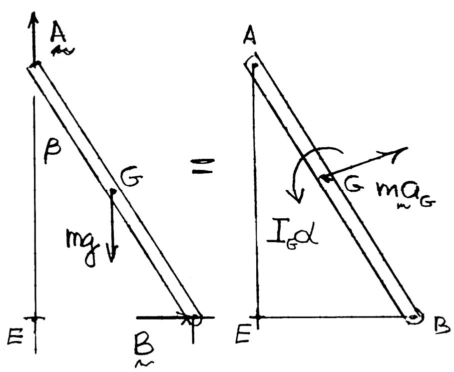
$$-(10.091)^2(-0.0625\mathbf{i} + 0.108253\mathbf{j})$$

$$= 0.125\alpha\mathbf{j} + 60.2301\mathbf{j} - 0.0625\alpha\mathbf{j} - 0.108253\alpha\mathbf{i}$$

$$+6.3643\mathbf{i} - 11.0232\mathbf{j}$$

$$\mathbf{a}_G = (-0.108253\alpha + 6.3643)\mathbf{i} + (0.0625\alpha + 49.2069)\mathbf{j}$$

Kinetics: Use rod AB as a free body.



$$+\circlearrowleft\ \Sigma\mathbf{M}_E = \Sigma(\mathbf{M}_E)_{\text{eff}}:$$

$$-mg\frac{L}{2}\sin\beta\mathbf{k} = I_G\boldsymbol{\alpha} + \mathbf{r}_{G/E}\times(m\mathbf{a}_G)$$

$$-(2)(9.81)(0.125)\sin 30°\mathbf{k}$$

$$= 0.0104667\boldsymbol{\alpha} + (0.0625\mathbf{i} + 0.10825\mathbf{j})\times(m\mathbf{a}_G)$$

$$-1.22625 = 0.0104667\alpha + 0.03125\alpha + 4.7730$$

$$0.0417167\alpha = -5.99925$$

$$\alpha = -143.808 \text{ rad/s}^2$$

$$\mathbf{a}_G = (-21.933 \text{ m/s}^2)\mathbf{i} + (40.2189 \text{ m/s}^2)\mathbf{j}$$

$\xrightarrow{+}\ \Sigma F_x = \Sigma(F_x)_{\text{eff}} = m(a_G)_x$: $\quad -B = (2)(-21.932) = 43.864$ N $\qquad \mathbf{B} = 43.9 \text{ N} \longrightarrow$ ◀

$+\uparrow\ \Sigma F_y = \Sigma(F_y)_{\text{eff}} = m(a_G)_y$: $\quad A - mg = (2)(40.4289) = 80.4378$

$A = (2)(9.81) + 80.4378 = 100.058 \qquad \mathbf{A} = 100.1 \text{ N}\uparrow$ ◀

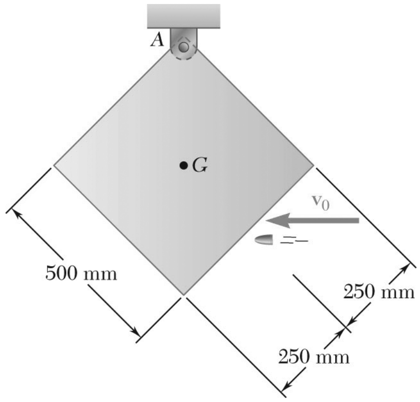
Check by considering

$$+\circlearrowleft\ \Sigma M_G = \Sigma M_{G\,\text{eff}}: \quad \circlearrowleft\ \Sigma M_G = (0.0625)A - 0.108253B = 1.5052 \text{ N}\cdot\text{m}\ \circlearrowright$$

$$\circlearrowleft\ \Sigma(M_G)_{\text{eff}} = I_G(-\alpha) = (0.0104667)(143.808) = 1.5052 \text{ N}\cdot\text{m}\ \circlearrowright$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.141

A 35-g bullet B is fired horizontally with a velocity of 400 m/s into the side of a 3-kg square panel suspended from a pin at A. Knowing that the panel is initially at rest, determine the components of the reaction at A after the panel has rotated 45°.

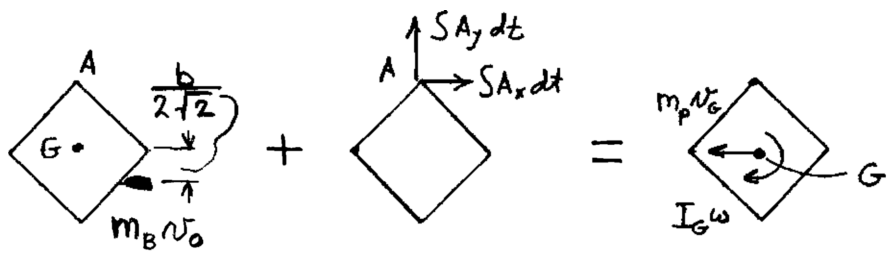
SOLUTION

Masses and moment of inertia.

$$m_B = 0.035 \text{ kg}$$
$$m_P = 3 \text{ kg}$$
$$b = 500 \text{ mm} = 0.5 \text{ m}$$
$$I_G = \frac{1}{6}m_P b^2 = \frac{1}{6}(3)(0.5)^2 = 0.125 \text{ kg}\cdot\text{m}^2$$

Note: The mass of the bullet is neglected in comparison with that of the plate after impact.

Analysis of impact: Use principle of impulse and momentum.



Kinematics: After impact the plate rotates about the pin at A.

$$v_G = \frac{b}{\sqrt{2}}\omega = \frac{0.5}{\sqrt{2}}\omega$$

+ Moments about A:

$$m_B v_0\left(\frac{b}{\sqrt{2}} + \frac{b}{2\sqrt{2}}\right) = I_G\omega + m_p v_G \frac{b}{\sqrt{2}}$$

$$\frac{3}{2\sqrt{2}} m_G v_0 b = (I_G + \frac{1}{2}m_P b^2)\omega$$

$$\frac{3}{2\sqrt{2}}(0.035)(400)(0.5) = \left[0.125 + \frac{1}{2}(3)(0.5)^2\right]\omega$$

$$\omega = 14.8492 \text{ rad/s}$$

$$v_G = \frac{0.5}{\sqrt{2}}(14.8492) = 5.25 \text{ m/s}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

Corresponding kinetic energy. $$T_1 = \frac{1}{2}I_G\omega^2 + \frac{1}{2}m_P v_G^2$$

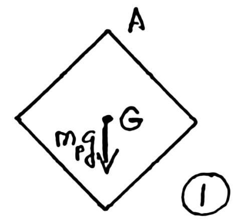
$$T_1 = \frac{1}{2}(0.125)(14.8492)^2 + \frac{1}{2}(3)(5.25)^2$$
$$= 55.125 \text{ J}$$

Plate rotates through $45°$.

Position 1: $\theta = 0°$

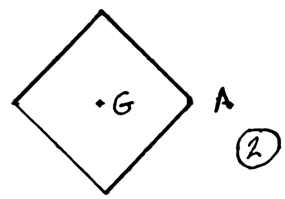
Use Point A as the datum for potential energy.

$$V_1 = -m_P g\frac{b}{\sqrt{2}}$$
$$= -(3)(9.81)\frac{0.5}{\sqrt{2}}$$
$$= -10.4051 \text{ J}$$



Position 2: $\theta = 90°$

$V_2 = 0$ since G is at level A.



$$T_2 = \frac{1}{2}I_G\omega_2^2 + \frac{1}{2}m_P(v_G)_2^2$$
$$= \frac{1}{2}(0.125)\omega_2^2 + \frac{1}{2}(3)\left(\frac{0.5}{\sqrt{2}}\omega_2\right)^2$$
$$= 0.25\omega_2^2$$

Principle of conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$
$$55.125 \text{ J} - 10.4051 \text{ J} = 0.25\omega_2^2 + 0$$
$$\omega_2^2 = 178.879 \text{ (rad/s}^2)$$
$$\omega_2 = 13.3746 \text{ rad/s}$$

Analysis at $90°$ rotation. $\boldsymbol{\alpha} = \alpha \circlearrowleft$

Kinematics: $$(a_G)_t = \frac{b}{\sqrt{2}}\alpha = \frac{0.5}{\sqrt{2}}\alpha \qquad (\mathbf{a}_G)_t = 0.35355\alpha \downarrow$$

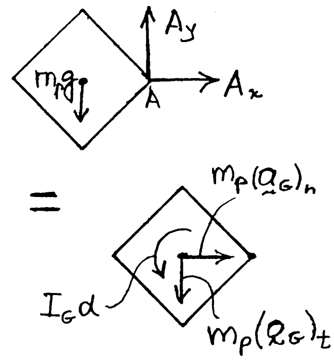
$$(a_G)_n = \frac{b}{\sqrt{2}}\omega^2$$
$$= \frac{(0.5)(178.879)}{\sqrt{2}} \qquad (\mathbf{a}_G)_n = 63.2434 \text{ m/s}^2 \longrightarrow$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.141 (Continued)

Kinematics: Use the free body diagram of the plate.



$$+\circlearrowleft\ \Sigma M_A = \Sigma(M_A)_{\text{eff}}:\quad m_P g\frac{b}{\sqrt{2}} = I_G\alpha + m_P(a_G)_t\frac{b}{\sqrt{2}}$$

$$= \left(I_G + \frac{1}{2}m_P b^2\right)\alpha$$

$$\frac{(3)(9.81)(0.5)}{\sqrt{2}} = \left[0.125 + \frac{1}{2}(3)(0.5)^2\right]\alpha$$

$$\boldsymbol{\alpha} = 20.810 \text{ rad/s}^2 \circlearrowleft$$

$$(\mathbf{a}_G)_t = 7.3574 \text{ m/s}^2 \downarrow$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}:\quad A_x = m_P(a_G)_n = (3)(63.2434)$$

$$\mathbf{A}_x = 189.7 \text{ N} \longrightarrow \blacktriangleleft$$

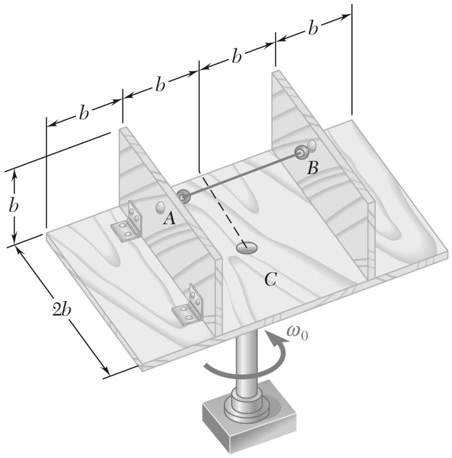
$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}:\quad A_y - m_P g = -m_P(a_G)_y$$

$$A_y - (3)(9.81) = (3)(-7.3574)$$

$$\mathbf{A}_y = 7.36 \text{ N} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

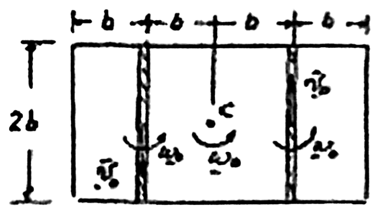


PROBLEM 17.142

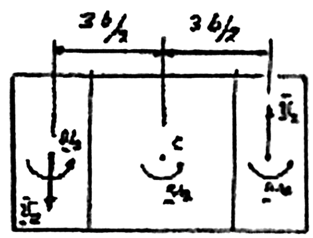
Two panels A and B are attached with hinges to a rectangular plate and held by a wire as shown. The plate and the panels are made of the same material and have the same thickness. The entire assembly is rotating with an angular velocity $\boldsymbol{\omega}_0$ when the wire breaks. Determine the angular velocity of the assembly after the panels have come to rest against the plate.

SOLUTION

Geometry and kinematics:



Panels in up position



Panels in down position

$$\bar{v}_0 = b\omega_0 \qquad \bar{v}_2 = \frac{3}{2}b\omega_0$$

Let ρ = mass density, t = thickness

Plate:

$$m_{\text{plate}} = \rho t(2b)(4b) = 8\rho t b^2$$

$$\bar{I}_{\text{plate}} = \frac{1}{12}(8\rho t b^2)[(2b)^2 + (4b)^2] = \frac{160}{12}\rho t b^4 = \frac{40}{3}\rho t b^4$$

Each panel:

$$m_{\text{panel}} = \rho t(b)(2b) = 2\rho t b^2$$

<u>Panel in up position</u>

$$(\bar{I}_{\text{panel}})_0 = \frac{1}{12}(2\rho t b^2)(2b)^2 = \frac{8}{12}\rho t b^4 = \frac{2}{3}\rho t b^4$$

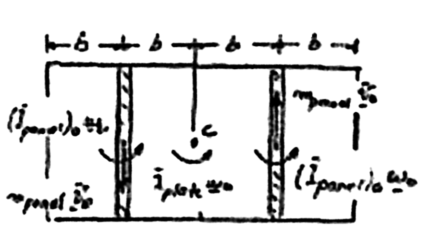
PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

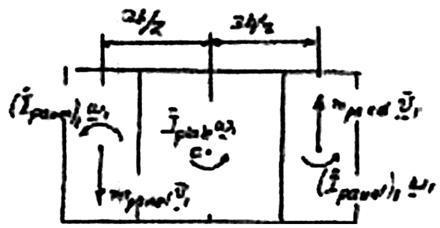
<u>Panel in down position</u>

$$(\overline{I}_{\text{panel}})_1 = \frac{1}{12}(2\rho t b^2)[b^2 + (2b)^2]$$
$$= \frac{10}{12}\rho t b^4$$
$$= \frac{5}{6}\rho t b^4$$

Conservation of angular momentum about the vertical spindle.



<u>Initial momenta</u>



<u>Final momenta</u>

$+\circlearrowleft$ Moments about C:

$$\overline{I}_{\text{plate}}\omega_0 + 2[(\overline{I}_{\text{panel}})_0\omega_0 + m_{\text{panel}}v_0(b)] = \overline{I}_{\text{plate}}\omega_1 + 2\left[(\overline{I}_{\text{panel}})_1\omega_1 + m_{\text{panel}}v_1\left(\frac{3b}{2}\right)\right]$$

$$\frac{40}{3}\rho t b^4\omega_0 + 2\left[\frac{2}{3}\rho t b^4\omega_0 + (2\rho t b^2)(b\omega_0)b\right] = \frac{40}{3}\rho t b^4\omega_0 + 2\left[\frac{5}{6}\rho t b^4\omega_1 + 2\rho t b^2\left(\frac{3}{2}b\omega_0\right)\left(\frac{3}{2}b\right)\right]$$

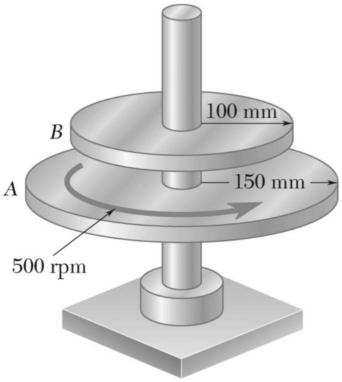
$$\left[\frac{40}{3} + \frac{4}{3} + 4\right]\rho t b^4\omega_0 = \left[\frac{40}{3} + \frac{10}{6} + 9\right]\rho t b^4\omega_1$$

$$\frac{56}{3}\omega_0 = 24\omega_1$$

$$\omega_1 = \frac{56}{(3)(24)} \qquad \omega_1 = \frac{7}{9}\omega_2 \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.143

Disks *A* and *B* are made of the same material and are of the same thickness; they can rotate freely about the vertical shaft. Disk *B* is at rest when it is dropped onto disk *A*, which is rotating with an angular velocity of 500 rpm. Knowing that disk *A* has a mass of 8 kg, determine (*a*) the final angular velocity of the disks, (*b*) the change is kinetic energy of the system.

SOLUTION

Disk *A*: $m_A = 8 \text{ kg} \quad r_A = 0.15 \text{ m}$

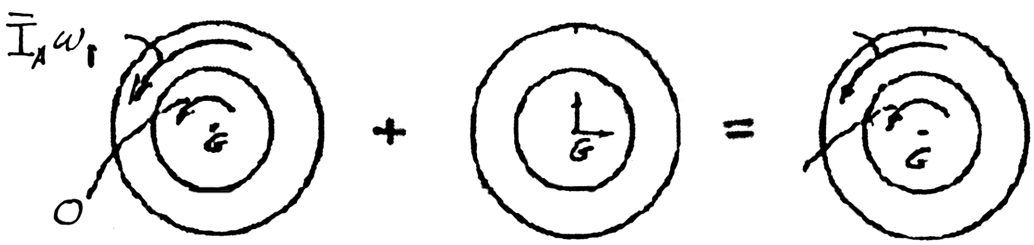
$$\bar{I}_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2}(8)(0.15)^2 = 0.0900 \text{ kg} \cdot \text{m}^2$$

Disk *B*: $r_B = 0.100 \text{ m}$

$$m_B = m_A \left(\frac{r_B}{r_A}\right)^2 = (8)\left(\frac{0.10}{0.15}\right)^2 = 3.5556 \text{ kg}$$

$$\bar{I}_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2}(3.5556)(0.10)^2 = 0.017778 \text{ kg} \cdot \text{m}^2$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

$+\circlearrowright$ Moments about *B*: $\bar{I}_A \omega_0 + 0 + 0 = \bar{I}_A \omega_2 + \bar{I}_B \omega_2$

$$\omega_2 = \frac{\bar{I}_A}{\bar{I}_A + \bar{I}_B}\omega_1 = \frac{0.09}{0.10778}\omega_1 = 0.83505\omega_1$$

Initial angular velocity of disk *A*: $\omega_1 = 500 \text{ rpm} = 52.36 \text{ rad/s}$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

PROBLEM 17.143 (Continued)

(*a*) Final angular velocity of system: $\omega_2 = (0.83505)(52.36)$

$\omega_2 = 43.723 \text{ rad/s}$ $\quad\quad \omega_2 = 418 \text{ rpm}$ ◀

Initial kinetic energy: $T_1 = \frac{1}{2}\bar{I}_A \omega_1^2$

$T_1 = \frac{1}{2}(0.09)(52.36)^2 = 123.37 \text{ J}$

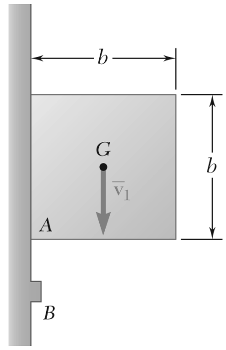
Final kinetic energy: $T_2 = \frac{1}{2}(\bar{I}_A + \bar{I}_B)\omega_2^2$

$T_2 = \frac{1}{2}(0.10778)(43.723)^2 = 103.02 \text{ J}$

(*b*) Change in energy: $T_2 - T_1 = -20.35 \text{ J}$ $\quad\quad \Delta T = -20.4 \text{ J}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

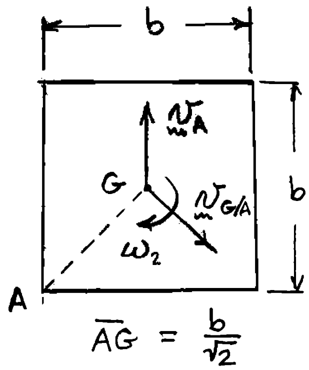


PROBLEM 17.144

A square block of mass m is falling with a velocity $\bar{\mathbf{v}}_1$ when it strikes a small obstruction at B. Knowing that the coefficient of restitution for the impact between corner A and the obstruction B is $e = 0.5$, determine immediately after the impact (*a*) the angular velocity of the block, (*b*) the velocity of its mass center G.

SOLUTION

Moments of inertia. $\bar{I} = \frac{1}{6}mb^2$



Kinematics. Before impact, block is translating.

$$\mathbf{v}_1 = v_1 \downarrow \qquad \omega_1 = 0$$

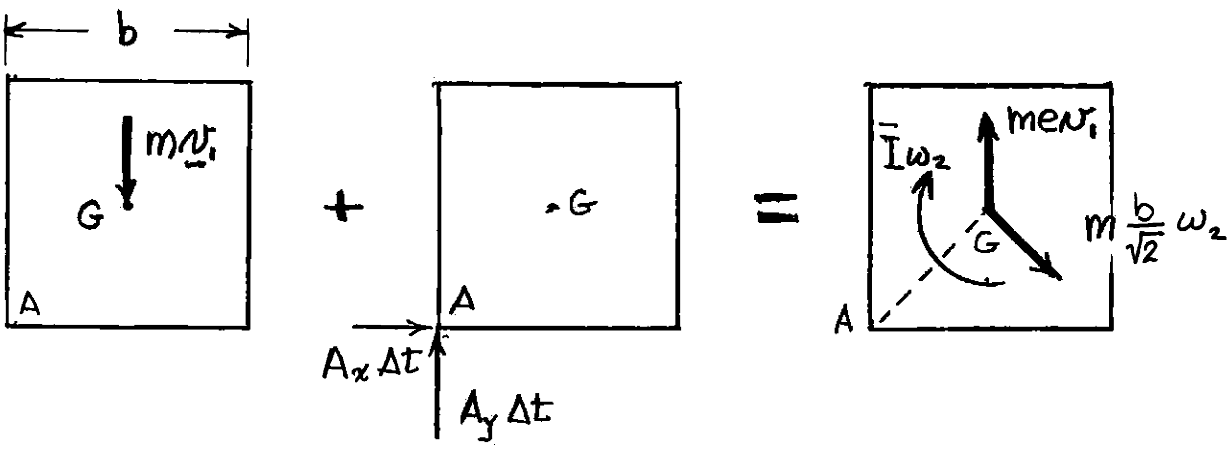
After impact,

$$\mathbf{v}_A = ev_1 \uparrow$$

$$\bar{\mathbf{v}}_2 = \mathbf{v}_A + \mathbf{v}_{G/A}$$

$$= [ev_1 \uparrow] + \left[\frac{b}{\sqrt{2}}\omega_2 \searrow 45°\right] \qquad \overline{AG} = \frac{b}{\sqrt{2}} \qquad (1)$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

+↻ Moments about A:

$$m\bar{v}_1\frac{b}{2} = \bar{I}\omega_1 - me\bar{v}_1\frac{b}{2} + m\frac{b}{\sqrt{2}}\omega_2\frac{b}{\sqrt{2}}$$

$$\bar{I} = \frac{1}{6}mb^2$$

$$m\bar{v}_1\frac{b}{2} = \frac{1}{6}mb^2\omega_2 - me\bar{v}_1\frac{b}{2} + m\left(\frac{b}{\sqrt{2}}\right)^2\omega_2$$

$$\frac{(1+e)\bar{v}_1 b}{2} = \frac{2}{3}b^2\omega_2$$

(*a*) Angular velocity.

$$\omega_2 = \frac{3}{4b}(1+e)v_1$$

$$\boldsymbol{\omega}_2 = 1.125\frac{v_1}{b} \circlearrowright \blacktriangleleft$$

(*b*) Velocity of the mass center.

From Eq. (1),

$$\bar{\mathbf{v}}_2 = e\bar{v}_1\uparrow + \left[\frac{b}{\sqrt{2}}\frac{3}{4b}(1+e)v_1 \searrow 45°\right]$$

$$= e\bar{v}_1\uparrow + \left[\frac{3}{4\sqrt{2}}(1+e)v_1\sin 45°\downarrow\right] + \left[\frac{3}{4\sqrt{2}}(1+e)v_1\cos 45°\longrightarrow\right]$$

$$= e\bar{v}_1\uparrow + \left[\frac{3}{8}(1+e)v_1\downarrow\right] + \left[\frac{3}{8}(1+e)v_1\longrightarrow\right]$$

$$= \left[\left(\frac{5}{8}ev_1 - \frac{3}{8}v_1\right)\uparrow\right] + \left[\frac{3}{8}(1+e)v_1\longrightarrow\right]$$

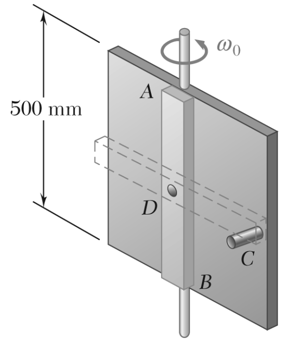
$$= \left[\left(\frac{5}{8}(0.5) - \frac{3}{8}\right)v_1\uparrow\right] + \frac{3}{8}(1+0.5)v_1\longrightarrow\Big]$$

$$= [-0.0625\uparrow] + [0.5625\longrightarrow]$$

$$\bar{\mathbf{v}}_2 = 0.566\text{ m/s} \searrow 6.34° \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.145

A 3-kg bar AB is attached by a pin at D to a 4-kg square plate, which can rotate freely about a vertical axis. Knowing that the angular velocity of the plate is 120 rpm when the bar is vertical, determine (*a*) the angular velocity of the plate after the bar has swung into a horizontal position and has come to rest against pin C, (*b*) the energy lost during the plastic impact at C.

SOLUTION

Moments of inertia about the vertical centroidal axis.

Square plate. $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(4)(0.500)^2 = 0.083333 \text{ kg} \cdot \text{m}^2$

Bar AB vertical. $\bar{I} = \text{approximately zero}$

Bar AB horizontal. $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(3)(0.500)^2 = 0.0625 \text{ kg} \cdot \text{m}^2$

Position 1. Bar AB is vertical. $I_1 = 0.083333 \text{ kg} \cdot \text{m}^2$

Angular velocity. $\omega_1 = 120 \text{ rpm} = 4\pi \text{ rad/s}$

Angular momentum about the vertical axis.

$$(H_O)_1 = I_1\omega_1 = (0.083333)(4\pi) = 1.04720 \text{ kg} \cdot \text{m}^2/\text{s}$$

Kinetic energy. $T_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(0.083333)(4\pi)^2 = 6.5797 \text{ J}$

Position 2. Bar AB is horizontal. $I_2 = 0.145833 \text{ kg} \cdot \text{m}^2$

$$(H_O)_2 = I_2\omega_2 = 0.145833\omega_2$$

Conservation of angular momentum. $(H_O)_1 = (H_O)_2$:

$$1.04720 = 0.145833\omega_2 \qquad \omega_2 = 7.1808 \text{ rad/s}$$

(*a*) Final angular velocity. $\omega_2 = 68.6 \text{ rpm}$ ◀

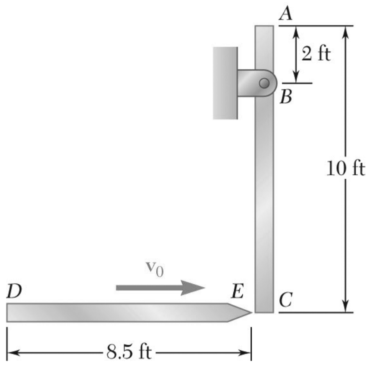
(*b*) Loss of energy.

$$T_1 - T_2 = T_1 - \frac{1}{2}I_2\omega_2^2 = 6.5797 - \frac{1}{2}(0.145833)(7.1808)^2$$

$T_1 - T_2 = 2.82 \text{ J}$ ◀

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



PROBLEM 17.146

A 1.8-lb javelin *DE* impacts a 10-lb slender rod *ABC* with a horizontal velocity of $\mathbf{v}_0$ = 30 ft/s as shown. Knowing that the javelin becomes embedded into the end of the rod at Point *C* and does not penetrate very far into it, determine immediately after the impact (*a*) the angular velocity of the rod *ABC* after the impact, (*b*) the components of the reaction at *B*. Assume that the javelin and the rod move as a single body after the impact.

SOLUTION

Masses and moments of inertia.

$$m_{AC} = \frac{W_{AC}}{g} = \frac{10 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.31056 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_{DE} = \frac{W_{DE}}{g} = \frac{1.8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.05590 \text{ lb} \cdot \text{s}^2/\text{ft}$$

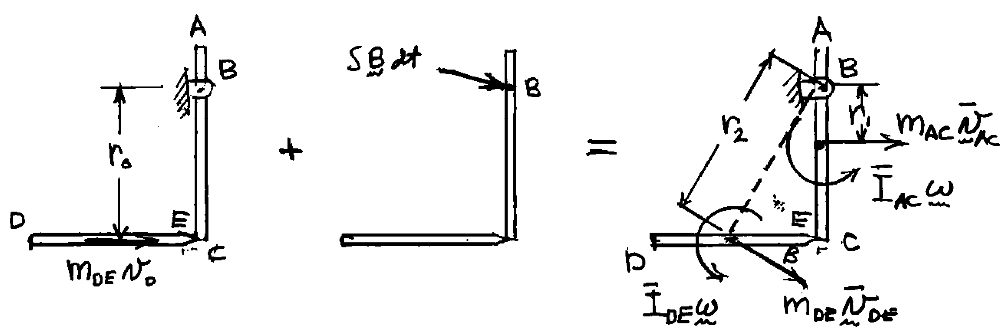
$$\bar{I}_{AC} = \frac{1}{12} m_{AC} L_{AC}^2 = \frac{1}{12}(0.31056)(10)^2 = 2.5880 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\bar{I}_{DE} = \frac{1}{12} m_{DE} L_{DE}^2 = \frac{1}{12}(0.05590)(8.5)^2 = 0.3365 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

(*a*) *Angular velocity immediately after the impact.*

Principle of impulse and momentum.

$$\boldsymbol{\omega}_{DE} = \boldsymbol{\omega}_{AB} = \boldsymbol{\omega} = \omega \circlearrowright$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta$_2$**

$+\circlearrowright$ Moments about *B*: $\quad m_{DE}v_0r_0 + 0 = \bar{I}_{AC}\omega + m_{AC}\bar{v}_{AC}r_1 + \bar{I}_{DE}\omega + m_{DE}\bar{v}_{DE}r_2$

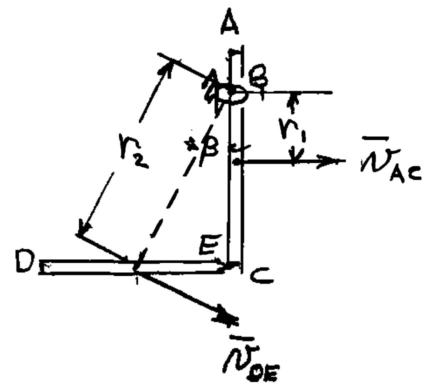
where $\quad v_0 = 30 \text{ ft/s} \quad r_0 = 10 \text{ ft} - 2 \text{ ft} = 8 \text{ ft}$

$$r_1 = 5 \text{ ft} - 2 \text{ ft} = 3 \text{ ft}$$

$$r_2 = \sqrt{(4.25 \text{ ft})^2 + (8 \text{ ft})^2} = 9.0588 \text{ ft}$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com



Kinematics: (Rotation about B)

$$\bar{v}_{AC} = r_1\omega$$

$$\bar{v}_{DE} = r_2\omega$$

$$\tan\beta = \frac{4.25\text{ ft}}{8\text{ ft}} \qquad \beta = 27.98°$$

$$m_{DE}v_0r_0 = \bar{I}_{AC}\omega + m_{AC}r_1^2\omega + \bar{I}_{DE}\omega + m_{DE}r_2^2\omega$$
$$= I_B\omega$$

where
$$I_B = \bar{I}_{AC} + m_{AC}r_1^2 + \bar{I}_{DE} + m_{DE}r_2^2$$
$$= 2.5880 + (0.31056)(3)^2 + 0.3365 + (0.05590)(9.0588)^2$$
$$= 10.3068 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\omega = \frac{m_{DE}v_0r_0}{I_B} = \frac{(0.05590 \text{ lb}\cdot\text{s}^2/\text{ft})(30 \text{ ft/s})(8 \text{ ft})}{10.3068 \text{ lb}\cdot\text{s}^2\cdot\text{ft}}$$
$$= 1.30167 \text{ rad/s}$$

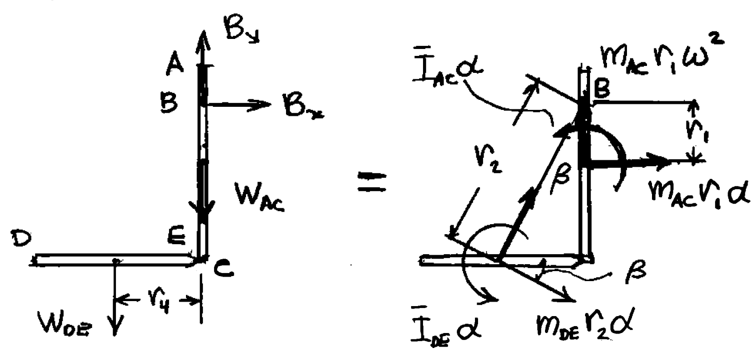
$\boldsymbol{\omega} = 1.302$ rad/s ↻ ◀

Accelerations: $\boldsymbol{\alpha} = \alpha$ ↻

$$\bar{a}_{AD} = [r_1\alpha \longrightarrow] + [r_1\omega^2 \uparrow]$$

$$\bar{a}_{DE} = [r_2\alpha \nwarrow \beta] + [r_2\omega^2 \nearrow \beta]$$

Free body and kinetic diagrams.



+↻ Moments about B:

$$W_{DE}r_4 = \bar{I}_{AC}\alpha + m_{AC}(\bar{a}_{AC})_t r_1 + \bar{I}_{DE}\alpha + m_{DE}(\bar{a}_{DE})_t r_2$$
$$= \bar{I}_{AC}\alpha + m_{AC}r_1^2\alpha + \bar{I}_{DE}\alpha + m_{DE}r_2^2\alpha$$
$$= I_B\alpha$$

$$\alpha = \frac{W_{DE}r_4}{I_B} = \frac{(1.8 \text{ lb})(4.25)}{10.3068 \text{ lb}\cdot\text{s}^2\cdot\text{ft}}$$
$$= 0.74223 \text{ rad/s}^2$$

$\boldsymbol{\alpha} = 0.742$ rad/s^2 ↻

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com

(*b*) *Components of reaction at B.*

$\Sigma F = \Sigma m\bar{a}$:

$$\mathbf{B} + [W_{AC}\downarrow] + [W_{DE}\downarrow] = [m_{AC} r_1 \alpha \longrightarrow] + [m_{AC} r_1 \omega^2 \uparrow] + [m_{DE} r_2 \alpha \searrow \beta] + [m_{DE} r_2 \omega^2 \nearrow \beta]$$

Component $\xrightarrow{+}$:

$$B_x = m_{AC} r_1 \alpha + m_{DE}(r_2 \cos\beta)\alpha + m_{DE}(r_2 \sin\beta)\omega^2$$
$$= (0.31056)(3)(0.74223) + (0.05590)(8)(0.74223) + (0.05590)(4.25)(1.30167)^2$$
$$= 0.6915 + 0.3319 + 0.4025$$

$$\mathbf{B}_x = 1.426 \text{ lb} \longrightarrow \blacktriangleleft$$

Component $+\uparrow$:

$$B_y - W_{AC} - W_{DC} = m_{AC} r_1 \omega^2 - m_{DE}(r_1 \sin\beta)\alpha + m_{DE}(r_2 \cos\beta)\omega^2$$
$$B_y - 10 - 1.8 = (0.31056)(3)(1.30167)^2 - (0.05590)(4.25)(0.74223) + (0.05590)(8)(1.30167)^2$$
$$B_y = 11.8 + 1.5785 - 0.1763 + 0.7577$$

$$\mathbf{B}_y = 13.96 \text{ lb} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. *No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.*

STUDENTS-HUB.com