

10) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{\sqrt{n^2+2}} \cdot \frac{1}{\frac{1}{n^2}}$$

$$\lim_{n \rightarrow \infty} \sqrt{\frac{n^2+n}{n^2+2}} = 1$$

Hence, $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$ div by LCT

22) $\sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}}$

$$\lim_{n \rightarrow \infty} \frac{n+1}{n^2 \sqrt{n}} \cdot \frac{n^{\frac{3}{2}}}{1} = \lim_{n \rightarrow \infty} \frac{n^{\frac{5}{2}} + n^{\frac{3}{2}}}{n^{\frac{5}{2}}} = 1$$

Hence, $\sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}}$ conv by LCT

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$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}} \text{ conv } P > 1 \text{ P-series}$$

36) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \text{ conv } P > 1 \text{ P-series}$$

$$\lim_{n \rightarrow \infty} \frac{n+2^n}{n^2 2^n} \cdot \frac{n^2}{1} = \lim_{n \rightarrow \infty} \frac{n+2^n}{2^n} = \lim_{n \rightarrow \infty} \frac{1 + 2^n \ln 2}{2^n \ln 2} = \lim_{n \rightarrow \infty} \frac{2^n (\ln 2)^2}{2^n (\ln 2)^2} = 1$$

Hence, $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$ conv by LCT

46) $\sum_{n=1}^{\infty} \tan \frac{1}{n}$

$$u = \frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ Harmonic series div}$$

$$\lim_{n \rightarrow \infty} \frac{\tan \frac{1}{n}}{\frac{1}{n}} = \lim_{u \rightarrow 0^+} \frac{\tan u}{u} = \lim_{u \rightarrow 0^+} \sec^2 u = 1$$

Hence, $\sum_{n=1}^{\infty} \tan \frac{1}{n}$ div by LCT

$$8] \sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{\sqrt{n^2+3}}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}+1}{\sqrt{n^2+3}} \cdot \frac{n^{\frac{1}{2}}}{1} = \lim_{n \rightarrow \infty} \frac{n+\sqrt{n}}{\sqrt{n^2+3}} = 1$$

Hence, $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{\sqrt{n^2+3}}$ div by LCT

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$$

div $p < 1$
p-series

$$15] \sum_{n=2}^{\infty} \frac{1}{\ln n}$$

$$\sum_{n=2}^{\infty} \frac{1}{n} \rightarrow \text{div}$$

$$\frac{1}{n} > \frac{1}{\ln n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\ln n} \cdot \frac{n}{1} = \lim_{n \rightarrow \infty} \frac{n}{\ln n} = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} n = \infty$$

Hence, $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ div by LCT

$$18] \sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$$

$$\frac{1}{n} < \frac{3}{n+\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ div}$$

Hence, $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$ div by DCT

27] $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$

$$\frac{1}{\ln(\ln n)} \Rightarrow \frac{1}{\ln n} > \frac{1}{n}$$

Hence, $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$

div by DCT

$$\sum_{n=3}^{\infty} \frac{1}{n} \text{ div}$$

28] $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^3}$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \text{ conv}$$

$$\lim_{n \rightarrow \infty} \frac{(\ln n)^n}{n^3} \cdot \frac{n^2}{1} = \lim_{n \rightarrow \infty} \frac{(\ln n)^n}{n} = \lim_{n \rightarrow \infty} \frac{2 \ln n (\frac{1}{n})}{1}$$

$$= 2 \lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0 \quad \text{Hence, } \sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^3} \text{ conv by LCT}$$

32] $\sum_{n=2}^{\infty} \frac{\ln(n+1)}{n+1}$

$$u = \ln(x+1)$$

$$du = \frac{1}{x+1} dx$$

$$\int u du = \frac{1}{2} u^2$$

$$\int_2^{\infty} \frac{\ln(x+1)}{x+1} = \lim_{b \rightarrow \infty} \left. \frac{1}{2} u^2 \right|_{\ln 3}^b$$

$$= \lim_{b \rightarrow \infty} \frac{1}{2} (b^2 - (\ln 3)^2) = \infty$$

Hence, $\sum_{n=2}^{\infty} \frac{\ln(n+1)}{n+1}$ div by Integral test

40] $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{3^n + 4^n}$

$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{3^n + 4^n}$ ~~$\lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{1 + (\frac{4}{3})^n}$~~

$\sum_{n=1}^{\infty} (\frac{3}{4})^n$

$r = \frac{3}{4} \in (-1, 1)$
Conv Geometric Series

$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{3^n + 4^n} = \lim_{n \rightarrow \infty} \frac{8^n + 12^n}{9^n + 12^n} = \lim_{n \rightarrow \infty} \frac{1 + (\frac{8}{12})^n}{1 + (\frac{9}{12})^n} = 1$

Hence, $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{3^n + 4^n}$ conv by LCT

43] $\sum_{n=2}^{\infty} \frac{1}{n!}$

$\frac{1}{n!} \leq \frac{1}{n(n-1)} \quad n \geq 2$

$\sum_{n=2}^{\infty} (\frac{1}{n-1} - \frac{1}{n}) \rightarrow$ conv by n^{th} Partial Sum test

$\frac{A}{n} + \frac{B}{n+1} = -\frac{1}{n} + \frac{1}{n+1}$

$S_n = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots$

$S_n = 1 - \frac{1}{n}$

Hence, $\sum_{n=2}^{\infty} \frac{1}{n!}$ conv by ~~Partial Sum test~~ DCT

$\lim_{n \rightarrow \infty} S_n = 1$

52] $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv $p > 1$
p-series

$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n^2} \cdot \frac{n^2}{1} = 1$

Hence, $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2}$ conv by LCT