$\boxed{6.1}$ Uniform Probability Distribution "Continuous"

- Discreate Prob. distributions like 1) binomial prob. distribution
  2) Poisson prob. Distribution

- Continuous Prob. distribution like 1) Uniform Prob. distribution
  2) Normal prob. distribution
  3) Exponential prob. distribution

- Whenever the prob. is proportional to the length of the interval, the random variable is uniformly distributed.

- Uniform prob. density function

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

- The expected value and variance for the uniform continuous prob. distribution are:

$$E(x) = \frac{a+b}{2} \quad \text{and} \quad Var(x) = \frac{(b-a)^2}{12}$$

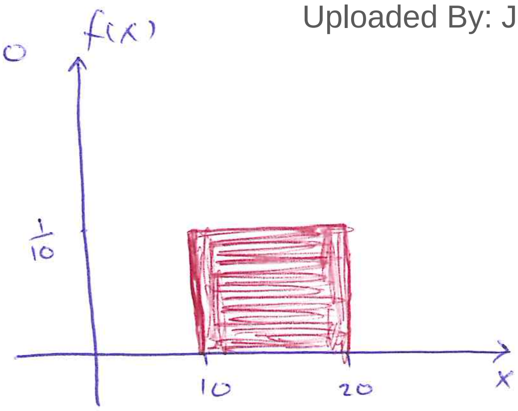
- The prob. of any single point is zero.

Example (Q2 page 229) The random variable x is known to be uniformly distributed between 10 and 20.

$\boxed{a}$ Show the graph of the prob. density function.

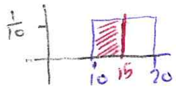
$$f(x) = \begin{cases} \frac{1}{20-10} & \text{for } 10 \le x \le 20 \\ 0 & \text{elsewhere} \end{cases}$$

$$= \begin{cases} \frac{1}{10} & \text{for } 10 \le x \le 20 \\ 0 & \text{elsewhere} \end{cases}$$



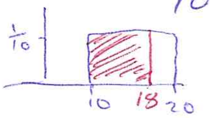
$\boxed{\acute{a}}$ Compute $P(x = 15) = 0$ "No area"

[b] Compute $P(x < 15)$



$P(x < 15) = P(10 \le x < 15) = \frac{1}{10}(15 - 10) = \frac{5}{10} = 0.5$

[c] Compute $P(12 \le x \le 18)$



$= \frac{1}{10}(18 - 12) = \frac{6}{10} = 0.6$

[d] Compute $E(x) = \frac{a+b}{2} = \frac{10+20}{2} = 15$

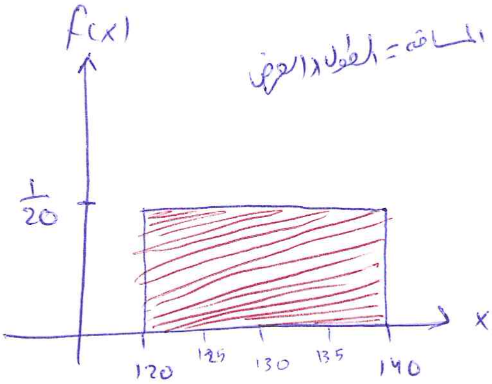
[e] Compute $Var(x) = \frac{(b-a)^2}{12} = \frac{(20-10)^2}{12} = \frac{10^2}{12} = \frac{100}{12} = 8.33$

Example: Consider the random variable x representing the flight time of an airplane traveling from Chicago to New York. Suppose x is known to be uniformly distributed between 120 min to 140 min.

[a] show the graph of the prob. density function

$$f(x) = \begin{cases} \frac{1}{140-120} & \text{for } 120 \le x \le 140 \\ 0 & \text{elsewhere} \end{cases}$$

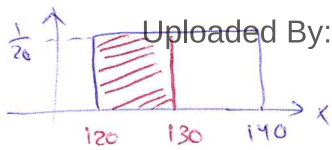
$$= \begin{cases} \frac{1}{20} & \text{for } 120 \le x \le 140 \\ 0 & \text{elsewhere} \end{cases}$$



المساحة = الطول × العرض

[b] what is the prob. that the flight time is between 120 and 130 minutes?

$P(120 \le x \le 130) = \frac{1}{20}(130 - 120) = \frac{10}{20} = 0.5$



[c] what is the prob. that the flight time is between 128 and 136 minutes?

$P(128 \le x \le 136) = \frac{1}{20}(136 - 128) = \frac{8}{20} = 0.40$

[d] $P(120 \le x \le 140) = \frac{1}{20}(140 - 120) = \frac{20}{20} = 1$

$\boxed{f(x) \ge 0 \text{ and } \sum f(x) = 1}$

Similar to discreate.

[e] What is the expected time to arrive New York?

$$E(x) = \frac{a+b}{2} = \frac{120+140}{2} = 130 \text{ minutes}$$

[f] what is the variance in minutes?

$$Var(x) = \frac{(b-a)^2}{12} = \frac{(140-120)^2}{12} = \frac{(20)^2}{12} = \frac{400}{12} = 33.33 \text{ minute}$$

[g] what is the prob. that x is exactly 135 minutes

$$p(x=135) = p(135 \le x \le 135) = \frac{1}{20}[135-135) = 0$$

Differences between

Continuous random Variables X	discrete random variables X
1) Prob. of single value is zero	• Prob. of single value takes any value between 0 and 1
2) The random variable X takes values in interval $=[a,b]$	• The random variable X takes values $0, 1, 2, 3, \ldots$
3) $P(a \le x \le b)$ = area under the graph between a and b $= p(a < x < b)$	• $P(a \le x \le b) \neq p(a < x < b)$

