$y = \theta$ ← $\Sigma F_x = m\ddot{x}_2$

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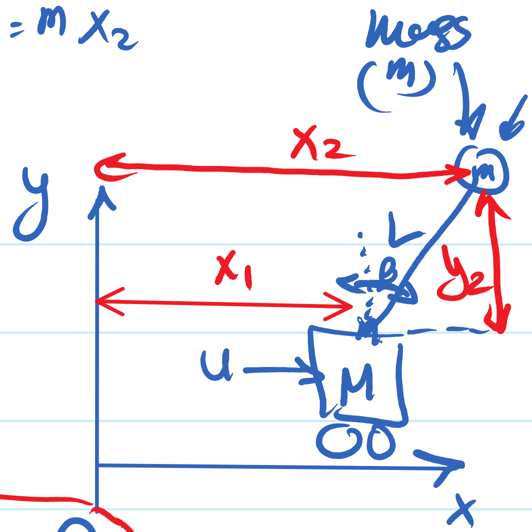


Ex: Find the SSR if the output

The selected DoFs (x_1, θ)

$x_2 = x_1 + L\sin\theta$

$\dot{x}_2 = \dot{x}_1 + L\dot{\theta}\cos(\theta)$

$$\ddot{x}_2 = \ddot{x}_1 + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta \quad \text{— (1)}$$

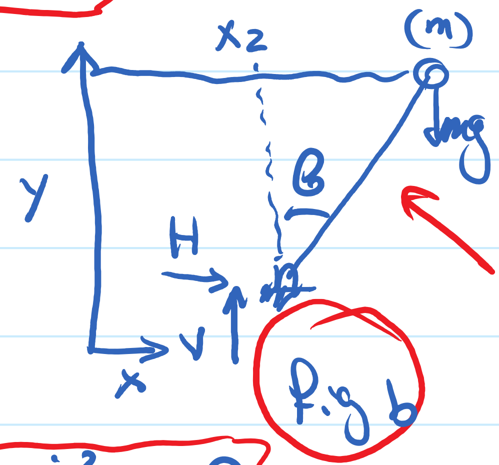
$y_2 = L\cos\theta \Rightarrow \dot{y}_2 = -L\dot{\theta}\sin\theta$

$$\ddot{y}_2 = -L\ddot{\theta}\sin\theta - L\dot{\theta}^2\cos\theta \quad \text{— (2)}$$

from fig (b)

$\Sigma F_x = m\ddot{x}_2$

$H = m\ddot{x}_2$ — (c)



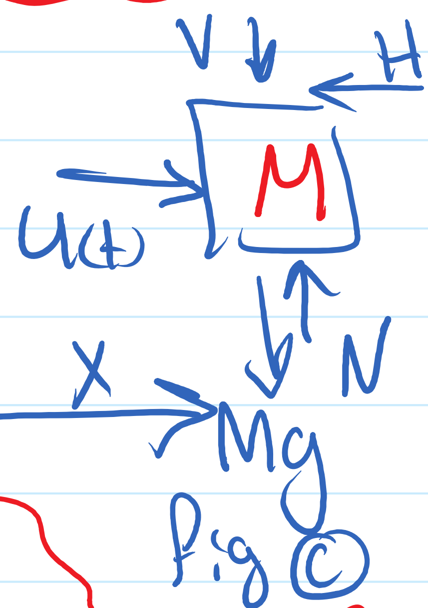
sub Eq (1) into Eq (c)

$$H = m\ddot{x}_1 + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta \quad \text{— (3)}$$

states
inputs
outputs

$\Sigma F_x = M\ddot{x}_1$

$U - H = M\ddot{x}_1$ — (d)



sub Eq (3) into Eq (d)

$$U(t) = (M+m)\ddot{x}_1 + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta \quad \text{— (i)}$$

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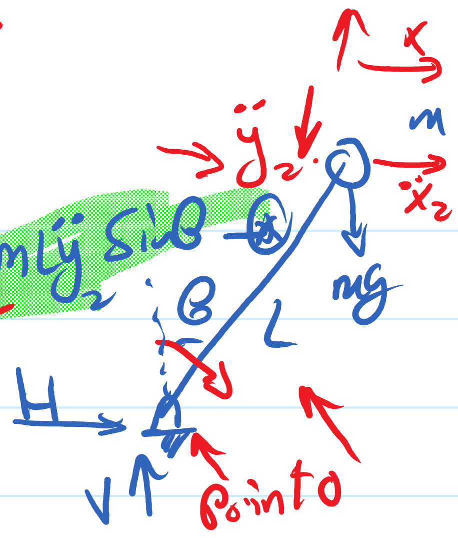
mg $\ddot{y}_2$ $\ddot{x}_2$ $m\ddot{x}_2 L\cos\theta$ $L\sin\theta\, m\ddot{y}_2$



$$\Sigma M = J_{eq}\ddot{\theta}$$

$$= mgL\sin\theta - m\ddot{x}_2 L\cos\theta - mL\ddot{y}_2\sin\theta \quad (*)$$

Sub $\ddot{x}_2$ (1) and $\ddot{y}_2$ (2) in Eq (*)

$$mgL\sin\theta = mL(\ddot{x}_1\cos\theta + L\ddot{\theta}\cos^2\theta - L\dot{\theta}^2\sin\theta\cos\theta$$
$$+ L\sin^2\theta + L\dot{\theta}^2\sin\theta\cos\theta)$$

$$mgL\sin\theta = mL\ddot{x}_1\cos\theta + mL^2\ddot{\theta}\cos^2\theta + mL^2\sin\theta \quad (ii)$$

$$m\ddot{x}_1\cos\theta + mL\ddot{\theta}\cos^2\theta - mg\sin\theta + mL\ddot{\theta}\sin\theta = 0$$

The model is nonlinear (Eq (i) and Eq (ii))
based on that, it is recommended to
linearize the model when $\theta(0) = 0$
$\cos\theta = 1$
$\sin\theta = \theta$

$$(M+m)\ddot{x}_1 + mL\ddot{\theta} = u(t) \quad \text{— (i)}$$
$$m\ddot{x}_1 + mL\ddot{\theta} - mg\theta = 0 \quad \text{(ii)}$$

} linear model when $\theta \simeq 0$

$$\Rightarrow \begin{bmatrix} (m+M) & (mL) \\ m & (mL) \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} u(t) \\ mg\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (m+M) & (mL) \\ m & (mL) \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} u(t) \\ mg\theta \end{bmatrix}$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} mL & -mL \\ -m & (M+m) \end{bmatrix} \begin{bmatrix} u(t) \\ mg\theta \end{bmatrix}$$

$$\Delta = (M+m)mL - m^2L = mML$$

$$\ddot{x}_1 = \frac{mL\,u(t) - mL * mg\theta}{mML}$$

$$\ddot{x}_1 = \frac{1}{M}\left[u(t) - mg\theta\right] \quad \text{— (i)}$$

$$\ddot{\theta} = \frac{-m\,u(t) + (M+m)mg\theta}{MmL}$$

$$\ddot{\theta} = \frac{\left[-u(t) + (M+m)g\theta\right]}{ML} \quad \text{— (ii)}$$

Now let the linear SSR

$q_1 = x_1 \Rightarrow \dot{q}_1 = \dot{x}_1 = q_2$, $q_2 = \dot{x}_1$ $\Rightarrow$ $\boxed{\dot{q}_1 = q_2}$ — (I)

$q_3 = \theta \Rightarrow \dot{q}_3 = \dot{\theta} = q_4$, $q_4 = \dot{\theta}$ $\Rightarrow$ $\boxed{\dot{q}_3 = q_4}$ — (I)

sub the states in Eq (i) and (ii)

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$$\dot{q}_2 = \frac{1}{M}\left[u(t) - mg\, q_3\right] \text{ — (III)}$$

$$\dot{q}_4 = \frac{1}{ML}\left[-u(t) + (M+m) g\, q_3\right] \text{ — (IV)}$$

$$\dot{q} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{M+m}{ML} & 0 \end{bmatrix} q + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{-1}{ML} \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} q$$