Brakes and Clutches

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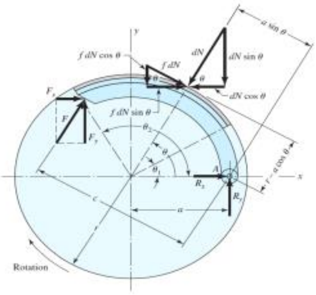
* Note that our referance is:
→ Origin at the center of the drum.
→ ⊕ x-axis is taken through the hinge pin.
→ ⊕ y-axis is always in the direction of the shoe.

16.2: Internal shoe drum brake:

* The pressure acting on a certain point on the frictional material at an angle (θ) from the pin is:



$$\boxed{p = \frac{p_a}{\sin\theta_a}\sin\theta} \quad [16.1]$$

where: p_a = max. pressure on the shoe

$$\theta_a = \begin{cases} \theta_2, & \text{for short shoe} \quad (\theta_2-\theta_1) < 90^\circ \\ 90^\circ, & \text{for long shoe} \quad (\theta_2-\theta_1) > 90^\circ \end{cases}$$

* The moment of the frictional forces (M_f) about the pin is:

$$\boxed{M_f = \frac{f p_a b r}{\sin\theta_a}\left[\left(-r\cos\theta\right)\Big|_{\theta_1}^{\theta_2} - a\left(\frac{1}{2}\sin^2\theta\right)\Big|_{\theta_1}^{\theta_2}\right]} \quad [16.2]$$

where: f = the friction coefficient

b = face width of the friction material

r = the radius of the drum.

a = the distance between the center of the drum & pin

* The moment of the normal of the normal forces (M_N) about the pin:

$$\boxed{M_N = \frac{p_a b r a}{\sin\theta_a}\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)\Big|_{\theta_1}^{\theta_2}} \quad [16.3]$$

* The actuating force (F) that must balance these moments is:

$$\boxed{F = \frac{M_N - M_f}{c}} \quad [16.4]$$

(Used for self energizing brake)

where: c: the normal distance between the pin and the line of the force.

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* If $M_f = M_N$, we obtain self locking thus no actuating force (F) is required to brake.

* For a self energizing action, the dimension (a) must be such that:

$$\boxed{M_N > M_f} \quad [16.5]$$

* The torque applied to the drum brake shoe is:

$$\boxed{T = \frac{f p_a b r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}} \quad [16.6]$$

* The hinge pin reactions for self energizing rotation are:

$$\boxed{\begin{aligned} R_x &= \frac{p_a b r}{\sin\theta_a}(A - fB) - F_x \\ R_y &= \frac{p_a b r}{\sin\theta_a}(B + fA) - F_y \end{aligned}} \quad [16.9]$$

where:

$$\boxed{\begin{aligned} A &= \left(\frac{1}{2}\sin^2\theta\right)_{\theta_1}^{\theta_2} \\ B &= \left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_{\theta_1}^{\theta_2} \end{aligned}} \quad [16.8]$$

---

* For de-energizing rotation, the actuating force becomes:

$$\boxed{F = \frac{M_N + M_f}{c}} \quad [16.7]$$

* The reactions on the pin become:

$$\boxed{\begin{aligned} R_x &= \frac{p_a b r}{\sin\theta_a}(A + fB) - F_x \\ R_y &= \frac{p_a b r}{\sin\theta_a}(B - fA) - F_y \end{aligned}} \quad [16.10]$$

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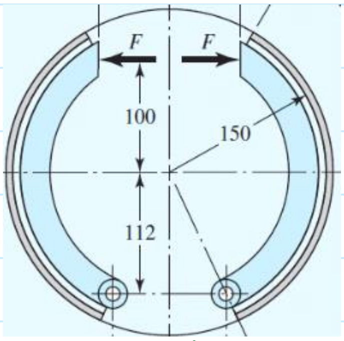
* For a brake with two symmetrical shoes:



$$\left(\frac{M_N}{P_a}\right)_{Left} = \left(\frac{M_N}{P_a}\right)_{Right}$$

$$\left(\frac{M_f}{P_a}\right)_{Left} = \left(\frac{M_f}{P_a}\right)_{Right}$$

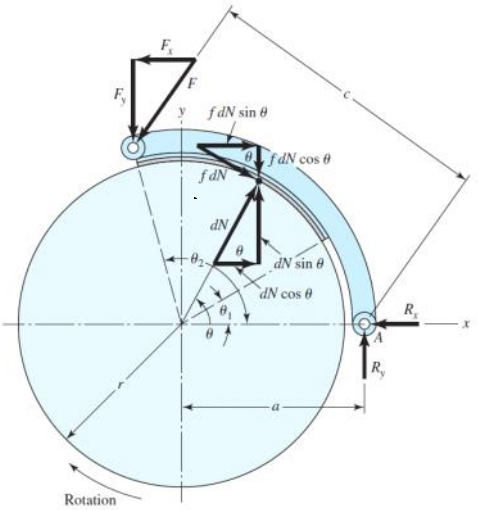
* Find $(P_a)_{Left}$ by substituting $(M_N)_{Left}$ & $(M_f)_{Left}$ in equation [16.7] after finding the force (F) from the right shoe.

* The braking capacity = total torque = $T_R + T_L$

16.3: External Shoe drum brake:

* Here, CW rotation produce de-energizing rotation.

* Same as previous section in the internal shoe for de-energizing rotation:



$$M_f = \frac{f\, p_a\, b\, r}{\sin\theta_a}\left[\left(-r\cos\theta\right)\Big|_{\theta_1}^{\theta_2} - a\left(\frac{1}{2}\sin^2\theta\right)\Big|_{\theta_1}^{\theta_2}\right] \quad [16.9]$$

$$M_N = \frac{p_a\, b\, r\, a}{\sin\theta_a}\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)\Big|_{\theta_1}^{\theta_2} \quad [16.10]$$

$$F = \frac{M_N + M_f}{c} \quad [16.11]$$

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* The reactions are:

$$\boxed{\begin{aligned} R_x &= \frac{p_a br}{\sin\theta_a}(A + fB) - F_x \\ R_y &= \frac{p_a br}{\sin\theta_a}(fA - B) + F_y \end{aligned}}$$

[16.12]

* The CCW rotation produces a self energizing rotation thus:

$$\boxed{F = \frac{M_N - M_f}{c}}$$

[16.13]

* The reactions are:

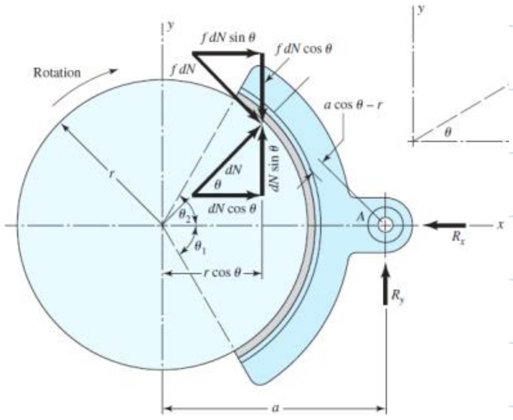
$$\boxed{\begin{aligned} R_x &= \frac{p_a br}{\sin\theta_a}(A - fB) - F_x \\ R_y &= \frac{p_a br}{\sin\theta_a}(-fA - B) + F_y \end{aligned}}$$

[16.14]

* Special Case: Symmetrical external drum shoe

→ Pivot is located so that $\sum M_f = 0$

→ Distance (a) is chosen by finding where $\sum M_f = 0$



* Symmetry means: $\boxed{\theta_1 = \theta_2}$

* The distance (a) is:

$$\boxed{a = \frac{4r\sin\theta_2}{2\theta_2 + \sin 2\theta_2}}$$

[16.15] Where: $a > r$

* The reactions on the pin when it is located at a distance (a) are:

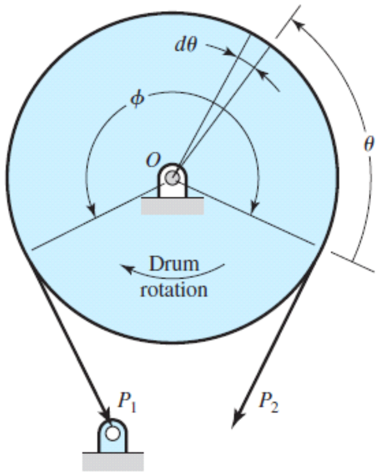
$$\boxed{R_x = \frac{p_a b r}{2}(2\theta_2 + \sin 2\theta_2)} \quad [16.16]$$

$$\boxed{R_y = \frac{p_a b r f}{2}(2\theta_2 + \sin 2\theta_2)} \quad [16.17]$$

* The torque is: $\boxed{T = a f N}$ [16.18]

where: $R_x = N$
$R_y = fN$

16.4: Band type clutches & brakes:



(a)

P_2 = actuating force
P_1 = pin reaction

* Because of friction:

$$\boxed{\frac{P_1}{P_2} = e^{f\phi}} \quad [16.19]$$

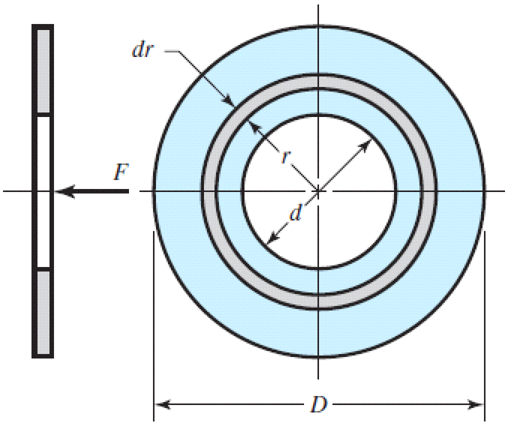
* The torque is: $\boxed{T = (P_1 - P_2)\frac{D}{2}}$ [16.20]

* The pressure on the element of area: $\boxed{p = \frac{P}{br} = \frac{2P}{bD}}$ [16.21]

* The max. pressure will occur at: $\boxed{p_a = \frac{2P_1}{bD}}$ [16.22]

16.5: Disk Clutch (Frictional Contact axial clutch)

① Uniform Wear:



* The max. pressure (p_a) occurs when (r) is minimum:

$\rightarrow r = \frac{d}{2}$ thus:

$$\boxed{p r = p_a \frac{d}{2}} \quad (a)$$

* The total normal force is:

$$\boxed{F = \frac{\pi p_a d}{2}(D - d)} \quad [16.23]$$

* This equation gives the actuating force for the selected max. pressure (p_a).

* This equation holds for any number of friction surfaces.

* The torque is: $\boxed{T = \frac{\pi f p_a d}{8}(D^2 - d^2)}$ [16.24]

* Another expression for torque: $\boxed{T = \frac{Ff}{4}(D + d)}$ [16.25]

* This equation gives the torque capacity for only a single friction surface.

② Uniform Pressure

* When uniform pressure (p_a) can be assumed over the area of the disk, the actuating force is:

$$\boxed{F = \frac{\pi p_a}{4}(D^2 - d^2)} \quad [16.26]$$

* Note that the torque is for a single pair of mating surfaces.

* The torque is:

$$\boxed{T = \frac{\pi f p_a}{12}(D^3 - d^3)} \quad [16.27]$$

or

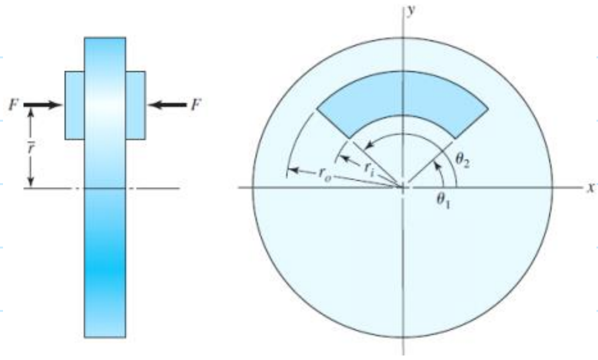
$$\boxed{T = \frac{Ff}{3}\frac{D^3 - d^3}{D^2 - d^2}} \quad [16.28]$$

* This value must be multiplied by the number of pairs or the surfaces in contact.

$\rightarrow T_{mul} = T \cdot N$

16.6: Disk Brake

* ($\bar{r}$) locates the line of action of force (F) that intersects the y-axis.



① Uniform Wear

* The actuating force is:

$$\boxed{F = (\theta_2 - \theta_1) p_a r_i (r_o - r_i)} \quad [16.33]$$

* The friction torque is: $$\boxed{T = \frac{1}{2}(\theta_2 - \theta_1) f p_a r_i (r_o^2 - r_i^2)} \quad [16.34]$$

* The radius of an equivalent shoe is: $$\boxed{r_e = \frac{r_o + r_i}{2}} \quad [16.35]$$

* The locating coordinate ($\bar{r}$) is: $$\boxed{\bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1} \frac{r_o + r_i}{2}} \quad [16.36]$$

② Uniform Pressure

* The actuating force is:

$$\boxed{F = \frac{1}{2}(\theta_2 - \theta_1) p_a (r_o^2 - r_i^2)} \quad [16.37]$$

* The friction torque is: $$\boxed{T = \frac{1}{3}(\theta_2 - \theta_1) f p_a (r_o^3 - r_i^3)} \quad [16.38]$$

* The radius of an equivalent shoe is: $$\boxed{r_e = \frac{2}{3} \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}} \quad [16.39]$$

* The locating coordinate ($\bar{r}$) is: $$\boxed{\bar{r} = \frac{2}{3} \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1}} \quad [16.40]$$

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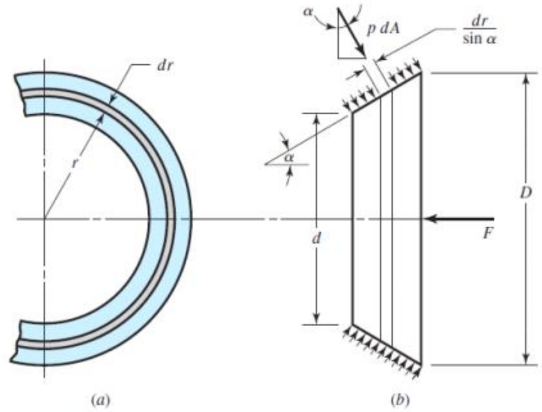
16.7: Cone Clutches and Brakes

① Uniform Wear

* The pressure relation is:

$$\boxed{p = p_a \frac{d}{2r}} \quad (a)$$



* The operating force is:

$$\boxed{F = \frac{\pi p_a d}{2}(D - d)} \quad [16.44]$$

* The torque is:

$$\boxed{\frac{\pi f p_a d}{8 \sin \alpha}(D^2 - d^2)} \quad [16.45] \quad \text{or} \quad \boxed{T = \frac{Ff}{4 \sin \alpha}(D + d)} \quad [16.46]$$

② Uniform Pressure ($p = p_a$)

* The actuating force is:

$$\boxed{F = \frac{\pi p_a}{4}(D^2 - d^2)} \quad [16.47]$$

* The torque is:

$$\boxed{T = \frac{\pi f p_a}{12 \sin \alpha}(D^3 - d^3)} \quad [16.48] \quad \text{or} \quad \boxed{T = \frac{Ff}{3 \sin \alpha} \frac{D^3 - d^3}{D^2 - d^2}} \quad [16.49]$$

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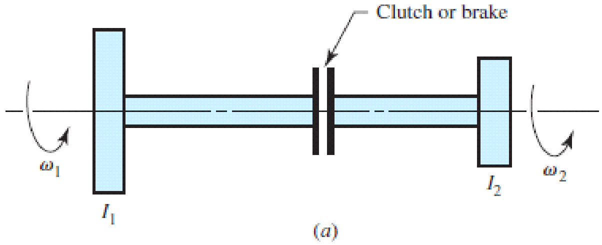
16.8: Energy Considerations



* Referring to this system as a clutch, during operation, the angular velocities change and eventually become equal. ($\omega_1 = \omega_2$)
* Assume T = constant.

$$I_1 \ddot{\theta}_1 = -T \quad (a)$$
$$I_2 \ddot{\theta}_2 = T \quad (b)$$

* We can determine $\dot{\theta}_1$ & $\dot{\theta}_2$ after any time using:

$$\dot{\theta}_1 = -\frac{T}{I_1} t + \omega_1 \quad (c)$$

$$\dot{\theta}_2 = \frac{T}{I_2} t + \omega_2 \quad (d)$$

Where: $\dot{\theta}_1 = \omega_1$, $\dot{\theta}_2 = \omega_2$ at $t = 0$

* The relative velocity is:

$$\boxed{\Delta\dot{\theta} = \dot{\theta}_1 - \dot{\theta}_2 = \omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right) t} \quad [16.50]$$

* Clutching operation is completed at the instant $\dot{\theta}_1 = \dot{\theta}_2$

* Let the time for the entire operation be (t_1) at $\Delta\dot{\theta} = 0$:

$$\boxed{t_1 = \frac{I_1 I_2(\omega_1 - \omega_2)}{T(I_1 + I_2)}} \quad [16.51]$$

* The rate of energy dissipation (work) during clutching is:

$$\boxed{u = T\left[\omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right) t\right]} \quad (e)$$

* The total energy dissipation during clutching or braking is:

$$\boxed{E = \frac{I_1 I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}} \quad [16.52]$$

* If I_1 & I_2 are in [lbf. in.s^2] then the energy absorbed by clutch in [in. lbf] is:

$$\boxed{H = \frac{E}{9336}}$$ [16.53]

* In SI units, I_1 & I_2 are in [Kg.m^2] and the energy is in [J].

* <u>For braking</u>:

→ Assume that the brake is applied at $t = t_1$

$\omega = \omega_1$

$V = V_1$

→ During time ($t = t_2$), the values are reduced to $\omega = \omega_2$

$V = V_2$

→ W_{KB} = frictions work & air resistace work.

W_{KM} = Motor's work = ⊖ve

$$\Sigma W = \Sigma E = \Sigma \frac{m}{2}(V_1^2 - V_2^2) + \Sigma \frac{I}{2}(\omega_1^2 - \omega_2^2) + \Sigma W(h_1 - h_2)$$

$W_{KB} = T(\varphi_2 - \varphi_1)$ <u>where</u>: φ = angular displacement of the drum.

* Since T = Constant → a = Const. → α = Const.

* The angular velocity & displacement are:

$$\boxed{\omega_2 = \omega_1 - \alpha(t_2 - t_1)}$$

$$\boxed{\varphi_2 = \varphi_1 + \left(\frac{\omega_1 + \omega_2}{2}\right)(t_2 - t_1) = \varphi_1 + \omega_1(t_2 - t_1) - \frac{\alpha}{2}(t_2 - t_1)^2}$$

16.9 : Temperature rise

* The temperature rise of the clutch or brake is:

$$\boxed{\Delta T = \frac{H}{C_p W} = (T_s - T_a)} \qquad (16\text{–}54)$$

ΔT = temperature rise, °F
C_p = specific heat capacity, Btu/(lbm · °F); use 0.12 for steel or cast iron
W = mass of clutch or brake parts, lbm
H = heat generated in [Btu]

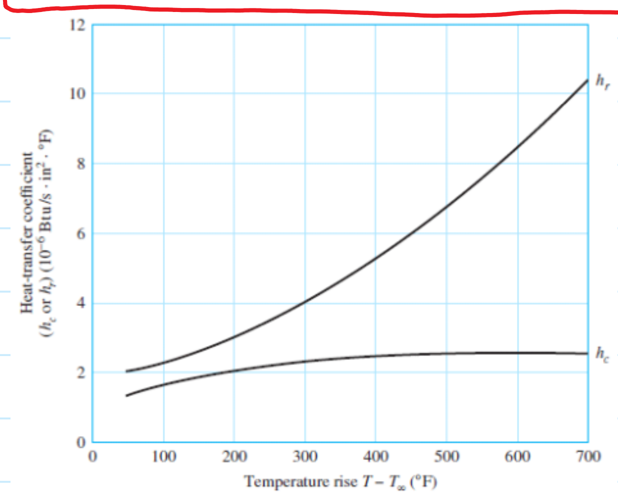
* In SI Units:

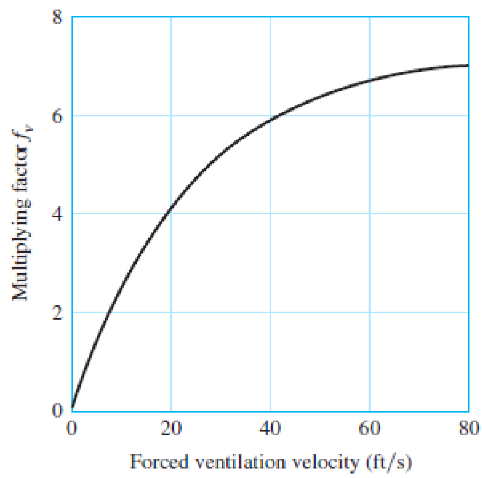
$$\boxed{\Delta T = \frac{E}{C_p m} = (T_s - T_a)} \qquad (16\text{–}55)$$

ΔT = temperature rise, °C
C_p = specific heat capacity; use 500 J/kg · °C for steel or cast iron
m = mass of clutch or brake parts, kg
E = heat generated in [J]

* The rate of heat transfer for clutch or brake is:

$$\boxed{H_{\text{loss}} = \hbar_{\text{CR}} A(T - T_\infty) = (h_r + f_v h_c) A(T - T_\infty)} \qquad (16\text{–}57)$$





* The rate of heat generation per unit area of friction is:

$$\boxed{H = f\,pV}$$

* The values of (pV) are given in the table below:

	PV	
Operating Conditions	(psi) (ft/min)	(KPa)(m/s)
Continuous poor heat dissp.	30,000	1050
Occasional, poor heat dissp.	60,000	2100
Continuous good heat dissp.	85,000	3000

Table 16–3

Characteristics of Friction Materials for Brakes and Clutches *Sources:* Ferodo Ltd., Chapel-en-le-frith, England; Scan-pac, Mequon, Wisc.; Raybestos, New York, N.Y. and Stratford, Conn.; Gatke Corp., Chicago, Ill.; General Metals Powder Co., Akron, Ohio; D. A. B. Industries, Troy, Mich.; Friction Products Co., Medina, Ohio.

Material	Friction Coefficient f	Maximum Pressure p_{max}, psi	Maximum Temperature Instantaneous, °F	Maximum Temperature Continuous, °F	Maximum Velocity V_{max}, ft/min	Applications
Cermet	0.32	150	1500	750		Brakes and clutches
Sintered metal (dry)	0.29–0.33	300–400	930–1020	570–660	3600	Clutches and caliper disk brakes
Sintered metal (wet)	0.06–0.08	500	930	570	3600	Clutches
Rigid molded asbestos (dry)	0.35–0.41	100	660–750	350	3600	Drum brakes and clutches
Rigid molded asbestos (wet)	0.06	300	660	350	3600	Industrial clutches
Rigid molded asbestos pads	0.31–0.49	750	930–1380	440–660	4800	Disk brakes
Rigid molded nonasbestos	0.33–0.63	100–150		500–750	4800–7500	Clutches and brakes
Semirigid molded asbestos	0.37–0.41	100	660	300	3600	Clutches and brakes
Flexible molded asbestos	0.39–0.45	100	660–750	300–350	3600	Clutches and brakes
Wound asbestos yarn and wire	0.38	100	660	300	3600	Vehicle clutches
Woven asbestos yarn and wire	0.38	100	500	260	3600	Industrial clutches and brakes
Woven cotton	0.47	100	230	170	3600	Industrial clutches and brakes
Resilient paper (wet)	0.09–0.15	400	300		$PV < 500\,000$ psi · ft/min	Clutches and transmission bands

Table 16–5

Friction Materials for Clutches

Material	Friction Coefficient Wet	Friction Coefficient Dry	Max. Temperature °F	Max. Temperature °C	Max. Pressure psi	Max. Pressure kPa
Cast iron on cast iron	0.05	0.15–0.20	600	320	150–250	1000–1750
Powdered metal* on cast iron	0.05–0.1	0.1–0.4	1000	540	150	1000
Powdered metal* on hard steel	0.05–0.1	0.1–0.3	1000	540	300	2100
Wood on steel or cast iron	0.16	0.2–0.35	300	150	60–90	400–620
Leather on steel or cast iron	0.12	0.3–0.5	200	100	10–40	70–280
Cork on steel or cast iron	0.15–0.25	0.3–0.5	200	100	8–14	50–100
Felt on steel or cast iron	0.18	0.22	280	140	5–10	35–70
Woven asbestos* on steel or cast iron	0.1–0.2	0.3–0.6	350–500	175–260	50–100	350–700
Molded asbestos* on steel or cast iron	0.08–0.12	0.2–0.5	500	260	50–150	350–1000
Impregnated asbestos* on steel or cast iron	0.12	0.32	500–750	260–400	150	1000
Carbon graphite on steel	0.05–0.1	0.25	700–1000	370–540	300	2100

*The friction coefficient can be maintained with ±5 percent for specific materials in this group.