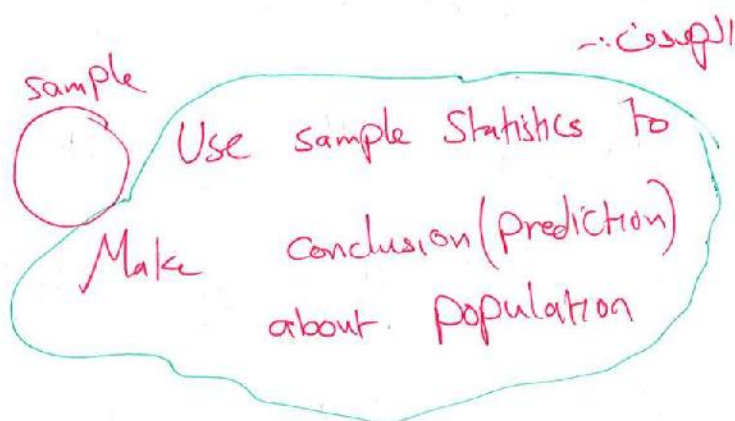
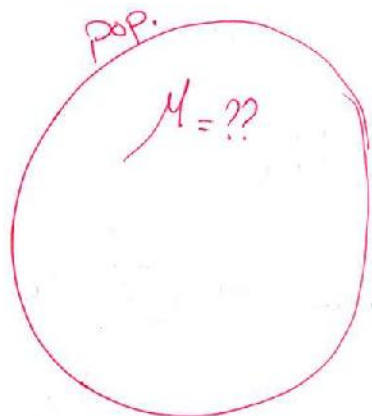


Summary: Chapter 9:

Hypothesis testing

9.1

We will use sample to test hypothesis about μ .



In any hypothesis testing, there are two hypotheses:

- [1] The Null hypothesis: H_0 : is the researcher claim
- [2] The Alternative hypothesis: H_a : is the opposite of H_0

9.2 Type I and Type II

conclusion Researcher	H_0 True	H_0 False
Reject H_0	Type I Error	Correct conclusion
Accept H_0	Correct conclusion	Type II error

Def * Def: Type I error: - Rejecting H_0 when H_0 is True
significance level α = the prob. of Making type I error

STUDENTS-HUB.com : Type II : Accepting H_0 when H_0 is False
 Uploaded By: 1210711@student.hkz.edu

9.3

معلوم σ known

9.4

غير معلوم σ Un known

In general, the steps for testing are:-

- ① Write the hypotheses H_0, H_a .
- ② Find the test statistics: a value of Z or t
- ③ Test using critical value or P-value Approach
- ④ Conclusion

σ : known
test statistics is

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

*** Hypothesis:-

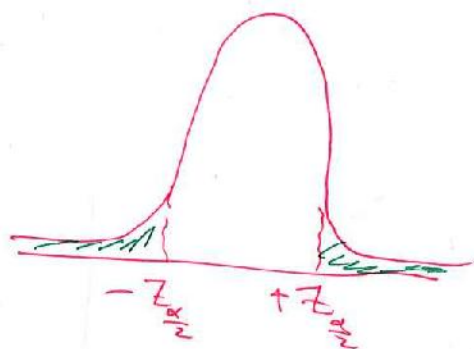
$$H_0: \mu = \mu_0$$

$$H_a: \mu \neq \mu_0$$

(two tailed test)

Test statistics :- $Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

Critical value :- $\pm Z_{\frac{\alpha}{2}}$



Conclusion :- Reject $H_0 \rightarrow |Z| \geq Z_{\frac{\alpha}{2}}$

Accept $H_0 \rightarrow |Z| < Z_{\frac{\alpha}{2}}$

Hypothesis:-

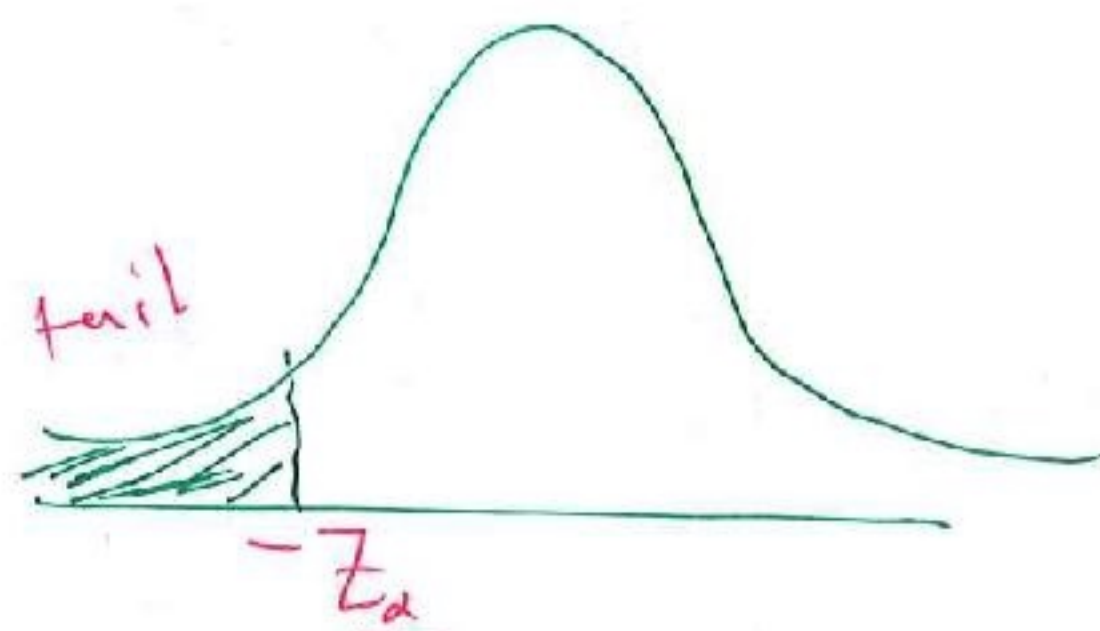
$$H_0 : \mu \geq \mu_0$$

$$H_a : \mu < \mu_0$$

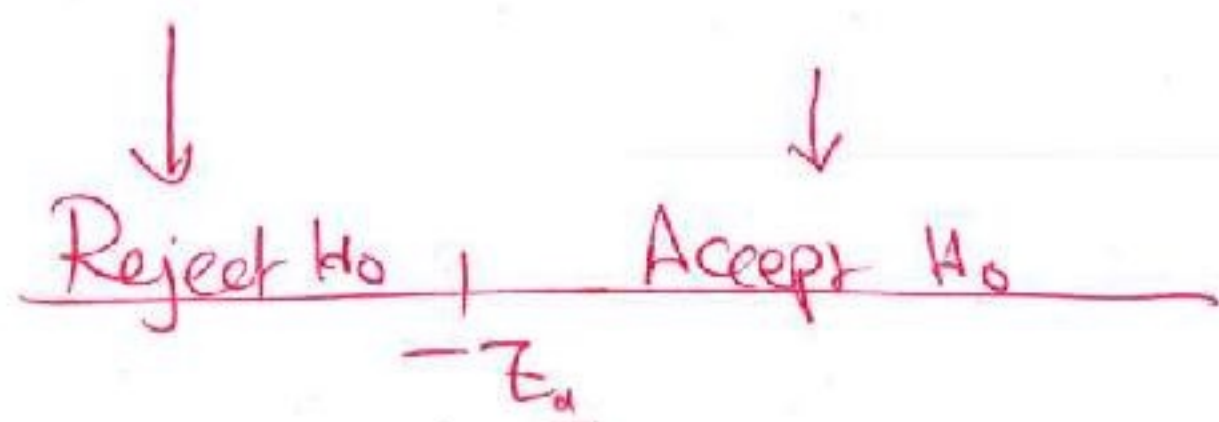
Lower tailed test

Test Statistics

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$



Critical value : $-Z_\alpha$



Conclusion

$Z < -Z_\alpha$ Reject H_0

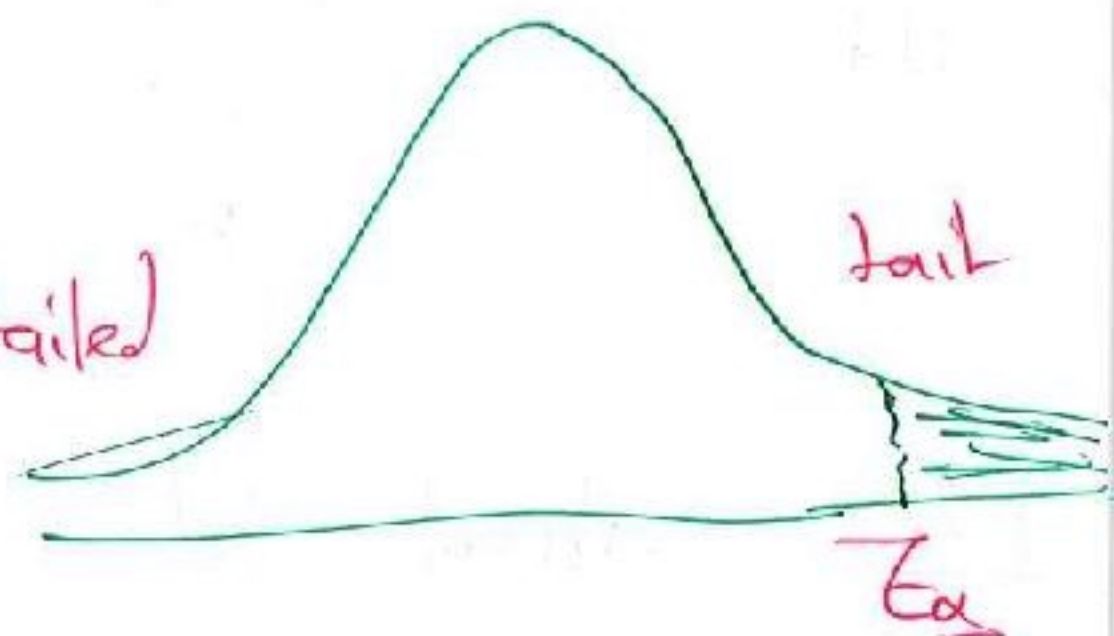
$Z > -Z_\alpha$ Accept H_0

Hypothesis :

$$H_0 : \mu \leq \mu_0$$

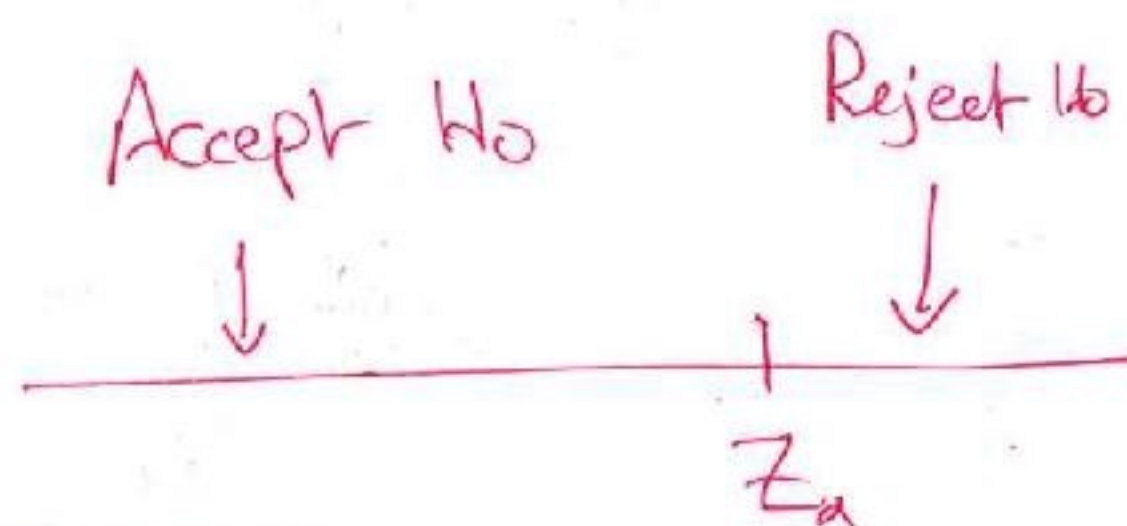
$$H_a : \mu > \mu_0$$

upper tailed test



Test Statistics :

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$



Critical value

C.V. = Z_α

Conclusion

$Z \geq Z_\alpha$ Reject H_0
 $Z < Z_\alpha$ Accept H_0

Ex. $H_0 = M \leq 20$

$H_a: M > 20$

Given that $n = 35$

$\bar{x} = 23$

$\sigma = 4.5$ (Known)

$\alpha = 10\%$

1 Find the test statistics.

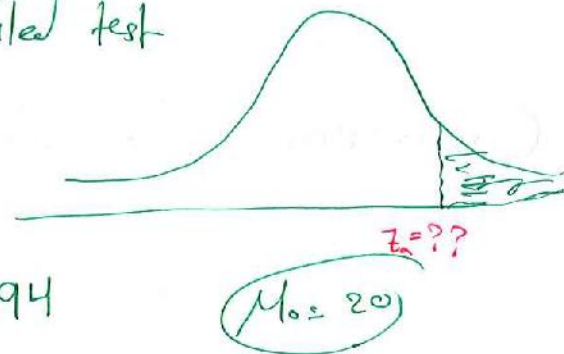
2 Find the critical value

3 What is your conclusion

Hypothesis $H_0 = M \leq 20$

$H_1 = M > 20$

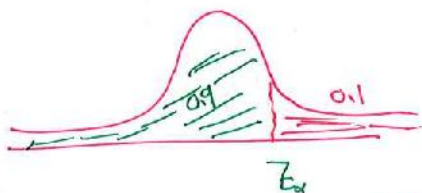
upper tailed test



1
$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{23 - 20}{\frac{4.5}{\sqrt{35}}} = 3.94$$

2 Critical value $Z_{\alpha} = ??$

$\alpha = 0.10 \longrightarrow Z_{0.1}$

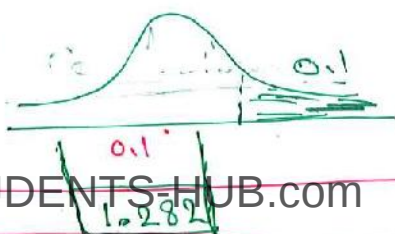


بالطريقة الأولى باستخدام Z table

لا احتاج الى قيمة n في جدول Z

	0.08	0.09
1.2	0.8997	0.9015

$Z_{\alpha} = Z_{0.1} = 1.28$



الطريقة الثانية باستخدام t table

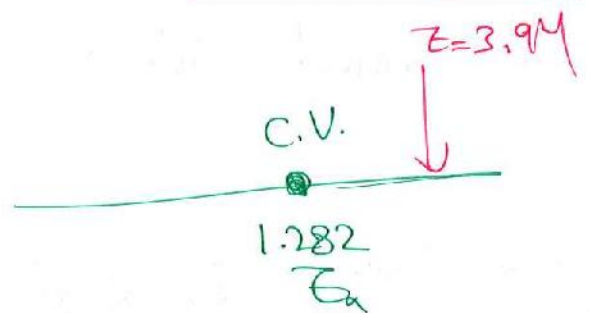
$t_{\alpha, Df=\infty} = Z_{\alpha}$

3. Conclusion:

$$Z > Z_{\alpha}$$

$$3.94 > 1.282$$

Reject H_0



Exp:

$$H_0: \mu = 80.42$$

$$H_a: \mu \neq 80.42$$

given that sample size = 100

$$\bar{X} = 81$$

$$\sigma = 15.2$$

$$\alpha = 1\%$$

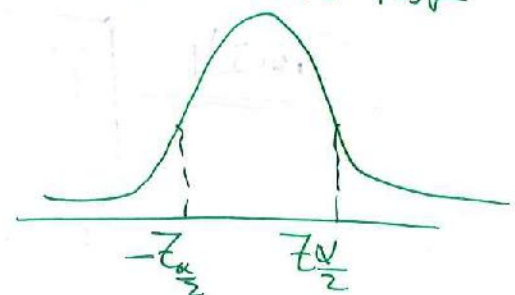
1. Find the test statistics

2. At a 1% significance level, what is your conclusion by using critical value approach

Solution • $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{81 - 80.42}{15.2/\sqrt{100}} = 0.38$ two tailed test

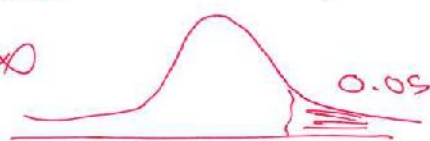
• Critical value $\pm Z_{\frac{\alpha}{2}}$

$$\alpha = 0.01 \rightarrow \frac{\alpha}{2} = 0.005$$



لايجاد $Z_{0.005}$

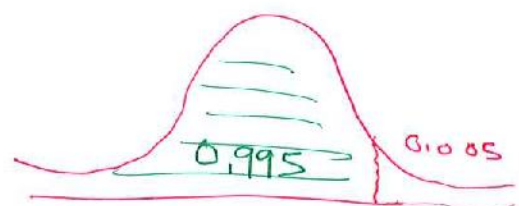
t table
 $D_f = \infty$



0.005

∞ 2.576

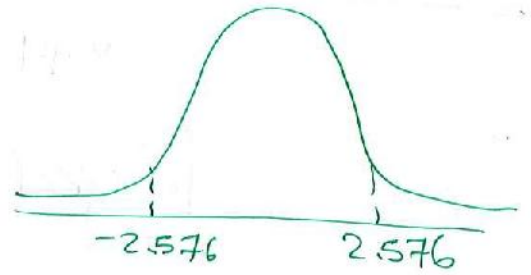
Z table plus



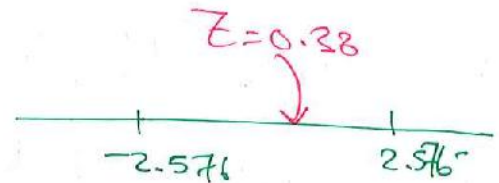
0.07 | 0.08

Critical value ± 2.576

• conclusion: $-Z_{\alpha/2} < Z < Z_{\alpha/2}$



Accept H_0 (Don't Reject H_0)



P-value Approach

P-value = probability (area) value found using Z-table based on the test statistics and type of test.

سؤال

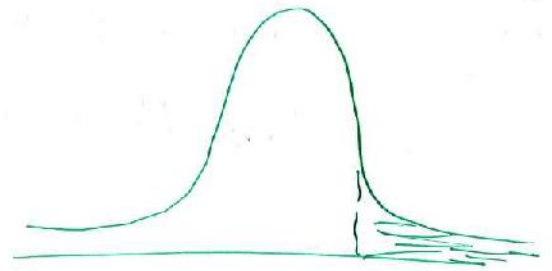
If P-value $\leq \alpha$ Reject

If P-value $> \alpha$ Accept

How to find P-value?? Use Z-table

* Upper tailed Test

- [1] Hypothesis: $H_0: \mu \leq \mu_0$
 $H_a: \mu > \mu_0$



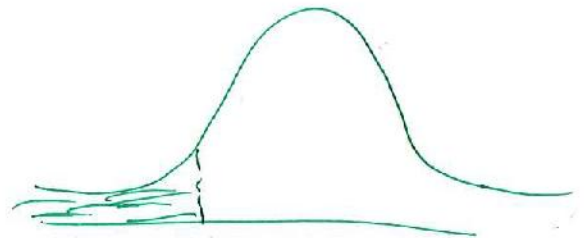
- [2] Test Statistics $Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

- [3] P-value = Area above Z

- [4] Conclusion

* Lower tailed Test

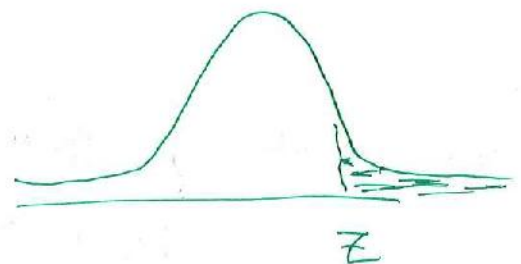
$$H_0: \mu \geq \mu_0$$
$$H_a: \mu < \mu_0$$



* P-value: Area below Z

* Two tailed Test

$$H_0: \mu = \mu_0$$
$$H_a: \mu \neq \mu_0$$



* P-value: $2(\text{Area above } Z)$ if $Z > 0$

$2(\text{Area below } Z)$ if $Z < 0$



Expt

$$H_0: \mu \leq 30.5$$

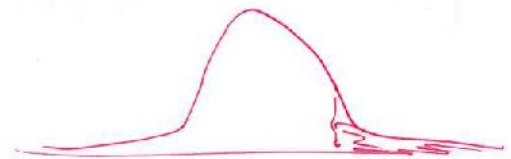
$$n = 70$$

$$H_a: \mu > 30.5$$

$$\bar{x} = 31.5$$

$$\sigma = 6.02$$

* Hypothesis : $H_0: \mu \leq 30.5$ * Upper tailed
 $H_a: \mu > 30.5$



* Test Statistics $z = \frac{31.5 - 30.5}{6.02/\sqrt{70}} = 1.39$

* P-value $P(Z \geq 1.39) = 1 - P(Z \leq 1.39)$
 $1 - 0.9177 = \boxed{0.0823}$

* If $\alpha = 10\%$, what is your conclusion

Since $P\text{-value} \leq \alpha$ Reject H_0

* What is your conclusion if $\alpha = 5\%$

Since $P\text{-value} > \alpha \rightarrow$ Don't Reject H_0 .

Ex: The Mean weight of high school student is 70 kg. A sample of 50 student is taken, this sample produced a mean of 73, Assume the pop. standard deviation is 20

[1] Use Significance level of 1% to test the claim that the mean weight is greater than 70 (Critical value)

[2] Use Significance level of 10% to test the claim that the Mean weight is different from 70 (P-value)

Short Answers

[1] * Hypothesis $H_0: M \leq 70$
 $H_a: M > 70$

* $Z = 1.06$

* Critical value $Z_{\alpha} = 2.326$

* Accept H_0



[2] * Hypothesis $H_0: M = 70$
 $M \neq 70$

* $Z = 1.06$

* P-value = 2 ($P \geq 1.06$)

$P > \alpha$

Exp:

*

$$H_0: M \geq 1220$$

$$H_a: M < 1220$$

$$n = 95$$

$$\alpha = 1\%$$

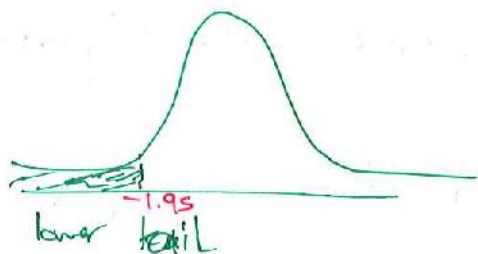
$$\bar{X} = 1210$$

$$\sigma = 50$$

1 Find test statistics.

2 Find P-value

3 What is your conclusion



$$* Z = \frac{\bar{X} - M_0}{\sigma / \sqrt{n}} = \frac{1210 - 1220}{50 / \sqrt{95}} = -1.95$$

* P-value : Area below (-1.95)

$$P(Z \leq -1.95) = P(Z \geq 1.95)$$

$$= 1 - P(Z \leq 1.95)$$

$$= ~~0.0256~~ 1 - 0.9744$$

$$P\text{-value} = 0.0256$$

* Conclusion

P-value $> \alpha$ Don't Reject H_0

0.0256 $> \alpha$ Accept H_0