## Gear Surface Durability:

Gear teeth surface deterioration:

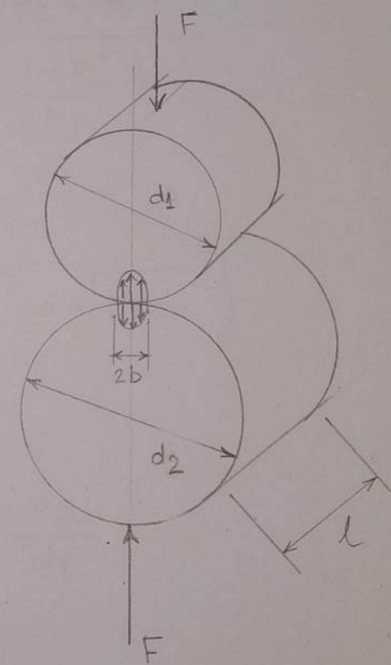
1- Abrasive wear:
Due to presence of foreign particles.

2- Scoring (lubrication failure):
At high speeds and inadequate lubrication → High temp. and high sliding friction. In addition to high pressure due to gear load.
⇒ High temp. and pressure → welding and tearing of surface material.

3- Pitting: failure due to repetition of contact stresses. (typical failure mode)

### Hertz Contact Stress.

Contact stress between two cylinders.




$$P_{max} = \frac{2F}{\pi b L}$$

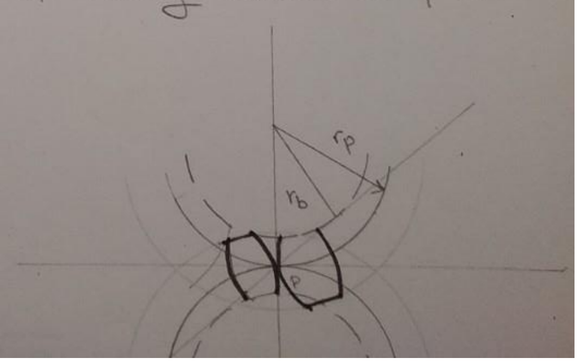
F = Force pressing two cylinders
L = Length of cylinders.

$$b = \sqrt{\frac{2F}{\pi L} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{(1/d_1) + (1/d_2)}}$$

Gear tooth contact stress:

Gears Involute are treated as cylinders of radii equal to radius of curvature of mating Involutes at pitch points

From Involute geometry:




$$R_1 = \frac{d_p \sin\phi}{2}, \quad R_2 = \frac{d_g \sin\phi}{2}$$

To use Hertz eq. we substitute.

| Hertz | Gears |
| --- | --- |
| $F$ | $F_t/\cos\phi$ |
| $P_{max}$ | $\sigma_H$ ($\sigma_c$) |
| $L$ | $b$ |
| $R_1$ | $\frac{d_p \sin\phi}{2}$ |
| $R_2$ | $\frac{d_g \sin\phi}{2}$ |

$$\Rightarrow \quad \sigma_H = 0.564 \sqrt{\frac{F_t\left[2/(d_p \sin\phi) + 2/(d_g \sin\phi)\right]}{\cos\phi \; b\left(\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_g^2}{E_g}\right)}}$$

Defining Elastic coefficient [$C_p$]:

$$C_p = 0.564 \sqrt{\frac{1}{\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_g^2}{E_g}}} \qquad \left[\sqrt{psi} \,,\, \sqrt{MPa}\right]$$

and Strength Geometry factor:

$$I = \frac{\sin\phi \cos\phi}{2} \, \frac{R}{R+1} \quad , \quad R = \frac{d_g}{d_p}$$

$I$ = dimensionless.

Pinion with external gear : $R = +ve$

Pinion with Internal gear : $R = -ve$.

Subst. $I$ and $R$ in $\sigma_H$:

$$\sigma_H = C_p \sqrt{\frac{F_t}{b \, d_p \, I}}$$

$C_p \rightarrow$ Table [14-5] $\quad C_p = \sqrt{MPa}$ , $\sqrt{kpsi}$

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AGMA Correction factor:

$C_o = K_o$ = overload factor → Table (notes)

$C_v = K_v$ = dynamic factor → Fig. (14-7).

$C_m = K_m$ = mounting factor → Table (14-6).

$$\Rightarrow \sigma_H = C_P \sqrt{\frac{F_t \, C_o \, C_m C_v}{b \, d_p \, I}}$$

Surface fatigue Strength:

AGMA Surface fatigue Strength for steel gears with different hardness ($H_B$) are estimated at life $= 10^7$ cycles and $R = 99\%$.

AGMA Fatigue strength ($S_c$) → Fig. (14-5) and Table (14-6).

Surface fatigue strength:

At any life and Reliability.

$$S_{fe} = S_c \frac{C_L \, C_t}{C_R}$$

$C_L$ = life factor → Fig. (14-15).

$C_R$ = reliability factor → Table (14-10)

Normally: It is desirable to make one surface harder than the other.
The surface subjected to higher stress → harder.

For steel gears:

Pinion is made harder than the gear:
→ Pinion is subjected to more fatigue cycles.
→ It is economical to make smaller member to higher hardness.

Surface fatigue is less critical than bending fatigue failure;
- produces slowly → give warning by gradually increasing gear noise.
- Surface failure is arbitrary → gears operate after their surface endurance limit.

Hardness - Ratio factor ($C_H$)

pinion is smaller → subjected to higher contact stress than gear

So normally pinion is made harder than the gear

In order to make balance for the life of both gears

Surface-hardened pinion is mated with through-hardened gear

$C_H$ = hardness ratio is only applied for the gear. to adjust surface strength for this effect.

$$C_H = 1 + A'(m_g - 1) \qquad m_g = \frac{\omega_P}{\omega_g} = \frac{N_g}{N_P}$$

$$A' = 8.98 \times 10^{-3}\left(\frac{H_{BP}}{H_{Bg}}\right) - 8.29 \times 10^{-3} \quad , \quad 1.2 \leq \frac{H_{BP}}{H_{Bg}} \leq 1.7$$

$H_{BP}$ = hardness of pinion (Brinell)

$H_{Bg}$ = " " gear

Fig. [14-12] $C_H$ Vs $m_g$ for differen $\frac{H_{BP}}{H_{Bg}}$

For $\frac{H_{BP}}{H_{Bg}} < 1.2 \rightarrow C_H = 1.0$

$\frac{H_{BP}}{H_{Bg}} > 1.7 \rightarrow A' = 0.00698$

If pinion surface hardness > C48 Rockwell is run with through hardened gear

$$C_H = 1 + B'(450 - H_{BG})$$

$$B' = 0.00075\, e^{[-0.0112 f_P]}$$

$f_P$ = Surface finish of pinion ($\mu$in)

[$C_H$ Vs $H_{BG}$ and $f_P$] Fig [14-13]

for $f_P > 64 \rightarrow C_H = 1.0$

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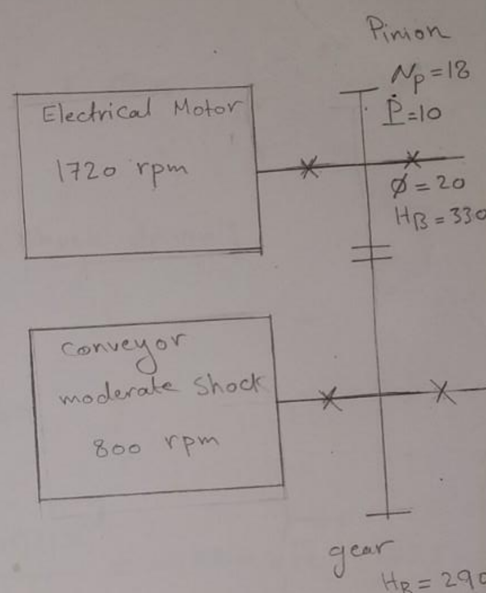
Factor of safety for surface failure:

$$n = \frac{S_{fe}}{\sigma_H} \qquad n = [1.1 - 1.5].$$

Example:

Estimate max. hp that can be transmitted with only a 1% chance of surface fatigue during 5 years of 40 hr/week 50 wk/yr operation. Face width $b = 1.25''$, $Q_v = 7.0$.




Solution:

1- $S_{fe} = S_c \dfrac{C_L}{C_R}$ Fig. [14-5] grade 1 $S_c = 322 H_B + 29100$ psi

Pinion: $S_c = 135.36$ Ksi
gear: $S_c = 122.5$ Ksi } Fig. (14-3).

$$Life = 1720 \times 60 \times \left(5\,yr \times 50 \frac{wk}{yr} \times 40 \frac{hr}{wk}\right)$$

$L = 1.032 \times 10^9$ cycles.

$$C_L = 2.466 (N)^{-0.056} = 2.466 (1.032 \times 10^9)^{-0.056} = 0.77$$

using the lower curve of Fig.(14-8) to be more conservative..

$C_R = 1.0 \rightarrow R = 99\%$.

Pinion: $S_{fe} = S_c \dfrac{C_L}{C_R} = 135.36 \times 0.77 = 104.22$ Ksi

gear: $Life = 800 \times 60 \times (5 \times 50 \times 40) = 4.8 \times 10^8$ cycles.

$C_L = 2.466 (4.8 \times 10^8)^{-0.056} = 0.81$

$C_R = 1.0 \rightarrow R = 99\%$.

$S_{fe} = S_c \dfrac{C_L}{C_R} = 122.5 \times 0.81 = 99.22$ Ksi.

2- Surface contact Stress.

$$\sigma_H = C_p \sqrt{\frac{F_t}{b I d_p} C_o C_m C_v}$$

$C_H = 1 + A'(m_g - 1)$

$m_g = 2.5$

$A' = 8.98 \times 10^{-3} \left(\frac{H_{Bp}}{H_{Bg}}\right) - 8.29 \times 10^{-3}$

$\frac{H_{Bp}}{H_{Bg}} = 1.13 < 1.2$

$C_p = 2300 \sqrt{psi}$ Table [14-8] for Steel - Steel.

$b = 1.25$ , $d_p = 1.8$ in.

$$V = \frac{\pi d_p n_p}{12} = \frac{\pi (\frac{18}{10}) 1720}{12} = 811 \text{ fpm.}$$

$C_v = 1.30$ Fig. (14-9) [$V = 811$ fpm and $Q_v = 7.0$]

$C_m = K_m = 1.19$

$C_o = K_o = 1.25$ [Uniform driver, moderate shock driven]

$$I = \frac{\sin\phi \cos\phi}{2} \frac{R}{R+1}$$

$$R = \frac{d_g}{d_p} = \frac{w_p}{w_g} = 2.15 \Rightarrow I = 0.109.$$

$$\sigma_H = 2300 \sqrt{\frac{F_t}{(1.25)(1.8)(0.109)} \frac{(1.25)(1.19)(1.3)}{}} = 6458.4 \sqrt{F_t} \text{ psi}$$

3- Pinion: $\sigma_H = (S_{fe})_p \Rightarrow 6458.4\sqrt{F_t} = 104.22$ Ksi $\Rightarrow F_t = 260.4$ lb

gear : $\sigma_H = (S_{fe})_g \Rightarrow 6458.4\sqrt{F_t} = 99.225$ Ksi $\Rightarrow F_t = 236.$ lb.

4- Power transmitted based on gear strength.

$$H = \frac{236 \times 811}{33,000} = 5.8 \text{ hp.}$$

For $L = 10^7$ cycles

based on gear weaker member

$(S_{fe})_g = 128$ kpsi

$\Rightarrow 7907\sqrt{F_t} = 128 \times 10^3 \longrightarrow F_t = 254$ lb

$\Rightarrow H = \frac{254 \times 811}{33000} = 6.24$ hp

Based on bending :

$H = 14.9$ hp.

Excess of bending capacity is desirable, because bending failure is sudden and total, Surface failure is gradual → causes noise → warn.

In order to compansate the loss of power from $(4.2 - 14.9)$hp, Steel gear is normaly treated in order to make the tooth surface harder than core teeth material.

## Spur Gear Design Procedure:

For given application it is necessary to Design Suitable [optimal] pair of gears.

### Observation of gear Design:

1- Usually, gear ratio, (power + speed) or (Torque - speed) of one shaft are defined.

2- Design parameters: pitch diameters, $P$ (m), Face width = b, materials, factor of safety.

3- Design decisions: mesh-accuracy, no. of cycles, pressure angle, operating temp., reliability.

4- Determine factor of safety for bending-fatigue and surface-fatigue failure.

5- Better strategy: Start with bending stresses → Increasing surface hardness increase wear resistance than bending strength.

Thus: If material withstand bending stress → Its surface can be treated to withstand wear. without design modifications.

Surface strength:

Steel: $S_c = 0.76(H_B) - 70$ MPa , $S_c = 0.4(H_B) - 10$ Kpsi

Double $H_B$ more than Double $S_c$.

$\sigma_H = \sqrt{\frac{F_t}{bId_p}}$ Doubling $\sigma_H$ → $F_t$ increases 4 times.

Bending strength:

Doubling ($H_B$) less than double ($S_t$) or ($S_e$)

Increasing $H_B$ → Increasing bending fatigue strength.

5- Increasing tooth size (coarser pitch) increasing bending strength more than surface strength.

High Hardness steel gears ($H_B > 500$).

Balance between surface fatigue and bending fatigue occurs near $P = 8$.

Coarses pitch fail in bending, finer teeth failing in surface fatigue.

6- Softer teeth:

Surface fatigue more critical for finer pitches.

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7- Larger pitch radius → reduces $F_t$, but also increasing pitch line vel
reasonable compromise must be determined.

Smaller radius → Smaller no. of teeth to avoid interference.

8- IF min. no. of teeth is required.

Start with min. number of teeth to avoid interference → Table [13-1].

Normally [ 18-teeth for 20° pinion, 12-teeth for 25° pinion ].

Then, Solve for P (m).

9- Face width: $$\frac{9}{P} < b < \frac{14}{P}$$

0- Harder gears are more costly to manufacture.

harder gears → Smaller, doing the same Job.

Smaller gears → housing and associated parts are smaller and ligh

Smaller gears → lower pitch line velocity, lower dynamic loading and rubbing velocities.

Result: harder gears → Reduce overall cost.

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Example (1)

Design standard spur gears to connect 100 hp, 3600 rpm motor to 900 rpm load shaft. Shock load is neglected.
Center distance is to be as small as possible
life of the gear set: 5 yrs of 2000 hr/yr operation
full load transmitted only 10% of time.
half power 90% of time.
$R = 99\%$

Solution:

1- Min. center distance, choose harder gears.

Pinion = 400 Bhn. , gear = 350 Bhn.

2- hard gears [difficult to manufacture by normal machining]
Use precision manufacturing $Q_v = 10, 11$

3- More common, $\phi = 20°$, full depth involute, $N_p = 18$.
Min. no. of pinion teeth to avoid interference.

$$N_p = \frac{2K}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$K = 1.0$ full depth teeth.

$$m = \frac{N_g}{N_p} = \frac{\omega_p}{\omega_g} = 4$$

$N_p = 15.44 \rightarrow$ Take $N_p = 18. \rightarrow N_g = 18 \times 4 = 72.$

Max. no. of gear teeth to avoid interferece.

$$N_g = \frac{N_p^2 \sin^2\phi - 4K^2}{4K - 2N_p \sin^2\phi} = 160 \quad , N_g < N_{g\,max}.$$

4- Life.
Pinion $= 5\text{ yr} \times 3600\text{ (rpm)} \times 2000 \frac{hr}{yr} \times 60 \frac{min}{hr} = 2.16 \times 10^9$ rev.
full load $= 10\%$ life $= 2.16 \times 10^8$ rev. $\rightarrow$ power $= 100$ hp
half load $= 50\%$ life $= 1.08 \times 10^9$ rev $\rightarrow$ power $= 50$ hp.

For min d → take max. $b \approx \frac{14}{P}$ (2)

$$\frac{9}{P} \leq b \leq \frac{14}{P}$$

6- Start with bending stress:

$$\sigma = \frac{F_t\, P\, K_o\, K_v\, K_m}{b\, J}$$

7- $V = \frac{\pi\, d_p\, n_p}{12} = \frac{\pi\left(\frac{18}{P}\right) 3600}{12} = \frac{16964.6}{P}$

Since V is unknown.

$K_v \cong 1.4$ rough estimation [must be corrected when b is determin

$K_m \approx 1.3$ [must be corrected when b is determined]

$J_p = 0.32$ [for $N_p = 18$, $N_g = 72$, precision gear, load sharing]

8- Life factor : full load

$K_L = 1.683\, N^{-0.032}$ $L_p = 2.16 \times 10^8$, $K_{Lp} = 0.91$

gear, $L = 5.4 \times 10^7$ → $K_{Lg} = 0.95$

Half load : $L_p = 1.94 \times 10^9$ → $K_{Lp} = 0.86$

9- Bending strength. $\cong 10^{8}\,10^{4}$ $K_{Lp} = 0.86$

$$S_e = S_t \frac{K_L\, K_t}{K_r}$$

$K_r = 1.0$ → $R = 99\%$, $K_t = 1.0$, $K_o = 1.0$ Uniform drive.

Pinion : $S_e = 0.91 \times 39.855 = 36.26$ kpsi

gear : $S_e = 0.95 \times 36 = 34.2$ kpsi

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$\sigma_p = \dfrac{S_{ep}}{n_b}$

Take $n_b = 1.5$

To Find $F_t$

$$F_t = \frac{33000\ hp}{V} = \frac{33000 \times 100}{16960} \underline{P} = 195\ \underline{P}.$$

$$\sigma_p = \frac{(195\ \underline{P})(1.3)(1.4)\underline{P}}{\frac{14}{\underline{P}}(0.32)} = \frac{36.26}{1.5}$$

$\Rightarrow \underline{P} = 6.73$

Taking the gear:

$J_g = 0.47 \quad [N_g = 72, \quad N_p = 18]$

$$\sigma_g = \frac{(195)\underline{P}\ (1.3)(1.4)\ \underline{P}}{\frac{14}{\underline{P}} \cdot (0.47)} = \frac{34.2}{1.5} \longrightarrow \underline{P} = 7.3$$

Take $\underline{P} = 8.0$ Standard.

$\Rightarrow \quad V = 2120.57$ ft/min.

$A = 50 + 56(1-B) = 84.16, \quad B = \dfrac{(12 - Q_v)^{2/3}}{4} = 0.39$

$$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B = 1.185$$

$b = \dfrac{14}{\underline{P}} = 1.75. \qquad \leqslant \dfrac{14}{\underline{P}}$

$C_{pf} = \dfrac{b}{10d} - 0.0375 + 0.0125\,b = 0.062 \qquad 1 < b < 17$

$C_c = 1.0$

$C_{pm} = 1.0$ Straddle mounted $S_1 = 0$ [mid-span]

$C_{ma} = A + Bb + Cb^2 = 0.0675 + 0.0128\, b + -0.926\times10^{-4}\, b^2$
$= 0.0896$

Precision enclosed gear $\rightarrow$ Table [14-9]

$C_e = 1.0$

$K_m = 1 + C_c(C_{fp}C_{pm} + C_{ma}C_e) = 1.1516$

$$\sigma_p = \frac{(195\ P)\ (1.152)(1.185)(1.0)(8)}{1.75\quad(0.32)} = 30.41 \text{ kpsi}$$

$$n_b = \frac{S_{cp}}{\sigma_p} = 1.192$$

Take $b = 1.75 \rightarrow \sigma_p = 30.41\text{kpsi}$

$C_{pf} = 0.062$, $K_v = 1.185$

$C_{ma} = 0.0896$

$C_m = K_m = 1.1516$

$$n_b = \frac{36.26}{30.41} \cong 1.2$$

gear

$$\sigma_g = \frac{(195\times8)(1.1516)(1.185)\times8}{1.75\times0.47} = 20.76 \text{ kpsi}$$

$S_{eg} = 34.2\text{kpsi}$

$$n = \frac{34.2}{20.76} = 1.65$$

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Contact Stress

(5)

$$\sigma = \sqrt{\frac{F_t\ C_o C_m C_v}{b\, I\, d_p}} \times C_p$$

$$I = \frac{\sin\phi \cos\phi}{2}\ \frac{R}{R+1} \qquad R = \frac{w_p}{w_g} = 4$$

$I = 0.128$

$d_p = \frac{18}{8} = 2.25"$

$b = 1.75$

$$\sigma = \sqrt{\frac{195 \times 8 \times 1.1516 \times 1.185}{1.75 \times 0.128 \times 2.57}} \times 2300 = 139.86 \text{ Kpsi}$$

$C_p = 2300 \longrightarrow$ Steel – Steel.

Fatigue contact strength.

$$S_{fe} = S_c\ \frac{C_L C_t C_H}{C_R}$$

$$\frac{H_{BP}}{H_{Bg}} = 1.16 < 1.2 \longrightarrow C_H = 1.0$$

$C_L = 2.466\ N^{-0.056}$.

Pinion : $C_L = 0.842. \longrightarrow S_{fep} = 0.842\ S_{cp}$

gear : $C_L = 0.91. \longrightarrow S_{feg} = 0.9\ S_{cg}$.

Choose factor of safety $n_c = 1.25$ for contact stress.

$\Rightarrow \frac{S_{fep}}{n_c} = \sigma_c \rightarrow S_{cp} = 207.6$ kpsi $\rightarrow H_B$

$\Rightarrow \frac{S_{feg}}{n_c} = \sigma_c \rightarrow S_{cg} = 171.1 \qquad \rightarrow H_B$

Using through hardened steel of grade 1 $\rightarrow S_c = 322\ H_B + 29100$

$S_{cp} = 322 \times 350 + 29100 = 141{,}8$ Kpsi $\rightarrow$ gear will fail in contact.

So surface must be hardened to $S_c = 208.8$ k[illegible] $H_{BP} = 554$

$H_{BP}/H_{Bg} = 1.25 \qquad H_{Bg} = 441$

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– Contact Ratio:

$r_p = \frac{N_p}{2P} = 1.125''$

$r_g = \frac{N_g}{2P} = \frac{72}{2\times 8} = 4.5''$

$r_{ap} = r_p + \frac{1}{P} = 1.25''$

$r_{ag} = 4.5 + \frac{1}{8} = 4.625''$

$r_{bp} = r_p \cos\phi = 1.057''$

$r_{bg} = r_g \cos\phi = 4.228''$

$P_b = p\cos\phi = \frac{\pi}{P}\cos\phi = 0.369''$ , $C = r_p + r_g = 5.625''$

$$C.R = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - C\sin\phi}{P_b}$$

$C.R = 1.67$

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