

8) $\sum_{n=1}^{\infty} (-1)^n \frac{10^n}{(n+1)!}$

{10.6}

$$\sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty} \frac{10^n}{(n+1)!}$$

$$\lim_{n \rightarrow \infty} \frac{10^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{10^n} = \lim_{n \rightarrow \infty} \frac{1}{n+2} = 0 < 1$$

$$\text{so } \sum_{n=1}^{\infty} U_n \text{ conv} \rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{10^n}{(n+1)!} \text{ conv}$$

20) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^n}$

$$U_n = \frac{n!}{2^n}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{n!}{2^n} = \infty \quad \text{div by } n^{\text{th}} \text{ term test}$$

$$\times \lim_{n \rightarrow \infty} \frac{U_{n+1}}{U_n} = \lim_{n \rightarrow \infty} \frac{(n+1)!}{2^{n+1}} \cdot \frac{2^n}{n!}$$

$$= \frac{1}{2} \lim_{n \rightarrow \infty} (n+1) = \infty > 1 \quad \text{div by } n^{\text{th}} \text{ term test}$$

13) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$

Haytham Shehadeh

$$U_n = \frac{\sqrt{n}+1}{n+1} > 0, \quad U_n > U_{n+1}, \quad \lim_{n \rightarrow \infty} U_n = 0$$

conv by AST

25) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+h}{n^2}$

$$\sum_{n=1}^{\infty} \frac{1+h}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} + \sum_{n=1}^{\infty} \frac{h}{n^2} \rightarrow \sum_{n=1}^{\infty} \frac{1+h}{n^2} \text{ div}$$

$\downarrow$ conv $\downarrow$ div

$$U_n = \frac{1+h}{n^2} > 0, \quad U_n > U_{n+1}, \quad \lim_{n \rightarrow \infty} U_n = 0 \quad \text{conv AST}$$

$$\text{so } \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+h}{n^2} \text{ conv cond}$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$$

$$\sum_{n=1}^{\infty} \frac{\ln n}{n - \ln n} = \sum_{n=1}^{\infty} \frac{\ln n}{n} - \sum_{n=1}^{\infty} 1 \rightarrow \text{div}$$

$$n - \ln n < n$$

$$\frac{1}{n - \ln n} > \frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ div}$$

$$V_n = \frac{\ln n}{n - \ln n} > 0$$

$$f(x) = \frac{\ln x}{x - \ln x}$$

$$f'(x) = \frac{x - \ln x}{x^2} - \frac{\ln x \left[1 - \frac{1}{x}\right]}{(x - \ln x)^2}$$

$$f'(x) = \frac{1 - \frac{1}{x} \ln x - \ln x + \frac{1}{x} \ln x}{x^2 (x - \ln x)^2} = \frac{(1 - \ln x)}{x^2 (x - \ln x)^2} < 0$$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n - \ln n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{1 - \frac{1}{n}} = 0$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n} \text{ conv by AST}$$

$$39) \sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$$

$$V_n = \frac{(2n)!}{2^n n! n}$$

$$\lim_{n \rightarrow \infty} \frac{(2n+2)!}{2^{n+1} (n+1)! (n+1)} \cdot \frac{2^n n! n}{(2n)!} = \lim_{n \rightarrow \infty} \frac{(2n+2)(2n+1)n}{2(n+1)(n+1)} = \infty \text{ div by } n^{\text{th}} \text{ term test}$$

$$42) \sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2+n} - n)$$

$$\lim_{n \rightarrow \infty} \sqrt{n^2+n} - n = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n} + n}{\sqrt{n^2+n} + n} = \lim_{n \rightarrow \infty} \frac{n^2+n - n^2}{\sqrt{n^2+n} + n} = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 \left[1 + \frac{1}{n}\right]} + n} = \lim_{n \rightarrow \infty} \frac{n}{n \sqrt{1 + \frac{1}{n}} + n}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n}} + 1} = \frac{1}{2} \neq 0$$

div by n^{th} term test

$$14) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3\sqrt{n+1}}{\sqrt{n+1}}$$

$$U_n = \frac{3\sqrt{n+1}}{\sqrt{n+1}} > 0$$

$$\lim_{n \rightarrow \infty} U_n = 3 \neq 0 \text{ div by } n^{\text{th}} \text{ term test}$$

$$19) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3+1}$$

$$\sum_{n=1}^{\infty} \frac{n}{n^3+1} \text{ conv}$$

$$\frac{n}{n^3+1} < \frac{1}{n^2}$$

$$\text{so } \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3+1} \text{ conv Abs}$$

$$29) \sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1} n}{n^2+1}$$

$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2+1}$$

$$u = \tan^{-1} x$$

$$du = \frac{dx}{1+x^2}$$

$$\lim_{b \rightarrow \infty} \int u \, du = \lim_{b \rightarrow \infty} \left. \frac{(\tan^{-1} x)^2}{2} \right|_1^b = \lim_{b \rightarrow \infty} \left[\frac{(\tan^{-1} b)^2}{2} - \frac{(\tan^{-1} 1)^2}{2} \right]$$

$$= \frac{3\pi^2}{32}$$

$$\text{so } \sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1} n}{n^2+1} \text{ conv Abs}$$

53) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{10^n}$

$$|E| < U_{n+1} = |a_{n+1}|$$

$$|E| < a_5 = \left| (-1)^{5+1} \frac{1}{10^5} \right|$$

$$|E| < 0.00001$$

54) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$

$$|E| < 0.001$$

$$\frac{n+1}{(n+1)^2+1} < 0.001$$

$$(n+1)1000 < (n+1)^2+1$$

$$1000n+1000 < n^2+2n+2$$

$$0 < n^2 - 998n - 998$$

$$n > \frac{998 \pm \sqrt{(-998)^2 - 4(1)(-998)}}{2(1)} \approx 998.99899$$

$$n \geq 999$$