Leen Sabri

Q7 $\sum_{n=1}^{\infty} \frac{n^2(n+2)!}{n!\,3^{2n}}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^2(n+3)!}{(n+1)!\,3^{2(n+1)}}}{\frac{n^2(n+2)!}{n!\,3^{2n}}} = \lim_{n\to\infty} \frac{(n+1)^2(n+3)!}{(n+1)!\,3^{2(n+1)}} \cdot \frac{n!\,3^{2n}}{n^2(n+2)!} = \lim_{n\to\infty} \frac{(n+1)^{\cancel{2}}(n+3)\cancel{(n+2)!}}{\cancel{(n+1)}\cancel{n!}\,\cancel{3^{2n}}\times 3^2} \cdot \frac{\cancel{n!}\ \cancel{3^{2n}}}{n^2\cancel{(n+2)!}} = \lim_{n\to\infty} \frac{(n+1)\times(n+3)}{n^2\,3^2} = \lim_{n\to\infty} \frac{n^2+3n+n+3}{3^2} = \frac{1}{3}\left(\lim_{n\to\infty}\frac{n^2+4n+3}{n}\right) = \frac{1}{3} > 1$

$(n+1)\,n!$ $\quad 3^{2n+2} = 3^{2n} \times 3^2$

$\therefore$ converg by Ratio Test

Q12 $\sum_{n=1}^{\infty}\left(\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\left(\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}} = \lim_{n\to\infty}\left(\ln\left(e^2+\frac{1}{n}\right)\right)^{\frac{n+1}{n}} = \lim_{n\to\infty}\left(\ln\left(e^2+\overset{0}{\frac{1}{n}}\right)\right)^{1+\frac{1}{n}} = 2 > 1 \therefore$ diverges by the Root Test

$1+0=1$

$\lim_{n\to\infty} \ln(e)^2$ [illegible]

Q16 $\sum_{n=2}^{\infty} \frac{1}{n^{1+n}} \Rightarrow \lim \sqrt[n]{\frac{1}{n^{1+n}}} = \lim_{n\to\infty} \frac{\sqrt[n]{1}}{\sqrt[n]{n\times n^n}} = \lim_{n\to\infty}\frac{1}{n\sqrt[n]{n}} = \frac{1}{\infty\cdot 1} = \frac{1}{\infty} = 0 < 1 \therefore$ converge by Root Test

Q20 $\sum_{n=1}^{\infty} \frac{n!}{10^n} \Rightarrow \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{(n+1)!}{10^{n+1}}}{\frac{n!}{10^n}} = \lim_{n\to\infty}\frac{(n+1)!}{10^{n+1}}\cdot\frac{10^n}{n!} = \lim_{n\to\infty}\frac{(n+1)\cancel{n!}}{10^n\cdot 10}\cdot\frac{\cancel{10^n}}{\cancel{n!}} = \lim_{n\to\infty}\frac{n+1}{10} = \infty > 1 \therefore$ diverges by Ratio test

Q28 $\sum_{n=1}^{\infty}\frac{(\ln n)^n}{n^n} \Rightarrow \lim_{n\to\infty}\sqrt[n]{\left(\frac{\ln n}{n}\right)^n} = \lim_{n\to\infty}\frac{\ln n}{n} = \frac{\infty}{\infty} = \lim_{n\to\infty}\frac{\frac{1}{n}}{1} = \lim_{n\to\infty}\frac{1}{n} = 0 < 1 \therefore$ converges by The Root Test.

Q38 $\sum_{n=1}^{\infty}\frac{n!}{n^n} \Rightarrow \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} = \lim_{n\to\infty}\frac{(n+1)!}{(n+1)^{n+1}}\cdot\frac{n^n}{n!} = \lim_{n\to\infty}\frac{\cancel{(n+1)}\cancel{n!}}{\cancel{(n+1)}(n+1)^n}\cdot\frac{n^n}{\cancel{n!}} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty}\frac{1}{\left(\frac{n+1}{n}\right)^n} = \lim_{n\to\infty}\frac{1}{\boxed{\left(1+\frac{1}{n}\right)^n}} = \frac{1}{e} < 1 \therefore$ converge by Ratio test.

$e^x \leftarrow$ ch10.1 في 5 رقم حسب قاعدة

Q43 $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!} \Rightarrow \lim\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{((n+1)!)^2}{(2(n+1))!}}{\frac{(n!)^2}{(2n)!}} = \lim_{n\to\infty}\frac{((n+1)!)^2}{(2(n+1))!}\cdot\frac{(2n)!}{(n!)^2} = \lim_{n\to\infty}\frac{(n+1)^2\cancel{(n!)^2}}{(2(n+1))\cancel{(2n)!}}\cdot\frac{\cancel{(2n)!}}{\cancel{(n!)^2}}$ $\quad$ by The Ratio Test

Q46 $a_1 = 1,\ a_{n+1} = \frac{1+\tan^{-1} n}{n}a_n \Rightarrow \lim\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{1+\tan^{-1} n}{n} = \frac{1+\frac{\pi}{2}}{\infty} = 0 < 1 \therefore$ converge by Ratio Test

Q60 $\sum_{n=1}^{\infty}\frac{n^n}{(2^n)^2} \Rightarrow \lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{2^2}\right)^n} = \lim_{n\to\infty}\frac{n}{4} = \infty > 1 \therefore$ diverges by the Root Test