# Chapter 5

## Time value of money

أهم نقطة بهذا الفصل وبكل الفصول القادمة كمان

present value (القيمة الحالية)

future value (القيمة المستقبلية)

القوة الشرائية لـ $10,000 اليوم رح أفقدهم بعد 5 سنين

⇐ قيمتها اليوم أعلى

* قيمة العملة بتقل مع مرور الزمن.

عشان أحسب القيمة الحالية لازم أعرف قيمتهم المستقبلية.

* العملة اليوم رح تشتري بعد 5 سنين ← خسرت الشراء

اليوم ممكن أحصل على منتجات العملة

⇐ فـ دائماً دائماً ال present value بكون أقل من ال future value

discounting

⇒ We calculate the present value of a certain amount of money that is expected to be received at same future time.

Future value: we calculate the future value for a certain amount of money that we have today (Compounding)

فائدة مركبة (زي اللي بالبنك)

الفائدة بضيفها ع آخر مبلغ (مش زي ال simple)

Simple interest

PV = $1000

r = 5%

n = 2 years

FV = ??

⇒ r = 1000 × 5% × 2 = $100

FV = 1000 + 100 = $1100

Compounded interest:

PV = $1000

r = 5%

n = 2 years

FV = ??

⇒ r = 5% × 1000 = 50

after 1 year = 1000 + 50 = 1050

after year 2:

r = 5% × 1050 = $52.5

(Simple زي اللي) 1000 مش ⟵ باخد الفايدة اللي طلعت

($1050) ⟵ في آخر سنة مش المبلغ اللي بدأت فيه

⇒ FV = 1000 + 52.5 = $1102.5

(1050 + 52.5)

* Basic cash flow patterns: أنماط التدفقات النقدية

① Single amount مبلغ واحد

② Annuity = equal cash flows (could be in/out) - equal amount of money. مبالغ متساوية

③ Mixed stream = unequal cash flows تدفقات / مبالغ غير متساوية.

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[1] Calculating future value of a single amount

↓

Compounding of interest

القانون

$$FV = PV\ (1+r)^{n}$$

n → # of periods

FV: future value, PV: present value, r: rate of return

$$S = P(1+r)^{t}$$

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e.g (Page 215)

PV = $800

r = 6% compounded annually.

n = 5 years

FV = ??

The higher (n, r) أعلى / أكبر → The higher FV

$$FV = PV\ (1+r)^{n}$$

$$= 800\ (1+0.06)^{5}$$

$$= \$1,070.58$$

## 2 Calculating the present value of single amount

$$FV = PV(1+r)^n \Rightarrow PV = \frac{FV}{(1+r)^n}$$

$\frac{FV}{(1+r)^n} \quad \frac{PV(1+r)^n}{(1+r)^n}$

⇒ The higher the $(n, r)$ ~~the longer~~ ⇒ the lower the present value

علاقة عكسية

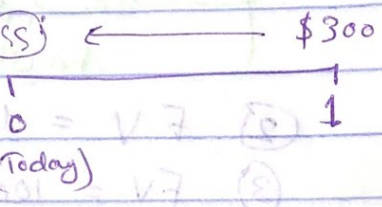
eg Page 217: $FV = \$300$ $n = 1$ year

$r = 6\%$

- discount rate : سعر الخصم
- interest rate - opportunity cost.
- rate of return.

$PV = ??$

$$PV = \frac{FV}{(1+r)^n}$$




$$PV = \frac{300}{(1.06)^1} = \$283.02$$

## 3 Calculating future of an annuity:

Annuity (equal cash flows)

- ordinary annuity. (في نهاية كل سنة (فترة))
  assumes that cash inflows/outflows happen at the end of each period.
- annuity due. (في بداية كل سنة (فترة))
  assumes that cash inflows/outflows happen at the beginning of each period

* Calculating future of an ordinary annuity (at the end)

eg (page 221):

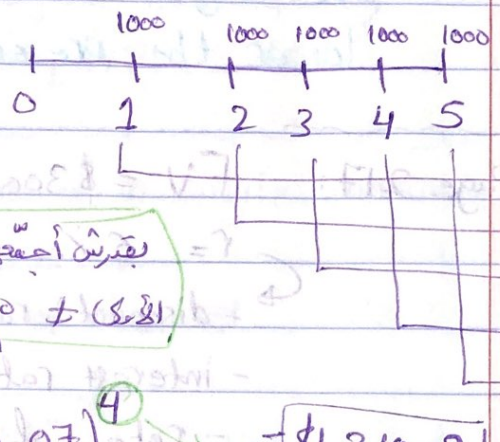
CF = $1000 / year.

ordinary annuity

n = 5 years.

r = 7%.

Future Value of Annuity ← FVA = ??




بقدرش أجمع بعض عشان ال 1000 في نهاية السنة، وممكن السنة 1000 + (عشان) في نهاية السنة

① $FV = PV(1+r)^n$

$= 1000(1+0.07)^4 = \$1,310.8$

بجمع

(عالج بعطي دائنة ع السنة الثانية وأتابع كل مثل) السنة (2) السنة

② $FV = 1000(1+0.07)^3 = \$1,225.04$

③ $FV = 1000(1+0.07)^2 = \$1,144.9$

④ $FV = 1000(1+0.07)^1 = \$1070$

⑤ $FV = 1000(1+0.07)^0 = \$1000$

منطقياً، 1000 في نهاية السنة الخامسة بساوي 1000 في نهاية السنة الخامسة

$\Rightarrow FVA = 1,310.8 + 1225.04 + 1144.9 + 1070 + 1000$

$= \$5750,74$

Dec 10, 2019
Tuesday

* $FVA = CF \left( \frac{(1+r)^n - 1}{r} \right)$

FVA: future value of an ordinary annuity
CF: cash flow (in/out)
r: required of return interest rate
n: number of periods.

eg page 221: CF = $1000 \ year
n = 5 years
r = 7 %.

تكملة مثال المحاضرة الماضية
لسه مختصرة بهاد القانون
(FVA)

$$FVA = CF \left( \frac{(1+r)^n - 1}{r} \right) = 1000 \left( \frac{(1+7\%) - 1}{7\%} \right)$$

= $5,750.74

لاحظ أني: نفس الجواب ☺

## [4] Calculating present value of an ordinary annuity:

* Example page 223: at the end of the year ⇒ ∴ ordinary
← لأنها في نهاية الفترة بتكون.

CF = $700 \ year
n = 5 years
r = 8 %.

⇒ PVA = ??
Present value of an annuity

زي time بس مرجّل
لسنة بيدا
PV

Time today 700 700 700 700 700
0 1 2 3 4 5

$ 648.15
$ 600.14
$ 555.68
$ 514.52
$ 476.41

$PV = \frac{FV}{(1+r)^n}$
(Single amount)

PVA = مجموعهم = $2,794.90

$$PVA = \frac{CF}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

$$= \frac{700}{0.08}\left(1 - \frac{1}{(1+0.08)^5}\right) = \$2,794.90$$

نفس الجواب

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* Annuity ordinary ⇐ بتكون بنهاية فترة معينة

Annuity due ⇐ بتكون بالبداية

بكون عندنا:

extra one period. (future value ال في حالة)

e.g: CF = $100 \ year (at the beginning of each year).

n = 3 years

r = 5%

FVA due = ??: (extra one period)

① $FV = PV(1+r)^n = 100(1+5\%)^3 = \$115.76$

② $FV = 100(1+5\%)^2 = \$110,25$

③ $FV = 100(1+5\%)^1 = \$105$

FVA due = ① + ② + ③ = $331.01

(n = 3 مش) n = 2 هنلاقي من ordinary لو

نقسم

* $FVA_{due} = CF\left(\frac{(1+r)^n - 1}{r}\right)(1+r)$

$FVA_{ordinary}$ . هادي مضروب

$= 100\left(\frac{(1+5\%)^5 - 1}{5\%}\right)(1+5\%) = \boxed{\$331.01}$

(نفس الجواب)

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6 Calculating present value of annuity due :

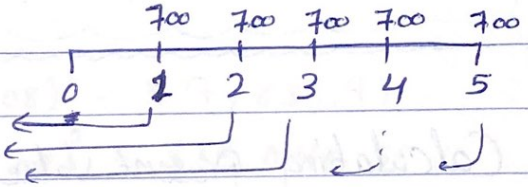
eg page 226 : CF = \$700 \ year (at the beginning of each year)

r = 8 %

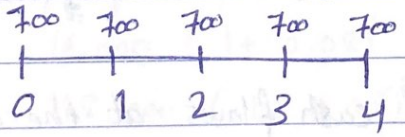
n = 5 years

PVA due ?

⇒ Ordinary annuity :




due :




PV لما due بدل لما يكون less one period

FV: extra 1 period

① $PV = \frac{FV}{(1+r)^n} = \frac{700}{(1+8\%)^0} = \$700$

② $PV = \frac{700}{(1+8\%)^1} = \$648.15$

③ $PV = \frac{700}{(1+8\%)^2} = \$600.14$

→

④ $PV = \frac{700}{(1+8\%)^3} = \$555.68$

⑤ $PV = \frac{700}{(1+8\%)^4} = \$514.52$

$PVA_{due} = ① + ② + ③ + ④ = \$3,018.49$

بالقانون : $PVA_{due} = \frac{CF}{r}\left(1+r\right)\left(1-\frac{1}{(1+r)^n}\right)$

n بعوّضها زي اللي بالسؤال (ما بتوه)

$= \frac{700}{0.08}\left(1+0.08\right)\left(1-\frac{1}{(1+8\%)^5}\right) = \$3,018.49$

* اذا كانت $n = \infty$ بقدر أوجد ال PV بس ما بقدر أوجد FV

## 7 Calculating present value of a perpetuity:

* perpetuity = equal cash flows at the end of each year. $n = \infty$

$$PV = \frac{CF}{r}$$

e.g : Preferred stock

$CF = D = \$100$

$r = 15\%$ $\quad n = \infty$

stock price today = ??

PV = ??

لأنها استنتجنا

$\Rightarrow PV = \frac{CF}{r} = \frac{100}{0.15} = \$666.67$

التفسير (من التونسي ( الدقيقة 50)):

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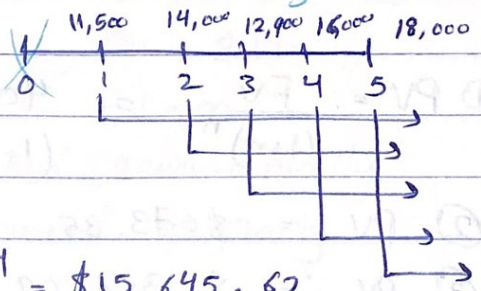
[8] Calculating future value of a mixed stream:

Single amount

eg / P.229 :




① $FV = PV(1+r)^n$

$(n = 4)$

$= 11,500(1+8\%)^4 = \$15,645.62$

② $FV = 14,000(1+0.08)^3 = \$17,635.97$

③ $FV = 12,900(1+0.08)^2 = \$15,048.56$

④ $FV = 16,000(1+0.08)^1 = \$17,280$

⑤ $FV = 18,000(1+0.08)^0 = \$18,000$

$\Rightarrow FV = ① + ② + ③ + ④ + ⑤ = \boxed{\$83,602.15}$

عشان بالسؤال ← الأرقام إلي معطيني إياها : كنت ال End of the year

نظام حسب ال 0

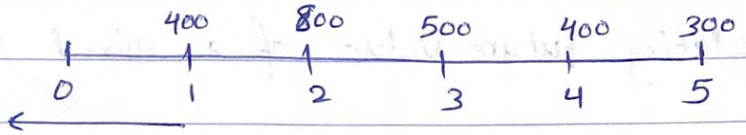
إذا حكالنا بال Beginning of the year ← مرجع رجوع (منقلش من $n=5$ لحد $n=1$)

## 9 Calculating present value of a mixed stream:

eg: Page 230 , r = 9%

n = 5 years

End of the year PV = ??




① $PV = \frac{FV}{(1+r)^n} = \frac{400}{(1+9\%)^1} = \$366.97$

② PV = $673.35

③ PV = $386.09

④ PV = $283.37

⑤ PV = $194.98

⇒ PV = ① + ② + ③ + ④ + ⑤

= $1,904.76

محاضرة التعويض Dec 10, 19

### * Compounding interest more frequently than annualy:

Before calculating future value or present value or present value some modifications are needed to be made:

② If interest was compounded quarterly, then

$\frac{r}{4}$ , $n \times 4 \Rightarrow (4n)$

① If interest was compounded semi-annualy, then

$\frac{r}{2}$ , $n \times 2 \Rightarrow (2n)$

③ If interest was compounded monthly, then:

$\frac{r}{12}$ , $n \times 12 \Rightarrow (12n)$

④ If interest was compounded weekly, then:

$\frac{r}{52}$ , $n \times 52 \Rightarrow (52n)$.

* Note: Started = r .

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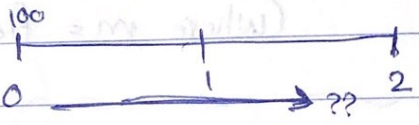
eg page 233: $n = 2$ year . $PV = \$100$

$r = 8\%$ . $FV = ??$

a- If interest was compounded semi-annually

b- If interest was compounded querterly.




[a]: $\frac{r}{2} = \frac{8\%}{2} = 4\%$ (2 = m = عدد المرات)

$n = 2 \times 2 = 4$ periods.

$\Rightarrow FV = 100\,(1 + 4\%) = \$116.99$

[b]: $\frac{r}{4} = \frac{8\%}{4} = 2\%$

$n = 4 \times 2 = 8$ periods

$\Rightarrow FV = 100\,(1 + 2\%) = \$117.7$

* كلما زاد ال Frequency ← ج يزيد ال FV

لأن تراكم الفوائد يزداد ( 117.7 > 116.99 )

↓ quorterly ↓ semi-annualy

## * Nominal interest rate versus effective annual rate:

(*) Nominal interest rate = Stated interest rate
= annual interest rate

r = 8% = annual interest rate
= Stated interest rate

(*) Effective annual rate = interest rate actually paid

$$\Rightarrow EAR = \left(1 + \frac{r}{m}\right)^m - 1$$

(where m = frequency).

e.g page 236: EAR = ??

r = 8% = nominal interest rate.

a- when interest is compounded annually?

b- " " " " semi-annually?

c- " " " " quarterly?

@ $EAR = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{8\%}{1}\right)^1 - 1 = \boxed{8\%}$

∴ ⇒ If interest was compounded annually (m=1), then EAR = nominal interest rate.

- (كلما ↓ m ↓ 8%)

(b) $EAR = \left(1 + \frac{8\%}{2}\right) - 1 = \boxed{8.16\%}$

(c) $EAR = \left(1 + \frac{8\%}{4}\right) - 1 = \boxed{8.24\%}$

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## * Special applications of time value of money:

* Loan ammortization schedual.

eg Page 241 :

Loan ammount = $6,000

r = 10 %.

n = 4 years

PMT = ??

↳ Payment cash flow.

in equal payment (دفعات متساوية) : annuity

$$PVA = \frac{CF}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

$$\$6{,}000 = \frac{CF}{0.1}\left(1 - \frac{1}{(1+0.1)^4}\right) \Rightarrow CF = \$1892.82$$

CF ↓ PMT

Loan PMT = interest + principal

المبلغ الي سددته = loan PMT - interest

| Year | Beg. Bal | loan Payment | interest (10%) | Principal | End. Balance |
|---|---|---|---|---|---|
| 1 | $6,000 | $1892.82 | $600 | $1292.82 | $4,707.18 |
| 2 | $4,707.18 | $1892.82 | $470.72 | $1,422.1 | $3,285.08 |
| 3 | $3,285.08 | $1892.82 | $328.51 | $1,564.31 | $1,720.77 |
| 4 | $1,720.77 | $1892.82 | $172.08 | $1,720.74 | -0- |

دفعات متساوية

لازم يكون صفر ولا يكون غلط

ملاحظات وقوانين و~~~~ المثال السابق :

* Interest = Beg. Balance × interest rate.
* principal = (Loan / PMT) - interest
* End. Balance = Beg. Balance - principal.
* ال End. Bal. بتنقل ال Beg. Bal في السنة التالية.

* أي زيادة في ال Assets تعتبر cash outflow

* تفسير P4-18 :

(b) expenses = fixed + variable
exp. are not all variable

(d)

| | | | |
|---|---|---|---|
| | $180,000 overstated of Profit | $230,625 | $285,500 understated of Profit. |
| بالمقارنة مع ~~~~ الطريقة الأولى | $187,325 | $295,480 | $383,640 |