BIRZEIT UNIVERSITY

Algorithm Analysis

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COMP242

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Mathematical Background

Logs and exponents

We will be dealing mostly with binary numbers (base 2)

Definition: $\log_X B = A$ means $X^A = B$

Mathematical Background

Properties of logs

We will assume logs to base 2 unless specified otherwise

$\log AB = \log A + \log B$ (note: $\log AB \neq \log A \bullet \log B$)

$\log A/B = \log A - \log B$ (note: $\log A/B \neq \log A / \log B$)

$\log A^B = B \log A$ (note: $\log A^B \neq (\log A)^B = \log^B A$)

SELF STUDY

Mathematical Background: Sums

$$f(n) = 1 + 2 + \ldots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$f(n) = 1 + 3 + 5 + \ldots + (2n-1) = \sum_{i=1}^{n} (2i-1) = n^2$$

$$f(n) = 1^2 + 2^2 + \ldots + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$f(n) = 1^4 + 5^4 + 9^4 + \ldots + (4n-3)^4 = \sum_{i=1}^{n} (4i-3)^4$$

Sum of squares: $\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6} \approx \frac{N^3}{3}$ for large N

Mathematical Background: Sums

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \ldots + x^n = \frac{x^{n+1} - 1}{x - 1} \quad (x \neq 1)$$

$$\sum_{k=0}^{n-1} x^k = \frac{x^n - 1}{x - 1} \quad (x \neq 1)$$

$$\sum_{k=0}^{n-1} 2^k = 2^n - 1$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1 - x} \quad \text{if } x < 1$$

Algorithm Analysis: Motivation

Problem

Design an algorithm

Algorithm

Analysis of algorithm

Solution 1

Solution 2

Solution 3

Solution 4

Solution 5

Algorithm Analysis: Motivation

To select the best algorithm from many you have to do Analyzing

- **Time:** is a function describing the amount of time an algorithm takes in terms of the amount of input to the algorithm.

> The better algorithm is the algorithm that run in less time

- **Space:** The number of memory cells which an algorithm needs.

> A good algorithm keeps this number as small as possible, too (consume less memory)

Algorithm Analysis: Motivation

In this course we will focus on time complexity instead of space.

How fast will your program run?

The running time of your program depends on:

1. Algorithm design
2. Input Size
3. Programming Language
4. The compiler you use
5. The OS on your Computer
6. Your computer Hardware (CPU, RAM, Busses)

How fast will your program run?

Time complexity: computational complexity that measures or estimates the time taken for running an algorithm.

Complexity can be viewed as the maximum number of primitive operations that a program may execute.

Common time complexities

$O(1)$	Constant time
$O(\log n)$	Logarithmic time
$O(\log^2 n)$	Log-squared time
$O(n)$	Linear time
$O(n^2)$	Quadratic time
$O(n^3)$	Cubic time
$O(n^i)$ for some i	Polynomial time
$O(2^n)$	Exponential time

Higher order functions of n are normally considered less efficient.

How fast will your program run?

Approximate time to run a program with ***n*** inputs on 1GHz machine:

Function	n = 10	n = 50	n = 100	n = 1000	n = 10^6
O(n log n)	35 ns	200 ns	700 ns	10000 ns	20 ms
O(n^2)	100 ns	2500 ns	10000 ns	1 ms	17 min
O(n^5)	0.1 ms	0.3 s	10.8 s	11.6 days	3x10^{13} years
O(2^n)	1000 ns	13 days	4 x 10^{14} years	*Too long!*	*Too long!*
O(n!)	4 ms	*Too long!*	*Too long!*	*Too long!*	*Too long!*

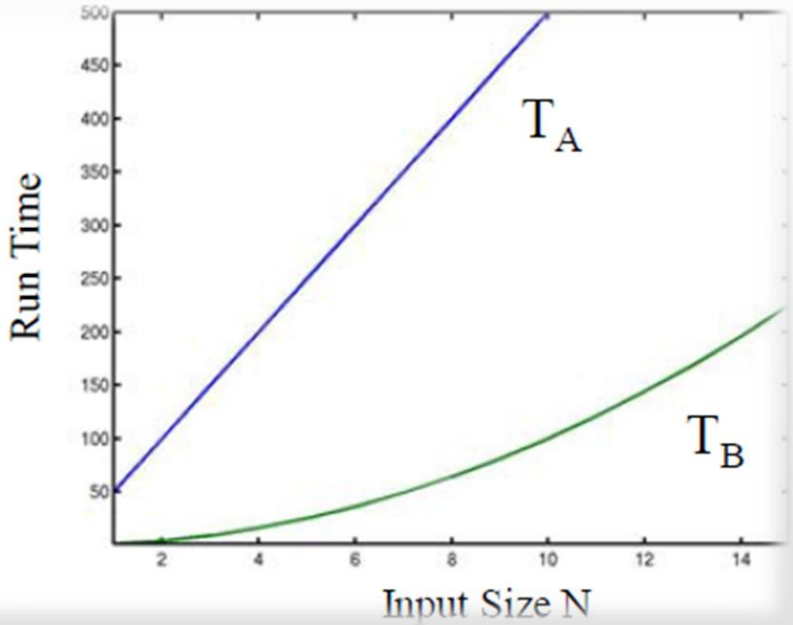
n: input size(data)

Algorithm Analysis: Motivation

Suppose you are given two algorithms A and B for solving a problem

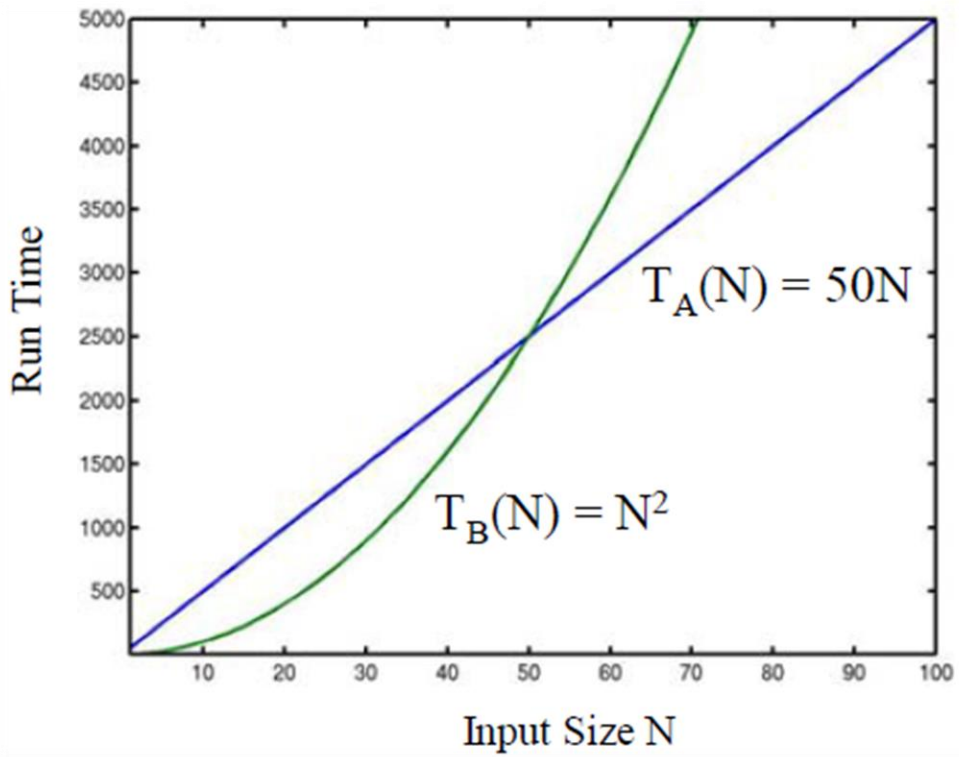
Here is the *running time* $T_A(N)$ and $T_B(N)$ of A and B as a function of input size N:

Which algorithm would you choose?



Algorithm Analysis: Motivation

For large N, the running time of A and B is:



Now which algorithm would you choose?

Algorithm Analysis: Running time Analysis

Suppose that T(n) is a function of time at a given input size n (size of data)

```
Algorithm Add (a,b){
  return a+b;          → 1 unit
}
```

In this case : T(n)=1

Constant

Running time is independent of the number of data items

```
Algorithm Add (a,n){
    s=0;                  → 1
     for i=1 to n do {    → n +1
       s=s + A[i];        → n
    }
    return s;             → 1
}
```

In this case : T(n)=2n+3

Polynomial function with degree 1
Linear function

Algorithm Analysis: Running time Analysis

Suppose that T(n) is a function of time at a given input size n (size of data)

```
Algorithm MatAdd (a,b,c){
    for i=1 to n do {   → n+1
       for j=1to n do { → n(n+1)
          c[i][j]=a[i][j]+B[i][j]; → n^2
        }
    }

}
```

In this case : $T(n)=2n^2+2n+1$

Polynomial function with degree 2

Highest degree of this polynomial is n^2 , so we called quadratic

Algorithm Analysis: Running time Analysis

Suppose that T(n) is a function of time at a given input size n (size of data)

one machine an algorithm may take

$T(n) = 15n^3 + n^2 + 4$

On another, $T(n) = 5n^3 + 4n + 5$

Both will belong to the same class of functions. Namely, "cubic functions of n".