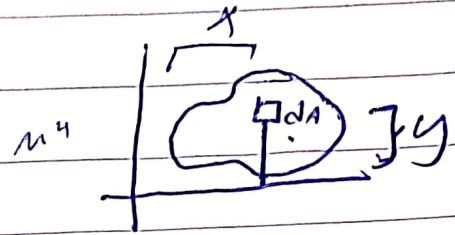


CHAPTER 9

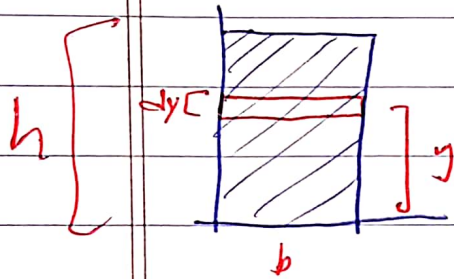
$$I_x = \int y^2 dA$$



$$I_y = \int x^2 dA$$

m^4

عند حساب التكامل
نأخذ شريحة موازية لل محور

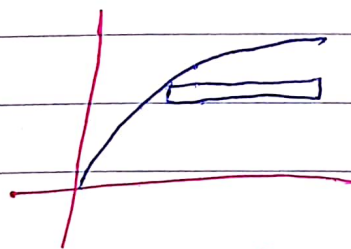


Find I_x

$$\int_0^h y^2 \cdot b \, dy$$

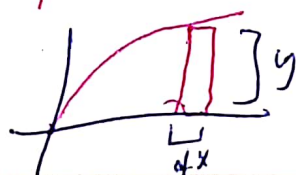
$$= \frac{y^3 b}{3} \Big|_0^h = \frac{bh^3}{3}$$

يمكن الاستفادة من $\frac{bh^3}{3}$
لتيجاد I_x حيث نأخذ شريحة
عمودية على المحور
أولاً انزلنا المحور



$$I_x = \int y^2 dA$$

or



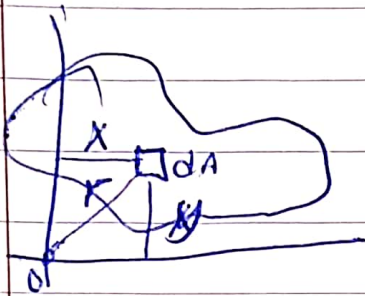
$$dI_x = \frac{dx y^3}{3} \rightarrow I_x = \int \frac{y^2}{3} dx$$

Diagram: Polar moment of inertia

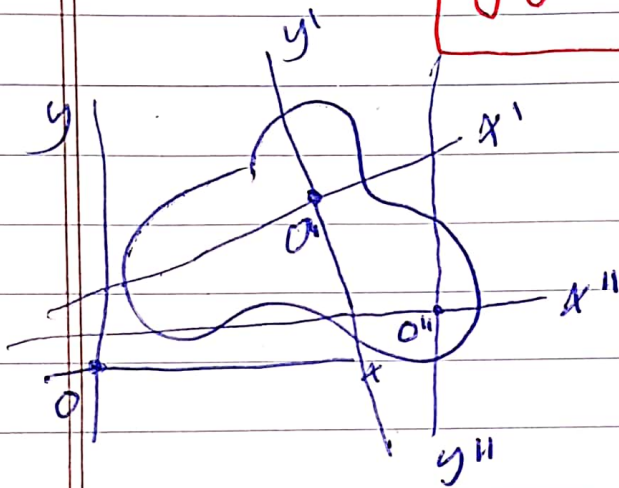
$$I_z = J_0 = \int r^2 dA$$

$$= \int x^2 + y^2 dA$$

$$= \int x^2 dA + \int y^2 dA$$



$$J_0 = I_x + I_y$$



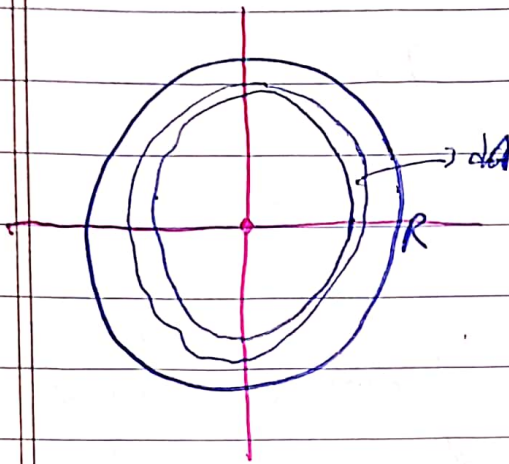
for any 2 perpendicular axes

$$J_0 = I_{axis 1} + I_{axis 2}$$

I_x & I_y are against bending moment

$I_z / J_0 =$ against twisting

Find J_0 / I_x / I_y for the circle



$$J_0 = \int r^2 dA$$

$$= \int_0^R r^2 2\pi R dr$$

$$= \int_0^R r^3 2\pi dr$$

$$= \frac{R^4}{4} \times 2\pi$$

J_0

$$J_0 = \frac{R^4 \pi}{2}$$

but $J_0 = I_x + I_y$

and area is symmetric around x and y and distributed same ly
then $I_x = I_y$

$$2I_x = \frac{R^4 \pi}{2}$$

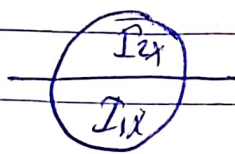
$$I_x = I_y = \frac{R^4 \pi}{4}$$

I_x / I_y نسبت الی
کلیات قیاسی

یعنی هیچ چیز خاصی

$$I_{\text{circle}} = \frac{R^4 \pi}{4}$$

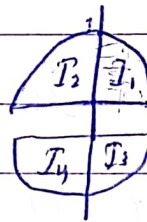
Find I_x



$I_1 = I_2$ because they have the same distribution

$$I_x = \frac{R^4 \pi}{8}$$

$$I_y = ?$$



$$I_1 = I_2 = I_3 = I_4 = \frac{I}{4}$$

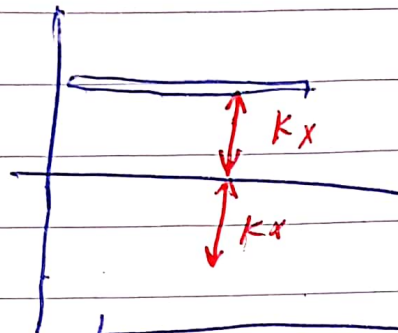
the same distribution

$$I_1 = \frac{1}{2} * (I_1 + I_3) \quad \text{notice the distribution}$$

$$I_y = \frac{R^4 \pi}{8}$$

Distribution given
بالتوزيع

Radius of Gyration of an Area



→ A to axis distance
also area A of
the area is k_x

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$k_o = \sqrt{\frac{J_o}{A}}$$

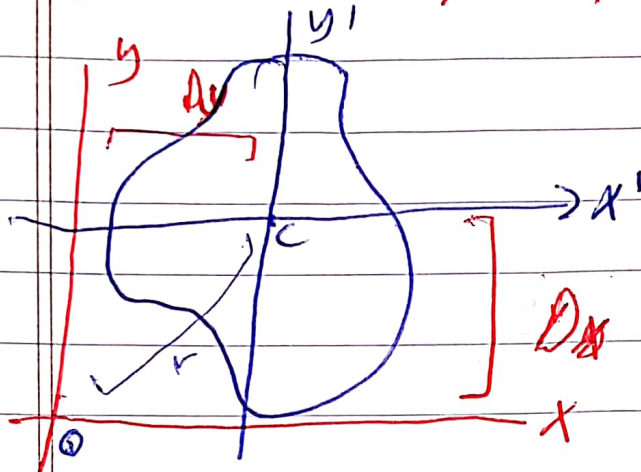
$$= \sqrt{\frac{I_x + I_y}{A}} = \sqrt{k_x^2 + k_y^2}$$

$$I_x = k_x^2 A$$

$$I_y = k_y^2 A$$

Parallel axis Theorem

The axis x' pass through centroid



$$I_x = I_{x'} + D_x^2 A$$

$$I_y = I_{y'} + D_y^2 A$$

$$I_o = I_c + D_r^2 A$$

الموجات = الموجات في المحاور
التي يمر بالسنترية + $A \times (\text{المسافة بين المحاور})^2$

$$K_x^2 = K_{x'}^2 + D_x^2$$

$$K_y^2 = K_{y'}^2 + D_y^2$$

easy to know,

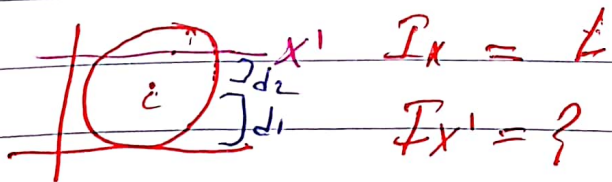
if you know

the ~~up~~ rules

UP

$$K_o^2 = K_c^2 + D_r^2$$

إذا لم يكن I_x المعطى عند السنترية



$$I_x = I$$

$$I_{x'} = ?$$

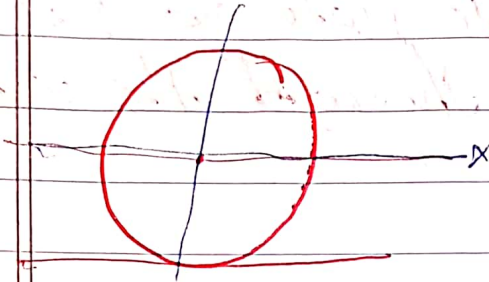
$$I_x = I_c + d_1^2 A$$

$$(I_c = I_x - d_1^2 A) \rightarrow I_{x'} = I_c + d_2^2 A$$

$$I_{x'} = I_x - d_1^2 A + d_2^2 A \quad \checkmark$$

يقوم على:

Example Find I_T of the circle
 $I_{Tangential}$



$$I_x = \frac{\pi R^4}{4}$$

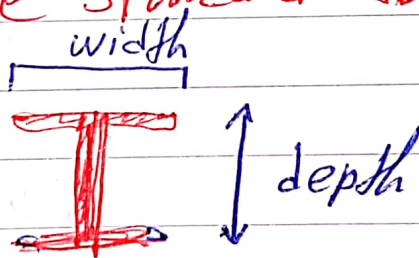
$$I_T = \frac{\pi R^4}{4} + AR^2$$

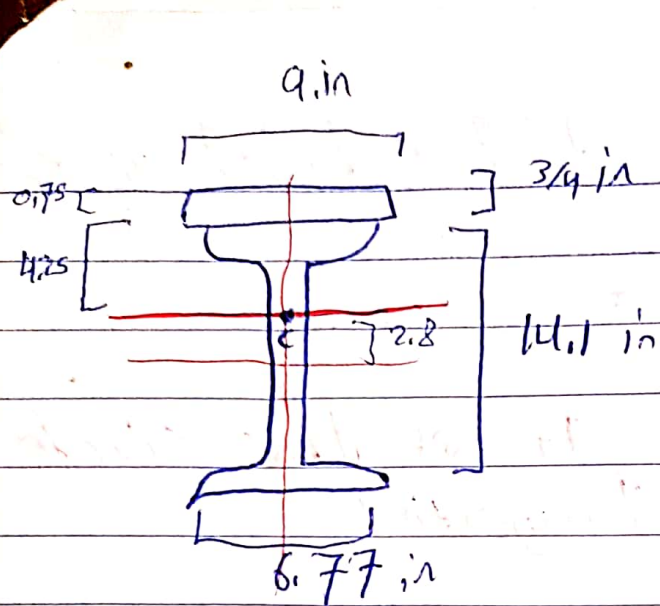
$$= \frac{\pi R^4}{4} + \pi R^4$$

$$= \pi R^4 * \frac{5}{4}$$

beams have standard shapes

I shape
S shape
C shape





W4x38

Find the moment of inertia and radius of gyration with to an axis that is parallel to plate and pass through the centroid

First find $\bar{y}$

$$= \frac{y_w \cdot A_w + y_p \cdot A_p}{\sum A} = \frac{0 + \frac{27}{4} \cdot (7.05 + 38)}{\frac{27}{4} + 11.2}$$

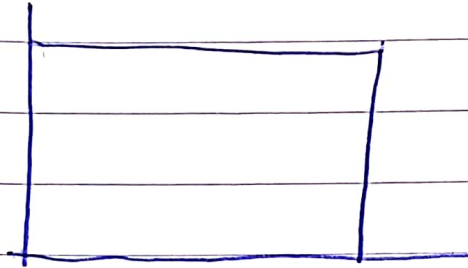
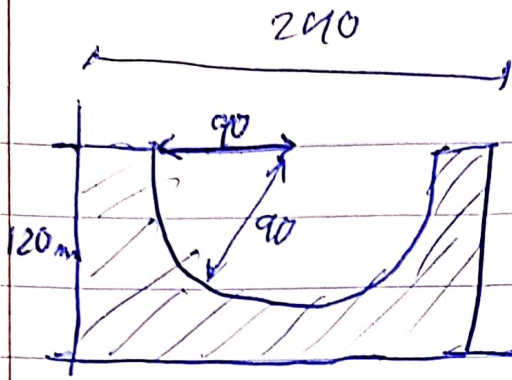
$$\bar{y} = 2.8 \text{ in}$$

$$I_{xw} = I_{w'x} + 2.8^2 \cdot 11.2 = 472.8 \text{ in}^4$$

$$I_{xp} = \frac{5^3 \cdot 9}{3} - \frac{4.25^3 \cdot 9}{3} = 3(5^3 - 4.25^3) = 144.7$$

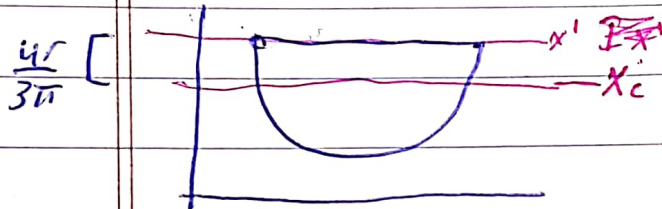
$$I_{x\text{total}} = 342.5 \quad 617.5$$

$$K_x = \sqrt{\frac{I_x}{A}} = 5.86$$



$$I_x = \frac{1}{3} \times 240 \times 120^3$$

$$= 138.24 \times 10^4$$



$$I_{x'} = \frac{1}{8} \pi r^4$$

$$I_{x_c} = \frac{1}{8} \pi r^4 - \left(\frac{4r}{3\pi} \right)^2 \times \frac{\pi r^2}{2}$$

$$I_x = I_{x_c} + \left(120 - \frac{4r}{3\pi} \right)^2 \frac{\pi r^2}{2}$$

$$= 92.3 \times 10^6$$

$$= \left(\frac{1}{8} \pi - \frac{16}{9\pi} \times \frac{\pi}{2} \right) r^4$$

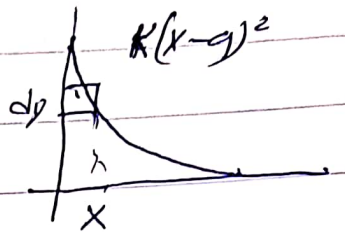
$$= \left(\frac{1}{8} \pi - \frac{8}{9\pi} \right) r^4$$

Home work 5

DS

$$I_x = \int y^2 dA$$

$$dA = dy * x$$



$$I_x = \int y^2 x dy$$

$$= \int_0^b y^2 * a(1 - \frac{\sqrt{y}}{\sqrt{b}}) dy$$

$$= a \int_0^b y^2 - \frac{y^{\frac{5}{2}}}{\sqrt{b}} dy$$

$$= a \left[\frac{y^3}{3} - \frac{2y^{\frac{7}{2}}}{7\sqrt{b}} \right]_0^b$$

$$= a \left(\frac{b^3}{3} - \frac{2b^3\sqrt{b}}{7\sqrt{b}} \right)$$

$$= a \left(\frac{b^3}{3} - \frac{2}{7}b^3 \right)$$

$$= a \left(\frac{b^3}{21} \right) = \frac{ab^3}{21}$$

$$ak = b \rightarrow k = \frac{b}{a^2}$$

$$y = k(x-a)^2$$

$$\sqrt{\frac{y}{k}} = |x-a|$$

$$\frac{a\sqrt{y}}{\sqrt{b}} = a-x$$

$$x = a - \frac{a\sqrt{y}}{\sqrt{b}}$$

$$x = a \left(1 - \frac{\sqrt{y}}{\sqrt{b}} \right)$$

Way 2

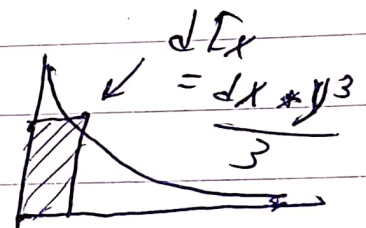
$$\int_0^a \frac{y^3}{3} dx = \frac{1}{3} \int_0^a k^3 (x-a)^6 dx$$

$$= \frac{1}{3} \left(\frac{k^3 (x-a)^7}{7} \right) \Big|_0^a$$

$$= k^3 \frac{a^7}{7} * \frac{1}{3}$$

$$ka^2 = b \rightarrow$$

$$\frac{b^3 a}{21}$$

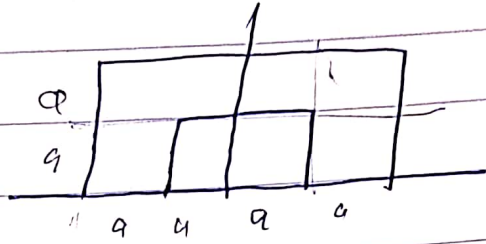


P27

hard to use a polar equation

so $I_p = I_x + I_y$

$$I_x = \frac{1}{3} * 4a * (2a)^3 - \frac{1}{3} * 2a * a^3$$



$$I_x = \frac{1}{3} * 32a^4 - \frac{2}{3} * a^4 = \frac{30}{3} a^4 = 10a^4$$

$$I_y = 2 * \left(\frac{1}{3} * 2a * (2a)^3 - \frac{1}{3} * a * a^3 \right)$$

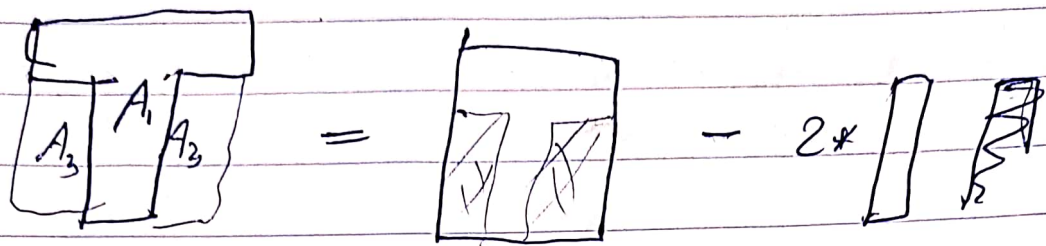
$$= 2 \left(\frac{1}{3} * 2 * 8 a^4 - \frac{1}{3} a^4 \right)$$

$$I_y = 10a^4$$

$$I_p = 10a^4 + 10a^4 = 20 a^4$$

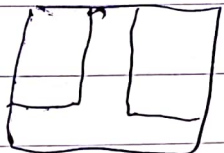
$$K_p^2 = \frac{I_p}{A} = \frac{20 a^4}{8a^2} = \frac{10}{3} a^2$$

D31



$$I_{x1} = \frac{1}{3} b h^3 - 2 * \frac{1}{3} b_3 h_3^3$$

$$\frac{1}{3} (24 * 30^3 - 2 * 8 * 24^3) = \frac{4}{3} * 142272 \text{ mm}^4$$

$I_{x2} =$  the same as I_{x1}

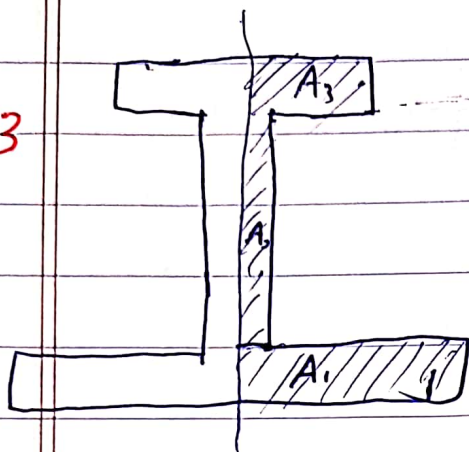
$$= 247680 \text{ mm}^4$$

$$I_x = 389952$$

$$\sqrt{K_x^2} = \frac{I_x}{A} = \frac{389952}{24*6 + 48*8 + 6*48} = \sqrt{477.88 \text{ mm}^2}$$

$$K_x = 21.86 \text{ mm}$$

P₃₃



because of the symmetry and Areas are distributed same/y around y on the left and right we cannot I_y for right part and multiply ~~with~~ by 2

$$I_{y1} = \frac{1}{3} * 6 * (24)^3 = 27648$$

$$I_{y2} = \frac{1}{3} * 48 * 4^3 = 1024$$

$$I_{y3} = \frac{1}{3} * 6 * 12^3 = 3456$$

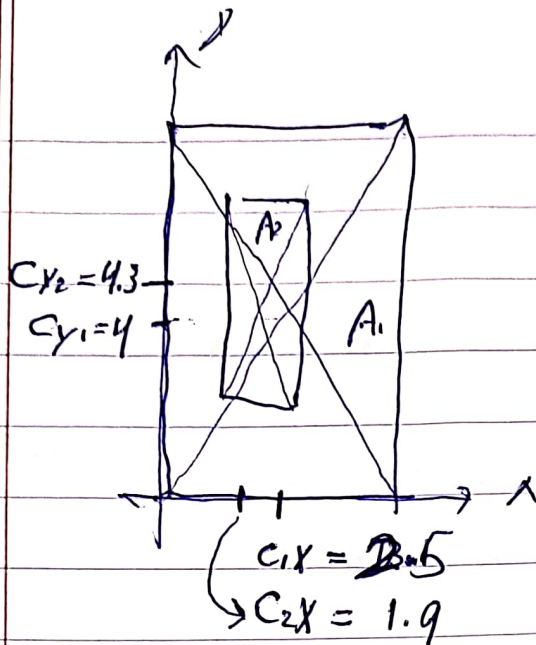
$$I_{y(\text{right})} = 32128$$

$$I_y = 64256$$

$$\sqrt{k_y^2} = \frac{64256}{24*6 + 48*8 + 6*48} = \sqrt{78.7 \text{ mm}^2}$$

$$\rightarrow k_y = 8.87 \text{ mm}$$

43



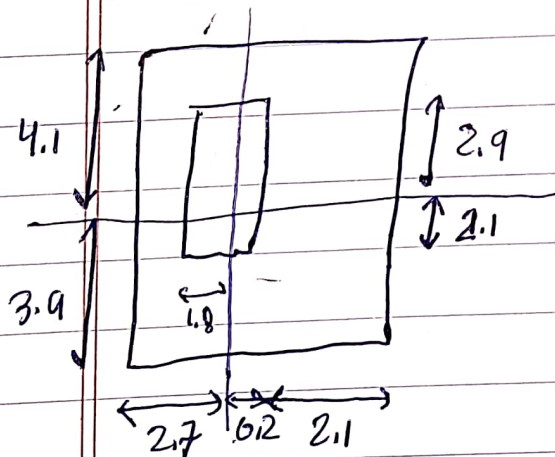
First find the centroid

$$C_x = \frac{A_1 C_{1x} - A_2 C_{2x}}{A_1 - A_2} = \frac{40 \times 2.5 - 10 \times 1.9}{40 - 10}$$

$$C_x = \bar{x} = 2.7$$

$$C_y = \frac{A_1 C_{1y} - A_2 C_{2y}}{A_1 - A_2} = \frac{40 \times 4 - 10 \times 4.3}{30}$$

$$\bar{y} = C_y = 3.9$$



$$I_{y(r)} = \frac{1}{3} \times 8 \times 2.3^3 - \frac{1}{3} \times 5 \times 0.2^3$$

$$= 32.4 \text{ mm}^4$$

$$I_y(qt) = \frac{1}{3} \times 8 \times 2.7^3 - \frac{1}{3} \times 5 \times 1.8^3$$

$$= 42.768 \text{ mm}^4$$

$$I_x = 75.17 \text{ in}^4$$

$$I_{x(upper)} = \frac{1}{3} (5 \times 4.1^3 - 2 \times 2.9^3) = 98.6 \text{ mm}^4$$

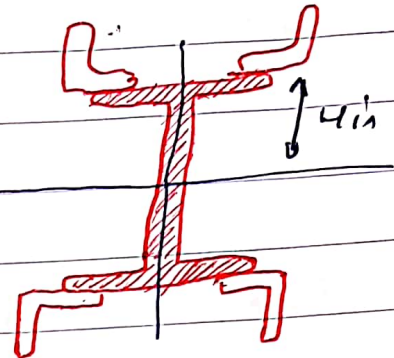
$$I_{x(lower)} = \frac{1}{3} (5 \times 3.9^3 - 2 \times 2.1^3) = 92.7 \text{ mm}^4$$

$$I_y = 191.3 \text{ in}^4$$

Prob 51

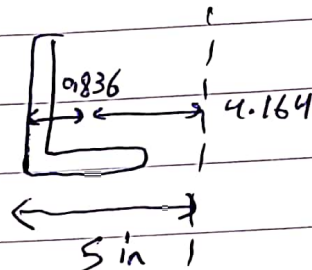
there is a symmetry around point c
"centroid of (w shape)"
then point c is the centroid

$$I_x = I_x \text{ of w shape} \\ + 4 * I_x \text{ of } \angle \text{ shape}$$



I_x of angle?

$$I_x = I_{x'} + A X^2 \\ = 1.23 + 1.44 * (4.164)^2$$



$$26.19 = I_{Ay} \quad \text{--- (1)}$$

I_x of angle = ?

$$= I_{x'} + A * 4^2 \\ = 1.23 + 1.44 * 4.836^2$$

$$34.9 = I_{Ax} \quad \text{--- (2)}$$

$$I_x = I_{xw} + 4 * I_{Ax}$$

$$= 110 + 4 * 34.9 = 249.6 \text{ in}^4$$

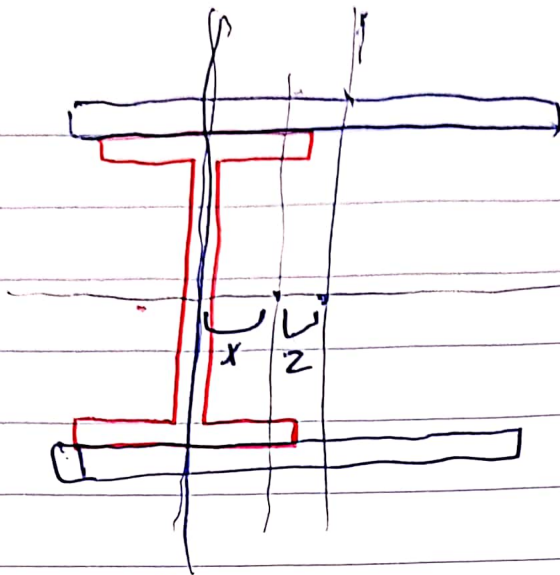
$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{249.6}{4 * 1.44 + 9.12}} = 4.095 \text{ in}$$

$$I_y = I_{yw} + 4 * I_{Ay} =$$

$$= 37.1 + 4 * 26.19 = 141.86 \text{ in}^4$$

$$k_y = \sqrt{\frac{I_y}{A}} = 3.09 \text{ in}$$

56



I_{wx} remains the same because the centroid has shifted on x axis not y axis

$$I_x = I_{wx} + I_{px} = I_{wy} + I_{py}$$

$$= 385 + \frac{2}{3} (26 * (8.03^3 - 7.03^3)) = 3353.4 \text{ in}^4$$

$$I_{ywc} = I_{wy'} + X^2 A_w$$

$$I_{py'} = 2929.3 \text{ in}^4$$

$$I_{pyc} = I_{py'} + A_p z^2$$

$$I_y = I_x \Rightarrow ?$$

$$3353.4 = I_{py'} + A_p z^2 + I_{wy'} + X^2 A_w$$

$$3353.4 = 2929 + 52 * z^2 + 26.7 + X^2 * 11.2$$

$$X = \bar{X} = \frac{0 * A_w + 11 * A_p}{\Sigma A} = \frac{52a}{63.2} = 0.823a$$

$$4473.6 = 2929 + 52 * (a - 0.823a)^2 + 26.7 + (0.823a)^2 * 11.2$$

$$447 = 0.03133 a^2 * 52$$

$$+ 11.2 * 0.6778 a^2$$

$$447.2 = 9.2152 a^2$$

$$a = \sqrt{48.5}$$

$$\rightarrow a = 6.7 \text{ in}$$