



Analyzing Algorithm Examples

General Rules of analyzing algorithm code:

Rule 1 — *for* loops:

The running time of a **for** loop is at most the running time of the statements inside the **for** loop (including tests) **times** the number of iterations.

Rule 2 — Nested loops:

Analyze these **inside out**. The total running time of a statement inside a group of nested loops is the running time of the statement multiplied by the product of the sizes of all the loops.

Rule 3 — Consecutive Statements:

These just add (which means that the maximum is the one that counts).

Rule 4 — *if/else*:

```
if( condition )  
    S1  
else  
    S2
```

The running time of an **if/else** statement is never more than the running time of the **test** plus the larger of the running times of **S1** and **S2**.

Rule 5 — *methods call*:

If there are method calls, these must be analyzed first.

Sorting Algorithm

1- Bubble Sort (revision) → $O(n^2)$

```
public static void bubble(int[] arr){  
    int temp;  
    for (int i = 0; i < arr.length-1; i++) {  
        for (int j = 0; j < arr.length-i-1 ; j++) {  
            if(arr[j+1]<arr[j]){  
                temp = arr[j];  
                arr[j] = arr[j+1];  
                arr[j+1] = temp;  
            }  
        }  
    }  
}
```





2- **Selection Sort** → $O(n^2)$: named selection because every time we select the smallest item.

```
public static void selection (int[] arr){
    int temp, minIndex;
    for (int i = 0; i < arr.length-1; i++) {
        minIndex = i;
        for (int j = i+1; j < arr.length ; j++) {
            if(arr[j]<arr[minIndex]){
                minIndex=j;
            }
        }
        if(i!= minIndex){
            temp = arr[i];
            arr[i] = arr[minIndex];
            arr[minIndex] = temp;
        }
    }
}
```

3- **Insertion sort** → $O(n^2)$:

```
public static void insertion (int[] arr){
    int j, current;
    for (int i = 1; i < arr.length; i++) {
        current = arr[i];
        j=i-1;
        while (j>=0 && arr[j]>current){
            arr[j+1] = arr[j];
            j--;
        }
        arr[j+1]=current;
    }
}
```



$O(n^2)$ sorting algorithms comparison:

(run demo @ <http://www.sorting-algorithms.com/>)

Bubble Sort	Selection Sort	Insertion Sort
Very inefficient	- Better than bubble sort - Running time is independent of ordering of elements	- Relatively good for small lists - Relatively good for partially sorted lists





Merge sort: recursive algorithm

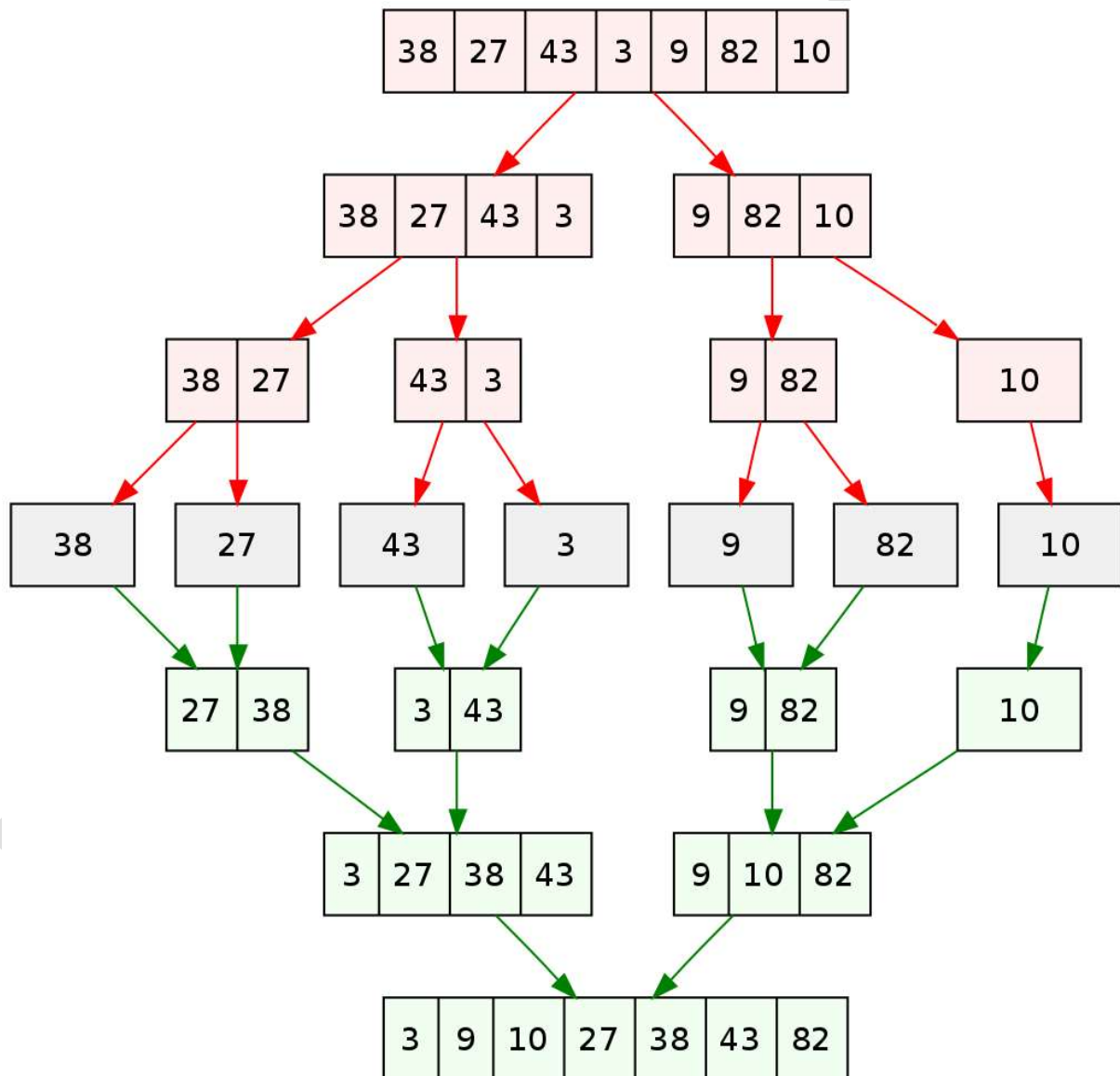
Merge: take 2 sorted arrays and merge them together into one.

Step 1 – if it is only one element in the list it is already sorted, return.

Step 2 – divide the list recursively into two halves until it can no more be divided.

Step 3 – merge the smaller lists into new list in sorted order.

Example:



**Java code:**

```
void sort(int nums[], int left, int right) {
    if (left < right) {
        int m = (left + right) / 2; // Find the middle point
        sort(nums, left, m); // Sort first halve
        sort(nums, m + 1, right); // Sort second halve
        merge(nums, left, m, right); // Merge the sorted halves
    }
}

void merge(int nums[], int left, int m, int right) {
    int n1 = m - left + 1;
    int n2 = right - m;

    int Left_arr[] = new int[n1];
    int Right_arr[] = new int[n2];

    for (int i = 0; i < n1; ++i)
        Left_arr[i] = nums[left + i];
    for (int j = 0; j < n2; ++j)
        Right_arr[j] = nums[m + 1 + j];

    int i = 0, j = 0;

    int k = left;
    while (i < n1 && j < n2) {
        if (Left_arr[i] <= Right_arr[j]) {
            nums[k] = Left_arr[i];
            i++;
        } else {
            nums[k] = Right_arr[j];
            j++;
        }
        k++;
    }

    while (i < n1) {
        nums[k] = Left_arr[i];
        i++;
        k++;
    }

    while (j < n2) {
        nums[k] = Right_arr[j];
        j++;
        k++;
    }
}
```

H.W: implement merge sort your own

**Searching elements** in an array:

7	2	5	8	1	10
---	---	---	---	---	----

a [2] = 5 : O(1)

find (8) : O(n)

delete (item) : O(n)

Case 1: unordered array:

3	7	20	32	45	55	60	75
---	---	----	----	----	----	----	----

find (60)

Finding Index

$$\left\lfloor \frac{7+0}{2} \right\rfloor = 3 \rightarrow a[3] = 32$$

$$\left\lfloor \frac{7+3}{2} \right\rfloor = 5 \rightarrow a[5] = 55$$

$$\left\lfloor \frac{7+5}{2} \right\rfloor = 6 \rightarrow a[6] = 60$$
Case 2: ordered array: -Binary search-

3	7	20	32	45	55	60	75
---	---	----	----	----	----	----	----

First Search : n

Second Search : $\frac{n}{2}$

Third Search : $\frac{n}{4}$

⋮

(i-1)th Search : 2

ith Search : $1 = \frac{n}{2^{i-1}}$

$2^{i-1} = n \rightarrow (i-1) = \log_2 n$

find (item) = O(log₂n)

n	log ₂ n
2	1
1024	10
1048576 (Million)	20
1099511627776 (Trillion)	40

Inserting and deleting items from ordered array

3	7	20	32	45	52	55	60	75
---	---	----	----	----	----	----	----	----

Insert (52)

Insert (item) = O (n)

Search (item) = O (log₂n)

3	7	20	32	45	52	60	75
---	---	----	----	----	----	----	----

Delete (55)

Delete (item) = O (n)

