Partial fraction

① cover Method

Exp (12) $\int \frac{2x+1}{x^2-7x+12}\,dx$

$\int \frac{2x+1}{\cancel{(x-4)}\cancel{(x-3)}}\,dx$

linear factor "distinct"

$A = \frac{2\boxed{4}+1}{\boxed{4}-3} = \frac{9}{1} = 9$

$\int \left(\frac{A}{x-4} + \frac{B}{x-3}\right) dx$

$B = \frac{2\boxed{3}+1}{\boxed{3}-4} = \frac{7}{-1} = -7$

$\int \left(\frac{9}{x-4} - \frac{7}{x-3}\right) dx$

$9\ln|x-4| - 7\ln|x-3| + c$

Q. $\int \frac{f(x)}{g(x)}\,dx$ f and g are poly.

- If $\deg f \geq \deg g$ we use long Division

$g(x)\overline{)\ f(x)\ }$

- $\int \frac{7}{(x-1)(x+1)^2}\,dx = \int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right)$

$(x+1)(x+1)$

$$\frac{1}{(x+1)(x+1)}$$

$$\int \frac{x}{x^3\,(x^2+1)^2} = \int\left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}\right)$$

$x \times x \times x$ $\qquad$ $(x^2+1)^2$

$$= \boxed{\int \frac{Ax^2+Bx+C}{x^3}}$$

$$\int \frac{x}{(x^2+1)\,(x^3+2)}\,dx = \int\left(\frac{Ax+B}{x^2+1} + \frac{Cx^2+Dx+E}{x^3+2}\right)dx$$

[illegible] $\qquad$ [illegible]

$$\int \frac{x^2}{(x-1)^3\,(x^2+1)^2}\,dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$

Exp (18)

$$\int_{-1}^{0} \frac{x^3}{x^2-2x+1}\,dx$$

long Division

$$= \int_{-1}^{0}\left[(x+2) + \frac{3x-2}{x^2-2x+1}\right]dx$$

$$\begin{array}{r|l} & x+2 \\ x^2-2x+1 & x^3 \\ & -(x^3-2x^2+x) \end{array}$$

$$= \int_{-1}^{0} \left((x+2) + \frac{3x-2}{x^2-2x+1} \right) dx$$

$$\begin{array}{r} -\cancel{x^3} + 2x^2 \pm x \\ \hline 2x^2 - x \\ -\cancel{2x^2} + 4x \pm 2 \\ \hline 3x - 2 \end{array}$$

$$= \int_{-1}^{0} (x+2)\,dx + \int_{-1}^{0} \frac{3x-2}{(x-1)^2}\,dx$$

$$= \frac{x^2}{2} + 2x \Big|_{-1}^{0} + \int_{-1}^{0} \left(\frac{A}{x-1} + \frac{B}{(x-1)^2} \right) dx$$

$$\frac{3x-2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$= \frac{A(x-1)}{(x-1)^2} + \frac{B}{(x-1)^2}$$

$$= \frac{A(x-1)+B}{(x-1)^2}$$

$$\boxed{3x-2 = A(x-1) + B}$$

• مقارنة معاملات

$$\boxed{3 = A}, \quad -2 = -A + B$$

$$= -3 + B$$

$$\boxed{1 = B}$$

• أو تعويض في

$$x = 1 \Rightarrow 3 - 2 = A(0) + B$$

$$\underline{1 = B}$$

$$x = 0 \Rightarrow 0 - 2 = A(0-1) + B$$

$$-2 = -A + B$$

$$-2 = -A + 1$$

$$\boxed{3 = A}$$

$$= (0+0) - \left(\frac{1}{2} - 2\right) + \int \left(\frac{3}{x-1} + \frac{1}{(x-1)^2} \right) dx$$

$$= \frac{3}{2} + 3\ln|x-1| \Big|_{-1}^{0} + \int_{-1}^{0} \frac{dx}{(x-1)^2}\,dx$$

• الاشتقاق

$$3 = A$$

$$= \frac{3}{2} + 3\cancel{\ln 1} - 3\ln 2 + \int_{-2}^{-1} \frac{du}{u^2}$$

$u = x - 1$

$du = dx$

$x = -1 \Rightarrow u = -2$

$x = 0 \Rightarrow u = -1$

$$= \frac{3}{2} - 3\ln 2 + \int_{-2}^{-1} u^{-2}\,du$$

$$= \frac{3}{2} - 3\ln 2 + \frac{u^{-1}}{-1} \Big|_{-2}^{-1}$$

$$= \frac{3}{2} - 3\ln - \frac{1}{u} \Big|_{-2}^{-1}$$

$$= \ldots = 2 - 3\ln 2$$

$$33\quad \int \frac{y^2+2y+1}{(y^2+1)^2}\,dy = \int \left(\frac{\overset{0}{A}y+\overset{1}{B}}{y^2+1} + \frac{\overset{2}{C}y+\overset{0}{D}}{(y^2+1)^2} \right) dy$$

$$\frac{y^2+2y+1}{(y^2+1)^2} = \frac{Ay+B}{y^2+1} + \frac{Cy+D}{(y^2+1)^2}$$

$$= \frac{(Ay+B)(y^2+1)}{(y^2+1)^2} + \frac{Cy+D}{(y^2+1)^2}$$

$$= \frac{(Ay+B)(y^2+1) + Cy+D}{(y^2+1)^2}$$

$$y^2+2y+1 = (Ay+B)(y^2+1) + Cy+D$$

مقارنة معاملات

$\boxed{1 = B+D}$

$y=0 \Rightarrow \boxed{1 = B+D}$ (1)

$D=0$

$$2y+2 = (Ay+B)(2y) + (y^2+1)A + C$$

$y=0 \qquad 2 = 0 + A + C$

$\boxed{2 = A+C}$

$$\boxed{2 = A + C}$$

$$C = 2$$

$$2y + 2 = 2Ay^2 + 2By + Ay^2 + A + C$$

$$\boxed{2 = A + C}$$

$$\boxed{2 = 4Ay + 2B + 2Ay}$$

$$y = 0 \Rightarrow 2 = 0 + 2B + 0 \Rightarrow B = 1$$

$$0 = 4A + 0 + 2A$$

$$0 = 6A \Rightarrow A = 0$$

$$\int \frac{y^2 + 2y + 1}{(y^2+1)^2}\,dy = \int \frac{1}{y^2+1}\,dy + \int \frac{2y}{(y^2+1)^2}\,dy$$

$u = y^2 + 1$

$du = 2y\,dy$

$$= \tan^{-1} y + \int \frac{du}{u^2}$$

$$= \tan^{-1} y + \int u^{-2}\,du$$

$$= \tan^{-1} y + \frac{u^{-1}}{-1} + C$$

$$\tan^{-1} y - \frac{1}{\quad} + C$$

$$= \tan^{-1} y - \frac{1}{\boxed{y^2+1}} + C$$

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38) $$\int \frac{x^2+x}{x^4-3x^2-4}\,dx = \int \frac{x^2+x}{(x^2-4)(x^2+1)}\,dx$$

irred.

$$= \int \frac{x^2+x}{(x-2)(x+2)(x^2+1)}\,dx$$

$$= \int \left(\frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1} \right) dx$$

$$\frac{x^2+x}{(x-2)(x+2)(x^2+1)} = \frac{\overset{3/10}{A}}{(x-2)} + \frac{\overset{-1/10}{B}}{x+2} + \frac{\overset{-1/5}{C}x+\overset{1/5}{D}}{x^2+1}$$

$$= \frac{A(x+2)(x^2+1)}{(x-2)(x+2)(x^2+1)} + \frac{B(x-2)(x^2+1)}{(x+2)(x-2)(x^2+1)} + \frac{(Cx+D)(x-2)(x+2)}{(x^2+1)(x-2)(x+2)}$$

$$x^2+x = A(x+2)(x^2+1) + B(x-2)(x^2+1) + (Cx+D)(x-2)(x+2)$$

$0 + 0$

$x=2 \Rightarrow 4+2 = A(4)(5)$

$6 = 20A$

$A = 3$

$6 = 20A$

$A = \frac{3}{10}$

$x=-2 \Rightarrow 4-2 = 0 + B(-4)(5) + 0$

$2 = -20B$

$B = \frac{-1}{10}$

$x=0 \Rightarrow 0+0 = 2A - 2B - 4D$

$0 = 2\,\frac{3}{10} - 2\left(\frac{-1}{10}\right) - 4D \Rightarrow 4D = \frac{8}{10} \Rightarrow D = \frac{1}{5}$

$x=1 \Rightarrow 1+1 = \underset{\frac{3}{10}}{A}(3)(2) + \underset{\frac{-1}{10}}{B}(-1)(2) + \left(C + \frac{1}{5}\right)(-3)$

$C = \frac{-1}{5}$

$$\int \frac{x^2+x}{x^4-3x^2-4}\,dx = \int \frac{\frac{3}{10}}{x-2}\,dx - \int \frac{\frac{1}{10}}{x+2}\,dx + \int \frac{\frac{-1}{5}x + \frac{1}{5}}{x^2+1}\,dx$$

$$= \frac{3}{10}\ln|x-2| - \frac{1}{10}\ln|x+2| + \frac{-1}{5}\frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{dx}{x^2+1}$$

$$= \frac{3}{10}\ln|x-2| - \frac{1}{10}\ln|x+2| - \frac{1}{5}\frac{1}{2}\ln(x^2+1) + \frac{1}{5}\tan^{-1}x + c$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

. . . .

(34) $\int \frac{x^4}{x^2-1}\,dx$

أعلى درجة

المقسوم عليه

$$\begin{array}{r|l} & x^2+1 \\ \hline x^2-1 & x^4 \\ & \cancel{-x^4+x^2} \quad + \\ \hline & x^2 \\ & \text{[illegible]} \quad + \end{array}$$

$\int \left(x^2+1 + \frac{1}{x^2-1}\right)dx$

$$\int \left(x^2+1 + \frac{1}{x^2-1} \right) dx$$

$$\begin{array}{l} x^2 \\ \underline{-x^2+1 \quad +} \\ \quad 1 \end{array}$$

ابهمنا

$$\int (x^2+1)\,dx + \int \frac{1}{(x-1)(x+1)}\,dx$$

$$\frac{x^3}{3} + x + \int \left(\frac{A}{x-1} + \frac{B}{x+1} \right) dx$$

$+1 \quad -1$

$$A = \frac{1}{\boxed{1}+1} = \frac{1}{2}$$

$$B = \frac{1}{\boxed{-1}-1} = \frac{-1}{2}$$

$$\frac{x^3}{3} + x + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$$