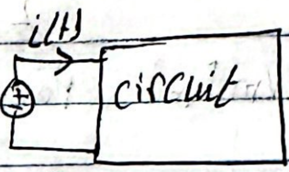
Chapter 10: Sinusoidal steady state Power Calculation:

* instantaneous power $P(t)$:



$$p(t) = v(t) \cdot i(t)$$

$$= V_m I_m \cos(\omega t + \theta v)\cos(\omega t + \theta i)$$

$$= \frac{1}{2} V_m I_m \left[\underbrace{\cos(2\omega t + \theta v + \theta i)}_{\text{Twice time frequency}} + \underbrace{\cos(\theta v - \theta i)}_{\text{constant}}\right]$$

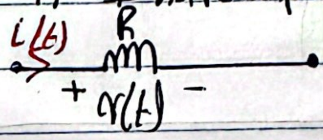
Rev.

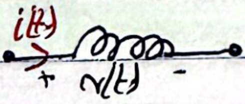
* $i(t) = I_m \cos(\omega t + \theta i)$
* $v(t) = V_m \cos(\omega t + \theta v)$.
* $\cos a \cos b = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$

* Average power: Real Power

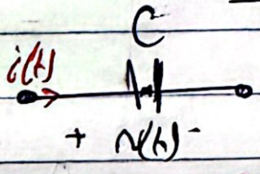
$$\boxed{P_{av} = \frac{1}{2} V_m I_m \cos(\theta v - \theta i)} \quad W.$$

* if I have a pure:





i leads v



$\theta v - \theta i = \text{Zero}$

$P_{av} = \frac{1}{2} V_m I_m$

$= \frac{1}{2} \frac{V_m^2}{R}$

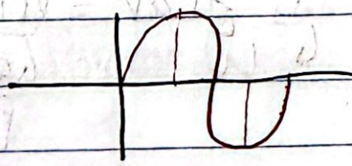
$= \frac{1}{2} I_m^2 R$

$\theta v - \theta i = 90$

$P_{av} = \text{Zero}$

$\theta v - \theta i = -90$

$P_{av} = \text{Zero}$



Rev:

~~* $P_{av} = \frac{1}{2} V_m I_m$~~

* $P_{av} = \underbrace{\frac{1}{2} V_m I_m}\ \underbrace{\cos(\theta_v - \theta_i)}_{\text{power factor}}\ W.$

R: $P_{av} = \frac{1}{2} V_m I_m$

L: $P_{av} = 0$

C: $P_{av} = 0$

* RMS value:

$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$, $\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$

$V_{rms} = V_{eff}$
$I_{rms} = I_{eff}$

22V, 50 HZ

$$\boxed{\begin{aligned} P_{av} &= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \\ &= V_{rms} I_{rms} \cos(\theta_v - \theta_i) \end{aligned}}$$

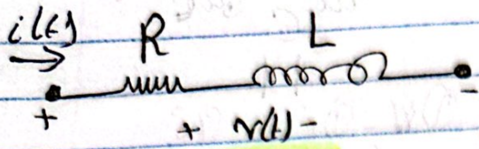
* Apparent Power & Power factor:-

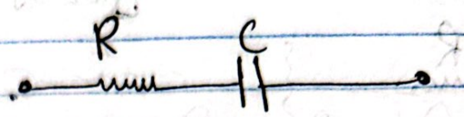
$\underset{W}{P_{av}} = \underbrace{V_{rms} I_{rms}}_{\text{apparent Power } \boxed{VA}}\ \underbrace{\cos(\theta_r - \theta_i)}_{\text{Power factor } \boxed{P.F}}$

R:
$\theta_v - \theta_i = 0$
$\cos(\theta_v - \theta_i) = 1$
$P.F = 1$
unity P.F

L:
$\theta_v - \theta_i = 90^\circ$
$\cos(\theta_v - \theta_i) = 0$
$P.F = 0$

C:
$\theta_v - \theta_i = -90^\circ$
$\cos(\theta_v - \theta_i) = 0$
$P.F = 0$





inductive load.

$0 < \theta_v - \theta_i < 90$

$\underbrace{0 < P.F < 1}$

lagging P.F

{i lags v}

Capacitive load

$-90° < \theta_v - \theta_i < 0$

$\underbrace{0 < P.F < 1}$

loading P.F

{i leads v}

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* Complex Power:-

$$\vec{S} = \vec{V}_{rms} \, \vec{I}_{rms} \quad , \ VA$$

$$= (V_{rms} \angle \theta_v)(I_{rms} \angle -\theta_i)$$

$$= \{V_{rms} I_{rms} \angle \theta_v - \theta_i\}$$

$$= \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P_{av}} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

P_{av}
P
Real Power
avg Power
W

Q
Reactive Power
VAR

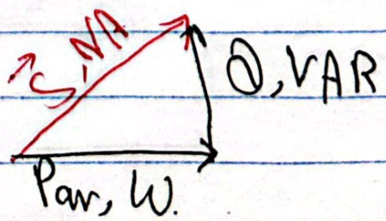
$\rightarrow \vec{S} = P_{av} + jQ$

$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$|\vec{S}| = \sqrt{(P_{av})^2 + Q^2} = V_{rms} I_{rms}$ apparent Power

$\angle \vec{S} = \tan^{-1} \frac{Q}{P_{av}}$

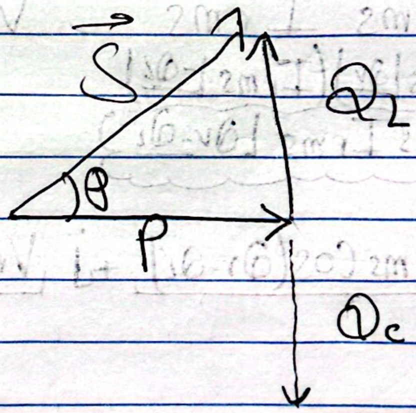


$jwL\ \Omega$ | $-j\frac{1}{wc} = \frac{1}{jwc}\ \Omega$

R | L | C

R	L	C
$\theta v - \theta i = 0$	$\theta v - \theta i = 90$	$\theta v - \theta i = -90°$
$\theta = \text{Zero}$	$\sin 90° = 1$	$\sin(-90°) = -1$
	$Q = V_{rms} I_{rms}$	$Q = -V_{rms} I_{rms}$
	$= \overset{+ve}{V_{rms}} \left(\frac{V_{rms}}{wL}\right)$	$= -\overset{-ve}{V_{rms}} (V_{rms}\ wc)$
	$\Rightarrow \frac{V_{rms}^2}{wL} = \frac{V_{rms}^2}{X_L}$	$= -V_{rms}^2 wc = \frac{V_{rms}^2}{Xc}$
	$= I_{rms}^2 (wL) = I_{rms}^2 (XL)$	$= \frac{-I_{rms}^2}{wc} = I_{rms}^2 Xc$

* Power triangle.



$PF = \cos\theta$

$\theta = \cos^{-1}(PF)$

+ve? −ve?

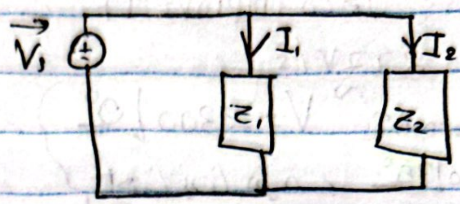
$\rightarrow \theta = +\cos^{-1}(PF)$, lagging PF

$\Rightarrow \theta = -\cos^{-1}(PF)$, leading PF

$\therefore PF = \cos\theta = \frac{P_{av}}{|S|}$

$\theta = \tan^{-1}\frac{Q}{P}$

* Conservation of AC Power



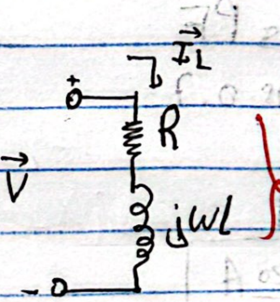
$$\vec{S}_{source} = \vec{V}_s \vec{I}_s^* = \vec{V}_s(\vec{I}_1^* + \vec{I}_2^*) = \vec{V}_s\vec{I}_1^* + \vec{V}_s\vec{I}_2 = \vec{S}_1 + \vec{S}_2$$

* $P_{av_T} = P_{av_1} + P_{av_2} + \dots + P_{av_n}$
* $Q_T = Q_1 + Q_2 + \dots + Q_n$
* $\vec{S}_T = \vec{S}_1 + \vec{S}_2 + \dots + \vec{S}_n$

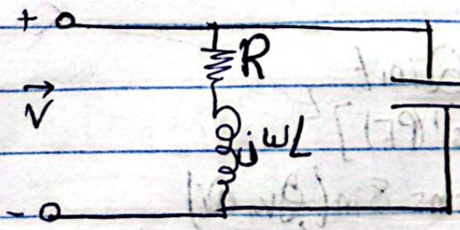
* Power Factor Correction:

$0 < \theta_v - \theta_i < 90°$

$PF \neq 1$



Q_L (+ve)



Q_c (-ve)

↳ $Q_c = Q_{final} - Q_{init}$

$$Q_c = \frac{V_{rms}^2}{X_c} = -\omega C V_{rms}^2$$

$$\rightarrow \boxed{C = \frac{-Q_c}{\omega V_{rms}^2}}$$

Ex page 43:-

load 1MW at 0.7 lagging . $V = 2300\,V_{rms}$.

$C_{min} = ??$ to improve PF = 0.9 lagging

$\omega = 377\,r/s$

في الدوسية $V = 2300\angle 0^\circ$

احنا بدنا نفرض $\vec{V} = 2300\angle 25^\circ$

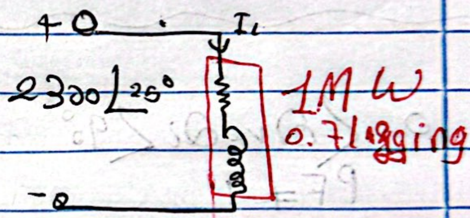
$\cos(\theta_v - \theta_i) = 0.7$

$\theta_v - \theta_i = \cos^{-1} 0.7$

$\theta_v - \theta_i = 45^\circ$, $\theta_i = \theta_v - 45^\circ$

$\theta_i = 25 - 45^\circ$

$\boxed{\theta_i = -20^\circ}$

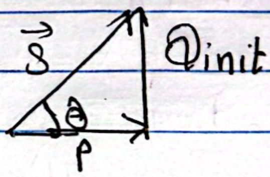


- $P_{Load} = V_{rms} I_{rms} PF$

$10^6 = 2300\, I_{rms}\, 0.7$

$|I_{rms}| = 621\ A$

→ $\boxed{I_{rms} = 621\angle -20\ A}$



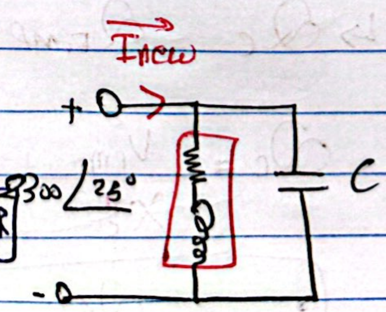
↳ $Q_c = Q_{final} - Q_{init}$

$Q_{init.} = P \tan[\cos^{-1}(PF)]$

$= V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$= 2300 \times 621 \sin(25 - (-20))$

$= \boxed{1.02\ MVAR}$



$Q_{final} = P \tan\{\cos^{-1} PF_{new}\}$

$= 1M \times \tan\{\cos^{-1} 0.9\} = \boxed{0.484\,MVAR}$

$Q_c = Q_{final} - Q_{init}$

$= 0.484 - 1.02$

$= \boxed{-0.536\ MVAR}$

→

$$C = \frac{-Q_c}{\omega (V_{rms})^2} = \frac{-0.536}{377\,(2300)^2} \quad \boxed{= 269\,\mu F}$$

$$I_{new} = \frac{P_{av}}{V_{rms}\, PF_{new}} = \frac{1M}{2300 \times 0.9} = \boxed{483\, A_{rms}}$$

$$\vec{S}_{final} = \vec{V}_{rms}\, \vec{I}_{rms}^{\,*}$$

$$= (2300\angle 25^\circ)(483\angle -\theta_i)$$

$\rightarrow \cos(\theta_v - \theta_i) = 0.9$

$\theta_v - \theta_i = 26$

$\theta_i = 25 - 26$

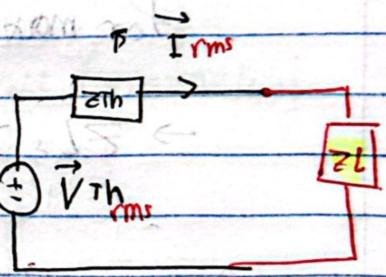
$= -1$

$\rightarrow 2300 \times 483 \angle 25 + \theta_i$

$\rightarrow 1.1109M \angle 26$

$= \boxed{1 + j0.484\, MVA}$

* Max. avg. Power Transfer



$\hookrightarrow Z_{Th} = R_{Th} + jX_{Th}$

$Z_L = R_L + jX_L$

$$\hookrightarrow \vec{I}_{rms} = \frac{\vec{V}_{Th}}{(R_{Th}+R_L) + j(X_{Th}+X_L)}$$

$$|\vec{I}_{rms}| = \frac{|V_{Th}|}{\sqrt{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}}$$

$$\rightarrow P_{av} = |\vec{I}_{rms}|^2 R_L$$

$$= \frac{V_{Th}^2 \, R_L}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

$$|\vec{I}_{rms}|^2 = \frac{V_{Th}^2}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

$$\frac{\partial P_{av}}{\partial X_L} = \frac{0 - 2V_{Th}^2 R_L (X_L + X_{Th})}{[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2]^2} = \text{zero}.$$

e.g

$$\frac{\partial P_{av}}{\partial R_L} = \frac{((R_{Th}+R_L)^2 + (X_{Th}+X_L)^2)\, V_{Th}^2 - 2V_{Th}^2 R_L (R_{Th}+R_L)}{[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2]^2} = \text{zero}$$

$\cancel{\frac{\partial P}{\partial X_L}} = 0 \longrightarrow$

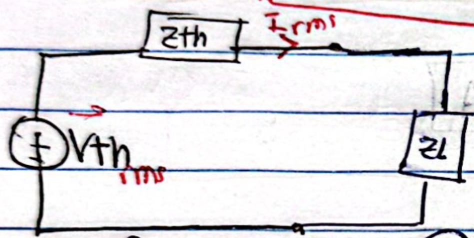
$$\frac{\partial P}{\partial X_L} = 0 \rightarrow \boxed{X_L = -X_{th}}$$

$$\frac{\partial P}{\partial R_L} = 0 \rightarrow (R_{th}+R_L)^2 + (X_{th}+X_L)^2 - 2R_L(R_{th}-R_L) = 0$$

$$\rightarrow (R_{th}+R_L)^{\cancel{2}} = 2R_L\cancel{(R_{th}+R_L)}$$

$$\boxed{R_L = R_{th}}$$

$$\boxed{Z_L = Z_{th}^*}$$



for max Power Transfer

$$\rightarrow Z_L = Z_{th}^* \quad \begin{cases} R_L = R_{th} \\ X_L = -X_{th} \end{cases}$$

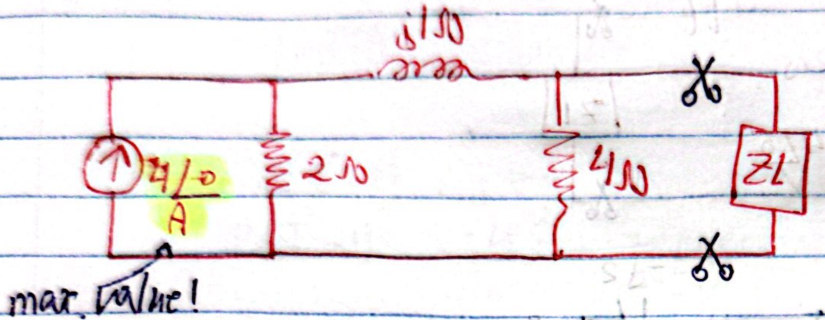
$$P_{max} = |\vec{I}|^2 R_L = \left(\frac{|V_{th}|}{2R_{Th}}\right)^2 * R_{th}$$

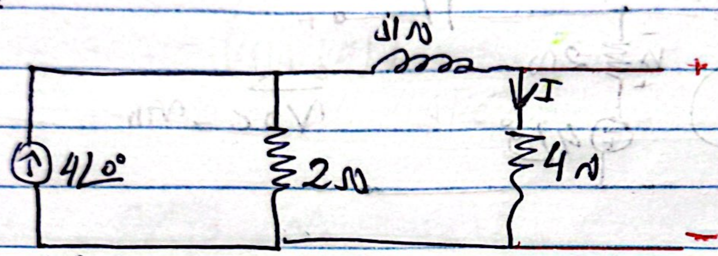
$$\because \vec{I} = \frac{\vec{V}_{th}}{Z_L + Z_{th}} = \frac{V_{th}}{R_{th} + jX_{th} + R_{Th} - jX_{Th}} = \frac{V_{th}}{2R_{Th}}$$

$$\boxed{P_{max} = \frac{|V_{th}|^2}{4R_{Th}}}$$, $V_{th} \rightarrow$ rms value!

$$\rightarrow P_{max} = \frac{|V_m|^2}{8R_{Th}}$$ max values

Example:
Find Z_L for maximum ~~avg~~ average power transfer.
Compute the maximum average power supplied to the load.

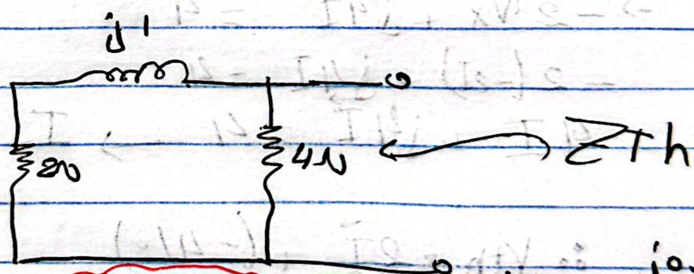




$V_{Th} = V_{o.c}$

$V_{Th} = R_4 \times I$

$V_{Th} = 4 \times \left(\frac{2}{2+4+j1} \times 4\right) = 5.28\angle -9.46°$ Volt (max value)

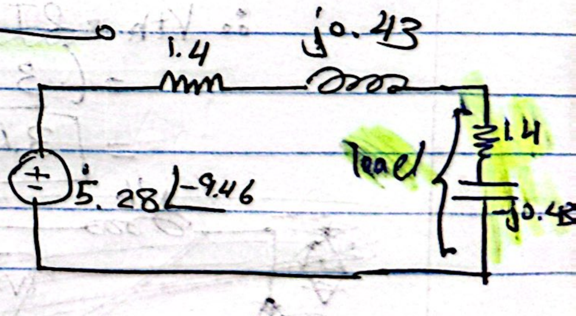


$Z_{Th} = (2+j1) \,//\, 4\Omega$

$= 1.4 + j0.43$

$\therefore Z_L = Z_{Th}^{*} = 1.4 - j0.43$

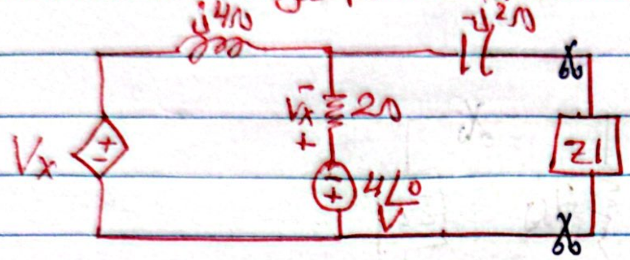
$R_{Th} = R_L$
$X_L = -X_{Th}$

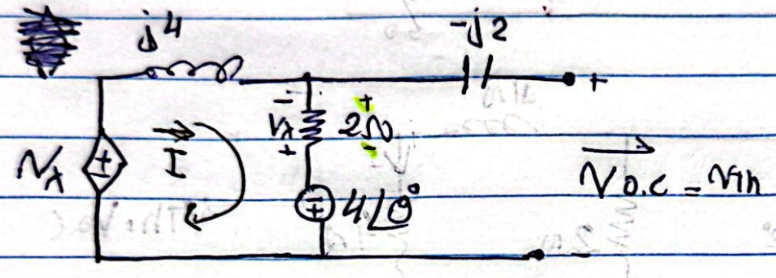


$P_{max} = \frac{V_{Th\,rms}^2}{4R_{Th}} \quad \frac{\left(\frac{5.28}{\sqrt{2}}\right)^2}{4 \times 1.4} = 2.489\ W$

Example:

Find Zl for maximum average Power transfer compute the maximum average Power supplied to Zl





$-V_x + j4I - V_x - 4\angle 0 = 0 \qquad , V_x = -2\vec{I}$

$\rightarrow -2V_x + j4\vec{I} = 4$

$-2(-2I) + j4I = 4$

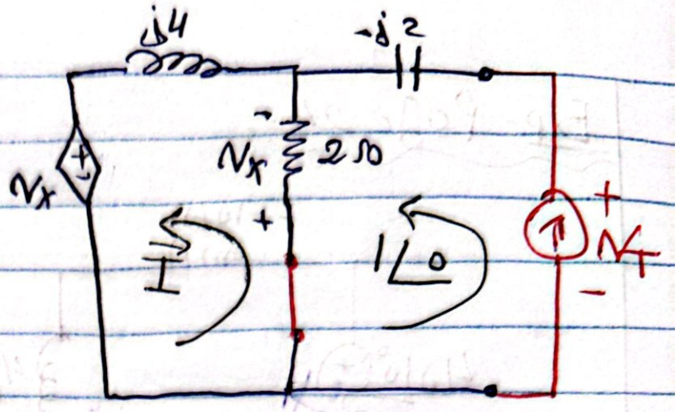
$4I + j4\vec{I} = 4 \rightarrow I = \frac{4}{4+j4} = 0.707\angle -45^\circ \text{ A (max)}$

$\therefore V_{th} = 2\vec{I} + (-4\angle 0)$

$= (-3 - j1) \text{ volt}$

$= \boxed{3.16\angle 198.43^\circ \text{ Volt (max)}}$

$Z_{Th} = \frac{\vec{V_T}}{\vec{I_T}}$



let $I_T = 1\angle 0°$

$KVL \rightarrow 2(\vec{I}-1) + j4\vec{I} + V_x = 0$, But $V_x = 2(I-1)$

$2(\vec{I}-1) + j4\vec{I} + 2(\vec{I}-1) = 0$

$(4+j4)I = 4$

$\rightarrow I = \frac{4}{4+j4} = 0.707\angle -45 \text{ A}$

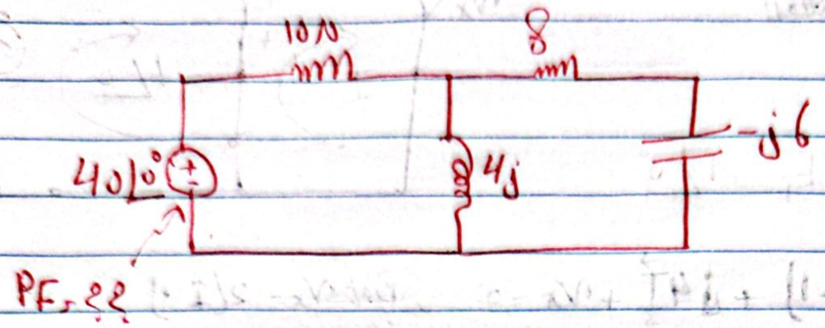
$\therefore V_T = V_{-j2} + V_2 = -j2+1 + 2(1-\vec{I}) = 2-j2 - 1.414\angle -45$

$= 1-j1$

$\therefore \boxed{Z_{th} = \frac{\vec{V_T}}{\vec{I_T}} = \frac{1-j1}{1}}$, $\boxed{Z_L = 1+j1\ \Omega}$

$P_{max} = \frac{V_{th}^2}{8R_{th}} = \frac{(3.16)^2}{8\times 1}$ $\boxed{= 1.248\text{ W}}$

Exp: Page 24:



$$Z_{tot} = \left[(8-j6)//4j\right] + 10$$

$$= \left[\frac{32j+24}{8-j2}\right] + 10 = 12.69\angle 20.62^\circ = 11.88 + j\,4.47$$

$$I = \frac{40\angle 0^\circ}{12.64\angle 20.62} = 3.152\angle -20.62\ A$$

$$PF = \cos(\theta_v - \theta_i)$$

$$= \cos(0 + 20.62) = 0.936 \text{ lagging}$$

OR

$$\vec{S} = \vec{V}_{rms} \cdot \vec{I}_{rms}^{*}$$

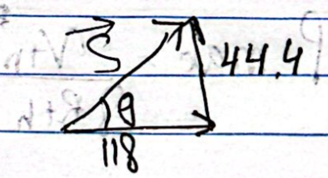
$$= (40\angle 0^\circ) \cdot (3.152\angle 20.62)$$

$$= 126.08\angle 20.62$$

$$= \underbrace{118}_{P \; W} + j\underbrace{44.4}_{Q \; VAR}$$

$$P = (3.152)^2 \times 11.88 = 118\ W$$

$$Q = (3.152)^2 \cdot (4.47) = 44.4\ VAR$$



$$PF = \cos\theta = \cos\left(\tan^{-1}\frac{44.4}{118}\right) = 0.935 \text{ lagging}$$

$$118 = (3.152)^2 \times 10 + I_2^2 \times 8$$

$$|I_2| = 1.526\ A$$

$$\vec{I}_2 = \frac{4j}{8-j2} = 3.152\angle -20.62$$

$$\boxed{= 1.526\angle 83.41^\circ}$$