

Ex. Solve.

$$(1) \begin{cases} y' = x - xy - y + 1, & y \neq 1 \\ y(0) = 0 \end{cases}$$

Ans. $(x+1)^2 + \ln(1-y)^2 = 1$

$$(2) (y-1) \sqrt{x^2+1} \frac{dy}{dx} + x^3 + x = y(x^3+x), y \neq 1$$

Ans. $y = \frac{1}{3}(x^2+1)^{3/2} + c.$

$$(3) \begin{cases} \left(x \cos^2\left(\frac{y}{x}\right) - y \right) dx + x dy = 0 \\ y(1) = \frac{\pi}{4} \end{cases} \quad ; x > 0$$

Ans. $\ln x + \tan\left(\frac{y}{x}\right) = 1.$

$$(4) (2xy + 2xy \ln y) dx + (2 + \ln y)(5 - x^2) dy = 0, y > e^{-1}, x \neq \pm\sqrt{5}$$

Ans. $\ln|5-x^2| + c = \ln y + \ln(1+\ln y)$

$$(5) \begin{cases} 2x \sin(x^2) \ln y dx - \frac{\cos(x^2)}{y} dy = 0, & y > 0 \\ y(0) = e \end{cases}$$

Ans. $\cos(x^2) \ln y = 1$

$$(6) \frac{dy}{dx} - (y-2x)^2 = 3$$

Ans. $\tan^{-1}(y-2x) - x = c.$