Simplified Steps for Regulator

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Method ①: When $n \leq 3$

① Check Controllability

② Define $K = [K_1 \ K_2 \ ---- \ K_n]$

③ Find $|SI - (A + BK)| = (S - \mu_1)(S - \mu_2) ---- (S - \mu_n)$

Where:

$\mu_1, \mu_2, --, \mu_n$: Desired poles that are found depending on system requirements (T_r or T_s or T_p, ζ or %OS)

$$\mu_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$$

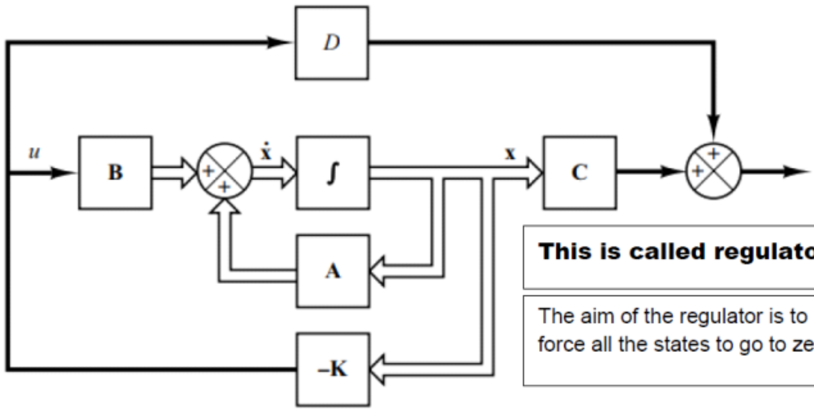
→ Assume $\mu_3, \mu_4, ----, \mu_n$ if the system needs it, where $\mu_{assumed} = -20, -30, ---$

→ Remember that : $T_s = \dfrac{4}{\zeta\omega_n}$

$$T_p = \frac{\pi}{\omega_n\sqrt{1 - \zeta^2}}$$

④ Solve for $k_1, k_2, ----, k_n$ by equating the coefficients of the similar powers on both sides.

⑤ Draw the control scheme



This is called regulator

The aim of the regulator is to force all the states to go to zero

Method (2):

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Case ①: The matrix (A) is in the canonical form

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & -a_{n-3} & \cdots & -a_1 \end{bmatrix}, B=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

① Check the controllability

② Find $A_{CL} = A - BK$

$$= A_{CL}=\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & 0 & 0 & 1 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & 1 \\ (-a_n-k_1) & (-a_{n-1}-k_2) & \cdots & \cdots & \cdots & (-a_1-k_n) \end{bmatrix}$$

Similar to $|SI-A|$

$S^n + a_1 S^{n-1} + \cdots + a_n$

③ From the desired poles:

$$(S-\mu_1)(S-\mu_2)\cdots(S-\mu_n) = S^n + \alpha_1 S^{n-1} + \alpha_2 S^{n-2} + \cdots + \alpha_n$$

④ Find A_{CL} of the desired poles:

$$A_{CL}=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & & & \\ & & & & \\ \vdots & & \ddots & & 1 \\ 0 & 0 & & \cdots & \\ -\alpha_n & -\alpha_{n-1} & -\alpha_{n-2} & \cdots & -\alpha_1 \end{bmatrix}.$$

⑤ let $A_{CL} = A_{CL\,desired}$

by solving : $K = [(\alpha_n - a_n) \quad (\alpha_{n-1} - a_{n-1}) \cdots (\alpha_1 - a_1)]$

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Case ②: The matrix (A) is not in Canonical form (Bass - Gaura Approach)

① Check the controllability

② Find (a_i)'s from $|SI - A| = S^n + a_1 S^{n-1} + a_2 S^{n-2} + \cdots + a_n$

$$\Rightarrow a_1 = \checkmark, \; a_2 = \checkmark, \; \cdots, \; a_n = \checkmark$$

③ Find the transformation matrix (T):

$$T = M\,W$$

where: M = Controllability matrix.

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & & \iddots & \iddots & 0 \\ & \iddots & \iddots & \cdots & \vdots \\ a_1 & 1 & \cdots & \cdots & \vdots \\ 1 & 0 & \cdots & \cdots & 0 \end{bmatrix}$$

④ Find $T^{-1} = \text{inv}(T)$

⑤ Find (α_i)'s from: $(S-\mu_1)(S-\mu_2)\cdots(S-\mu_n)$

$$= S^n + \alpha_1 S^{n-1} + \alpha_2 S^{n-2} + \cdots + \alpha_n$$

⑥ Find the gain matrix (K):

$$K = [(\alpha_n - a_n) \;\; (\alpha_{n-1} - a_{n-1}) \;\; \cdots \;\; (\alpha_1 - a_1)] \cdot T^{-1}$$

⑦ Find the input (U): $U = -KX$

⑧ Find the closed loop eigenvalues:

$$A_{CL} = A - BK$$

Eigenvalues: $|\lambda I - A_{CL}| = \text{eig}(A_{CL})$

⑨ Draw the control scheme

Simplified Steps for Tracking System:

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Case ①: No integral action ($e_{ss}(\infty) = 0$)

① Check the type number

$|SI - A| = 0$
- → If there is no (S) = 0 → type 0
- → // // // one (S) = 0 → type 1
- → // // // two (S) = 0 → type 2

* See the table to check if $e_{ss}(\infty) = 0$ depending on the input.

TABLE 7.2 Relationships between input, system type, static error constants, and steady-state errors

		Type 0		Type 1		Type 2	
Input	Steady-state error formula	Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

② Check the controllability

$M = [B \;\; AB \;\; \cdots \;\; A^{n-1}B]$

if $\det(M) \neq 0 \Rightarrow$ Controllable

③ Find K matrix using pole placement (Regulator Method ②)

Case ① → Check if matrix (A) is in canonical form **or**

Case ② → Check if matrix (A) is not in canonical form (Bass Gura)

* Use the steps in the regulator to find K

④ $u = -[K_1 \; K_2 \cdots K_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + K_1 r$ → Since $y = x_1$

$= -K_1x_1 + -K_2x_2 + -K_3x_3 + \cdots + -K_nx_n + K_1 r$

⑤ Draw the control scheme.

Case 2: Integral Action

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Case 2: With integral action ($e_{ss}(\infty) \neq 0$)

1. Check the type number

$$|SI - A| = 0$$

→ If there is no (S) = 0 → type 0
→ " " " one (S) = 0 → type 1
→ " " " two (S) = 0 → type 2

* See the table to check if $e_{ss}(\infty) \neq 0$ depending on the input

2. Find $\hat{A}$ & $\hat{B}$: $\hat{A} = \begin{bmatrix} A & \vec{0} \\ -C & 0 \end{bmatrix}$, $\hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$

3. Check the controllability of $\hat{A}$ & $\hat{B}$

$$M = [\hat{B} \ \hat{A}\hat{B} \ \cdots \ \hat{A}^{n-1}\hat{B}]$$

If $\det(M) \neq 0 \Rightarrow$ Controllable

4. Find K matrix using pole placement (Regulator Method 2)

Case 1 → Check if matrix $(\hat{A})$ is in canonical form

or

Case 2 → Check if matrix $(\hat{A})$ is not in canonical form (Bass Gaura)

5. $\bar{K} = [K_1 \ K_2 \ \cdots \ K_{n-1}]$

$$u = -\bar{K}X + K_i \xi$$

Such that $K_i = K_n$

6. Draw the control scheme