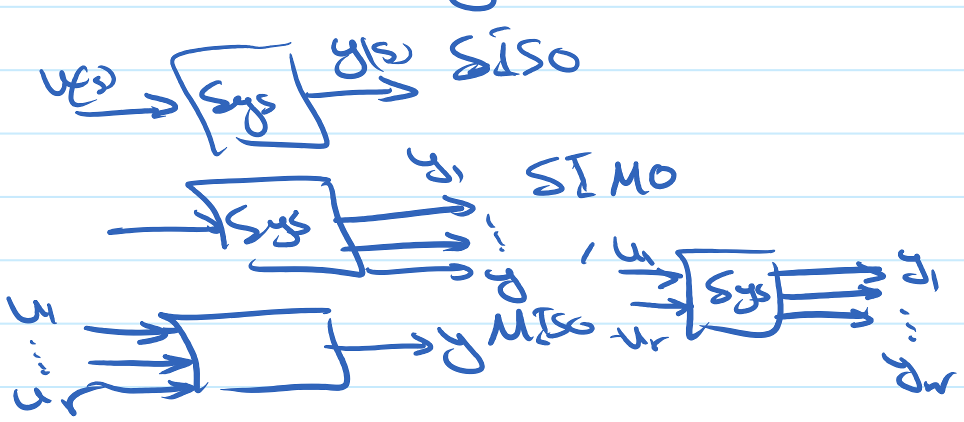
Modern Control Theory. The modern trend in engineering systems is toward greater complexity, due mainly to the requirements of complex tasks and good accuracy. Complex systems may have multiple inputs and multiple outputs and may be time varying. Because of the necessity of meeting increasingly stringent requirements on the performance of control systems, the increase in system complexity, and easy access to large scale computers, modern control theory, which is a new approach to the analysis and design of complex control systems, has been developed since around 1960. This new approach is based on the concept of state. The concept of state by itself is not new, since it has been in existence for a long time in the field of classical dynamics and other fields.

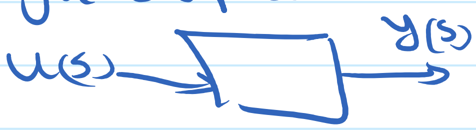
linear non-linear

Modern Control Theory Versus Conventional Control Theory. Modern control theory is contrasted with conventional control theory in that the former is applicable to multiple-input, multiple-output systems, which may be linear or nonlinear, time invariant or time varying, while the latter is applicable only to linear time-invariant single-input, single-output systems. Also, modern control theory is essentially time-domain approach and frequency domain approach (in certain cases such as H-infinity control), while conventional control theory is a complex frequency-domain approach. Before we proceed further, we must define state, state variables, state vector, and state space.

Modern Control
* Linear Sys + nonlinear Sys
* it can deal with SISO, SIMO, MISO and MIMO Sys.



Conventional Control
* Linear Sys.
* it can work with SISO Sys :- single input single output



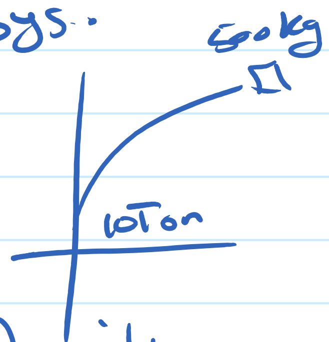
Modern Control

* time domain

* LTI, LTV

LTV := linear time varient Sys.

rocket

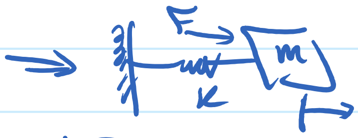


* it can deal with uncertainty

Conventional Control

* s-domain (root locus
Freq domain

* LTI := linear time invarient sys



LTI

$\Sigma F = m\ddot{x}$

$m\ddot{x} + kx = F$ ← linear

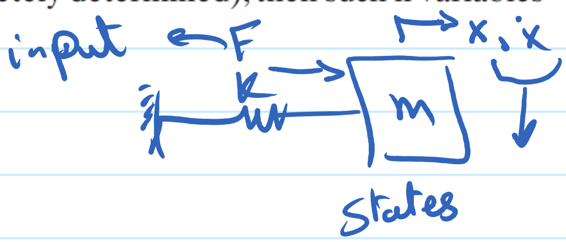
m, k constants

* it can't deal with uncertainty

State. The state of a dynamic system is the smallest set of variables (called *state variables*) such that knowledge of these variables at $t = t_0$, together with knowledge of the input for $t \geq t_0$, completely determines the behavior of the system for any time $t \geq t_0$.

Note that the concept of state is by no means limited to physical systems. It is applicable to biological systems, economic systems, social systems, and others.

State Variables. The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system. If at least n variables $x_1, x_2, \ldots, x_n$ are needed to completely describe the behavior of a dynamic system (so that once the input is given for $t \geq t_0$ and the initial state at $t = t_0$ is specified, the future state of the system is completely determined), then such n variables are a set of state variables.



State Vector. If n state variables are needed to completely describe the behavior of a given system, then these n state variables can be considered the n components of a vector $\mathbf{x}$. Such a vector is called a *state vector*. A state vector is thus a vector that determines uniquely the system state $\mathbf{x}(t)$ for any time $t \geq t_0$, once the state at $t = t_0$ is given and the input $u(t)$ for $t \geq t_0$ is specified.

State Space. The n-dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis, $\ldots$, x_n axis, where $x_1, x_2, \ldots, x_n$ are state variables, is called a *state space*. Any state can be represented by a point in the state space.

State-Space Equations. In state-space analysis we are concerned with three types of variables that are involved in the modeling of dynamic systems: input variables, 1 output variables, 2 and state variables. 3 As we shall see in Section 2–5, the state-space representation for a given system is not unique, except that the number of state variables is the same for any of the different state-space representations of the same system.

The dynamic system must involve elements that memorize the values of the input for $t \geq t_1$. Since integrators in a continuous-time control system serve as memory devices, the outputs of such integrators can be considered as the variables that define the internal state of the dynamic system. Thus the outputs of integrators serve as state variables. The number of state variables to completely define the dynamics of the system is equal to the number of integrators involved in the system.

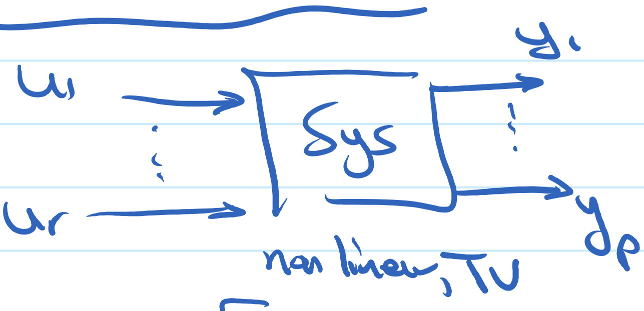
* State Space Rep. (SSR) :-

① Non linear, MIMO, time Varient

n :- # of states

r :- # of inputs

p :- # of outputs



$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ state vector

states Eq's

$$\dot{x}_1(t) = f_1(x_1, \ldots, x_n, u_1, \ldots, u_r, t)$$

$$\dot{x}_2 = f_2(x_1, \ldots, x_n, u_1, \ldots, u_r, t)$$

$$\vdots$$

$$\dot{x}_n = f_n(x_1, \ldots, x_n, u_1, \ldots, u_r, t)$$

$u = \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$ input vector

$y = \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$ output vector

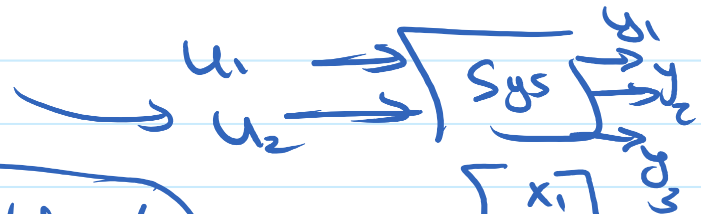
output Eqs

$$y_1(t) = g_1(x_1, \ldots, x_n, u_1, \ldots, u_r, t)$$

$$\vdots$$

$$y_p(t) = g_p(x_1, \ldots, x_n, u_1, \ldots, u_r, t)$$

Ex :-



$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\dot{x}_1 = \lg(x_1) + x_2^2 + u_1 + t \rightarrow f_1(x_1, \ldots, x_n, u_1, \ldots, u_r, t)$$

$$\dot{x}_2 = x_1 + \cos(x_2) + u_1^2 + u_2 + t^3 \rightarrow f_2$$

$$y_1 = x_1 + t \leftarrow y_1$$

$$y_2 = x_1 + 3t^2 \leftarrow y_2$$

$$y_3 = x_1^3 + x_2^2 + u_1 + u_2 \leftarrow y_3$$

$$f(\vec{x},\vec{u},t) = \begin{bmatrix} f_1(x_1,\dots x_n,u_1,\dots u_r,t) \\ \vdots \\ f_n(x_1,\dots,x_n,u_1,\dots u_r,t) \end{bmatrix}$$

n-Eqs

$$g(\vec{x},\vec{u},t) = \begin{bmatrix} g_1(x_1,\dots,x_n,u_1,\dots,u_r,t) \\ \vdots \\ g_p(x_1,\dots,x_n,u_1,\dots,u_r,t) \end{bmatrix}$$

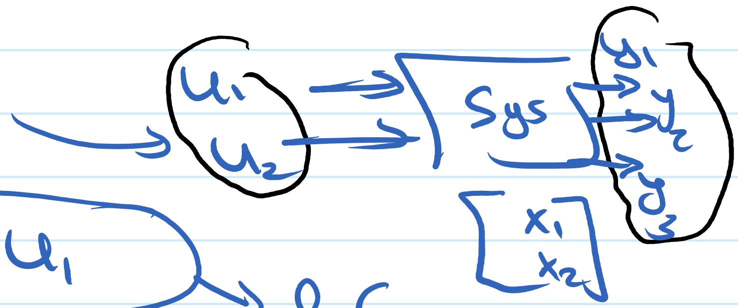
output Eqs

p Eqs

* Non linear, MIMO, TI (time invariant)

$$f(x,u) = \begin{bmatrix} f_1(x_1,\dots,x_n,u_1,\dots,u_r) \\ \vdots \\ f_n(x_1,\dots x_n,u_1,\dots u_r) \end{bmatrix}$$

$$g(x,u) = \begin{bmatrix} g_1(x_1,\dots x_n,u_1,\dots u_r) \\ \vdots \\ g_p(x_1,\dots,x_n,u_1,\dots u_r) \end{bmatrix}$$



$$\dot{x}_1 = \log(x_1) + x_2^2 + u_1 \rightarrow f_1(x_1,\dots,x_n,u_1,\dots,u_r)$$

$$\dot{x}_2 = x_1 + \cos(x_2) + u_1^2 + u_2 \rightarrow f_2$$

$$y_1 = x_1 \leftarrow g_1$$

$$y_2 = x_1 \leftarrow g_2$$

$$y_3 = x_1^3 + x_2^2 + u_1 + u_2 \leftarrow g_3$$

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⑧ MIMO Sys, linear (time-Varient) TV

$$\left.\begin{aligned} \dot{x}(t) &= A(t)\,x(t) + B(t)\,u(t) \\ y(t) &= C(t)\,x(t) + D(t)\,u(t) \end{aligned}\right\} \leftarrow \text{linear}$$

A :- dynamic matrix, system matrix

$A \in R^{n\times n}$ n :- # of states

$B \subset R^{n\times r}$ r :- # of inputs (input matrix)

$C \in R^{p\times n}$ p :- # of outputs (output matrix)

$D \in R^{p\times r}$ forward matrix

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \dot{x} = \underbrace{\begin{bmatrix} 3t & 2 & 4 \\ 5 & 6 & 10 \\ t^2 & 3 & 2 \end{bmatrix}}_{A(t)\ 3\times 3} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_3 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} t^2 & 7 \\ 5 & 8 \\ 6 & 10 \end{bmatrix}}_{B(t)\ 3\times 2} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 5 & 10 & 2 \\ 9 & 1 & 3 \end{bmatrix}}_{C(t)\ 2\times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix}}_{D(t)\ 2\times 2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$p = 2$ $n = 3$

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* linear sys, MIMO, TI (time invariant

$\dot{x}(t) = A\,x(t) + B\,u(t)$

$y(t) = C\,x(t) + D\,u(t)$

$$\dot{x} = \underbrace{\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 \\ 10 \end{bmatrix}}_{B} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{D} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{C} u$$

$n = 2\ (x_1, x_2)$

$r = 1\ (u)$

$p = 2\ (y_1, y_2$

~~MI~~MO SIMO

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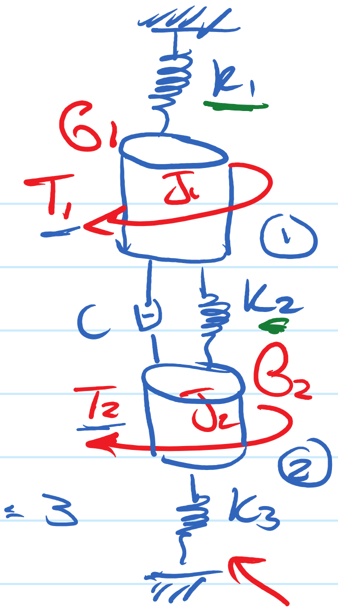
Ex:- Find the State Space Rep:-



if the outputs $y_1 = \theta_1$, $y_2 = \theta_2$

$y_3 = \dot{\theta}_1$

DoFs = 2

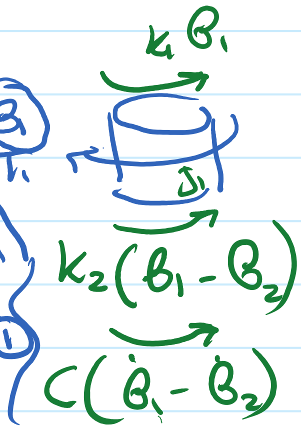
$n = 2 * DOF_s = 2 * 2 = 4$, $r = 2$, $P = 3$

$\dot{x} = \underset{4\times4}{A}x + \underset{4\times2}{B}u$ $\quad x \in R^{4\times1}$

$y = \underset{3\times4}{C}x + \underset{3\times2}{D}u$ $\quad y \in R^{3\times1}$

$u \in R^{2\times1}$

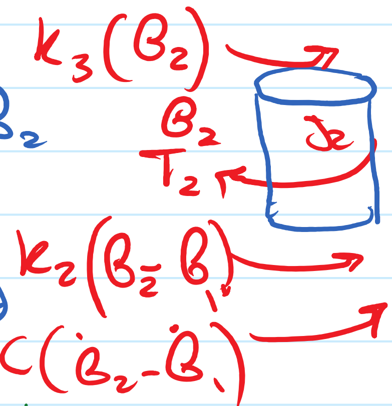
$\sum M = J_1 \ddot{\theta}_1$



$T_1 - k_1\theta_1 - k_2(\theta_1-\theta_2) - c(\dot{\theta}_1-\dot{\theta}_2) = J_1\ddot{\theta}_1$

① $\frac{T_1}{J_1} - \frac{(k_1+k_2)}{J_1}\theta_1 + \frac{k_2\theta_2}{J_1} - \frac{c\dot{\theta}_1}{J_1} + \frac{c\dot{\theta}_2}{J_1} = \ddot{\theta}_1$ — ①

$\sum M = J_2\ddot{\theta}_2$



$T_2 - k_3\theta_2 - k_2(\theta_2-\theta_1) - c(\dot{\theta}_2-\dot{\theta}_1) = J_2\ddot{\theta}_2$

② $\frac{T_2}{J_2} - \frac{(k_3+k_2)}{J_2}\theta_2 + \frac{k_2\theta_1}{J_2} - \frac{c\dot{\theta}_2}{J_2} + \frac{c\dot{\theta}_1}{J_2} = \ddot{\theta}_2$ — ②

let the states are:

$x_1 = \theta_1 \Rightarrow \dot{x}_1 = \dot{\theta}_1$ $\quad \boxed{\dot{x}_1 = x_2}$ — ⓐ

$x_2 = \dot{\theta}_1$

$x_3 = \theta_2 \Rightarrow \dot{x}_3 = \dot{\theta}_2$ $\quad \boxed{\dot{x}_3 = x_4}$ — ⓑ

$x_4 = \dot{\theta}_2$

Sub the states in Eq ① and Eq ②

$\dot{x}_4 = \ddot{\theta}_2$ $\quad \dot{x}_2 = \ddot{\theta}_1$

$\dot{x}_1 = x_2$ (a) $\dot{x}_3 = x_4$ (b)

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$x_1 = \theta_1 \quad x_2 = \dot{\theta}_1 \leftarrow$

$x_3 = \theta_2 \quad x_4 = \dot{\theta}_2$

$$\frac{T_1}{J_1} - \frac{(k_1+k_2)}{J_1}\theta_1 + \frac{k_2\theta_2}{J_1} - \frac{c\dot{\theta}_1}{J_1} + \frac{c\dot{\theta}_2}{J_1} = \ddot{\theta}_1 \quad (1)$$

$$\boxed{\frac{T_1}{J_1} - \frac{(k_1+k_2)}{J_1}x_1 + \frac{k_2}{J_1}x_3 - \frac{c}{J_1}x_2 + \frac{c}{J_1}x_4 = \dot{x}_2} \quad (c)$$

$$\frac{T_2}{J_2} - \frac{(k_3+k_2)}{J_2}\theta_2 + \frac{k_2\theta_1}{J_2} - \frac{c\dot{\theta}_2}{J_2} + \frac{c\dot{\theta}_1}{J_2} = \ddot{\theta}_2 \quad (2)$$

$$\boxed{\frac{T_2}{J_2} - \left(\frac{k_2+k_3}{J_2}\right)x_3 + \frac{k_2}{J_2}x_1 - \frac{c}{J_2}x_4 + \frac{c}{J_2}x_2 = \dot{x}_4} \quad (d)$$

$$\dot{X} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{(k_1+k_2)}{J_1} & -\frac{c}{J_1} & \frac{k_2}{J_1} & \frac{c}{J_1}\\ 0 & 0 & 0 & 1\\ \frac{k_2}{J_2} & \frac{c}{J_2} & -\frac{(k_2+k_3)}{J_2} & -\frac{c}{J_2}\end{bmatrix}}_{A,\ 4\times 4}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \underbrace{\begin{bmatrix}0 & 0\\ \frac{1}{J_1} & 0\\ 0 & 0\\ 0 & \frac{1}{J_2}\end{bmatrix}}_{B,\ 4\times 2}\begin{bmatrix}T_1\\ T_2\end{bmatrix}$$

$y_1 = \theta_1 = x_1$ (i)

$y_2 = \theta_2 = x_3$ (ii)

$y_3 = \dot{\theta}_1 = x_2$ (iii)

$$y = \begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} = \underbrace{\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}}_{3\times 4} \underset{4}{X} + \underbrace{\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}}_{3\times 2}\begin{bmatrix}T_1\\ T_2\end{bmatrix}$$

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$\dot{z}_1 = \dot{\theta}_1 = z_3$

$\boxed{\dot{z}_1 = z_3}$ — (a)

$x_1 = \theta_1 \rightarrow z_1 = \theta_1$

$x_2 = \dot{\theta}_1 \Rightarrow z_2 = \theta_2$

$x_3 = \theta_2 \quad z_3 = \dot{\theta}_1$

$x_4 = \dot{\theta}_2 \quad z_4 = \dot{\theta}_2$

$\dot{z}_2 = z_4 = \dot{\theta}_2 \Rightarrow \boxed{\dot{z}_2 = z_4}$ — (b)

∘ D

New

A ✓ B ✓, C ✓, D ✓

$y_1 = \theta_1 = z_1, \; y_2 = \theta_2 = z_2$

$y_3 = \dot{\theta}_1 = z_3$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u$$

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$m\ddot{x}+c\dot{x}+kx=f$

Dynamical Sys SISO → TF

$\frac{1}{ms^2+cs+k}$

Dynamical Sys MIMO, SIMO, MISO, SISO

SSR1

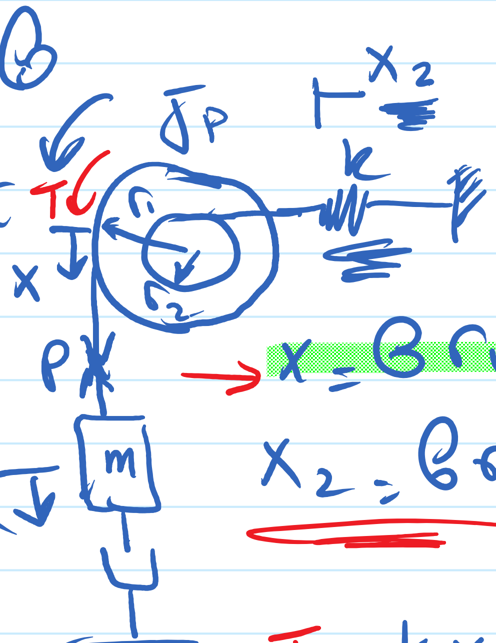
SSR2

SSRn

c, k, J_0, θ

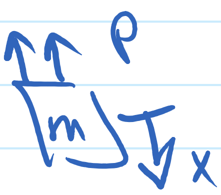
r_1, r_2

Find SSR if the output $y = x$



$x = \theta r_1$

$x_2 = \theta r_2$

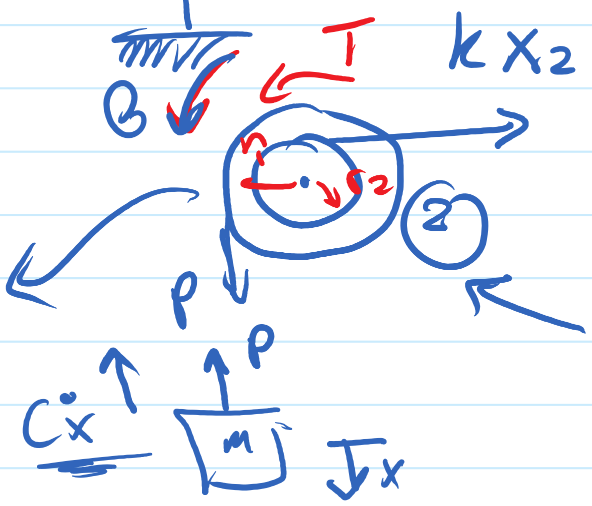


$\Sigma F = m\ddot{x}$

$-c\dot{x} - P = m\ddot{x}$

$P = -(m\ddot{x} + c\dot{x})$ — ①

$\Sigma M = J_0\ddot{\theta}$



$T + Pr_1 - kx_2 r_2 = J_0\ddot{\theta}$

$T - (m\ddot{x} + c\dot{x})r_1 - k\theta r_2^2 = J_0\ddot{\theta}$

$T - (m\ddot{\theta}r_1 + c\dot{\theta}r_1)r_1 - k\theta r_2^2 = J_0\ddot{\theta}$

$$(mr_1^2 + J)\ddot{\theta} = -c r_1^2 \dot{\theta} - k r_2^2 \theta + T$$

$y = x$

$n = 2 \quad r = 1 \quad p = 1$

DOF = 1
↳ 1 diff eq.

let the states are → $z_1 = \theta \Rightarrow \dot{z}_1 = \dot{\theta}$; $\boxed{\dot{z}_1 = z_2}$ (i)

$z_2 = \dot{\theta} \Rightarrow$

$$\boxed{\dot{z}_2 = \frac{-c r_1^2 z_2}{(m r_1^2 + J)} - \frac{k r_2^2 z_1}{(m r_1^2 + J)} + \frac{T}{(m r_1^2 + J)}} \text{ (ii)}$$

$$\dot{Z} = A Z + B T$$
$$y = C Z + D T$$

u = Torque input

$$Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

(i) row 1

$$\dot{Z} = \underbrace{\begin{bmatrix} 0 & 1 \\ \frac{-k r_2^2}{(m r_1^2 + J)} & \frac{-c r_1^2}{(m r_1^2 + J)} \end{bmatrix}_{2\times 2}}_{A} Z + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{(m r_1^2 + J)} \end{bmatrix}_{2\times 1}}_{B} T$$

$$y = x = \theta r_1 = r_1 z_1 \text{ (iii)}$$

$$y = \begin{bmatrix} r_1 & 0 \end{bmatrix} Z + \begin{bmatrix} 0 \end{bmatrix} T$$

Eigenvalues and Eigenvectors:

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$\vec{q}$ eigen vector
λ eigen value

Definition: Let A be an $n \times n$ matrix.

① An eigenvalue of A matrix is a nonzero vector $\vec{q}$ in R^n such that $A\vec{q} = \lambda \vec{q}$, for some scalar λ.

② An eigenvalue of A is a scalar λ such that the equation $A\vec{q} = \lambda\vec{q}$ has a non-trivial solution

x: if $A\vec{q} = \lambda\vec{q}$ for $\vec{q}$ we say that λ is the eigen value for $\vec{q}$ and that $\vec{q}$ is an eigen vector for λ

Notes:
- Eigen vectors are by definition are non zero vectors
- Eigenvalues may be equal to zero

$A\vec{q} = \lambda\vec{q}$

Ex: $\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

find eigen values and eigen vectors? Also study the stability of the sys

$\Rightarrow A\vec{q} = \lambda\vec{q} \Rightarrow \underset{n\times n}{A}\underset{n\times 1}{\vec{q}} - \lambda\vec{q} = \vec{0}$

$(A - \lambda I)\vec{q} = \vec{0} \Leftarrow \vec{q} = \vec{0}$ ~~α~~

$(A - \lambda I) = \vec{0} \Rightarrow \boxed{(\lambda I - A) = 0}$ — ⊛

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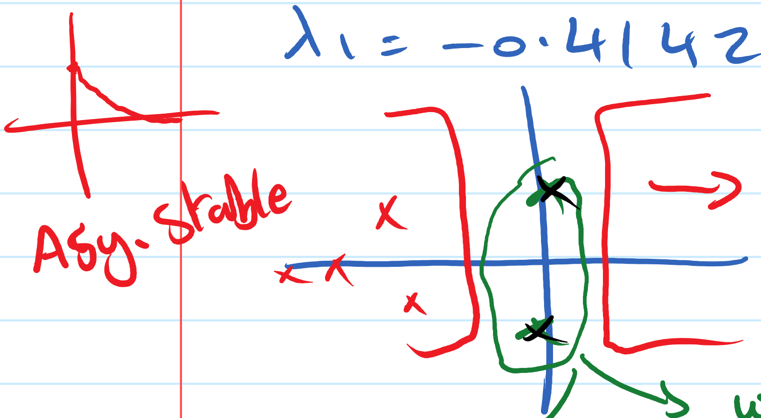
$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow |(\lambda I - A)| = 0$$

$$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ -1 & \lambda - 2 \end{bmatrix}$$

$$|(\lambda I - A)| = 0 \Rightarrow \lambda^2 - 2\lambda - 1 = 0$$

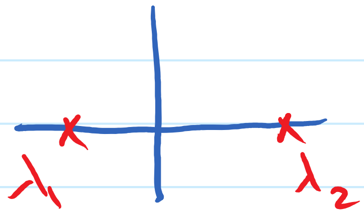
$\lambda_1 = -0.4142$ $\lambda_2 = 2.4142$



Asy. stable

unstable

s-domain



unstable

without dublication

m. stable

$S_{1,2} = \pm j$

with dublication

$S_{1,2} = \pm j$

$S_{3,4} = \pm j$

unstable m.stable

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$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow [\lambda I - A] = \begin{bmatrix} \lambda & -1 \\ -1 & \lambda-2 \end{bmatrix} \quad \text{—} \ (*)$$

For $\lambda_1 = -0.4142$

$$\vec{q}_1 = \begin{bmatrix} q_{11} \\ q_{12} \end{bmatrix}$$

$$[\lambda_1 I - A]\vec{q}_1 = \vec{0}$$

$$\begin{bmatrix} -0.4142 & -1 \\ -1 & -2.4142 \end{bmatrix} \begin{bmatrix} q_{11} \\ q_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

— if the rank = 2 so we have two Eqs with two Variables

— if the rank = 1 so we have one eq with 2 Variables

$$\det(\lambda_1 I - A) \begin{cases} \neq 0 & \text{Full rank} \\ = 0 & \text{not full rank} \end{cases}$$

$$\det(\lambda_1 I - A) = -0.4142 \times -2.4142 - 1 = 0$$

So rank = 1

let take 1st row

$$-0.4142\, q_{11} - q_{12} = 0 \quad \text{—} \ (1)$$

let $q_{11} = 1$

$$\Rightarrow -0.4142(1) = q_{12}$$

$$\vec{q}_1 = \begin{bmatrix} 1 \\ -0.4142 \end{bmatrix} \Rightarrow \hat{q}_1 = \frac{1}{\sqrt{(1)^2 + (-0.4142)^2}} \begin{bmatrix} 1 \\ -0.4142 \end{bmatrix}$$

normalization of vector

normalized vector

for $\lambda_2 = 2.4142$

$[\lambda_2 I - A]\vec{q}_2 = \vec{0} \Rightarrow [\lambda I - A] = \begin{bmatrix} \lambda & -1 \\ -1 & \lambda - 2 \end{bmatrix}$

$[\lambda_2 I - A] = \begin{bmatrix} 2.4142 & -1 \\ -1 & 0.4142 \end{bmatrix}$

$\begin{bmatrix} 2.4142 & -1 \\ -1 & 0.4142 \end{bmatrix} \begin{bmatrix} q_{21} \\ q_{22} \end{bmatrix} = \vec{0}$

$\det(\lambda_2 I - A) = -2.4142 \times 0.4142 - 1 = 0$

let take second row rank(1)

$-q_{21} + 0.4142\, q_{22} = 0$ let $q_{22} = 1$

$q_{21} = 0.4142$ $\vec{q}_2 = \begin{bmatrix} 0.4142 \\ 1 \end{bmatrix}$

$\vec{\hat{q}}_2 = \dfrac{1}{\sqrt{(0.4142)^2 + (1)^2}} \begin{bmatrix} 0.4142 \\ 1 \end{bmatrix}$

in mat lab

- rank(A) " to check the rank of matrix A
- det(A) " to check the determinant of A.

lamda = eig(A) " eigenvalues "

[q lamda] = eig A " eigenvectors + eigenvalues "

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