

5.5 Indefinite Integrals and Substitution Method

(106)

Example: Evaluate the indefinite integrals:

$$\begin{aligned} \textcircled{1} \int (x^3 + x)^5 (3x^2 + 1) dx & \quad u = x^3 + x \\ & \quad du = (3x^2 + 1) dx \\ & = \int u^5 du = \frac{u^6}{6} + C = \frac{1}{6} (x^3 + x)^6 + C \end{aligned}$$

$$\begin{aligned} \textcircled{2} \int (3x + 2)(3x^2 + 4x)^4 dx & \quad u = 3x^2 + 4x \\ & \quad du = (6x + 4) dx \\ & \quad \frac{du}{2} = (3x + 2) dx \\ \frac{1}{2} \int u^4 du & = \frac{1}{2} \frac{u^5}{5} + C \\ & = \frac{1}{10} (3x^2 + 4x)^5 + C \end{aligned}$$

$$\begin{aligned} \textcircled{3} \int 7\sqrt{7x-1} dx & \quad u = 7x-1 \\ & \quad du = 7 dx \\ \int u^{\frac{1}{2}} du & = \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{3} (7x-1)^{\frac{3}{2}} + C \end{aligned}$$

$$\begin{aligned} \textcircled{4} \int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2} & \quad u = 1 + \sqrt{x} \\ & \quad du = \frac{dx}{2\sqrt{x}} \\ 2 \int u^{-2} du & = \frac{-2}{u} + C = \frac{-2}{1+\sqrt{x}} + C \end{aligned}$$

$$\begin{aligned} \textcircled{5} \int \frac{-6 \tan^2 x \sec^2 x}{(2 + \tan^3 x)^3} dx & \quad u = 2 + \tan^3 x \\ & \quad du = 3 \tan^2 x \sec^2 x dx \end{aligned}$$

$$\text{STUDENTS-HUB.com} \quad -2 \int \frac{du}{u^3} = -2 \frac{u^{-2}}{-2} + C = \frac{1}{(2 + \tan^3 x)^2} + C$$

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Example: Solve the IVP $\frac{ds}{dt} = 12 + (3t^2 - 1)^3$, $s(1) = 3$

$$\begin{aligned} s(t) & = \int 12 + (3t^2 - 1)^3 dt = 12t + \int u^3 du \\ & \quad u = 3t^2 - 1 \\ & \quad du = 6t dt \\ s(t) & = 2 \frac{u^4}{4} + C = \frac{(3t^2 - 1)^4}{2} + C \\ s(1) & = \frac{2^4}{2} + C = 3 \Leftrightarrow C = -5 \\ s(t) & = \frac{1}{2} (3t^2 - 1)^4 - 5 \end{aligned}$$

Example (a) $\int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$

(b) $\int \cos^2 x dx = \int \frac{1 + \cos 2x}{2} dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$