Chap 2 : limits & continuity

$\lim_{x \to x_0} f(x) = L$ iff $\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = L$

Th:

- The sandwich Theorem: if $g(x) \leq f(x) \leq h(x)$
  if ~~Then~~ $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$
  Then $\lim_{x \to c} f(x) = L$

- $f(x)$ is continious at x_0 iff : $\lim_{x \to x_0} f(x_0)$ exist and $f(x_0)$ exist and $\lim_{x \to x_0} f(x) = f(x)$

$f(x)$ is ~ ~ $[a, b]$ iff f is cont at every point of $[a, b]$

Exp - $\sin(x)$ / $\cos(x)$ / $|x|$ / and any polynomials are cont on $\mathbb{R}$
- all Rational function $\frac{f(x)}{g(x)}$ are cont on $\mathbb{R}$ except The zeros of $g(x)$

Discontinuity : when $f(x)$ is Discontinual in a specific point we use a Removable Disconting point and we creat an Continious extension : $f(x)$ is Discont at x_0 and $\lim_{x \to x_0} = a$ Then

$$F(x) = \begin{cases} f(x), & x \neq x_0 \\ a, & x = x_0 \end{cases}$$

The Intermediate Value Theorem IVT

* suppose That $f(x)$ is cont on $[a,b]$ and $f(a) \leq y_0 \leq f(b)$
Then $\exists$ a number $x_0 \in [a,b]$ s.t $f(x_0) = y_0$

Imp: If They ask if $f(x)$ has a zero at $[a,b]$
we find $f(a), f(b)$ and if $y_0 = 0$ is between
Then There is a zero

To draw a Rational function we use Asymptotes

Asymptotes are 3 Kinds: → H.Asy : line $y = b$
s.t : $\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$

→ V.Asy line : $x = a$
s.t: $\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$

→ oblique.Asy: when $\frac{g(x)}{h(x)}$
$g(x)$ degree > $h(x)$ degree

Notes

$$\lim_{x \to \pm\infty} \frac{\sin x}{x} = 0$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$