SOLVING PROBLEMS ON YOUR OWN

In this lesson we introduced the *vector product* or *cross product* of two vectors. In the following problems, you may want to use the vector product to compute the *moment of a force about a point* and also to determine the *perpendicular distance* from a point to a line.

We defined the moment of the force **F** about the point O of a rigid body as

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \tag{3.11}$$

where **r** is the position vector *from O to any point* on the line of action of **F**. Since the vector product is not commutative, it is absolutely necessary when computing such a product that you place the vectors in the proper order and that each vector have the correct sense. The moment $\mathbf{M}_O$ is important because its magnitude is a measure of the tendency of the force **F** to cause the rigid body to rotate about an axis directed along $\mathbf{M}_O$.

1. Computing the moment M_O of a force in two dimensions. You can use one of the following procedures:

a. Use Eq. (3.12), $M_O = Fd$, which expresses the magnitude of the moment as the product of the magnitude of **F** and the *perpendicular distance* d from O to the line of action of **F** [Sample Prob. 3.1].

b. Express **r** and **F** in component form and formally evaluate the vector product $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$ [Sample Prob. 3.2].

c. Resolve **F** into components respectively parallel and perpendicular to the position vector **r**. Only the perpendicular component contributes to the moment of **F** [Sample Prob. 3.3].

d. Use Eq. (3.22), $M_O = M_z = xF_y - yF_x$. When applying this method, the simplest approach is to treat the scalar components of **r** and **F** as positive and then to assign, by observation, the proper sign to the moment produced by each force component. For example, applying this method to solve Sample Prob. 3.2, we observe that both force components tend to produce a clockwise rotation about B. Therefore, the moment of each force about B should be represented by a negative scalar. We then have for the total moment

$$M_B = -(0.16\text{ m})(400\text{ N}) - (0.20\text{ m})(693\text{ N}) = -202.6\text{ N} \cdot \text{m}$$

2. Computing the moment M_O of a force F in three dimensions. Following the method of Sample Prob. 3.4, the first step in the process is to select the most convenient (simplest) position vector **r**. You should next express **F** in terms of its rectangular components. The final step is to evaluate the vector product $\mathbf{r} \times \mathbf{F}$ to determine the moment. In most three-dimensional problems you will find it easiest to calculate the vector product using a determinant.

3. Determining the perpendicular distance d from a point A to a given line. First assume that a force **F** of known magnitude F lies along the given line. Next determine its moment about A by forming the vector product $\mathbf{M}_A = \mathbf{r} \times \mathbf{F}$, and calculate this product as indicated above. Then compute its magnitude M_A. Finally, substitute the values of F and M_A into the equation $M_A = Fd$ and solve for d.

3.1 A 90-N forc
that the leng
the moment
horizontal a

3.2 A 90-N forc
that the len
the momen
component

3.3 A 90-N for
that the le
force abou

3.4 A crate of
(*a*) the m
(*b*) the sm
magnitude

3.5 A crate o
(*a*) the m
(*b*) the sm
magnitud
and point
vertical f
site sense

3.6 A 300-N
(*a*) Com
into hor
part *a*, d
action o

3.7 A 400
(*a*) Co
into c
that li
force

PROBLEMS

3.1 A 90-N force is applied to the control rod AB as shown. Knowing that the length of the rod is 225 mm and that $\alpha = 25°$, determine the moment of the force about point B by resolving the force into horizontal and vertical components.

3.2 A 90-N force is applied to the control rod AB as shown. Knowing that the length of the rod is 225 mm and that $\alpha = 25°$, determine the moment of the force about point B by resolving the force into components along AB and in a direction perpendicular to AB.

3.3 A 90-N force is applied to the control rod AB as shown. Knowing that the length of the rod is 225 mm and that the moment of the force about B is 13.5 N · m clockwise, determine the value of α.

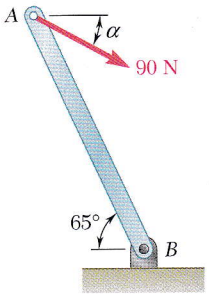

Fig. P3.1, P3.2, and P3.3

3.4 A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about E, (*b*) the smallest force applied at B that creates a moment of equal magnitude and opposite sense about E.

3.5 A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about E, (*b*) the smallest force applied at A that creates a moment of equal magnitude and opposite sense about E, (*c*) the magnitude, sense, and point of application on the bottom of the crate of the smallest vertical force that creates a moment of equal magnitude and opposite sense about E.

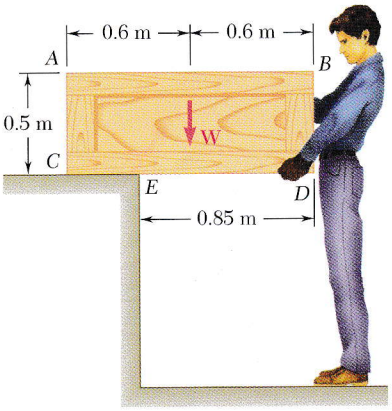

Fig. P3.4 and P3.5

3.6 A 300-N force **P** is applied at point A of the bell crank shown. (*a*) Compute the moment of the force **P** about O by resolving it into horizontal and vertical components. (*b*) Using the result of part *a*, determine the perpendicular distance from O to the line of action of **P**.

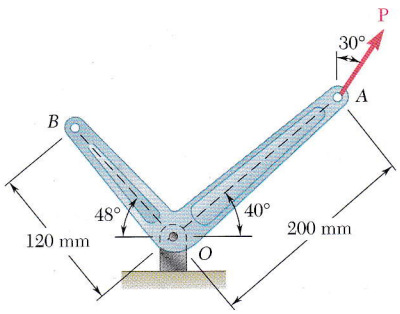

Fig. P3.6 and P3.7

3.7 A 400-N force **P** is applied at point A of the bell crank shown. (*a*) Compute the moment of the force **P** about O by resolving it into components along line OA and in a direction perpendicular to that line. (*b*) Determine the magnitude and direction of the smallest force **Q** applied at B that has the same moment as **P** about O.

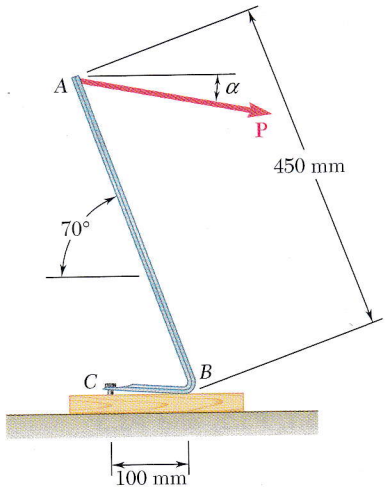

Fig. P3.8

3.8 It is known that a vertical force of 200 N is required to remove the nail at C from the board. As the nail first starts moving, determine (a) the moment about B of the force exerted on the nail, (b) the magnitude of the force **P** that creates the same moment about B if $\alpha = 10°$, (c) the smallest force **P** that creates the same moment about B.

3.9 and 3.10 It is known that the connecting rod AB exerts on the crank BC a 2.5-kN force directed down and to the left along the centerline of AB. Determine the moment of the force about C.

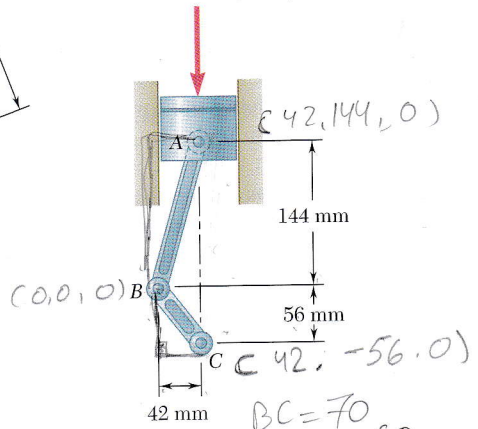

Fig. P3.9

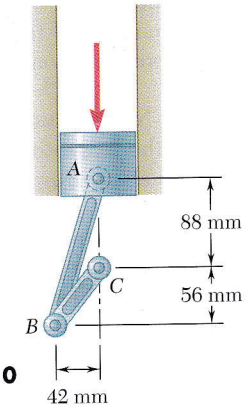

Fig. P3.10

3.11 A winch puller AB is used to straighten a fence post. Knowing that the tension in cable BC is 1040 N and length d is 1.90 m, determine the moment about D of the force exerted by the cable at C by resolving that force into horizontal and vertical components applied (a) at point C, (b) at point E.

3.12 It is known that a force with a moment of 960 N · m about D is required to straighten the fence post CD. If $d = 2.80$ m, determine the tension that must be developed in the cable of winch puller AB to create the required moment about point D.

3.13 It is known that a force with a moment of 960 N · m about D is required to straighten the fence post CD. If the capacity of winch puller AB is 2400 N, determine the minimum value of distance d to create the specified moment about point D.

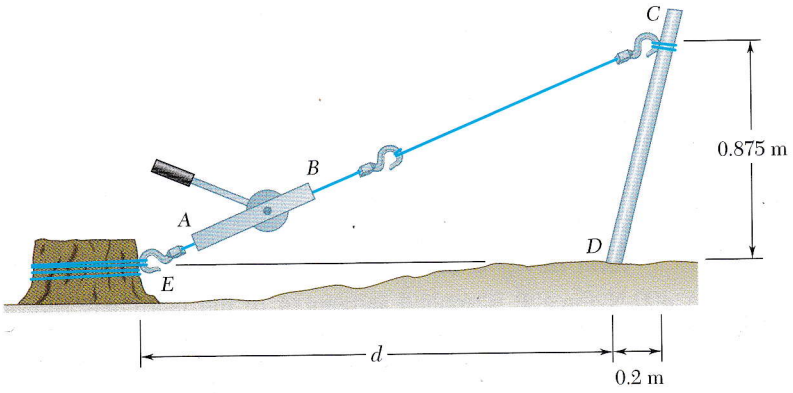

Fig. P3.11, P3.12, and P3.13

3.14 A mechanic uses a piece of pipe AB as a lever when tightening an alternator belt. When he pushes down at A, a force of 485 N is exerted on the alternator at B. Determine the moment of that force about bolt C if its line of action passes through O.

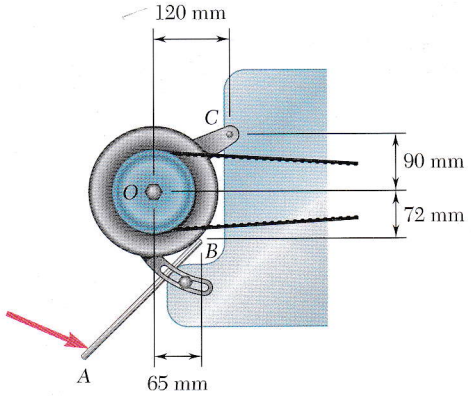

Fig. P3.14

3.15 Form the vector products $\mathbf{B} \times \mathbf{C}$ and $\mathbf{B}' \times \mathbf{C}$, where $B = B'$, and use the results obtained to prove the identity

$$\sin \alpha \cos \beta = \tfrac{1}{2} \sin (\alpha + \beta) + \tfrac{1}{2} \sin (\alpha - \beta).$$

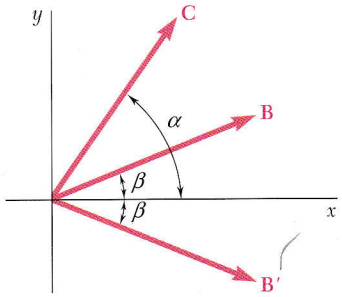

Fig. P3.15

3.16 The vectors $\mathbf{P}$ and $\mathbf{Q}$ are two adjacent sides of a parallelogram. Determine the area of the parallelogram when (*a*) $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, (*b*) $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

3.17 A plane contains the vectors $\mathbf{A}$ and $\mathbf{B}$. Determine the unit vector normal to the plane when $\mathbf{A}$ and $\mathbf{B}$ are equal to, respectively, (*a*) $\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $4\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$, (*b*) $3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $-2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$.

3.18 A line passes through the points (20 m, 16 m) and (−1 m, −4 m). Determine the perpendicular distance d from the line to the origin O of the system of coordinates.

3.19 Determine the moment about the origin O of the force $\mathbf{F} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ that acts at a point A. Assume that the position vector of A is (*a*) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, (*b*) $\mathbf{r} = -8\mathbf{i} + 6\mathbf{j} - 10\mathbf{k}$, (*c*) $\mathbf{r} = 8\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$.

3.20 Determine the moment about the origin O of the force $\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ that acts at a point A. Assume that the position vector of A is (*a*) $\mathbf{r} = 3\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$, (*b*) $\mathbf{r} = \mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$, (*c*) $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$.

3.21 The wire AE is stretched between the corners A and E of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about O of the force exerted by the wire (*a*) on corner A, (*b*) on corner E.

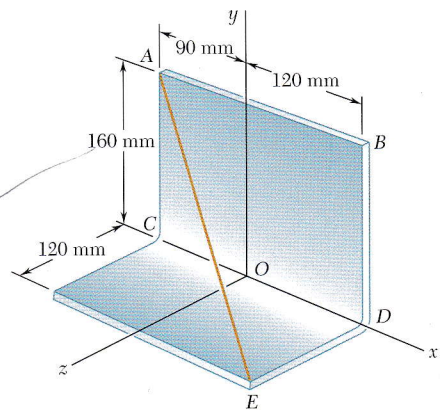

Fig. P3.21

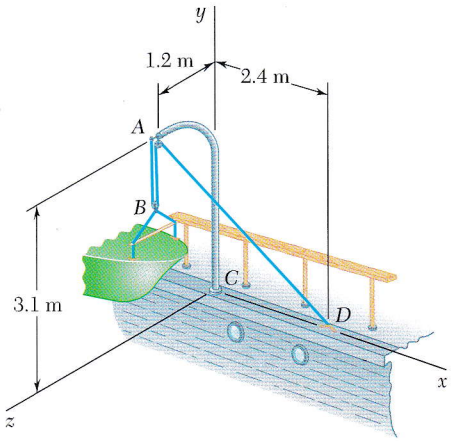

Fig. P3.22

3.22 A small boat hangs from two davits, one of which is shown in the figure. The tension in line *ABAD* is 369 N. Determine the moment about *C* of the resultant force $\mathbf{R}_A$ exerted on the davit at *A*.

3.23 A 2-m-long fishing rod *AB* is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 30 N. Determine the moment about *A* of the force exerted by the line at *B*.

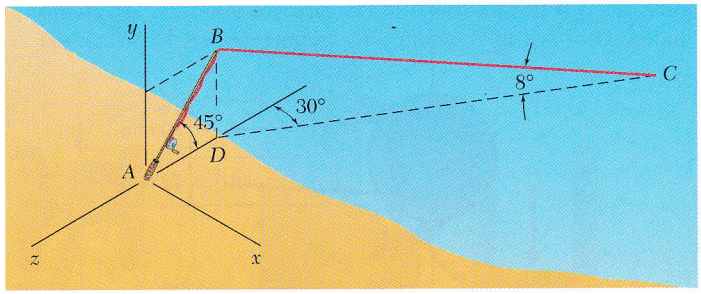

Fig. P3.23

3.24 A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable *BD* is 900 N, determine the moment about point *O* of the force exerted by the cable at *B*.

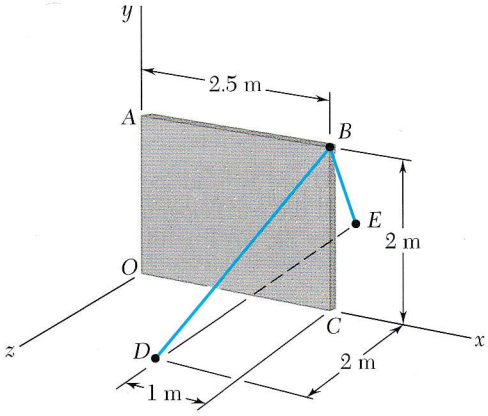

Fig. P3.24

3.25 A 200-N force is applied as shown to the bracket *ABC*. Determine the moment of the force about *A*.

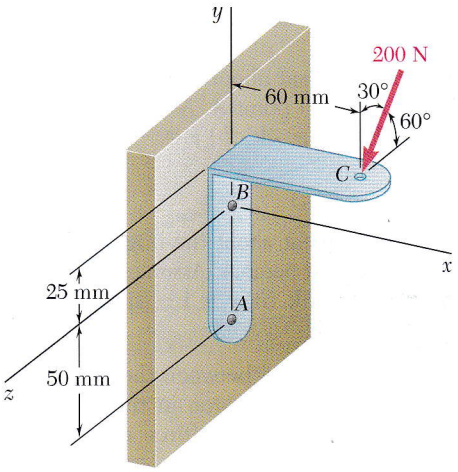

Fig. P3.25

3.26 The 6-m
from th
cal wall
moment

3.27 In Prob
to wire

3.28 In Prob
to wire

3.29 In Prob
to porti

3.30 In Prob
to a lin

3.31 In Prob
to a lin

3.32 In Prob
to cabl

3.33 In Prob
to cabl

3.34 A gard
the fou
passes
minimi
needed

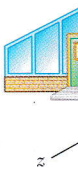

Fig. P3

3.26 The 6-m boom AB has a fixed end A. A steel cable is stretched from the free end B of the boom to a point C located on the vertical wall. If the tension in the cable is 2.5 kN, determine the moment about A of the force exerted by the cable at B.

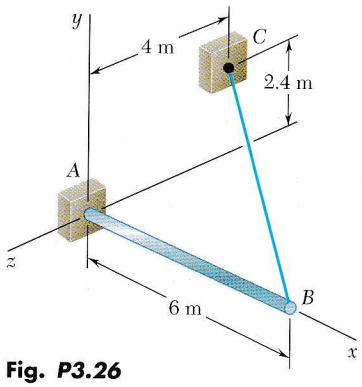

Fig. P3.26

3.27 In Prob. 3.21, determine the perpendicular distance from point O to wire AE.

3.28 In Prob. 3.21, determine the perpendicular distance from point B to wire AE.

3.29 In Prob. 3.22, determine the perpendicular distance from point C to portion AD of the line $ABAD$.

3.30 In Prob. 3.23, determine the perpendicular distance from point A to a line drawn through points B and C.

3.31 In Prob. 3.23, determine the perpendicular distance from point D to a line drawn through points B and C.

3.32 In Prob. 3.24, determine the perpendicular distance from point O to cable BD.

3.33 In Prob. 3.24, determine the perpendicular distance from point C to cable BD.

3.34 A gardener wishes to connect a water supply pipe from point C on the foundation of a 30-m-long greenhouse to a water main that passes through points A and B. Determine (*a*) the value of L which minimizes the length of supply pipe needed, (*b*) the length of pipe needed.

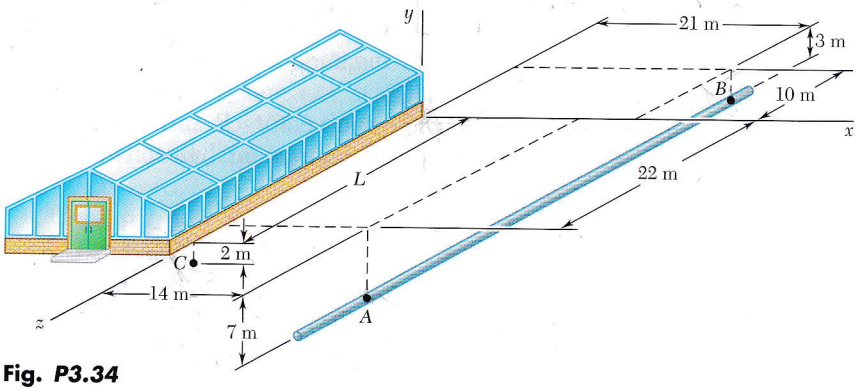

Fig. P3.34