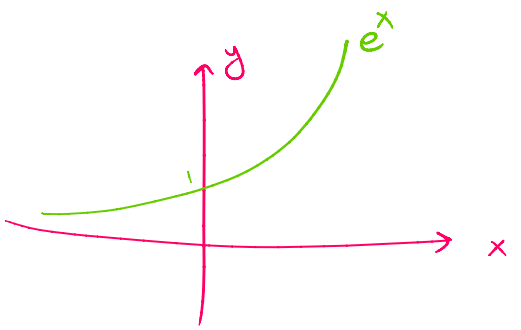
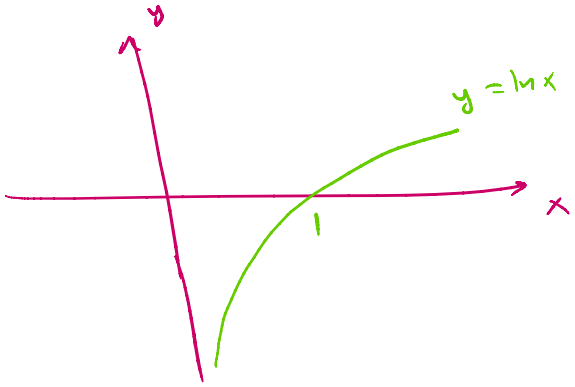
7.8

Monday, December 20, 2021 12:44 PM

Relative Rates of Growth



$x \to \infty \Rightarrow e^x \to \infty$



$x \to \infty \Rightarrow \ln x \to \infty$

$f(x)$, $g(x)$ positive for large x and

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = L$$

(1) If $L = \infty$ then f grows faster than g as $x \to \infty$

then g grows slower than f as $x \to \infty$

(2) If $L = 0$ then g grows faster than f as $x \to \infty$

then f = slower = g = as $x \to \infty$

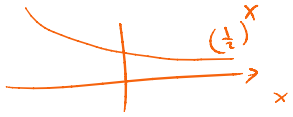
(3) If $0 < L < \infty$ then f and g grow at same rate as $x \to \infty$

$L = \frac{1}{\ln 2}$

Exp Which faster as $x \to \infty$

① y^x, e^x

$-1 < a < 1 \Rightarrow \lim_{x\to\infty} a^x = 0$



$b > 1 \Rightarrow \lim_{x\to\infty} b^x = \infty$

$$\lim_{x\to\infty} \frac{y^x}{e^x} = \lim_{x\to\infty} \left(\frac{y}{e}\right)^x$$

$e \approx 2.718$

$= \infty$

y^x grows faster than e^x as $x \to \infty$

or e^x " slower " y^x " "

② $\left(\frac{3}{2}\right)^x$, e^x

$\frac{3}{2} = 1.5$

$e \approx 2.718$

$$\lim_{x\to\infty} \frac{\left(\frac{3}{2}\right)^x}{e^x} = \lim_{x\to\infty} \left(\frac{3}{2e}\right)^x = 0$$

$\frac{3}{2e} < 1$ ∴

$\left(\frac{3}{2}\right)^x$ grows slower than e^x as $x \to \infty$

$\left(\frac{3}{2}\right)^x$ grows slower than e^x as $x \to \infty$

or e^x = faster = $\left(\frac{3}{2}\right)^x$ = --

③ $\log_2 x$, $\ln x$

$$\lim_{x\to\infty} \frac{\log_2 x}{\ln x} = \lim_{x\to\infty} \frac{\frac{\ln x}{\ln 2}}{\ln x} = \frac{1}{\ln 2}$$

$\Rightarrow$ $\log_2 x$, $\ln x$ grow at same rate as $x \to \infty$

④ 2^x , x^2 , e^x

أسرع من

2^x 2.2718^x

2^x e^x

$$\lim \frac{2^x}{x^2} \quad \frac{\infty}{\infty} \quad = \lim_{x\to\infty} \frac{2^x \ln 2}{2x} \quad \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{2^x}{x^2} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{2^x \ln 2}{2x} \overset{\infty}{=} \lim_{x\to\infty} \frac{2^x (\ln 2)^2}{2} = \infty$$

2^x grows faster than x^2

$x^2 , 2^x , e^x$