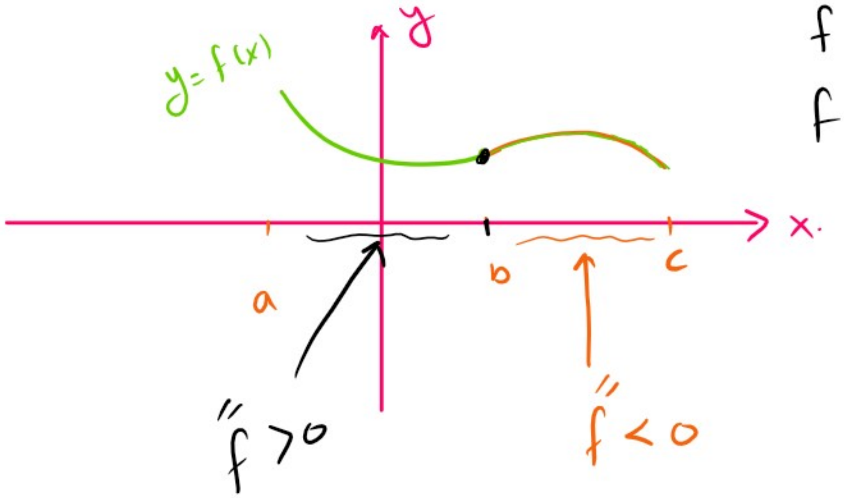
10.2 Part 2 Concavity and Points of Inflections

Saturday, June 4, 2022 12:06 PM

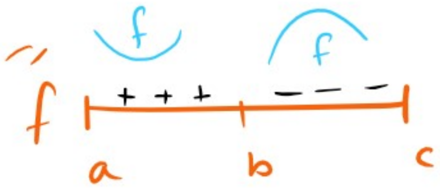


f concave up on (a, b)

f concave down on (b, c)

(b, f(b)) is inflection Point

or Point of Diminishing Return



- To find inflection Point

check

f'' = 0 or f'' undefined

صفر في المقام

Extreme Values القيم القصوى

Abs. : Absolute

Max

Min

Abs. → Rel

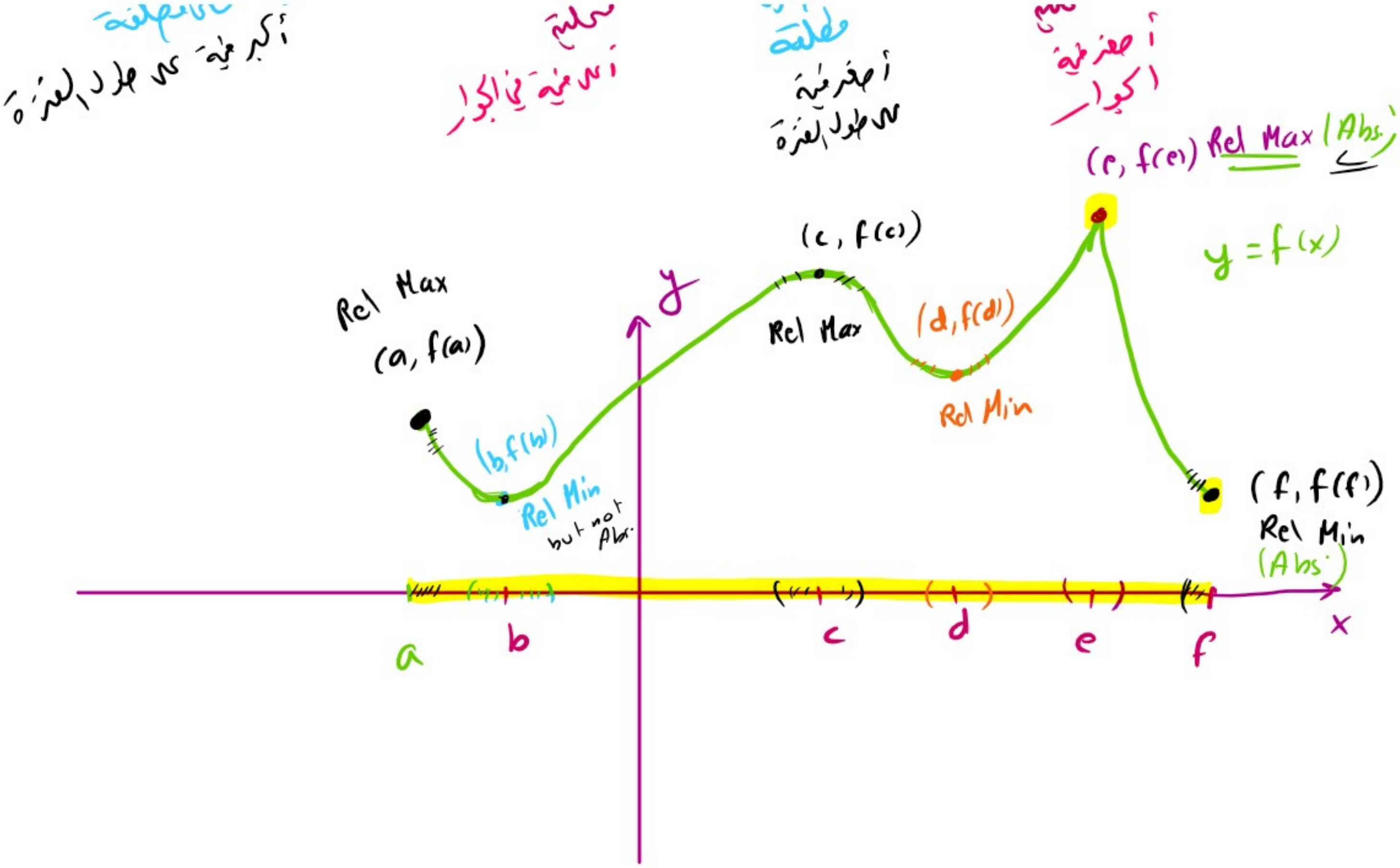
Abs → Rel

قيمة عظمى مطلقة
أكبر قيمة على مجال ...

قيمة عظمى محلية
أكبر قيمة ...

قيمة صغرى مطلقة
أصغر قيمة

قيمة صغرى محلية
أصغر قيمة
...



Def (2nd Derivative Test for Extreme Values)

Assume x_0 is critical value for f(x).

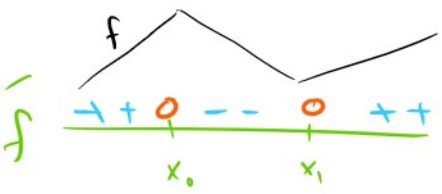
Assume $f''(x)$ exists.

① If $f''(x_0) < 0$ then f has Rel Max of $f(x_0)$

② If $f''(x_0) > 0$ " " " " Min of $f(x_0)$

③ If $f''(x_0) = 0$ or $f''(x_0)$ undefined $\}$ then This Test Fails ⇒

Apply 1st Derivative Test



f has Rel Max of $f(x_0)$

f " " Min of $f(x_1)$

Exp $f(x) = \frac{x^3}{3} - x^2 - 3x + 2$ Find

① Rel. Max/Min using 1st Derivative Test

$f' = x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = 3, \ x = -1}$ → critical values

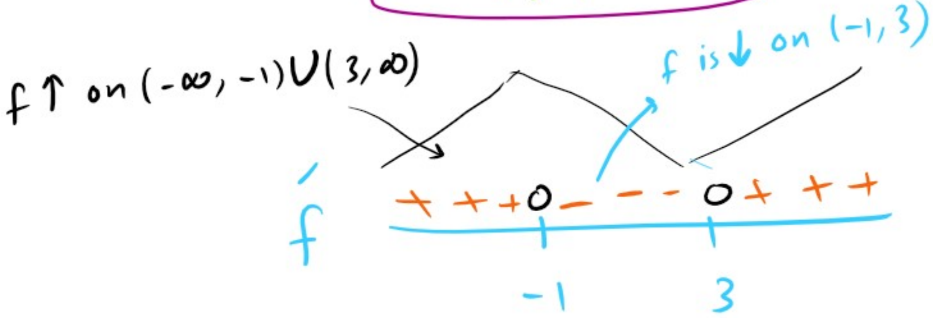
$f(x) = \frac{x^3}{3} - x^2 - 3x + 2$

$f(-1) = \frac{-1}{3} - 1 + 3 + 2$

$= 4 - \frac{1}{3} = \frac{11}{3}$

$f(3) = \cancel{\frac{27}{3}} - \cancel{9} - 9 + 2$

$= -7$



f has Rel Max of $f(-1) = \frac{11}{3}$

f has Rel Min of $f(3) = -7$

f has Rel Min of $f(3) = -7$

② Critical Point

$(3, f(3)) = (3, -7)$

$(-1, f(-1)) = (-1, \frac{11}{3})$

③ Rel Max / Min using 2^{nd} Derivative Test

$f' = x^2 - 2x - 3 \Rightarrow$ critical values $x_1 = 3$

$x_2 = -1$

✓ $\boxed{f'' = 2x - 2}$

$\Rightarrow f''(x_1) = f''(3) = 2(3) - 2 = 4 > 0$

$\Rightarrow$ f has Rel Min occurs at $x = 3$

or f has Rel Min of $f(3) = -7$

$\Rightarrow f''(x_2) = f''(-1) = 2(-1) - 2 = -4 < 0$

$\Rightarrow$ f has Rel Max of $f(-1) = \frac{11}{3}$

occurs at $x = -1$

④ Inflection Point $\Rightarrow$ check

- f'' undefined "$\emptyset$"
- $f'' = 0$

$f''(x) = 2x - 2$

$2x - 2 = 0$

$2x = 2$

$\boxed{x = 1}$

inflection value

inflection

Inflection Point is $\underline{\underline{(1, f(1))}} = (1, -\frac{5}{3})$

see 6

$$\boxed{f(x) = \frac{x^3}{3} - x^2 - 3x + 2}$$

$f(1) = \frac{1}{3} - 1 \cancel{-3} \cancel{+2}$

$= \frac{1}{3} - 2$

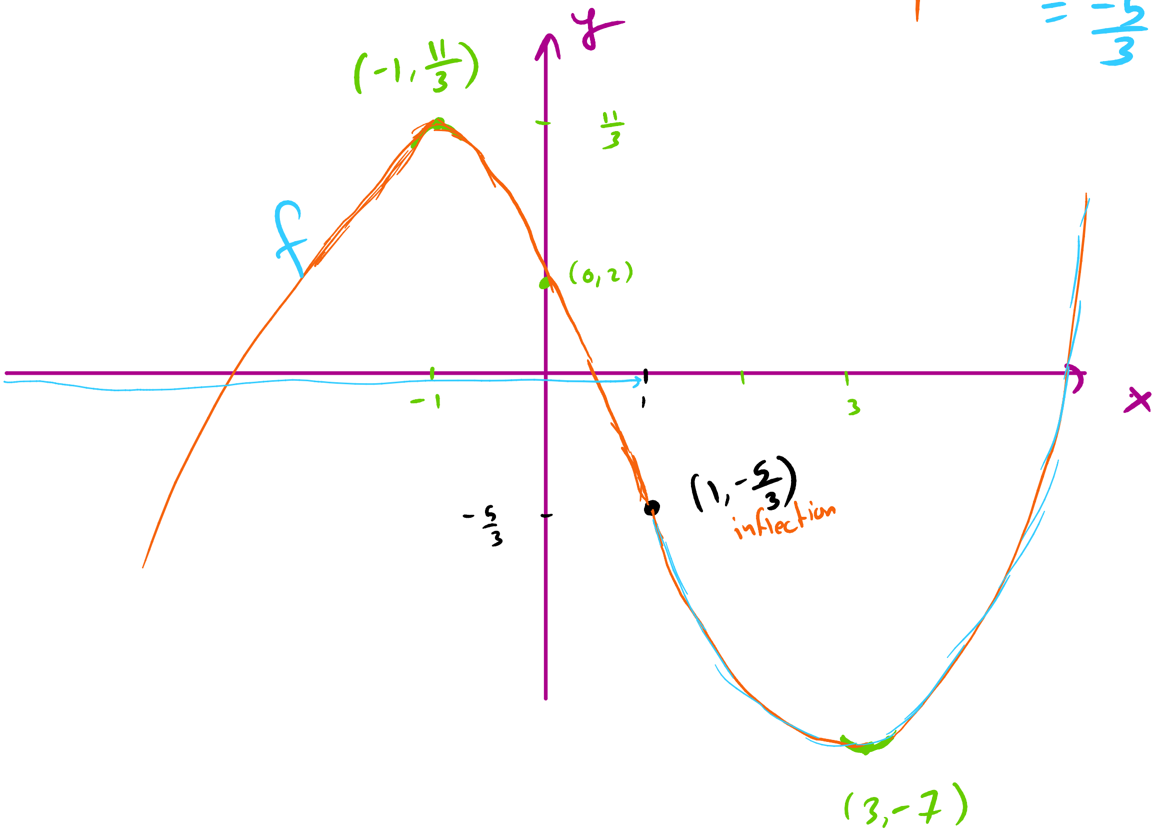
$= \frac{-5}{3}$

y-intercept

$x = 0 \Rightarrow y = 2$

$(0, 2)$

⑤ Sketch f(x)



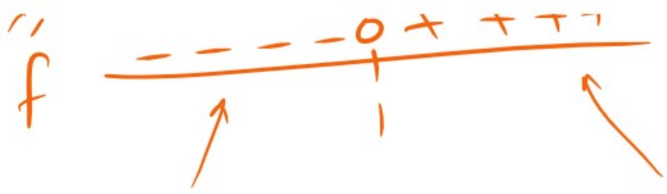
⑥ Find intervals where f is concave up and down

$f'' = 0 \Rightarrow 2x - 2 = 0$

$x = 1$

f'' – – – – – 0 + + + +

1



$f''(0) = 2(0) - 2 = -2$

$f''(2) = 2(2) - 2 = 2$

f is concave up on $(1, \infty)$

f ″ ″ down on $(-\infty, 1)$

Exp $f(x) = 3x^4 - 4x^3 - 2$ Find

① critical Values ⇒ check $\begin{cases} f' = 0 \\ f' \text{ undefined } \emptyset \end{cases}$

$$f'(x) = 12x^3 - 12x^2$$
$$= \underline{12x^2}(x-1) = 0$$
$$\boxed{x=0}, \quad \boxed{x=1}$$

critical value

$f(0) = 0 - 0 - 2$
$= -2$

$f(1) = 3\boxed{1}^4 - 4\boxed{1}^3 - 2$
$= 3 - 4 - 2$
$= -3$

② critical Points

$(0, f(0)) = (0, -2)$

$(1, f(1)) = (1, -3)$

③ Intervals where f is increasing / Decreasing

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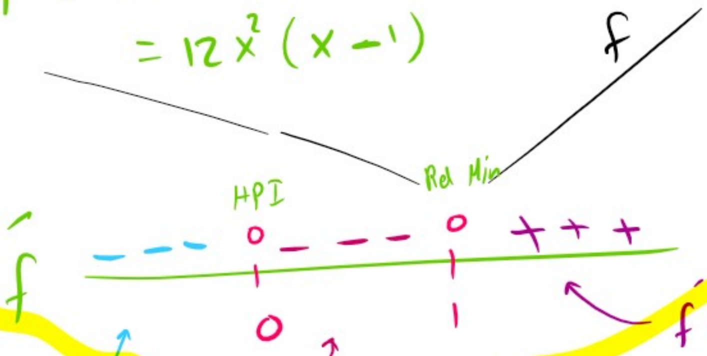
$f'(x) = 12x^3 - 12x^2$

$= 12x^2(x-1)$

→ 1st Derivative Test



$f'(2) = 12\underbrace{\boxed{2}^2}_{+}\left(\underbrace{\boxed{2}-1}_{+}\right) > 0$

$f'(-1) = 12\boxed{-1}^2\left(\boxed{-1}-1\right) = 12(-2) = -24 < 0$

$f'(\frac{1}{2}) = 12\underbrace{\boxed{\frac{1}{2}}^2}_{+}\left(\underbrace{\boxed{\frac{1}{2}}-1}_{-}\right) < 0$

f is decreasing on $(-\infty, 1)$ ✓

f is increasing on $(1, \infty)$

④ Classify the critical Points

Rel Max   Rel Min   HPI

f has Rel Min of $f(1) = -3$ occrs at $x = 1$

f has HPI of $f(0) = -2$ ocens at $x = 0$

⑤ Use 2nd Derivative Test to find Rel Max/Min.

$f' = 12x^3 - 12x^2$ critical values $x_1 = 0$

$f' = 12x^3 - 12x^2$

critical values $x_1 = 0$

$x_2 = 1$

$f'' = 36x^2 - 24x$

- $f''(x_1) = f''(0) = 0 - 0 = 0$ Test Fails $\rightarrow$ use 1^{st} Derivative Test $\Rightarrow$ f has HPI at $x=0$
- $f''(x_2) = f''(1) = 36\boxed{1}^2 - 24\boxed{1} = 36 - 24 > 0$ $\Rightarrow$

f has Rel Min of $f(1) = -3$ occurs at $x = 1$

⑥ Inflection Point/s $\Rightarrow$ check $\begin{cases} f'' \text{ undefined } \emptyset \\ f'' = 0 \end{cases}$

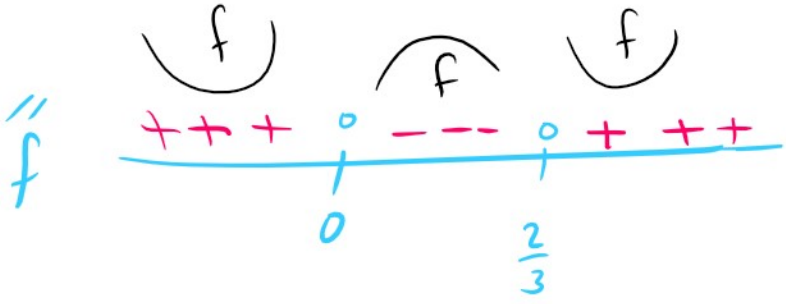
$f'' = 36x^2 - 24x$

دالة تربيعية

$36x^2 - 24x = 0$

$12x[3x - 2] = 0$

$\boxed{x=0}$, $\boxed{x = \frac{2}{3}}$



f has two inflections Points $\Leftarrow$

$(0, f(0)) = (0, -2)$

$(\frac{2}{3}, f(\frac{2}{3})) = (\frac{2}{3}, \frac{-70}{27})$

⑦ sketch $f(x)$