9.6 The Chain Rule and Power Rule

Sunday, May 22, 2022 8:07 AM

Composite function

الاقتران المركب

$$(f \circ g)(x) = f(g(x))$$

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Exp $f(x) = 2x^2+1$, $g(x) = \sqrt{x}$

Find ① $(f \circ g)(3)$

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 2\boxed{\sqrt{x}}^2 + 1$$

$$= 2x + 1$$

$$(f \circ g)(3) = 2\boxed{3} + 1$$

$$= 6 + 1$$

$$= 7$$

② $(g \circ f)(3)$

$$(g \circ f)(x) = g(f(x)) = g(2x^2+1) = \sqrt{2x^2+1}$$

$$(g \circ f)(3) = \sqrt{2\boxed{3}^2+1} = \sqrt{2(9)+1} = \sqrt{18+1} = \sqrt{19}$$

③ Is $(g \circ f)(3) = (f \circ g)(3)$ in this Exp?

$\sqrt{19} \neq 7$

NO since in general $(f \circ g)(x) \neq (g \circ f)(x)$

Q. Find y' if ① $y = (f \circ g)(x)$

$y = f(x)$

$\frac{dy}{dx} = \frac{df}{dx}$

$= f\left(g(x)\right)$

Chain Rule

قاعدة السلسلة

$y' = f'\left(g(x)\right) g'(x)$

$y' = \frac{dy}{dx} = \frac{df}{dg} \frac{dg}{dx}$

② $y = (g \circ f)(x)$

$= g\left(f(x)\right)$

$y' = g'\left(f(x)\right) f'(x)$

$$y = g(f(x))\, f'(x)$$

Power Rule

$$③\ y = [f(x)]^n$$

$$y' = n\,[f(x)]^{n-1}\, f'(x)$$

Exp Find y' if ① $y = (x^3 - 2x^2 + 1)^5$

$$y' = 5\,(x^3 - 2x^2 + 1)^4\,(3x^2 - 4x)$$

Find $y'(1) = 5(\boxed{1}^3 - 2\boxed{1}^2 + 1)^4 (3\boxed{1}^2 - 4\boxed{1})$

$= 5\,(1 - 2 + 1)^4\,(3 - 4)$

$= 5\,(0)^4\,(-1)$

$= 0$

② $y = \sqrt[2]{(3 - 2x^2)^1}$

$= (3 - 2x^2)^{\boxed{\frac{1}{2}}}$

$$y' = \frac{1}{2}\,(3 - 2x^2)^{\frac{1}{2} - 1}\,(0 - 4x)$$

$$= \cancel{\tfrac{1}{2}}\,(-\cancel{4}^{\,2}x)\,(3-2x^2)^{-\frac{1}{2}}$$

$$= -2x \;\frac{1}{(3-2x^2)^{\frac{1}{2}}}$$

$$= \frac{-2x}{\sqrt[2]{(3-2x^2)^{1}}}$$

Exp Find $p' = \dfrac{dp}{dq}$ if

① $p = \sqrt[3]{(3q^2+3q-5)^{1}} = (3q^2+3q-5)^{\frac{1}{3}}$

$$p' = \frac{1}{3}(3q^2+3q-5)^{\frac{1}{3}-1}\,(6q+3)$$

$\frac{1}{3} - 1$

$\frac{1}{3} - \frac{3}{3}$

$\frac{1-3}{3}$

$\frac{-2}{3}$

$$= \cancel{\tfrac{1}{3}}\;\cancel{(3)}\,(2q+1)\,(3q^2+3q-5)^{-\frac{2}{3}}$$

$$= (2q+1)\;\frac{1}{(3q^2+3q-5)^{\frac{2}{3}}}$$

$$= \frac{2q+1}{\sqrt[3]{(3q^2+3q-5)^2}}$$

Find $p'(1) = \frac{2\boxed{1}+1}{\sqrt[3]{3\boxed{1}^2+3\boxed{1}-5}} = \frac{2+1}{\sqrt[3]{3+3-5}} = \frac{3}{\sqrt[3]{1}} = 3$

2 $p = \frac{1}{5\sqrt[2]{(q^2+1)^5}}$ $= \frac{1}{5}\frac{1}{(q^2+1)^{\frac{5}{2}}}$

$$= \frac{1}{5}(q^2+1)^{-\frac{5}{2}}$$

$$p' = \frac{1}{\cancel{5}}\left(-\frac{\cancel{5}}{2}\right)\left(\underbrace{q^2+1}\right)^{-\frac{5}{2}-1} 2q$$

$-\frac{5}{2}-1$

$-\frac{5}{2}-\frac{2}{2}$

$\frac{-5-2}{2}$

$\frac{-7}{2}$

$$= -\frac{1}{\cancel{2}}(q^2+1)^{-\frac{7}{2}}(\cancel{2}q)$$

$$= -q(q^2+1)^{-\frac{7}{2}}$$

$$= \frac{-q}{\quad} = \frac{-q}{\quad}$$

$$= \frac{-7}{(q^2+1)^{\frac{7}{2}}} = \frac{1}{\sqrt[2]{(q^2+1)^7}}$$

Exp (Application)

Find the instantaneous rate of change of the demand (x')

$x = 98(2p+1)^{-\frac{1}{2}} - 1$ with respect to

the price when $p = 24$ where the

demand x is in hundred units

1 unit x = 100 Block

$$x' = \frac{dx}{dp} = 98\left(-\frac{1}{2}\right)(2p+1)^{-\frac{1}{2}-1}(2) - 0$$

$$= -98(2p+1)^{-\frac{3}{2}}$$

$$-\frac{1}{2} - 1 = -\frac{1}{2} - \frac{2}{2} = \frac{-1-2}{2} = -\frac{3}{2}$$

$$= \frac{-98}{(2p+1)^{\frac{3}{2}}}$$

$$x'(24) = \frac{-98}{\ldots} = \frac{-98}{\ldots} = \frac{-98}{(49)^{\frac{3}{2}}}$$

$$x'(24) = \frac{-98}{(2\boxed{24}+1)^{\frac{3}{2}}} = \frac{-98}{(48+1)^{\frac{3}{2}}} = \frac{-98}{(49)^{\frac{3}{2}}}$$

$$= \frac{-98}{(7^2)^{\frac{3}{2}}} = \frac{-98}{7^{2(\frac{3}{2})}} = \frac{-98}{7^3}$$

$$= \frac{-\cancel{49}(2)}{\cancel{(7)(7)}(7)} = \frac{-2}{7}$$

- what does $-\frac{2}{7}$ mean?

$$x'(24) = \left.\frac{dx}{dp}\right|_{p=24} = -\frac{2}{7}$$

This means if the price p changes by \$1 then the demand changes by $-\frac{2}{7}$ units

$$-\frac{2}{7}(100) = \frac{-200}{7}$$

If $p \uparrow$ by 1 \$ then demand $\downarrow$ by $\frac{200}{7}$