

3.8 Related Rates Equations

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 $\uparrow = +$, $\downarrow = -$

Suppose we are pumping air into a spherical balloon:

The Volume (V) and radius (r) increase over time t :

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} \quad (\text{chain Rule})$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

If we know r and $\frac{dV}{dt}$, then we can find $\frac{dr}{dt}$ $\leftarrow$ how fast the radius increases over time

Example 1 Water runs into a conical tank at the rate $9 \text{ m}^3/\text{min}$.

The tank stands point down and has a height 10 m and base radius 5 m . How fast is the water level rising when the water is 6 m deep?

The variables are: V, x, y For red with

relation: $V = \frac{1}{3} \pi x^2 y$

We must eliminate x because $\nearrow$ we have no information about x and $\frac{dx}{dt}$. Similar triangles $\Rightarrow$

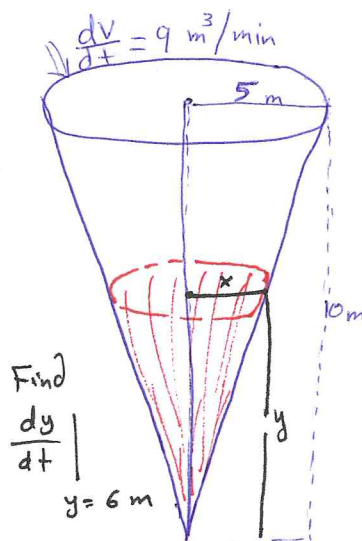
$$\frac{x}{y} = \frac{5}{10} \Leftrightarrow \boxed{x = \frac{y}{2}}$$

$$\Rightarrow V = \frac{1}{3} \pi \left(\frac{y}{2}\right)^2 y = \frac{\pi}{12} y^3$$

STUDENTS-HUB.COM $\frac{dV}{dt} = \frac{\pi}{4} y^2 \frac{dy}{dt} \Leftrightarrow 9 = \frac{\pi}{4} (6)^2 \frac{dy}{dt}$

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$$\Leftrightarrow \frac{dy}{dt} = \frac{1}{\pi} \approx 0.32 \text{ m/min (the level is rising)}$$

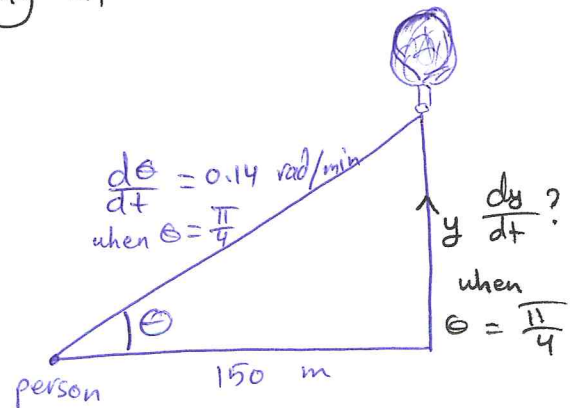


Example 2 A balloon rising straight up from a level field that is tracked by a person whose is 150 m from the lift off point. At the moment the person's elevation angle is $\frac{\pi}{4}$, the angle is increasing at rate of 0.14 rad/min . How fast is the balloon rising at that moment?

The Variables are θ and y with

relation $\tan \theta = \frac{y}{150}$

$$y = 150 \tan \theta$$



$$\frac{dy}{dt} = 150 \sec^2 \theta \frac{d\theta}{dt}$$

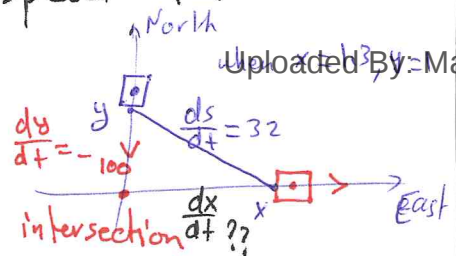
$$= 150 (\sqrt{2})^2 (0.14) \quad \theta = \frac{\pi}{4}$$

$$= 42 \text{ m/min} \quad \text{"the balloon is rising"}$$

Example 3 A police car, approaching a right-angled intersection from the north, is chasing a speeding car moving straight east. when the police is 1 km north of the intersection and the car is 1.3 km to the east, the police determine, using the radar, that the distance between them is increasing at rate 32 km/hr. If the police is moving at 100 km/hr at the instance of measurement, what is the speed of the car?

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The variables are x, y, s and related by $s^2 = x^2 + y^2$



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$$2s \frac{ds}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\sqrt{x^2 + y^2} \frac{ds}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$$

$$\Leftrightarrow \sqrt{(1.3)^2 + (1)^2} (32) = (1.3) \frac{dx}{dt} + (1)(-100)$$

$$\Leftrightarrow \frac{dx}{dt} = 117.3 \text{ km/hr}$$

car's speed.