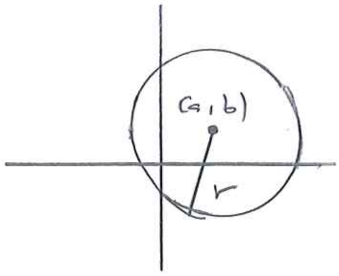
Circle, ellipse and parabolas, Hyperbola in the plane.

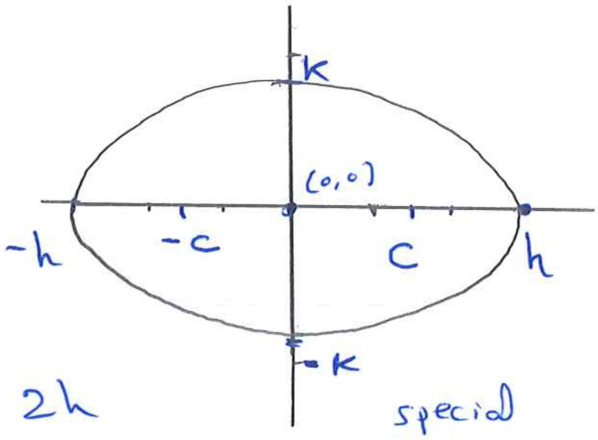
Circle : $(x-a)^2+(y-b)^2=r^2$

center (a,b) with radius $= r$



Ellipse with the horizontal major axis:

$$\frac{(x-a)^2}{h^2}+\frac{(y-b)^2}{k^2}=1$$



special case $(a,b)=$ origin

1) Center (a,b)

2) length of the major axis is $2h$

3) length of the minor axis is $2k$

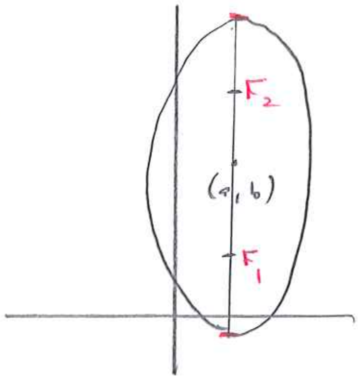
4) Distance between the center and either focus is C

with $C^2=h^2-k^2$, $h>k>0$

Focus : $(a\pm c,\ b)$

Ellipse with the vertical major axis:

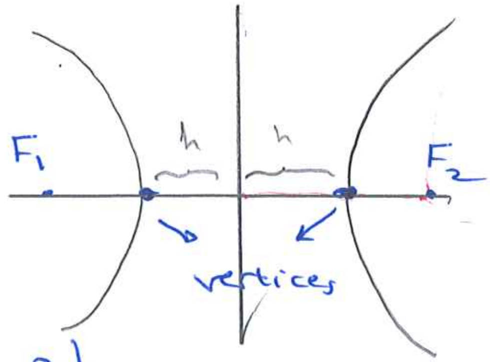
$$\frac{(x-a)^2}{k^2}+\frac{(y-b)^2}{h^2}=1$$



(1), (2), (3) and (4) the same as above. Focus $(a,\ b\pm c)$

Hyperbola with the horizontal transverse axis:

$$\frac{(x-a)^2}{h^2} - \frac{(y-b)^2}{k^2} = 1$$



1) Center (a, b)

2) Distance between the vertices is $2h$

3) Distance between the foci is $2k$

4) $c^2 = h^2 + k^2$. Focus $(a \pm c, b)$

Hyperbola with the vertical transverse axis

$$\frac{(x-a)^2}{k^2} - \frac{(y-b)^2}{h^2} = 1$$

---

parabola with the Horizontal axis:

$$(y-b)^2 = 4p(x-a), \quad p \neq 0$$

vertex (a, b), Focus $(a+p, b)$

---

parabola with vertical axis

$$(x-a)^2 = 4p(y-b), \quad p \neq 0$$

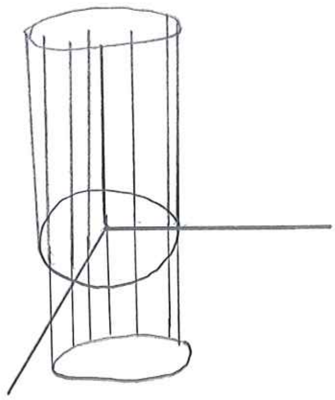
12.6 Cylinders and Quadric Surfaces:

Cylinders is a surface that is generated by moving a straight line along a given planner $\boxed{\text{Curve}}$ while holding the line parallel to a given fixed line.

generating curve

Example: $x^2+y^2=1$, z.

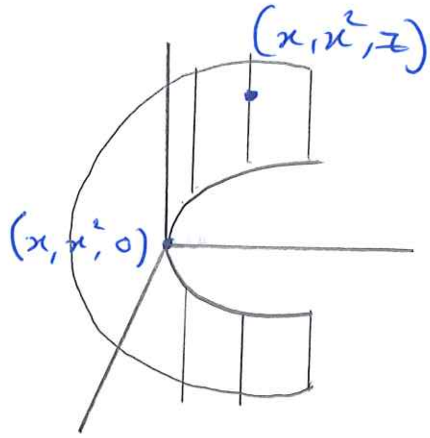
We have a circular cylinder where the generating curves are circles.



Example Find an equation for the cylinder made by lines parallel to the z-axis that pass through the parabola

$y=x^2$, $z=0$

This is called the cylinder $y=x^2$.



Quadric surfaces "أسطح رباعية"

A quadric surface is the graph in space of a second-degree equation in x, y, and z.

We focus on $Ax^2 + By^2 + Cz^2 + Dz = E$

where A, B, C, D and E are constants.

spheres are special cases "بيضاوي"

Example (1) The ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$

If $x = 0$, $\dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$, ellipse in yz-plane

If $y = 0$, $\dfrac{x^2}{a^2} + \dfrac{z^2}{c^2} = 1$, ellipse in xz-plane

If $z = 0$, $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, ellipse in xy-plane

If $a = b = c \iff x^2 + y^2 + z^2 = a^2$ (sphere)

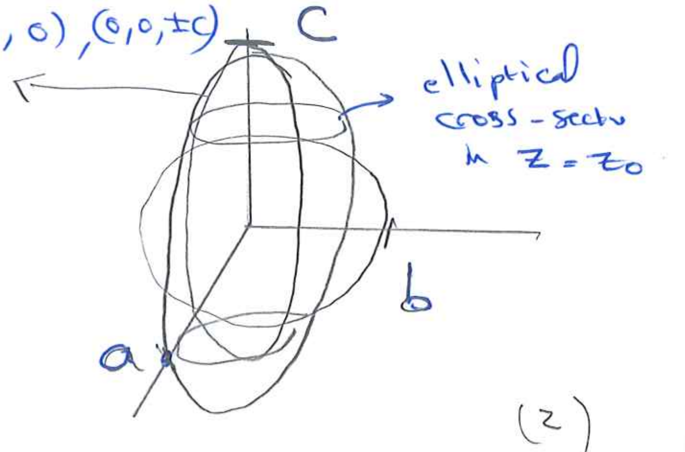
Coordinate axes at $(\pm a, 0, 0)$, $(0, \pm b, 0)$, $(0, 0, \pm c)$

بالكرة ينتج

الطابة الأمريكية

American football

$\dfrac{x^2}{a^2} + \dfrac{z^2}{c^2} = 1$



Example ②: The Hyperbolic paraboloid

قطع مكافئ قطعي

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c} \quad , c > 0$$

If $x = 0$: $\frac{y^2}{b^2} = \frac{z}{c} \Rightarrow z = \frac{c}{b^2} y^2$

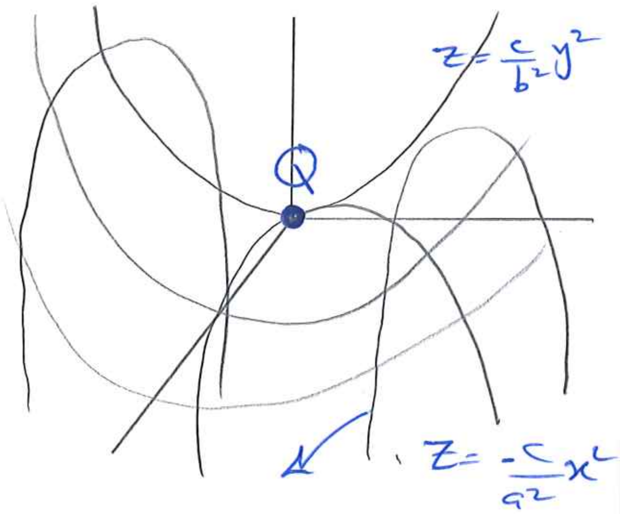
(parabola in yz-plane)

If $y = 0$: $z = -\frac{c}{a^2} x^2$. (parabola in xz-plane)

If $z = z_0 > 0$ (i.e. constant)

we have hyperbola

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z_0}{c}$$



$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, \quad z_0 = c$$

The point Q is called

saddle point

الشكل مثل سرج الفرس.

قطع مكافئ بيضاوي الشكل

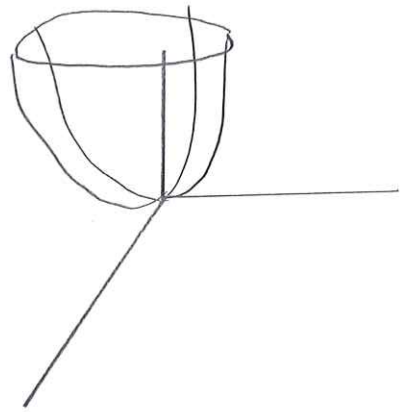
③ Elliptical ParaBoloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

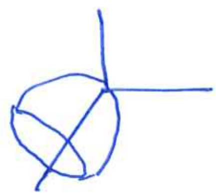
If $x = 0 \Rightarrow z = \frac{c}{b^2} y^2$ (parabola) in yz-plane)

If $y = 0 \Rightarrow z = \frac{c}{a^2} x^2$ (parabola) in xz-plane

If $z = c \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse in xy-plane)



Example: $\frac{y^2}{a^2} + \frac{z^2}{b^2} = \frac{x}{c}$



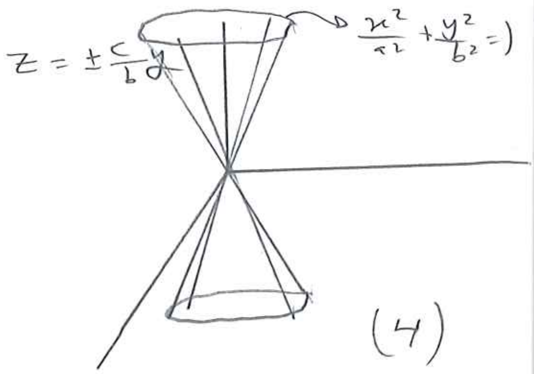
④ Elliptical Cone $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$

مخروط بيضاوي الشكل

If $x = 0 \Rightarrow z^2 = \frac{c^2}{b^2} y^2 \Rightarrow z = \pm \frac{c}{b} y$ (Line)

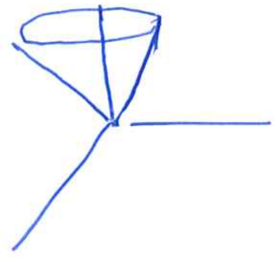
If $y = 0 \Rightarrow z = \pm \frac{c}{a} x$ (lines)

If $z = c \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse)



Example: ⓐ $x^2 + y^2 = z^2 \Rightarrow$ Circular Cone

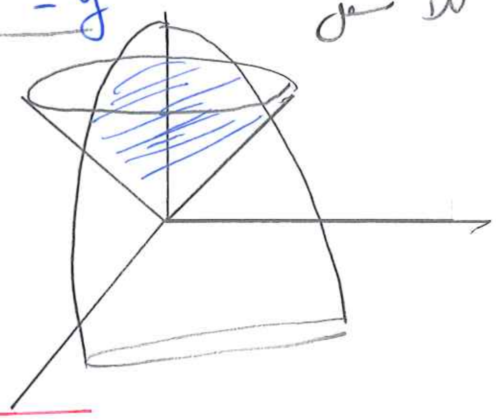
ⓑ $z = \sqrt{x^2+y^2} \geq 0$



ⓒ $x^2 + y^2 = -z^2 \Leftrightarrow x^2 + y^2 + z^2 = 0$ (Point $(0,0,0)$)

ⓓ sketch the common region:

$\underbrace{z = \sqrt{x^2+y^2}}_{\text{elliptical cone.}}$, $\underbrace{z = 9 - x^2 - y^2}$ $\rightarrow$ elliptical paraboloid [illegible]



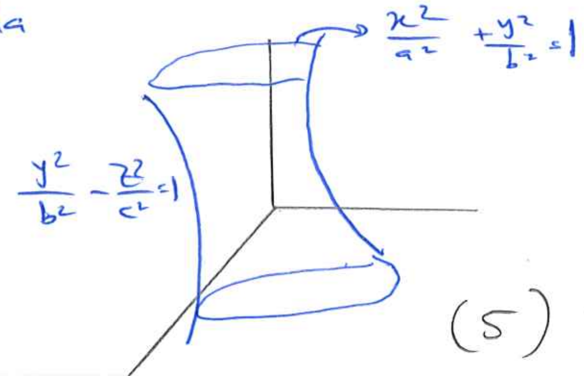
⑤ Hyperboloid of one Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

If $x = 0$: $\frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ (part of hyperbola in yz-plane)

If $y = 0$: $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$ (hyperbola

If $z = 0$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse)



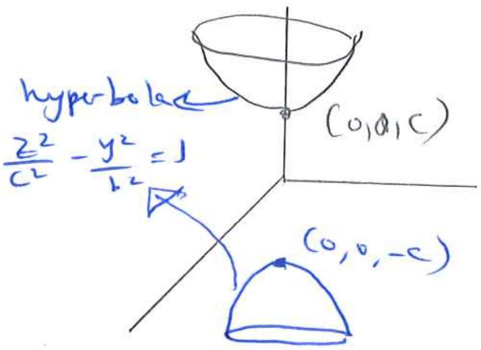
⑥ Hyperboloid of Two sheets

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

If $z = c\sqrt{2} \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse)

If $x = 0 \Rightarrow \frac{z^2}{c^2} - \frac{y^2}{b^2} = 1$

(hyperbola in yz-plane)



If $y = 0 \Rightarrow \frac{z^2}{c^2} - \frac{x^2}{a^2} = 1$ (hyperbola in xz plane)

Q1) Discussion:

1) $x^2 + y^2 + 4z^2 = 10$ ⓓ (ellipsoid)

2) $z^2 + 4y^2 - 4x^2 = 4$ ⓘ (hyperboloid of one sheet)

3) $9y^2 + z^2 = 16$ ⓐ (cylinder)

4) $y^2 + z^2 = x^2$ ⓖ (circular cone)

5) $x = y^2 - z^2$ l (hyperbolic paraboloid)

6) $x = -y^2 - z^2$ e (elliptical paraboloid)

7) $x^2 + 2z^2 = 8$ b (cylinder)

8) $z^2 + x^2 - y^2 = 1$ j (hyperboloid of one sheet)

9) $x = z^2 - y^2$ k (hyperbolic paraboloid)

10) $z = -4x^2 - y^2$ f (elliptical paraboloid)

11) $x^2 + 4z^2 = y^2$ h (cone)

12) $9x^2 + 4y^2 + 2z^2 = 36$ c (ellipsoid)