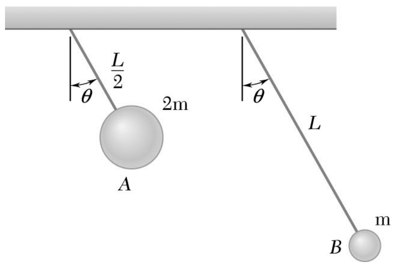
CHAPTER 16



PROBLEM 16.CQ1

Two pendulums, *A* and *B*, with the masses and lengths shown are released from rest. Which system has a larger mass moment of inertia about its pivot point?

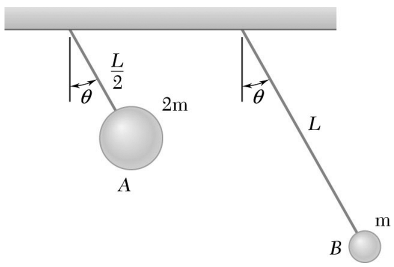
(*a*) *A*

(*b*) *B*

(*c*) They are the same.

SOLUTION

Answer: (*b*)



PROBLEM 16.CQ2

Two pendulums, *A* and *B*, with the masses and lengths shown are released from rest. Which system has a larger angular acceleration immediately after release?

(*a*) *A*

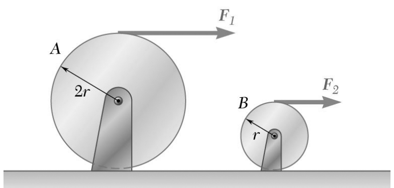
(*b*) *B*

(*c*) They are the same.

SOLUTION

Answer: (*a*)

PROBLEM 16.CQ3



Two solid cylinders, A and B, have the same mass m and the radii $2r$ and r respectively. Each is accelerated from rest with a force applied as shown. In order to impart identical angular accelerations to both cylinders, what is the relationship between F_1 and F_2?

(a) $F_1 = 0.5F_2$

(b) $F_1 = F_2$

(c) $F_1 = 2F_2$

(d) $F_1 = 4F_2$

(e) $F_1 = 8F_2$

SOLUTION

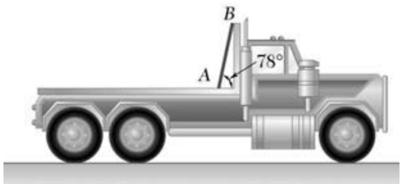
Answer: (c)

$$Fr = I\alpha$$

$$\alpha = \frac{FR}{\frac{1}{2}mR^2} = \frac{F_1(2r)}{\frac{1}{2}m(2r)^2} = \frac{F_2 r}{\frac{1}{2}mr^2}$$

$$\frac{F_1}{mr} = \frac{2F_2}{mr}$$

$\mathbf{F}_1 = 2F_2$ ◀

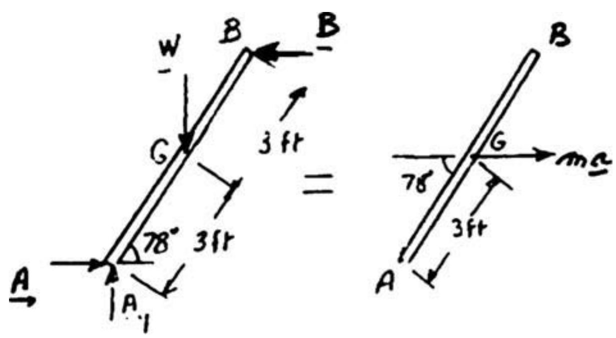


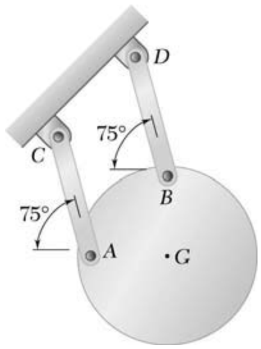
PROBLEM 16.F1

A 6-ft board is placed in a truck with one end resting against a block secured to the floor and the other leaning against a vertical partition. Draw the FBD and KD necessary to determine the maximum allowable acceleration of the truck if the board is to remain in the position shown.

SOLUTION

Answer:



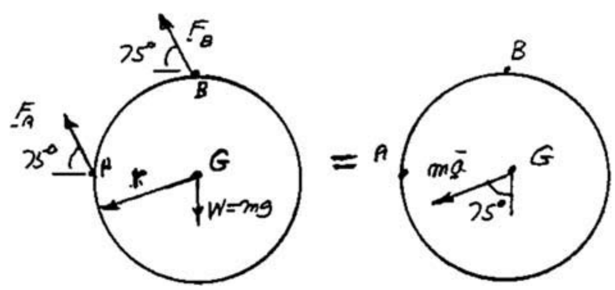


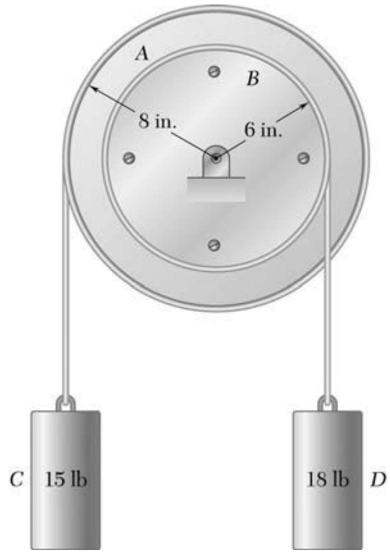
PROBLEM 16.F2

A uniform circular plate of mass 3 kg is attached to two links *AC* and *BD* of the same length. Knowing that the plate is released from rest in the position shown, in which lines joining *G* to *A* and *B* are, respectively, horizontal and vertical, draw the FBD and KD for the plate.

SOLUTION

Answer:



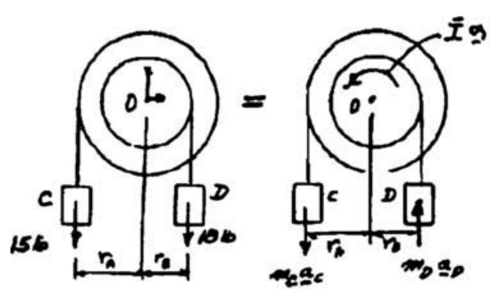


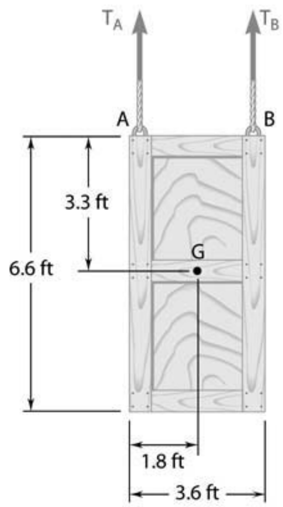
PROBLEM 16.F3

Two uniform disks and two cylinders are assembled as indicated. Disk *A* weighs 20 lb and disk *B* weighs 12 lb. Knowing that the system is released from rest, draw the FBD and KD for the whole system.

SOLUTION

Answer:



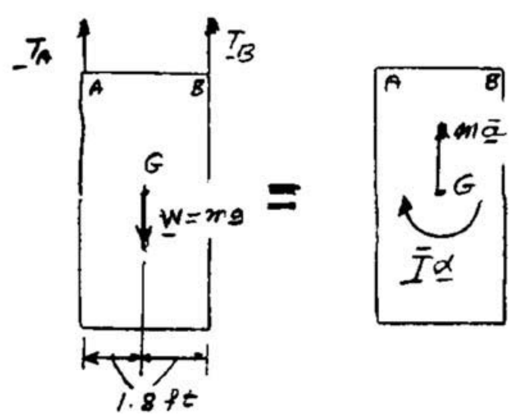


PROBLEM 16.F4

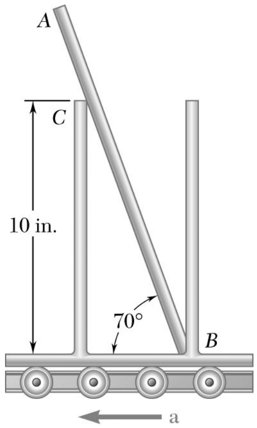
The 400-lb crate shown is lowered by means of two overhead cranes. Knowing the tension in each cable, draw the FBD and KD that can be used to determine the angular acceleration of the crate and the acceleration of the center of gravity.

SOLUTION

Answer:

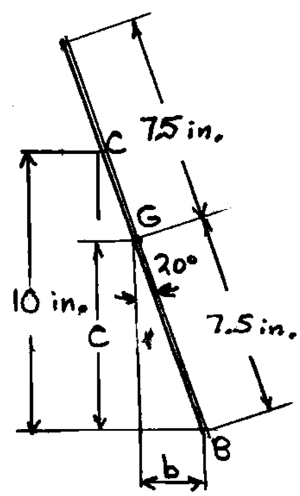


PROBLEM 16.1



A conveyor system is fitted with vertical panels, and a 15-in. rod AB weighing 5 lb is lodged between two panels as shown. If the rod is to remain in the position shown, determine the maximum allowable acceleration of the system.

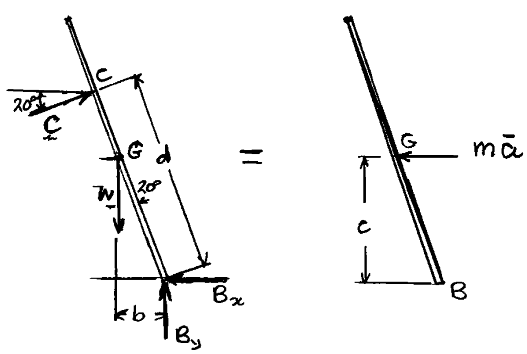
SOLUTION



Geometry:
$$d = \frac{10 \text{ in.}}{\cos 20°} = 10.642 \text{ in.}$$
$$b = \frac{1}{2}(15 \text{ in.})\sin 20° = 2.5652 \text{ in.}$$
$$c = \frac{1}{2}(15 \text{ in.})\cos 20° = 7.0477 \text{ in.}$$

Mass:
$$m = \frac{W}{g} = \frac{5 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.15528 \text{ slug}$$

Kinetics:



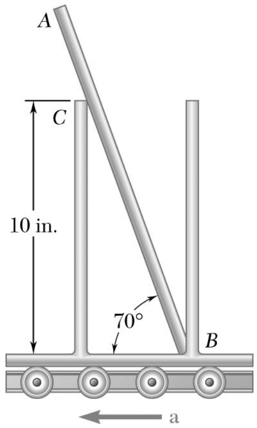
$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad Cd - Wb = -m\bar{a}c$$

Maximum allowable acceleration.

This occurs at loss of contact when $C = 0$.

$$ma = \frac{Wb}{c} = \frac{(5 \text{ lb})(2.5652 \text{ in.})}{7.0477 \text{ in.}} = 1.8199 \text{ lb}$$

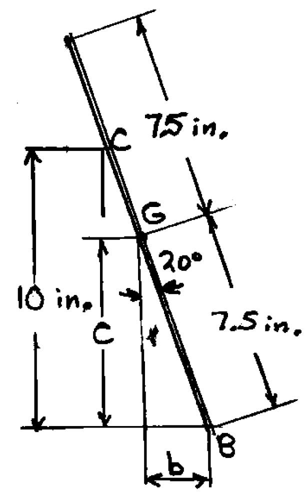
$$a = \frac{ma}{m} = \frac{1.8199 \text{ lb}}{0.15528 \text{ slug}} \qquad a = 11.72 \text{ ft/s} \blacktriangleleft$$



PROBLEM 16.2

A conveyor system is fitted with vertical panels, and a 15-in. rod *AB* weighing 5 lb is lodged between two panels as shown. Knowing that the acceleration of the system is 3 ft/s^2 to the left, determine (*a*) the force exerted on the rod at *C*, (*b*) the reaction at *B*.

SOLUTION



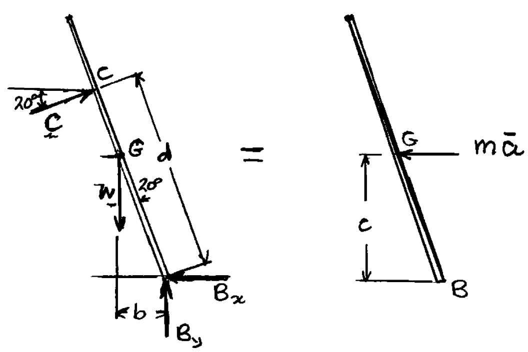
Geometry: $\overline{CB} = d = \dfrac{10 \text{ in.}}{\cos 20°} = 10.642 \text{ in.}$

$$b = \frac{1}{2}(15 \text{ in.})\sin 20° = 2.5652 \text{ in.}$$

$$c = \frac{1}{2}(15 \text{ in.})\cos 20° = 7.0477 \text{ in.}$$

Mass: $m = \dfrac{W}{g} = \dfrac{5 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.15528 \text{ slug}$

Kinetics: $m\bar{\mathbf{a}} = (0.15528 \text{ slug})(3 \text{ ft/s}^2) = 0.46588 \text{ lb} \longleftarrow$



(*a*) Force at *C*.

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad Cd - Wb = -m\bar{a}c$$

$$C = \frac{Wb}{d} - \frac{m\bar{a}c}{d} = \frac{(5 \text{ lb})(2.5652 \text{ in.})}{10.642 \text{ in.}} - \frac{(0.46588 \text{ lb})(7.0477 \text{ in.})}{10.642}$$

$$C = 0.89669 \text{ lb}$$

$\mathbf{C} = 0.897 \text{ lb} \measuredangle 20°$ ◀

PROBLEM 16.2 (Continued)

(*b*) Reaction at *B*.

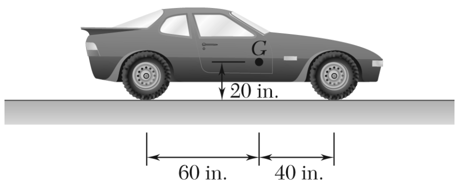
$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad B_y - W + C\sin 20° = 0$

$$B_y = 5 \text{ lb} - (0.89669 \text{ lb})\sin 20° = 4.6933 \text{ lb}$$

$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad B_x - C\cos 20° = ma$

$$B_x = (0.89669 \text{ lb})\cos 20° + 0.46588 = 1.3085 \text{ lb}$$

$$\mathbf{B} = 1.3085 \text{ lb} \longleftarrow + 4.6933 \text{ lb} \uparrow \qquad \mathbf{B} = 4.87 \text{ lb} \nwarrow 74.4° \blacktriangleleft$$

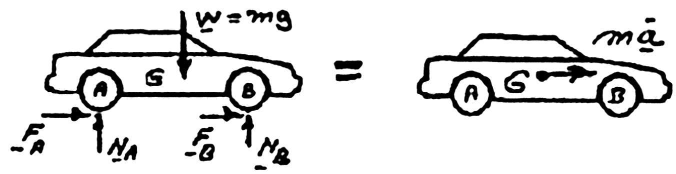


PROBLEM 16.3

Knowing that the coefficient of static friction between the tires and the road is 0.80 for the automobile shown, determine the maximum possible acceleration on a level road, assuming (*a*) four-wheel drive, (*b*) rear-wheel drive, (*c*) front-wheel drive.

SOLUTION

(*a*) Four-wheel drive:



$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0 \qquad N_A + N_B = W = mg$$

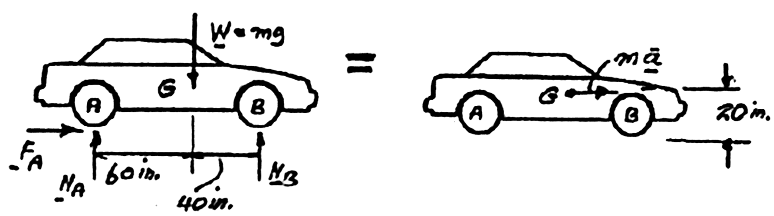
Thus: $$F_A + F_B = \mu_k N_A + \mu_k N_B = \mu_k (N_A + N_B) = \mu_k W = 0.80mg$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F_A + F_B = m\bar{a}$$

$$0.80mg = m\bar{a}$$

$$\bar{a} = 0.80g = 0.80(32.2 \text{ ft/s}^2) \qquad \bar{\mathbf{a}} = 25.8 \text{ ft/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) Rear-wheel drive:



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad (40 \text{ in.})W - (100 \text{ in.})N_A = -(20 \text{ in.})m\bar{a}$$

$$N_A = 0.4W + 0.2m\bar{a}$$

Thus: $$F_A = \mu_k N_B = 0.80(0.4W + 0.2m\bar{a}) = 0.32mg + 0.16m\bar{a}$$

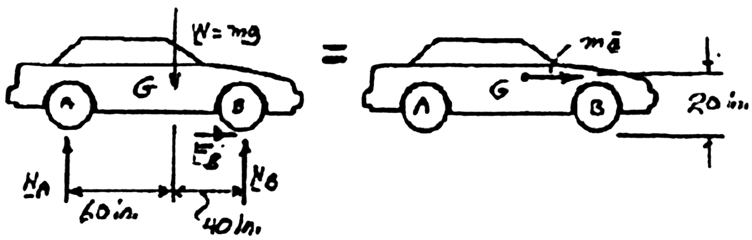
$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F_A = m\bar{a}$$

$$0.32mg + 0.16m\bar{a} = m\bar{a}$$

$$0.32g = 0.84\bar{a}$$

$$\bar{a} = \frac{0.32}{0.84}(32.2 \text{ ft/s}^2) \qquad \bar{\mathbf{a}} = 12.27 \text{ ft/s}^2 \longrightarrow \blacktriangleleft$$

(*c*) Front-wheel drive:



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad (100\text{ in.})N_B - (60\text{ in.})W = -(20\text{ in.})m\bar{a}$$

$$N_B = 0.6W - 0.2m\bar{a}$$

Thus: $F_B = \mu_k N_B = 0.80(0.6W - 0.2m\bar{a}) = 0.48mg - 0.16m\bar{a}$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F_B = m\bar{a}$$

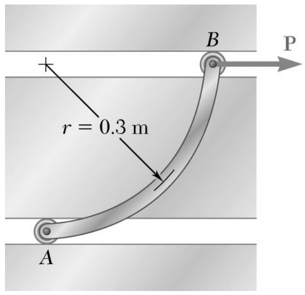
$$0.48mg - 0.16m\bar{a} = m\bar{a}$$

$$0.48g = 1.16\bar{a}$$

$$\bar{a} = \frac{0.48}{1.16}(32.2\text{ ft/s}^2)$$

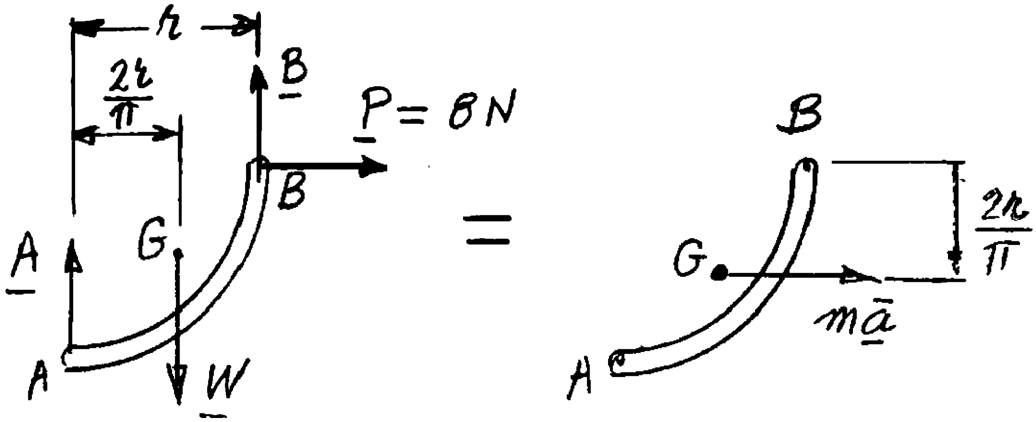
$\bar{\mathbf{a}} = 13.32\text{ ft/s}^2 \longrightarrow$ ◀

PROBLEM 16.4



The motion of the 2.5-kg rod AB is guided by two small wheels which roll freely in horizontal slots. If a force **P** of magnitude 8 N is applied at B, determine (a) the acceleration of the rod, (b) the reactions at A and B.

SOLUTION



(a)

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P = m\bar{a}$$

$$\bar{a} = \frac{P}{m} = \frac{8 \text{ N}}{2.5 \text{ kg}} = 3.20 \text{ m/s}^2$$

$$\bar{\mathbf{a}} = 3.20 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

(b)

$$+\uparrow \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad W\left(r - \frac{2r}{\pi}\right) - Ar = m\bar{a}\left(\frac{2r}{\pi}\right)$$

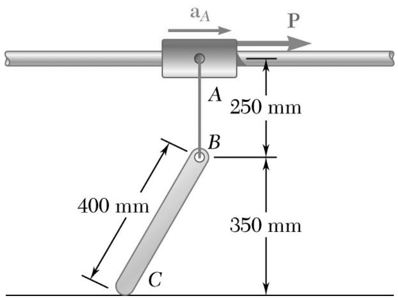
$$A = W\left(1 - \frac{2}{\pi}\right) - m\bar{a}\left(\frac{2}{\pi}\right) = mg\left(1 - \frac{2}{\pi}\right) - P\left(\frac{2}{\pi}\right)$$

$$= (2.5 \text{ kg})(9.81 \text{ m/s}^2)\left(1 - \frac{2}{\pi}\right) - (8 \text{ N})\left(\frac{2}{\pi}\right)$$

$$= 8.912 \text{ N} - 5.093 \text{ N} = 3.819 \text{ N} \qquad \mathbf{A} = 3.82 \text{ N}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A + B - W = 0$$

$$B = W - A = (2.5)(9.81) - 3.819, \qquad \mathbf{B} = 20.71 \text{ N}\uparrow \blacktriangleleft$$

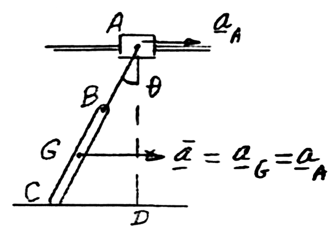


PROBLEM 16.5

A uniform rod BC of mass 4 kg is connected to a collar A by a 250-mm cord AB. Neglecting the mass of the collar and cord, determine (a) the smallest constant acceleration $\mathbf{a}_A$ for which the cord and the rod lie in a straight line, (b) the corresponding tension in the cord.

SOLUTION

<u>Geometry and kinematics</u>:



Distance between collar and floor $= AD = 250 \text{ mm} + 350 \text{ mm} = 600 \text{ mm}$

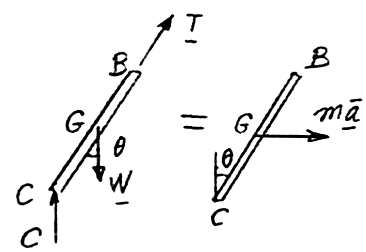
When cord and rod lie in a straight line:

$$AC = AB + BC = 250 \text{ mm} + 400 \text{ mm} = 650 \text{ mm}$$

$$\cos\theta = \frac{AD}{AC} = \frac{600 \text{ mm}}{650 \text{ mm}}$$

$$\theta = 22.62^\circ$$

<u>Kinetics</u>



(a) <u>Acceleration at A</u>.

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W(CG)\sin\theta = m\bar{a}(CG)\cos\theta$$

$$m\bar{a} = mg\tan\theta$$

$$\bar{a} = g\tan\theta = (9.81 \text{ m/s}^2)\tan 22.62^\circ$$

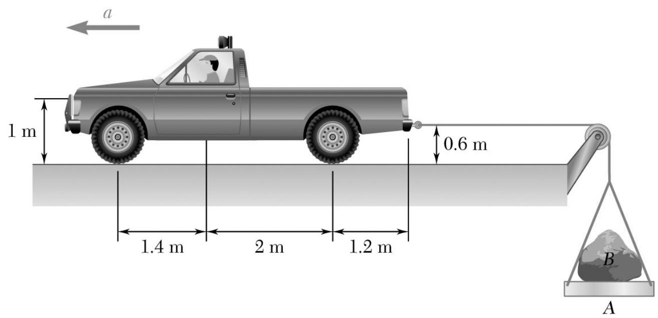
$\mathbf{a}_A = \bar{\mathbf{a}} = 4.09 \text{ m/s}^2 \longrightarrow$ ◀

(b) <u>Tension in the cord</u>.

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad T\sin\theta = m\bar{a} = mg\tan\theta$$

$$T = \frac{mg}{\cos\theta} = \frac{(4 \text{ kg})(9.81)}{\cos 22.62^\circ}$$

$T = 42.5 \text{ N}$ ◀



PROBLEM 16.6

A 2000-kg truck is being used to lift a 400-kg boulder B that is on a 50-kg pallet A. Knowing the acceleration of the rear-wheel drive truck is 1 m/s^2, determine (*a*) the reaction at each of the front wheels, (*b*) the force between the boulder and the pallet.

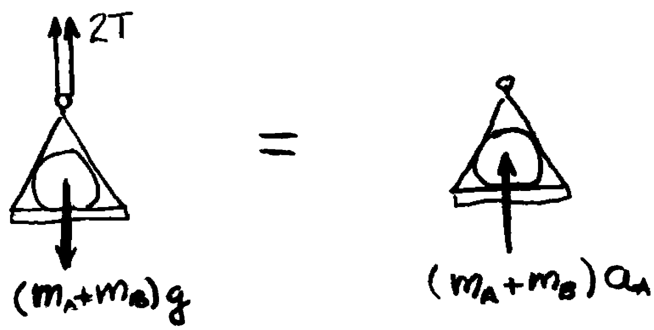
SOLUTION

Kinematics: Acceleration of truck: $\mathbf{a}_T = 1 \text{ m/s}^2 \longleftarrow$.

When the truck moves 1 m to the left, the boulder B and pallet A are raised 0.5 m.

Then, $\mathbf{a}_A = 0.5 \text{ m/s}^2 \uparrow$ $\quad$ $\mathbf{a}_B = 0.5 \text{ m/s}^2 \uparrow$

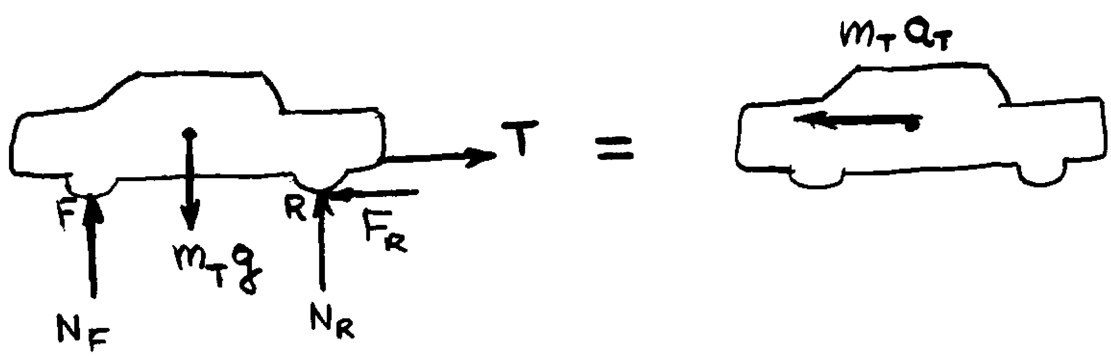
<u>Kinetics</u>: Let T be the tension in the cable.



Pallet and boulder: $+\!\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad 2T - (m_A + m_B)_g = (m_A + m_B)a_B$

$$2T - (450 \text{ kg})(9.81 \text{ m/s}^2) = (450 \text{ kg})(0.5 \text{ m/s}^2)$$

$$T = 2320 \text{ N}$$



PROBLEM 16.6 (Continued)

Truck: $\stackrel{+}{\circlearrowright} M_R = \Sigma(M_R)_{\text{eff}}: \quad -N_F(3.4\text{ m}) + m_T g(2.0\text{ m}) - T(0.6\text{ m}) = m_T a_T(1.0\text{ m})$

$$N_F = \frac{(2.0\text{ m})(2000\text{ kg})(9.81\text{ m/s}^2)}{3.4\text{ m}} - \frac{(0.6\text{ m})(2320\text{ N})}{3.4\text{ m}} + \frac{(1.0\text{ m})(2000\text{ kg})(1.0\text{ m/s})}{3.4\text{ m}}$$
$$= 11541.2\text{ N} - 409.4\text{ N} - 588.2\text{ N}$$
$$= 10544\text{ N}$$

$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N_F + N_R - m_T g = 0$

$$10544\text{ N} + N_R - (2000\text{ kg})(9.81\text{ m/s}^2) = 0$$
$$N_R = 9076\text{ N}$$

$\xleftarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F_R - T = m_T a_T$

$$F_R = 2320\text{ N} + (2000\text{ kg})(1.0\text{ m/s}^2)$$
$$= 4320\text{ N}$$

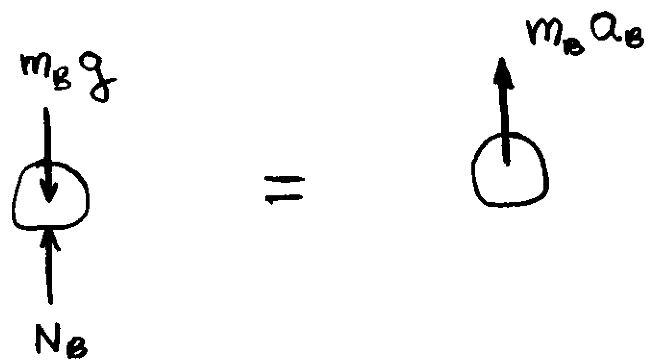
(*a*) *Reaction at each front wheel*:

$\frac{1}{2}N_F\uparrow$: $\qquad$ $5270\text{ N}\uparrow$ ◀

Reaction at each rear wheel:

$\frac{1}{2}F_R \longleftarrow + \frac{1}{2}N_R\uparrow$ $\qquad$ 5030 N $\searrow$ 64.5°

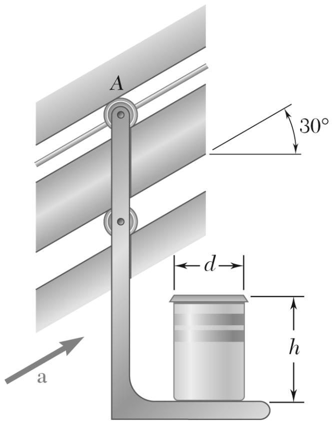
(*b*) *Force between boulder and pallet.*



Boulder $\quad +\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N_B + M_B g - m_B a_B$

$$N_B = (400\text{ kg})(9.81\text{ m/s}^2) + (400\text{ kg})(0.5\text{ m/s}^2)$$
$$= 4124\text{ N}$$

4120 N (compression) ◀



PROBLEM 16.7

The support bracket shown is used to transport a cylindrical can from one elevation to another. Knowing that $\mu_s = 0.25$ between the can and the bracket, determine (*a*) the magnitude of the upward acceleration **a** for which the can will slide on the bracket, (*b*) the smallest ratio *h*/*d* for which the can will tip before it slides.

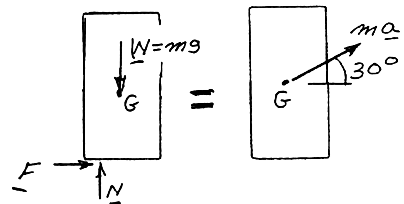
SOLUTION

(*a*) Sliding impends

$$\xrightarrow{+} \Sigma F_y = \Sigma(F_x)_{\text{eff}}: \qquad F = ma\cos 30°$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - mg = ma\sin 30°$$

$$N = m(g + a\sin 30°)$$



$$\mu_s = \frac{F}{N}$$

$$0.25 = \frac{ma\cos 30°}{m(g + a\sin 30°)}$$

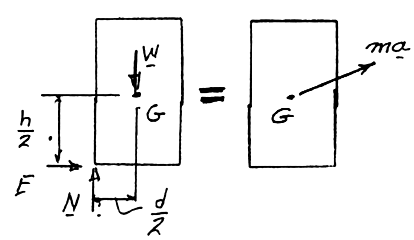
$$g + a\sin 30° = 4a\cos 30°$$

$$\frac{a}{g} = \frac{1}{4\cos 30° - \sin 30°}$$

$\mathbf{a} = 0.337g \measuredangle 30°$ ◀

(*b*) Tipping impends

$$\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F\left(\frac{h}{2}\right) - N\left(\frac{d}{2}\right) = 0$$

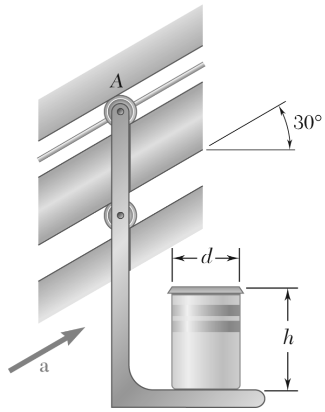


$$\frac{F}{N} = \frac{d}{h}$$

$$\mu = \frac{F}{N}; \quad 0.25 = \frac{d}{h};$$

$\frac{h}{d} = 4.00$ ◀

PROBLEM 16.8

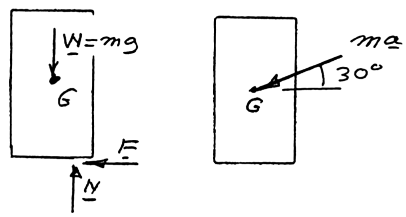


Solve Problem 16.7, assuming that the acceleration **a** of the bracket is directed downward.

PROBLEM 16.7 The support bracket shown is used to transport a cylindrical can from one elevation to another. Knowing that $\mu_s = 0.25$ between the can and the bracket, determine (*a*) the magnitude of the upward acceleration **a** for which the can will slide on the bracket, (*b*) the smallest ratio *h*/*d* for which the can will tip before it slides.

SOLUTION

(*a*) Sliding impends:



$$\overset{+}{\leftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \qquad F = ma\cos 30°$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - mg = -ma\sin 30°$$

$$N = m(g - a\sin 30°)$$

$$\mu_s = \frac{F}{N}$$

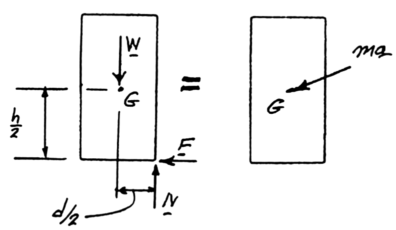
$$0.25 = \frac{ma\cos 30°}{m(g - a\sin 30°)}$$

$$g - a\sin 30° = 4a\cos 30°$$

$$\frac{a}{g} = \frac{1}{4\cos 30° + \sin 30°} = 0.25226$$

$\mathbf{a} = 0.252g \angle 30°$ ◀

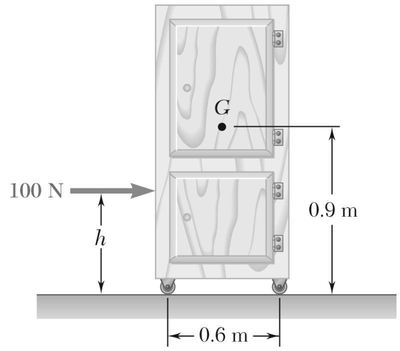
(*b*) Tipping impends:



$$\circlearrowleft^{+} \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F\left(\frac{h}{2}\right) = W\left(\frac{d}{2}\right) \quad \frac{F}{N} = \frac{d}{h}$$

$$\mu = \frac{F}{N}; \qquad 0.25 = \frac{d}{h};$$

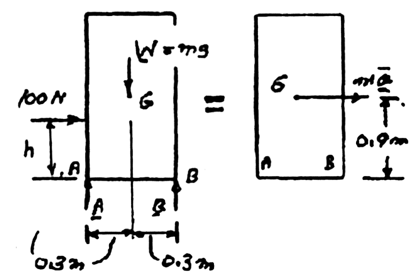
$\dfrac{h}{d} = 4.00$ ◀



PROBLEM 16.9

A 20-kg cabinet is mounted on casters that allow it to move freely $(\mu = 0)$ on the floor. If a 100-N force is applied as shown, determine (*a*) the acceleration of the cabinet, (*b*) the range of values of *h* for which the cabinet will not tip.

SOLUTION



(*a*) Acceleration

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad 100 \text{ N} = m\bar{a}$$

$$100 \text{ N} = (20 \text{ kg})\bar{a}$$

$$\bar{\mathbf{a}} = 5.00 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) For tipping to impend: $A = 0$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad (100 \text{ N})h - mg(0.3 \text{ m}) = m\bar{a}(0.9 \text{ m})$$

$$(100 \text{ N})h - (20 \text{ kg})(9.81 \text{ m/s}^2)(0.3 \text{ m}) = (100 \text{ N})(0.9 \text{ m})$$

$$h = 1.489 \text{ m}$$

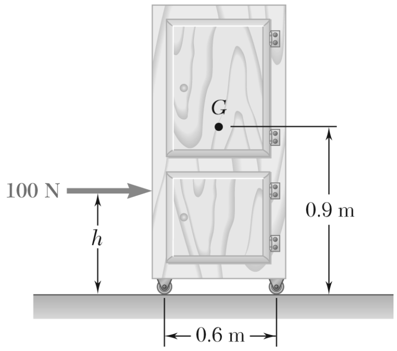
For tipping to impend: $B = 0$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad (100 \text{ N})h + mg(0.3 \text{ m}) = m\bar{a}(0.9 \text{ m})$$

$$(100 \text{ N})h + (20 \text{ kg})(9.81 \text{ m/s}^2)(0.3 \text{ m}) = (100 \text{ N})(0.9 \text{ m})$$

$$h = 0.311 \text{ m}$$

Cabinet will not tip: $0.311 \text{ m} \le h \le 1.489 \text{ m}$ ◀



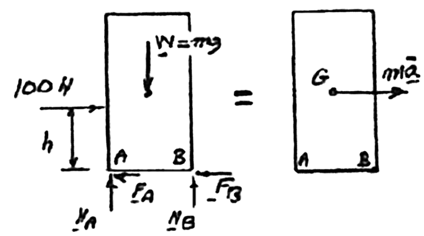
PROBLEM 16.10

Solve Problem 16.9, assuming that the casters are locked and slide on the rough floor $(\mu_k = 0.25)$.

PROBLEM 16.9 A 20-kg cabinet is mounted on casters that allow it to move freely $(\mu = 0)$ on the floor. If a 100-N force is applied as shown, determine (*a*) the acceleration of the cabinet, (*b*) the range of values of *h* for which the cabinet will not tip.

SOLUTION

(*a*) Acceleration.



$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = mg$$

But, $F = \mu N$, thus $F_A + F_B = \mu(mg)$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad 100\text{ N} - (F_A + F_B) = m\bar{a}$$

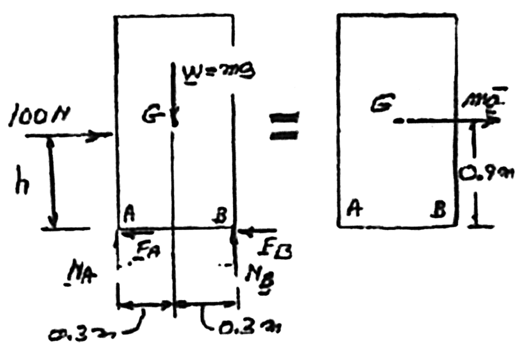
$$100\text{ N} - \mu mg = m\bar{a}$$

$$100\text{ N} - 0.25(20\text{ kg})(9.81\text{ m/s}^2) = (20\text{ kg})\bar{a}$$

$$\bar{a} = 2.548\text{ m/s}^2$$

$\bar{\mathbf{a}} = 2.55\text{ m/s}^2 \longrightarrow$ ◀

(*b*) Tipping of cabinet.



$$W = mg = (20\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 196.2\text{ N}$$

For tipping to impend $\circlearrowright$: $\quad N_A = 0.$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad (100\text{ N})h - W(0.3\text{ m}) = m\bar{a}(0.9\text{ m})$$

$$(100\text{ N})h - (196.2\text{ N})(0.3\text{ m}) = (20\text{ kg})(2.548\text{ m/s}^2)(0.9\text{ m})$$

$$h = 1.047\text{ m}$$

PROBLEM 16.10 (Continued)

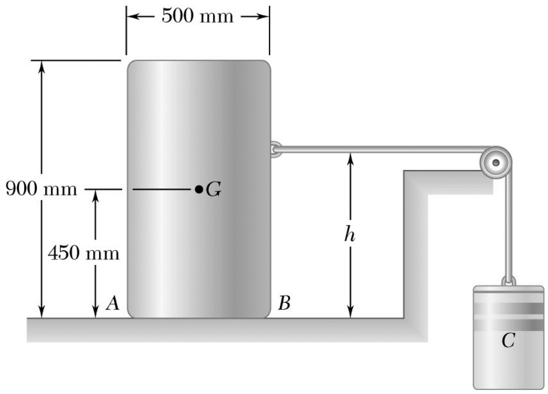
For tipping to impend: $N_B = 0$

$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad (100 \text{ N})h + W(0.3 \text{ m}) = m\bar{a}(0.9 \text{ m})$$

$$(100 \text{ N})h + (196.2 \text{ N})(0.3 \text{ m}) = (20 \text{ kg})(2.548 \text{ m/s}^2)(0.9 \text{ m})$$

$$h = -0.130 \text{ m} \quad \text{(impossible)}$$

Cabinet will not tip: $h \leq 1.047$ m ◀



PROBLEM 16.11

A completely filled barrel and its contents have a combined mass of 90 kg. A cylinder C is connected to the barrel at a height h = 550 mm as shown. Knowing $\mu_s = 0.40$ and $\mu_k = 0.35$, determine the maximum mass of C so the barrel will not tip.

SOLUTION

Kinematics: Assume that the barrel is sliding, but not tipping.

$$\alpha = 0 \qquad \mathbf{a}_G = a \longrightarrow$$

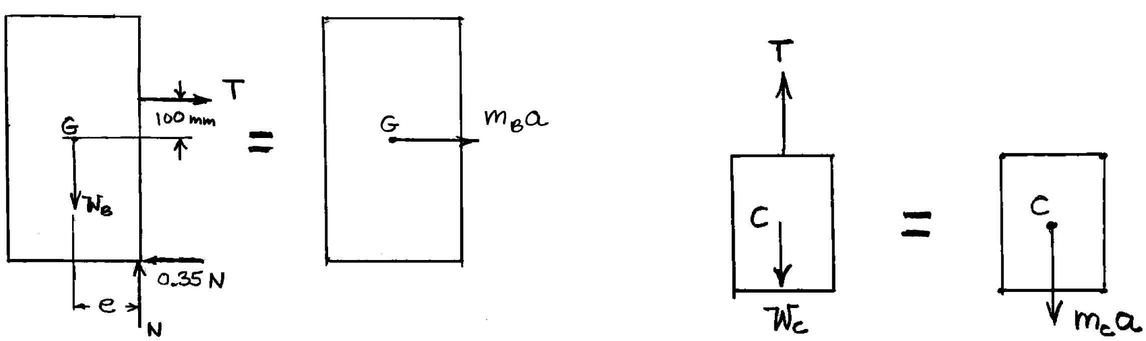
Since the cord is inextensible, $\mathbf{a}_C = a \downarrow$

Kinetics: Draw the free body diagrams of the barrel and the cylinder. Let T be the tension in the cord.

The barrel is sliding. $F_F = \mu_k N = 0.35\,N$

Assume that tipping is impending, so that the line of action of the reaction on the bottom of the barrel passes through Point B.

$$e = 250 \text{ mm}$$



For the barrel. $+\uparrow \Sigma F_y = 0$: $N - W_B = 0 \qquad N = W_B = m_B g = 882.90$ N

$$+\circlearrowright \Sigma M_G = 0: \quad Ne - 100T - (450)(0.35\,N) = 0$$

$$T = \frac{e - (450)(0.35)}{100} N = \frac{250 - 157.5}{100}(882.90) = 816.68 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = m_B a: \quad T - 0.35\,N = m_B a$$

$$a = \frac{816.68}{90} - 0.35\frac{882.90}{90} = 5.6407 \text{ m/s}^2$$

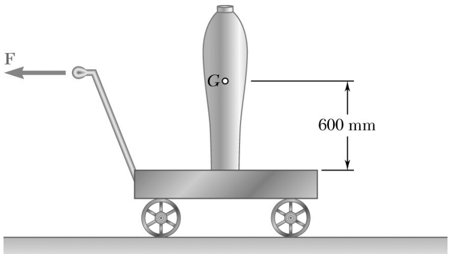
PROBLEM 16.11 (Continued)

For the cylinder: $+\!\downarrow \Sigma F = m_C a: \quad W_C - T = m_C a$

$$m_C g - T = m_C a$$

$$m_C = \frac{T}{g-a} = \frac{816.68}{9.81 - 5.6407} = 195.88 \text{ kg}$$

$$m_C = 195.9 \text{ kg} \blacktriangleleft$$

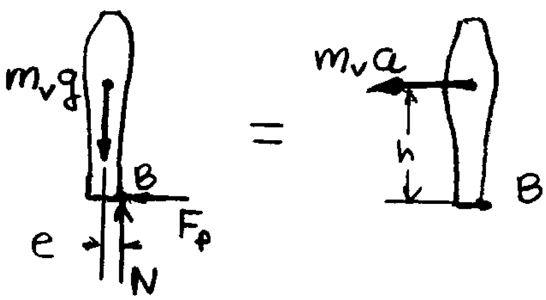


PROBLEM 16.12

A 40-kg vase has a 200-mm-diameter base and is being moved using a 100-kg utility cart as shown. The cart moves freely ($\mu = 0$) on the ground. Knowing the coefficient of static friction between the vase and the cart is $\mu_s = 0.4$, determine the maximum force **F** that can be applied if the vase is not to slide or tip.

SOLUTION

Vase:



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - m_V g = 0$$

$$N = m_V g = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

For impending sliding, $\qquad F_f = \mu_S N$

$$F_f = (0.4)(392.4 \text{ N}) = 156.96 \text{ N}$$

$$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F_f = m_V a$$

$$a = \frac{F_f}{m_V} = \frac{156.96 \text{ N}}{40} = 3.924 \text{ m/s}^2$$

This is the limiting value of a for sliding.

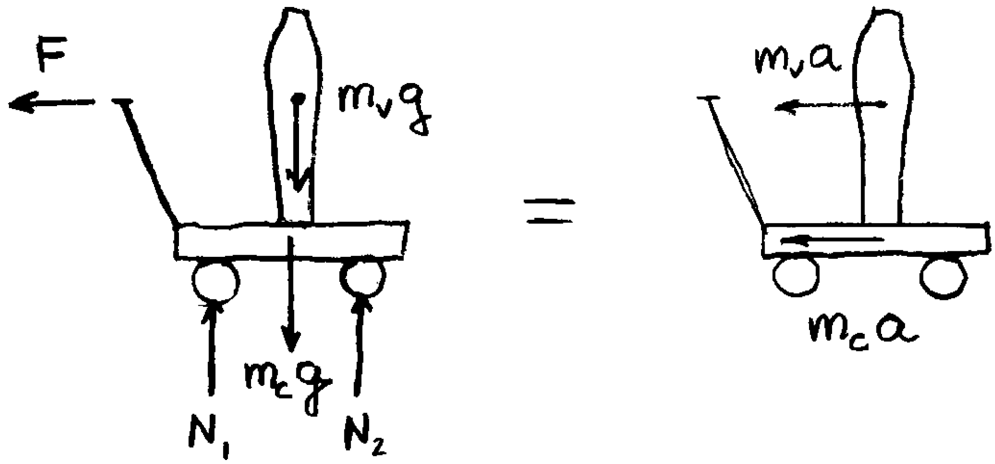
$$+\circlearrowleft M_B = \Sigma (M_B)_{\text{eff:}} \quad m_V g e = m_V a h$$

$$a = \frac{e}{h} g = \frac{100 \text{ mm}}{600 \text{ mm}} (9.81 \text{ m/s}^2) = 1.635 \text{ m/s}^2$$

This is the limiting value of a for tipping.

The smaller value governs. $\qquad a = 1.635 \text{ m/s}^2$

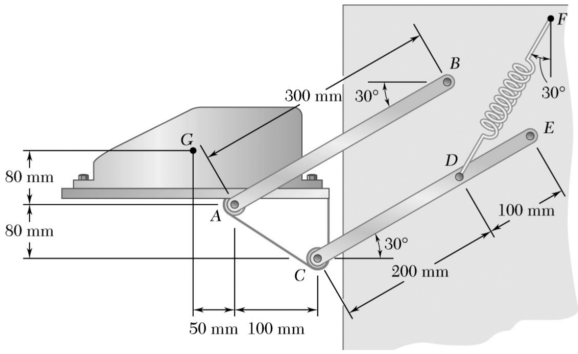
Cart and vase:



$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m_C a + m_V a$

$$F = (100 \text{ kg})(1.635 \text{ m/s}^2) + (40 \text{ kg})(1.635 \text{ m/s}^2)$$

$F = 229 \text{ N}$ ◀



PROBLEM 16.13

The retractable shelf shown is supported by two identical linkage-and-spring systems; only one of the systems is shown. A 20-kg machine is placed on the shelf so that half of its weight is supported by the system shown. If the springs are removed and the system is released from rest, determine (*a*) the acceleration of the machine, (*b*) the tension in link *AB*. Neglect the weight of the shelf and links.

SOLUTION

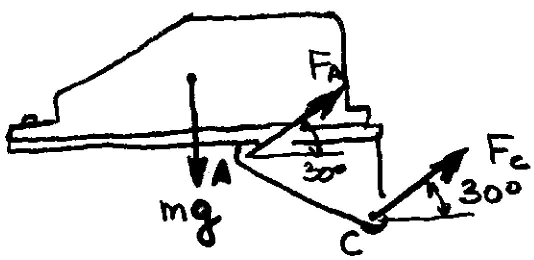
The links *AB* and *CE* keep the line *AC* on the shelf parallel to the fixed line *BE*. Thus the shelf moves in curvilinear translation.

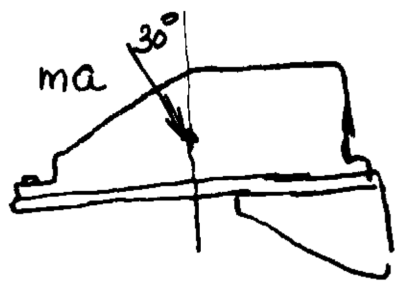
$$\alpha = 0 \qquad \text{and} \qquad \mathbf{a}_G \text{ is perpendicular to } \overrightarrow{AB}$$

Since link *AB* is a massless two-force member, the force at *A* is along link *AB*.

Since link *CDE* is massless and the spring *DF* is removed, the force at *C* is directed along the link *CDE*.

Consider the kinetics of the shelf.





Force perpendicular to link *AB*. $\measuredangle$ 30°

$$mg\cos 30° = m\bar{a}$$

$$\bar{a} = g\cos 30° = (9.81 \text{ m/s})\cos 30° = 8.4957 \text{ m/s}^2$$

(*a*) *Acceleration of the machine*: $\bar{\mathbf{a}} = 8.50 \text{ m/s}^2 \nwarrow 60°$ ◀

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (F_A\cos 30°)(0.080) + (F_A\sin 30°)(0.100) - mg(0.150)$$

$$= (ma\sin 30°)(0.160) + (ma\cos 30°)(0.150)$$

$$0.11928F_A - 0.150\,mg = -0.04990\,mg\cos 30°$$

PROBLEM 16.13 (Continued)

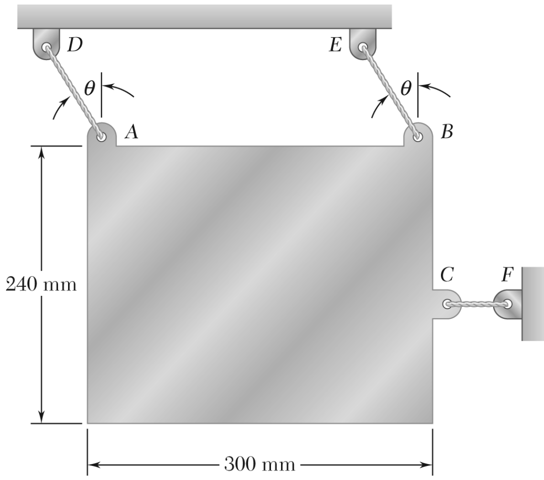
(*b*) *Tension in link AB.* $F_A = 89525\,mg$

Taking *mg* to be half the weight of the machine,

$$mg = \frac{1}{2}(20\text{ kg})(9.81\text{ m/s}^2) = 98.1\text{ N}$$

$$F_A = (0.89522)(98.1\text{ N})$$

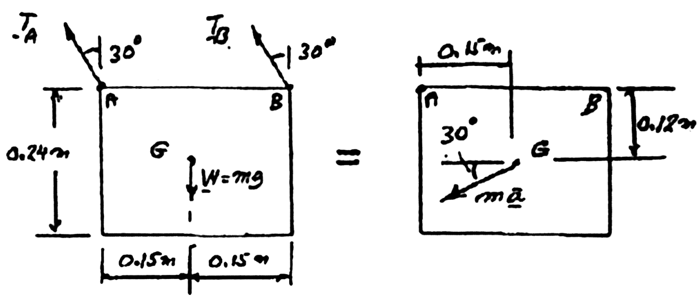
$F = 87.8$ N ◀



PROBLEM 16.14

A uniform rectangular plate has a mass of 5 kg and is held in position by three ropes as shown. Knowing that $\theta = 30°$, determine, immediately after rope *CF* has been cut, (*a*) the acceleration of the plate, (*b*) the tension in ropes *AD* and *BE*.

SOLUTION



(*a*) Acceleration $\quad +\swarrow 30° \Sigma F = \Sigma F_{\text{eff}}: \quad mg \sin 30° = m\bar{a}$

$$\bar{a} = 0.5g = 4.905 \text{ m/s}^2 \qquad \bar{\mathbf{a}} = 4.91 \text{ m/s}^2 \swarrow 30° \blacktriangleleft$$

(*b*) Tension in ropes

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad (T_B \cos 30°)(0.3 \text{ m}) - mg(0.15 \text{ m}) = -m\bar{a}(\cos 30°)(0.12 \text{ m}) - m\bar{a}(\sin 30°)(0.15 \text{ m})$$

$$0.2598T_B - (5 \text{ kg})(9.81 \text{ m/s}^2)(0.15 \text{ m}) = -(5 \text{ kg})(4.905 \text{ m/s}^2)(0.1039 + 0.075)$$

$$0.2598T_B - 7.3575 = -4.388$$

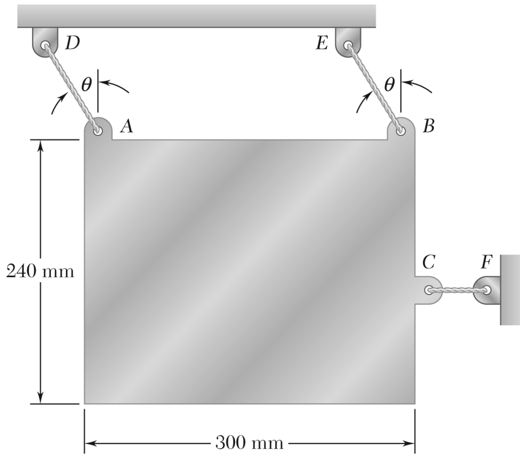
$$T_B = +11.43 \text{ N} \qquad T_{BE} = 11.43 \text{ N} \blacktriangleleft$$

$$+\searrow 10° \Sigma F = \Sigma F_{\text{eff}}: \quad T_A + 11.43 \text{ N} - mg \cos 30° = 0$$

$$T_A + 11.43 \text{ N} - (5 \text{ kg})(9.81) \cos 30° = 0$$

$$T_A + 11.43 \text{ N} - 42.48 \text{ N} = 0$$

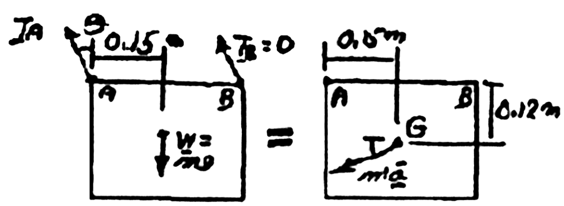
$$T_A = 31.04 \text{ N} \qquad T_{AD} = 31.0 \text{ N} \blacktriangleleft$$



PROBLEM 16.15

A uniform rectangular plate has a mass of 5 kg and is held in position by three ropes as shown. Determine the largest value of θ for which both ropes AD and BE remain taut immediately after rope CF has been cut.

SOLUTION



$$\swarrow \Sigma F = \Sigma F_{\text{eff}}: \quad mg\sin\theta = m\bar{a}$$

$$\bar{a} = g\sin\theta$$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad mg(0.15\text{ m}) = m\bar{a}\cos\theta(0.12\text{ m}) + m\bar{a}\sin\theta(0.15\text{ m})$$

$$\cancel{m}\cancel{g}(0.15) = \cancel{m}(\cancel{g}\sin\theta)(0.12\cos\theta + 0.15\sin\theta)$$

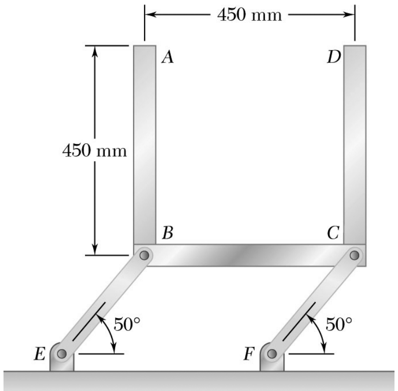
$$1 = 0.8\sin\theta\cos\theta - \sin^2\theta$$

$$1 - \sin^2\theta = 0.8\sin\theta\cos\theta$$

$$\cos^2\theta = 0.8\sin\theta\cos\theta$$

$$1 = 0.8\frac{\sin\theta}{\cos\theta}$$

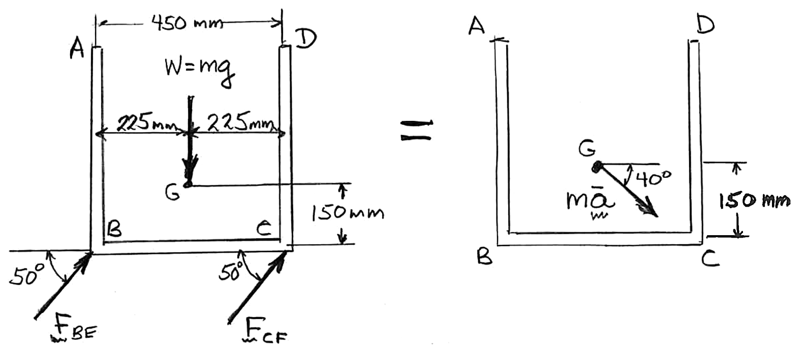
$$\tan\theta = 1.25; \qquad \theta = 51.3° \blacktriangleleft$$



PROBLEM 16.16

Three bars, each of mass 3 kg, are welded together and are pin-connected to two links *BE* and *CF*. Neglecting the weight of the links, determine the force in each link immediately after the system is released from rest.

SOLUTION



Mass center of *ABCD* is at *G*

$$\bar{y} = \frac{\Sigma m_i \bar{y}_i}{\Sigma m_i} = \frac{3(0.225) + 3(0.225) + 3(0)}{9} = 0.15 \text{ m}$$

$$W = mg$$

$$\searrow 40° \quad \Sigma F = \Sigma F_{\text{eff}}: \quad mg\cos 50° = m\bar{a}$$

$$(9.81 \text{ m/s}^2)\cos 50° = \bar{a}$$

$$\bar{\mathbf{a}} = 6.3057 \text{ m/s}^2 \searrow 40°$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}:$$

$$(F_{CF}\sin 50°)(0.450 \text{ m}) - (9 \text{ kg})(9.81 \text{ m/s}^2)(0.225 \text{ m}) = -m\bar{a}\sin 40°(0.225 \text{ m}) - m\bar{a}\cos 40°(0.150 \text{ m})$$

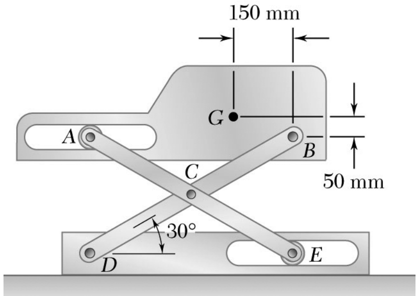
$$0.34472\, F_{CF} - 19.8653 = -m\bar{a}(0.14463 + 0.11491)$$

$$0.34472\, F_{CF} - 19.8653 = -9(6.3057)(0.25953)$$

$$F_{CF} = +14.8998 \text{ N} \qquad F_{CF} = +14.90 \text{ N compression} \blacktriangleleft$$

$$\measuredangle 50° \quad \Sigma F = \Sigma F_{\text{eff}}: \quad F_{BE} + 14.9 \text{ lb} - (9 \text{ kg})(9.81)\sin 50° = 0$$

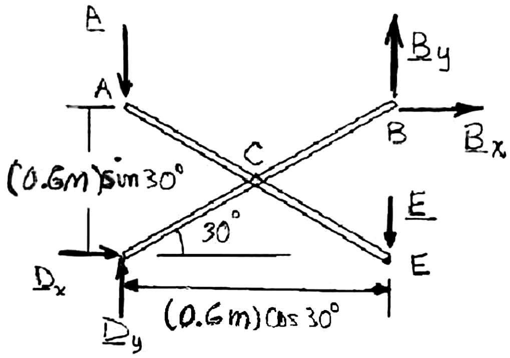
$$F_{BF} = +52.734 \text{ N} \qquad F_{BE} = +52.7 \text{ N compression} \blacktriangleleft$$

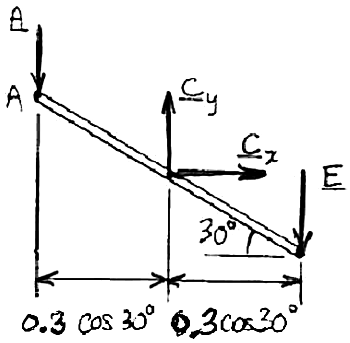


PROBLEM 16.17

Members ACE and DCB are each 600 mm long and are connected by a pin at C. The mass center of the 10-kg member AB is located at G. Determine (*a*) the acceleration of AB immediately after the system has been released from rest in the position shown, (*b*) the corresponding force exerted by roller A on member AB. Neglect the weight of members ACE and DCB.

SOLUTION





Analysis of linkage

Since members ACE and DCB are of negligible mass, their effective forces may also be neglected and the methods of statics may be applied to their analysis.

Free body: *Entire linkage*:

$$+\circlearrowleft \Sigma M_D = 0:$$

$$(B_y - E)(0.6\cos 30°) - B_x(0.6\sin 30°) = 0$$

$$(B_y - E)\cos 30° - B_x \sin 30° = 0 \qquad (1)$$

Free body: *member ACE*

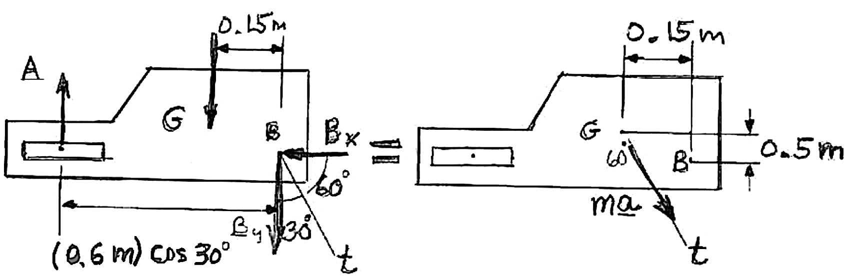
$$+\circlearrowleft \Sigma M_C = 0:$$

$$A(0.3\cos 30°) - E(0.3\cos 30°) = 0; \quad E = A$$

Carrying into Eq. (1):

$$(B_y - A)\cos 30° - B_x \sin 30° = 0 \qquad (2)$$

Equations of Motion for Member AB



(*a*) $+\searrow 60°\quad \Sigma F_t - \Sigma(F_t)_{\text{eff}}$:

$$(B_y - A)\cos 30° - B_x \sin 30° + W\cos 30° = m\bar{a}$$

Recalling equation (2), we have,

$$W\cos 30° = m\bar{a} \qquad \bar{a} = \frac{W}{m}\cos 30° = g\cos 30°$$

$$\bar{a} = (9.81\text{ m/s}^2)\cos 30°$$

$\bar{\mathbf{a}} = 8.50\text{ m/s}^2 \searrow 60°$ ◀

(*b*) $+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}$:

$$W(0.15\text{ m}) - A(0.6\text{ m})\cos 30° = (m\bar{a}\sin 60°)(0.15\text{ m}) - (m\bar{a}\cos 60°)(0.05\text{ m})$$

But, $\quad m\bar{a} = W\cos 30°$

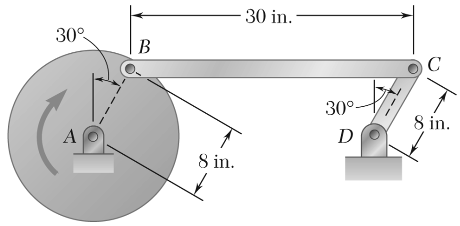
$$0.15W - 0.6A\cos 30° = W\cos 30°(0.15\sin 60° - 0.05\cos 60°)$$

$$A = W\left(\frac{1}{4\cos 30°} - \frac{1}{4}\sin 60° + \frac{1}{12}\cos 60°\right) = 0.11384W = 0.11384\,mg$$

Recalling that $m = 10$ kg so that $mg = 98.1$ N,

$$A = 0.11384(98.1) = 11.168\text{ N}$$

$\mathbf{A} = 11.17\text{ N}\uparrow$ ◀

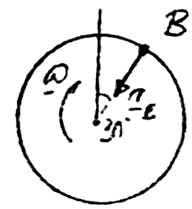


PROBLEM 16.18

The 15-lb rod *BC* connects a disk centered at *A* to crank *CD*. Knowing that the disk is made to rotate at the constant speed of 180 rpm, determine for the position shown the vertical components of the forces exerted on rod *BC* by the pins at *B* and *C*.

SOLUTION

We first determine the acceleration of Point *B* of disk.



$$\omega = 180 \text{ rpm} = 18.85 \text{ rad/s}$$

$$\text{Since } \omega = \text{constant}$$

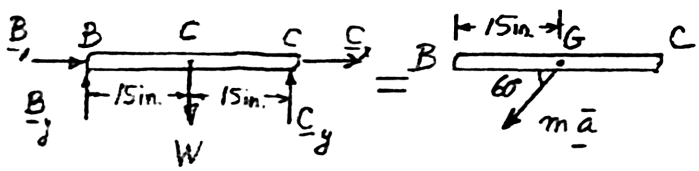
$$a_B = (a_B)_n = r\omega^2$$

$$= \left(\frac{8}{12}\text{ ft}\right)(18.85 \text{ rad/s})^2$$

$$\mathbf{a}_B = 236.9 \text{ ft/s}^2 \swarrow 60^\circ$$

Since rod *BC* is in translation.

$$\bar{\mathbf{a}} = \bar{\mathbf{a}}_B = 236.9 \text{ ft/s}^2 \swarrow 60^\circ$$



<u>Vertical components of forces at *B* and *C*.</u>

$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad C_y(30 \text{ in.}) - W(15 \text{ in.}) = -m\bar{a}\sin 60^\circ(15 \text{ in.})$$

Since $\quad W = 15 \text{ lb} \quad \text{and} \quad m\bar{a} = \dfrac{15 \text{ lb}}{32.2}(236.9) = 110.36 \text{ lb}$

$$30C_y - (15)(15) = -110.36\sin 60^\circ(15) = -95.57(15)$$

$$2C_y = -95.57 + 15 = -80.57, \quad C_y = -40.285 \text{ lb}$$

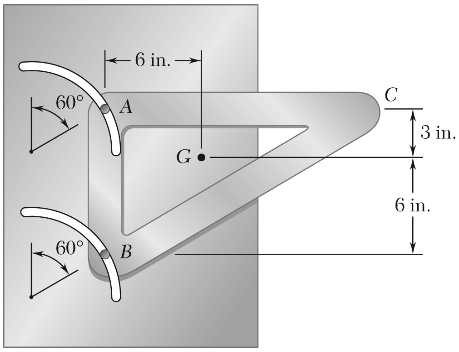
$$\mathbf{C}_y = 40.3 \text{ lb}\downarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad B_y - W + C_y = -m\bar{a}\sin 60^\circ$$

$$B_y = W - C_y - \frac{15}{32.2}(236.9)\sin 60^\circ = 15 + 40.285 - 95.57$$

$$B_y = -40.285 \text{ lb}$$

$$\mathbf{B}_y = 40.3 \text{ lb}\downarrow \blacktriangleleft$$

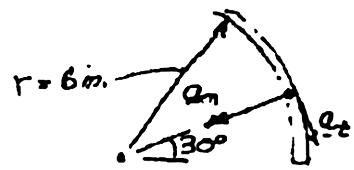


PROBLEM 16.19

The triangular weldment ABC is guided by two pins that slide freely in parallel curved slots of radius 6 in. cut in a vertical plate. The weldment weighs 16 lb and its mass center is located at Point G. Knowing that at the instant shown the velocity of each pin is 30 in./s downward along the slots, determine (a) the acceleration of the weldment, (b) the reactions at A and B.

SOLUTION

<u>Slot</u>: $v = 30 \text{ in./s} \searrow$

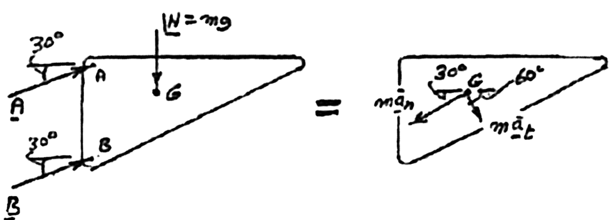


$$a_n = \frac{v^2}{r} = \frac{(30 \text{ in./s})^2}{6 \text{ in.}} = 150 \text{ in./s}^2$$

$$\mathbf{a}_n = 12.5 \text{ ft/s}^2 \swarrow 30°$$

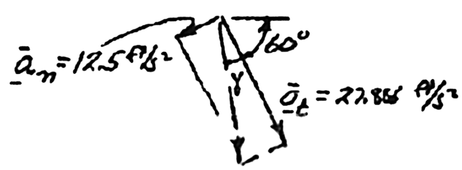
$$\mathbf{a}_t = a_t \searrow 60°$$

<u>Weldment is in translation</u> $\qquad \bar{\mathbf{a}}_n = 12.5 \text{ ft/s}^2$



$$\searrow 60° \; \Sigma F = \Sigma F_{\text{eff}}: \quad mg\cos 30° = ma_t$$

$$\bar{\mathbf{a}}_t = 27.886 \text{ ft/s}^2 \searrow 60°$$



(a) <u>Acceleration</u>

$$\beta = \tan^{-1}\frac{\bar{a}_n}{\bar{a}_t} = \tan^{-1}\frac{12.5}{27.886} = 24.14°$$

$$\bar{a}^2 = a_t^2 + a_n^2$$

$$= (27.886)^2 + (12.5)^2$$

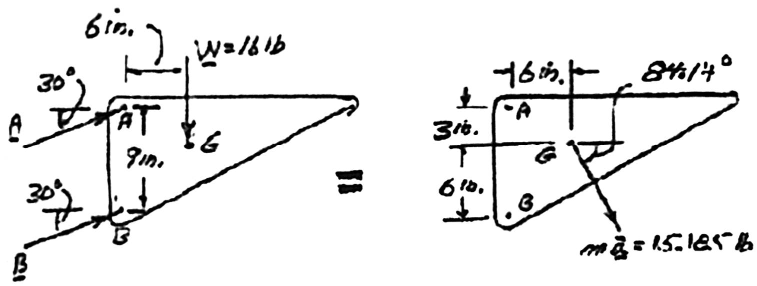
$$\bar{\mathbf{a}} = 30.56 \text{ ft/s}^2 \searrow 84.1°$$

$$\bar{\mathbf{a}} = 30.6 \text{ ft/s}^2 \searrow 84.1° \blacktriangleleft$$

PROBLEM 16.19 (Continued)

(*b*) Reactions

$$m\bar{a} = \frac{16\ \text{lb}}{32.2\ \text{ft/s}^2}(30.56\ \text{ft/s}^2) = 15.185\ \text{lb}$$



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}:$$

$$B\cos 30°(9\ \text{in.}) - (16\ \text{lb})(6\ \text{in.}) = (15.185\ \text{lb})(\cos 84.14°)(3\ \text{in.}) - (15.185\ \text{lb})(\sin 84.14°)(6\ \text{in.})$$

$$7.794B - 96 = +4.651 - 90.634$$

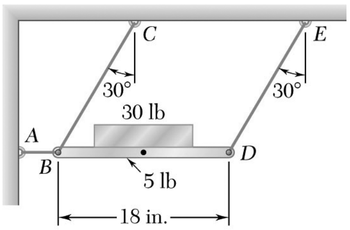
$$B = +1.285\ \text{lb} \qquad \mathbf{B} = 1.285\ \text{lb} \measuredangle 30° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A\cos 30° + B\cos 30° = m\bar{a}\cos 84.14°$$

$$A\cos 30° + (1.285\ \text{lb})\cos 30° = (15.185\ \text{lb})\cos 84.14°$$

$$A\cos 30° + 1.113\ \text{lb} = 1.550\ \text{lb}$$

$$A = +0.505\ \text{lb} \qquad \mathbf{A} = 0.505\ \text{lb} \measuredangle 30° \blacktriangleleft$$

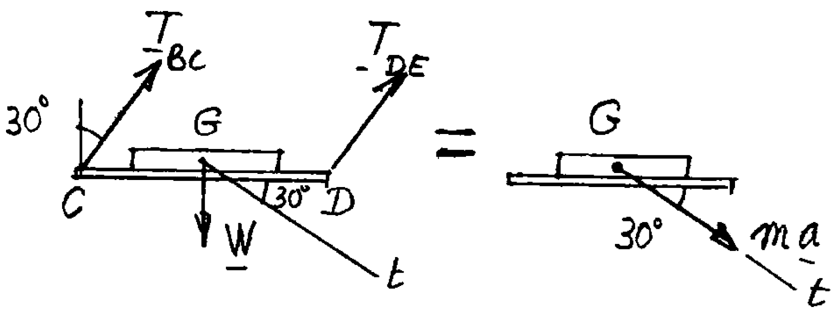


PROBLEM 16.20

The coefficients of friction between the 30-lb block and the 5-lb platform BD are $\mu_s = 0.50$ and $\mu_k = 0.40$. Determine the accelerations of the block and of the platform immediately after wire AB has been cut.

SOLUTION

Assume that the block does not slide relative to the platform. Draw the free body diagram of the platform and block.

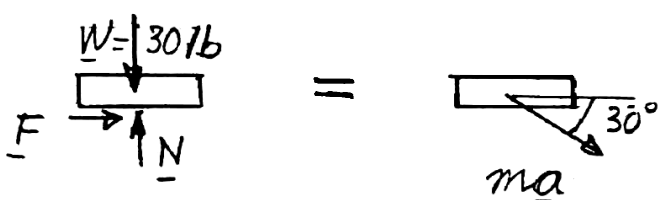


$$+\searrow \Sigma F_t = \Sigma (F_t)_{\text{eff}}: \quad W \sin 30° = ma$$

$$a = \frac{W}{m}\sin 30° = \frac{1}{2}g = 16.1 \text{ ft/s}^2$$

$$a = 16.1 \text{ ft/s}^2 \searrow 30°$$

Check whether or not the block will slide relative to the platform. Draw the free body diagram of the block alone.



$$ma = m\left(\frac{1}{2}g\right) = \frac{1}{2}W = 15 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = (15 \text{ lb}) \cos 30° = 12.99 \text{ lb}$$

$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad 30 \text{ lb} - N = (15 \text{ lb}) \sin 30°$$

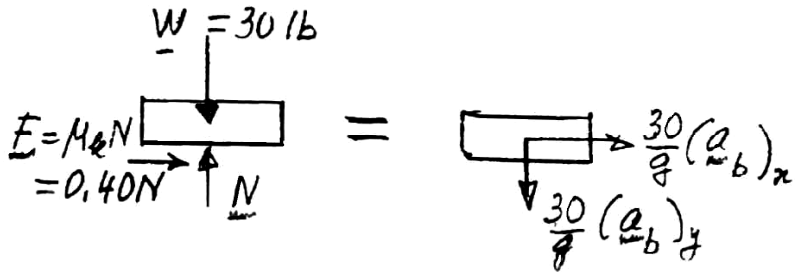
$$N = 30 - 7.5 = 22.5 \text{ lb}$$

Thus, $F_m = \mu_s N = 0.50\ (22.5\ \text{lb}) = 11.25\ \text{lb}$

Since $F > F_m$, <u>the block will slide</u>

Now assume that the block slides relative to the platform.

Equations of motion for block: (assuming sliding)



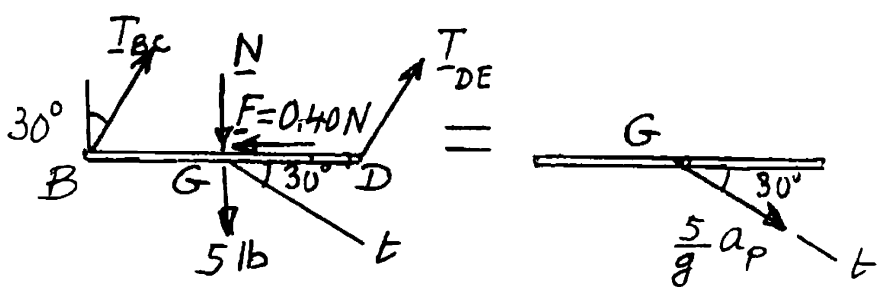
$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad 30 - N = \frac{30}{g}(a_b)_y$$

$$(a_b)_y = g\left(1 - \frac{N}{30}\right) \tag{1}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad 0.40\ \text{N} = \frac{30}{g}(a_b)_x$$

$$(a_b)_x = g\left(\frac{N}{75}\right) \tag{2}$$

Equations of motion for platform.



$$+\searrow \Sigma F_t = \Sigma (F_t)_{\text{eff}}: \quad (N - 5)\sin 30° - (0.40\ \text{N})\cos 30° = \frac{5}{g}a_P$$

$$a_p = g(0.5 + 0.030718\ \text{N}) \tag{3}$$

If contact is maintained between block and platform, we must have

$$(a_b)_y = (a_p)_y = a_p \sin 30° \tag{4}$$

PROBLEM 16.20 (Continued)

Substituting from (1) and (3) into (4):

$$g\left(1-\frac{N}{30}\right)=g(0.5+0.030718\text{ N})\sin 30°$$

$$(0.015359+0.033333)N=0.75$$

$$N=15.403\text{ lb}$$

Substituting for N in (2) and (1):

$$(a_b)_x=(32.2\text{ ft/s}^2)\,\frac{15.403}{75},\quad (\mathbf{a}_b)_x=6.61\text{ ft/s}^2\longrightarrow$$

$$(a_b)_y=(32.2\text{ ft/s}^2)\left(1-\frac{15.403}{30}\right),\quad (\mathbf{a}_b)_y=15.67\text{ ft/s}^2\downarrow$$

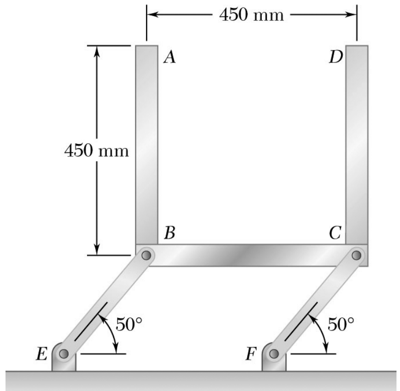
$$\mathbf{a}_b=17.01\text{ ft/s}^2 \searrow 67.1° \blacktriangleleft$$

Substituting for N in (3):

$$a_P=(32.2\text{ ft/s}^2)(0.5+0.030718\times 15.403)=31.335\text{ ft/s}^2$$

$$\mathbf{a}_P=31.3\text{ ft/s}^2 \searrow 30° \blacktriangleleft$$

Note: Since $N>0$, we check that contact between block and platform is maintained.



PROBLEM 16.21

Draw the shear and bending-moment diagrams for the vertical rod *AB* of Problem 16.16.

PROBLEM 16.16 Three bars, each of mass 3 kg, are welded together and are pin-connected to two links *BE* and *CF*. Neglecting the weight of the links, determine the force in each link immediately after the system is released from rest.

SOLUTION

From the solution of Problem 16.16, the acceleration of all points of vertical rod *AB* is

$$\mathbf{a} = 6.3057\ \text{m/s}^2 \searrow 40°$$

or $$\mathbf{a} = 4.8304\ \text{m/s}^2 \longrightarrow +4.0532\ \text{m/s}^2 \downarrow$$

Mass of rod *AB*: $m = 3\ \text{kg}$

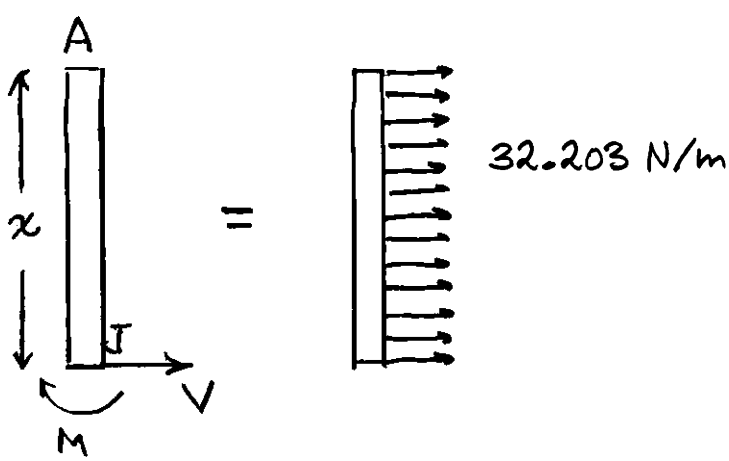
Mass per unit length: $$\frac{m}{l} = \frac{3\ \text{kg}}{0.450\ \text{m}} = 6.6667\ \text{kg/m}$$

Effective force per length: $$\frac{m}{l}\mathbf{a}$$

$$(6.6667\ \text{kg/m})(4.8304\ \text{m/s}^2) \longrightarrow + (6.6667\ \text{kg/m})(4.0532\ \text{m/s}^2)$$

$$32.203\ \text{N/m} \longrightarrow + 27.021\ \text{N/m} \downarrow$$

Only the horizontal component contributes to the shear and bending moment. Let x be a vertical coordinate (positive down) with its origin at *A*. Draw the free body diagram of the portion of the rod *AB* lying above the section defined by x.



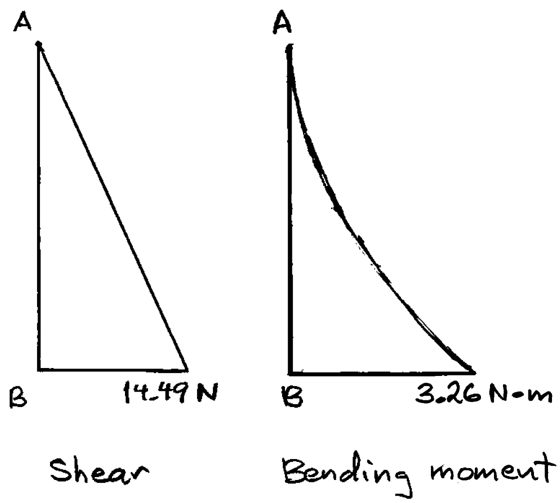
PROBLEM 16.21 (Continued)

$$\xrightarrow{+} \Sigma F = \Sigma(F_j)_{\text{eff}}: \quad V = 32.203x$$

$$\circlearrowleft + \Sigma M_J = \Sigma(M_j)_{\text{eff}}: \quad M = (32.203x)\frac{x}{2}$$

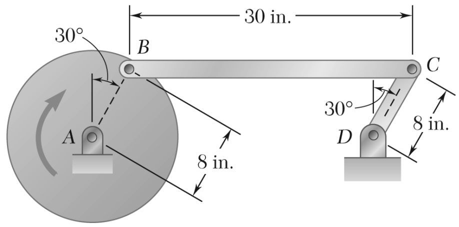
$$= 16.101x^2$$

Shear and bending moment diagrams.



$V_{\text{max}} = (32.203 \text{ N/m})(0.450 \text{ m})$ $\quad V_{\text{max}} = 14.49 \text{ N}$ ◀

$M_{\text{max}} = (16.101 \text{ N/m})(0.450 \text{ m})^2$ $\quad M_{\text{max}} = 3.26 \text{ N} \cdot \text{m}$ ◀



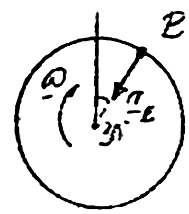
PROBLEM 16.22*

Draw the shear and bending-moment diagrams for the connecting rod BC of Problem 16.18.

PROBLEM 16.18 The 15-lb rod BC connects a disk centered at A to crank CD. Knowing that the disk is made to rotate at the constant speed of 180 rpm, determine for the position shown the vertical components of the forces exerted on rod BC by the pins at B and C.

SOLUTION

We first determine the acceleration of Point B of disk:



$$\omega = 180 \text{ rpm} = 18.85 \text{ rad/s}$$

Since $\omega = \text{constant}$

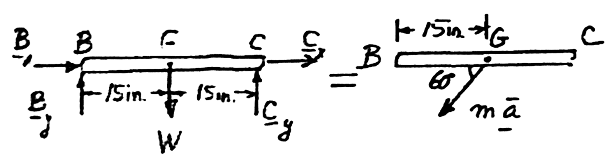
$$a_B = (a_B)_n = r\omega^2$$

$$= \left(\frac{8}{12}\text{ ft}\right)(18.85 \text{ rad/s})^2$$

$$\mathbf{a}_B = 236.9 \text{ ft/s}^2 \measuredangle 60°$$

Since rod BC is in translation.

$$\bar{\mathbf{a}} = \mathbf{a}_B = 236.9 \text{ ft/s}^2 \measuredangle 60°$$



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad C_y(30 \text{ in.}) - W(15 \text{ in.}) = -m\bar{a}\sin 60°(15 \text{ in.})$$

Since $W = 15$ lb and $m\bar{a} = \dfrac{15 \text{ lb}}{32.2}(236.9) = 110.36$ lb:

$$30C_y - (15)(15) = -110.36\sin 60°(15) = -95.57(15)$$

$$2C_y = -95.57 + 15 = -80.57, \quad C_y = -40.285 \text{ lb}$$

$$\mathbf{C}_y = 40.3 \text{ lb} \downarrow$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad B_y - W + C_y = -m\bar{a}\sin 60°$$

$$B_y = W - C_y - \frac{15}{32.2}(236.9)\sin 60° = 15 + 40.285 - 95.57$$

$$B_y = -40.285 \text{ lb} \qquad \mathbf{B}_y = 40.3 \text{ lb} \downarrow$$

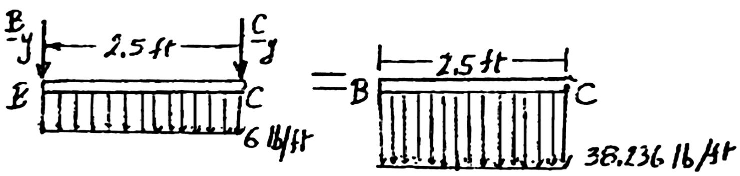
$$a_y = 236.7\sin 60° \qquad \mathbf{a}_y = 205.2 \text{ ft/s}^2 \downarrow$$

$$\text{Distributed weight per unit length} = w = \frac{15\text{ lb}}{\left(\frac{30}{12}\right)\text{ft}} = 6\text{ lb/ft}$$

$$\text{Distributed mass per unit length} = \frac{w}{g} = \frac{6}{g} = \frac{6}{32.2}\text{lb}\cdot\text{s}^2/\text{ft}^2$$

$$\text{Vertical component of effective forces} = \frac{w}{g}a_y = \frac{6}{32.2}(205.2)$$

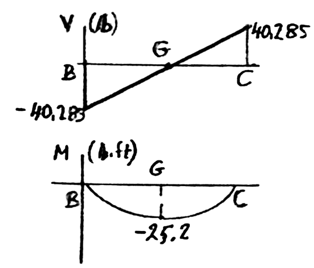
$$= 38.236\text{ lb/ft}$$



$$+\!\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad B_y + C_y + (2.5\text{ ft})(6\text{ lb/ft}) = (2.5\text{ ft})(38.236\text{ lb/ft})$$

$$B_y + C_y = 80.59\text{ lb}$$

From symmetry. $B_y = C_y = 40.285\text{ lb}$ $\mathbf{B}_y = \mathbf{C}_y = 40.3\text{ lb}\downarrow$

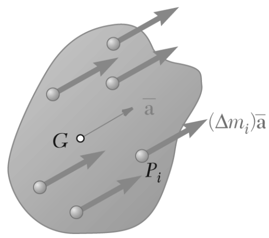


Maximum value of bending moment occurs at G, where $V = 0$:

$$|M|_{\max} = \text{Area under } V\text{-diagram from } B \text{ to } G = \frac{1}{2}(40.285\text{ lb})(1.25\text{ ft})$$

$|M|_{\max} = 25.2\text{ lb}\cdot\text{ft}$ ◀

$V_B = -40.3\text{ lb}$ ◀

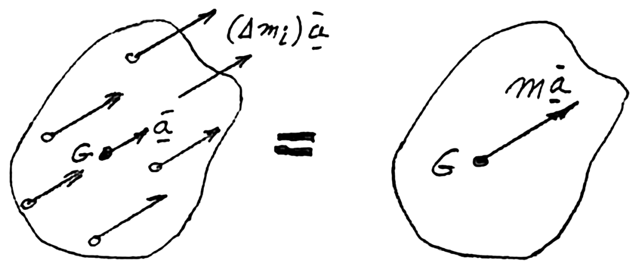


PROBLEM 16.23

For a rigid slab in translation, show that the system of the effective forces consists of vectors $(\Delta m_i)\bar{\mathbf{a}}$ attached to the various particles of the slab, where $\bar{\mathbf{a}}$ is the acceleration of the mass center G of the slab. Further show, by computing their sum and the sum of their moments about G, that the effective forces reduce to a single vector $m\bar{\mathbf{a}}$ attached at G.

SOLUTION

Since slab is in translation, each particle has same acceleration as G, namely $\bar{\mathbf{a}}$. The effective forces consist of $(\Delta mi)\bar{\mathbf{a}}$.



The sum of these vectors is: $\Sigma(\Delta m_i)\bar{\mathbf{a}} = (\Sigma\Delta m_i)\bar{\mathbf{a}}$

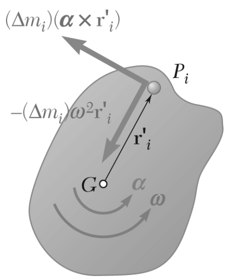
or since $\Sigma\Delta m_i = m,$

$$\Sigma(\Delta m_i)\bar{\mathbf{a}} = m\bar{\mathbf{a}}$$

The sum of the moments about G is: $\Sigma r_i' \times (\Delta m_i)\bar{\mathbf{a}} = (\Sigma\Delta m_i r_i') \times \bar{\mathbf{a}}$ (1)

But, $\Sigma\Delta m_i r_i' = m\bar{r}' = 0,$ because G is the mass center. It follows that the right-hand member of Eq. (1) is zero. Thus, the moment about G of $m\bar{a}$ must also be zero, which means that its line of action passes through G and that it may be attached at G.

PROBLEM 16.24

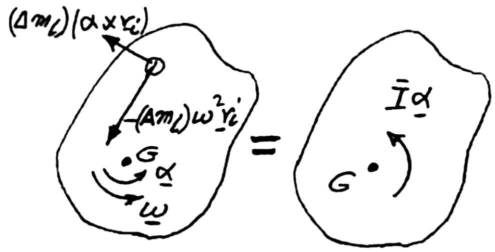


For a rigid slab in centroidal rotation, show that the system of the effective forces consists of vectors $-(\Delta m_i)\boldsymbol{\omega}^2\mathbf{r}'_i$ and $(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}'_i)$ attached to the various particles P_i of the slab, where $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are the angular velocity and angular acceleration of the slab, and where $\mathbf{r}'_i$ denotes the position vector of the particle P_i relative to the mass center G of the slab. Further show, by computing their sum and the sum of their moments about G, that the effective forces reduce to a couple $\bar{I}\boldsymbol{\alpha}$.

SOLUTION

For centroidal rotation: $$\mathbf{a}_i = (\mathbf{a}_i)_t + (\mathbf{a}_i)_n = \boldsymbol{\alpha}\times\mathbf{r}'_i - \omega^2\mathbf{r}'_i$$

Effective forces are: $$(\Delta m_i)a_i = (\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i) - (\Delta m_i)\omega^2 r'_i(\Delta m_i)(\alpha\times\mathbf{r}_i)$$



$$\Sigma(\Delta m_i)\mathbf{a}_i = \Sigma(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}'_i) - \Sigma(\Delta m_i)\omega^2 r'_i$$

$$= \alpha\times\Sigma(\Delta m_i)\mathbf{r}'_i - \omega^2\Sigma(\Delta m_i)r'_i$$

Since G is the mass center, $\Sigma(\Delta m_i)r'_i = 0$

effective forces reduce to a couple, Summing moments about G,

$$\Sigma(\mathbf{r}'_1\times\Delta m_i\mathbf{a}_i) = \Sigma[\mathbf{r}'_i\times(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}'_i)] - \Sigma\mathbf{r}'_i\times(\Delta m_i)\omega^2 r'_i$$

But, $$\mathbf{r}'_i\times(\Delta m_i)\omega^2\mathbf{r}'_i = \omega^2(\Delta m_i)(\mathbf{r}'_i\times\mathbf{r}'_i) = 0$$

and, $$\mathbf{r}'_i\times(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}'_i) = (\Delta m_i)r_1'^2\boldsymbol{\alpha}$$

Thus, $$\Sigma(\mathbf{r}'_i\times\Delta m_i\mathbf{a}_i) = \Sigma(\Delta m_i)r_1^2\alpha = \left[\Sigma(\Delta m_i)r_i'^2\right]\alpha$$

Since $\Sigma(\Delta m)r_i'^2 = \bar{I}$, the moment of the couple is $\bar{I}\boldsymbol{\alpha}$.

PROBLEM 16.25

The rotor of an electric motor has an angular velocity of 3600 rpm when the load and power are cut off. The 50-kg rotor, which has a centroidal radius of gyration of 180 mm, then coasts to rest. Knowing that kinetic friction results in a couple of magnitude $3.5\ \text{N}\cdot\text{m}$ exerted on the rotor, determine the number of revolutions that the rotor executes before coming to rest.

SOLUTION

$$\bar{I} = m\bar{k}^2$$

$$= (50)(0.180)^2$$

$$= 1.62\ \text{kg}\cdot\text{m}^2$$

$$M = \bar{I}\alpha: \quad 3.5\ \text{N}\cdot\text{m} = (1.62\ \text{kg}\cdot\text{m}^2)\alpha$$

$$\alpha = 2.1605\ \text{rad/s}^2 \ \text{(deceleration)}$$

$$\omega_0 = 3600\ \text{rpm}\left(\frac{2\pi}{60}\right)$$

$$= 120\pi\ \text{rad/s}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$0 = (120\pi\ \text{rad/s})^2 + 2(-2.1605\ \text{rad/s}^2)\theta$$

$$\theta = 32.891\times 10^3\ \text{rad}$$

$$= 5235.26\ \text{rev}$$

or $\theta = 5230\ \text{rev}$ ◀

PROBLEM 16.26

It takes 10 min for a 6000-lb flywheel to coast to rest from an angular velocity of 300 rpm. Knowing that the radius of gyration of the flywheel is 36 in., determine the average magnitude of the couple due to kinetic friction in the bearings.

SOLUTION

$$m = \frac{W}{g} = \frac{6000}{32.2} = 186.335 \text{ lb} \cdot \text{s}^2/\text{ft} \qquad \bar{k} = 36 \text{ in.} = 3 \text{ ft}$$

$$\bar{I} = m\bar{k}^2 = (186.336)(3)^2 = 1677 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\omega_0 = 300 \text{ rpm}\left(\frac{2\pi}{60}\right) = 10\pi \text{ rad/s}$$

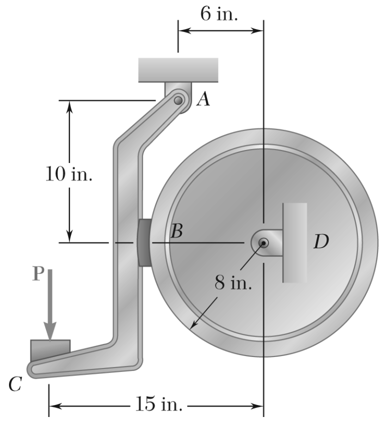
$$\omega = \omega_0 + \alpha t$$

$$0 = 10\pi \text{ rad/s} + \alpha(600 \text{ s})$$

$$\alpha = -0.05236 \text{ rad/s}^2$$

$$M = \bar{I}\alpha = (1677 \text{ lb} \cdot \text{s}^2 \cdot \text{ft})(-0.05236 \text{ rad/s}^2) = 87.81 \text{ lb} \cdot \text{ft}$$

$$M = 87.8 \text{ lb} \cdot \text{ft} \blacktriangleleft$$



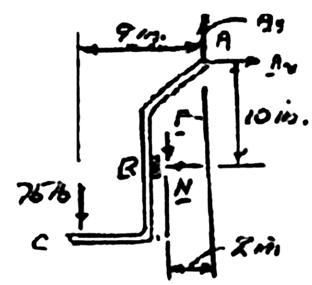
PROBLEM 16.27

The 8-in.-radius brake drum is attached to a larger flywheel that is not shown. The total mass moment of inertia of the drum and the flywheel is $14 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.35. Knowing that the angular velocity of the flywheel is 360 rpm counterclockwise when a force **P** of magnitude 75 lb is applied to the pedal C, determine the number of revolutions executed by the flywheel before it comes to rest.

SOLUTION

Lever ABC: Static equilibrium (friction force $\downarrow$)

$$F = \mu_k N = 0.35N$$



$$+\circlearrowleft \Sigma M_A = 0: \quad N(10 \text{ in.}) - F(2 \text{ in.}) - (75 \text{ lb})(9 \text{ in.}) = 0$$

$$10N - 2(0.35N) - 675 = 0$$

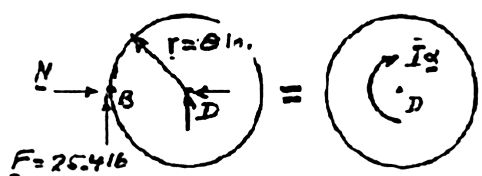
$$N = 72.58 \text{ lb}$$

$$F = \mu_k N$$

$$= 0.35(72.58 \text{ lb})$$

$$= 25.40 \text{ lb}$$

Drum:



$$r = 8 \text{ in.} = \frac{2}{3} \text{ ft}$$

$$\omega_0 = 360 \text{ rpm}\left(\frac{2\pi}{60}\right)$$

$$\omega_0 = 12\pi \text{ rad/s}$$

$$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$(25.4 \text{ lb})\left(\frac{2}{3}\text{ft}\right) = (14 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$$

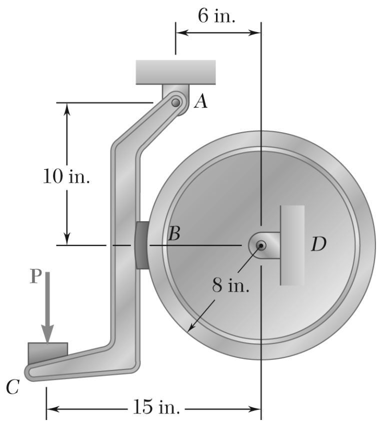
$$\alpha = 1.2097 \text{ rad/s}^2 \text{ (deceleration)}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta: \quad 0 = (12\pi \text{ rad/s})^2 + 2(-1.2097 \text{ rad/s}^2)\theta$$

$$\theta = 587.4 \text{ rad}$$

$$\theta = 587.4 \text{ rad}\left(\frac{1}{2\pi}\right) = 93.49 \text{ rev}$$

$\theta = 93.5 \text{ rev}$ ◀



PROBLEM 16.28

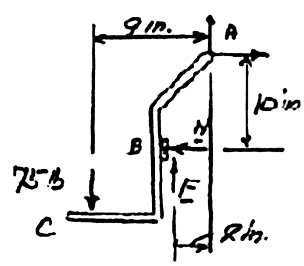
Solve Problem 16.27, assuming that the initial angular velocity of the flywheel is 360 rpm clockwise.

PROBLEM 16.27 The 8-in.-radius brake drum is attached to a larger flywheel that is not shown. The total mass moment of inertia of the drum and the flywheel is $14\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.35. Knowing that the angular velocity of the flywheel is 360 rpm counterclockwise when a force **P** of magnitude 75 lb is applied to the pedal C, determine the number of revolutions executed by the flywheel before it comes to rest.

SOLUTION

Lever ABC: Static equilibrium (friction force $\uparrow$)

$$F = \mu_k N = 0.35N$$



$$+\circlearrowleft \Sigma M_A = 0: \quad N(10\text{ in.}) + F(2\text{ in.}) - (75\text{ lb})(9\text{ in.}) = 0$$

$$10N + 2(0.35N) - 675 = 0$$

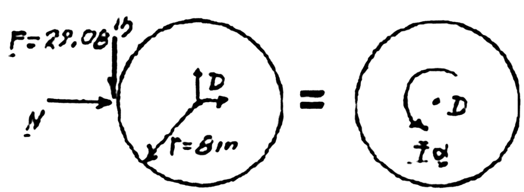
$$N = 63.08\text{ lb}$$

$$F = \mu_k N$$

$$= 0.35(63.08\text{ lb})$$

$$= 22.08\text{ lb}$$

Drum:



$$r = 8\text{ in.} = \frac{2}{3}\text{ft}$$

$$\omega_0 = 360\text{ rpm}\left(\frac{2\pi}{60}\right)$$

$$\omega_0 = 12\pi\text{ rad/s}$$

$$+\circlearrowright \Sigma M_D = \Sigma(M_D)_{\text{eff}} = 0: \quad Fr = \bar{I}\alpha$$

$$(22.08\text{ lb})\left(\frac{2}{3}\text{ft}\right) = (14\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$$

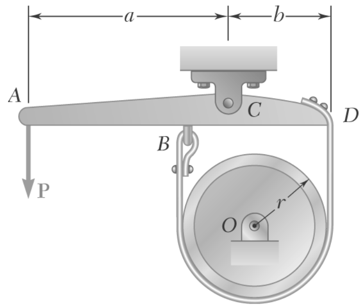
$$\alpha = 1.0515\text{ rad/s}^2 \quad \text{(deceleration)}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta: \quad 0 = (12\pi\text{ rad/s})^2 + 2(-1.0515\text{ rad/s}^2)\theta$$

$$\theta = 675.8\text{ rad}$$

$$\theta = 675.8\text{ rad} = 107.56\text{ rev}$$

$$\theta = 107.6\text{ rev} \blacktriangleleft$$

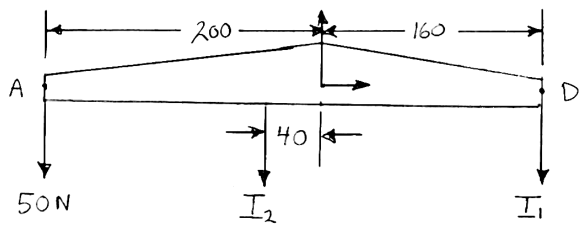


PROBLEM 16.29

The 100-mm-radius brake drum is attached to a flywheel which is not shown. The drum and flywheel together have a mass of 300 kg and a radius of gyration of 600 mm. The coefficient of kinetic friction between the brake band and the drum is 0.30. Knowing that a force **P** of magnitude 50 N is applied at *A* when the angular velocity is 180 rpm counterclockwise, determine the time required to stop the flywheel when $a = 200$ mm and $b = 160$ mm.

SOLUTION

Equilibrium of lever *AD*



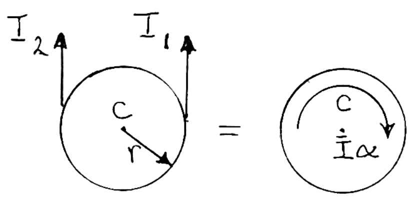
(Dimensions in mm)

$+\circlearrowleft \Sigma M_C = 0;$

$(50\text{ N})(200) + T_2(40) - T_1(160) = 0$

$4T_1 - T_2 = 250\text{ N}$ (1)

Equation of Motion for flywheel and drum



$\bar{I} = m\bar{k}^2$

$= (300\text{ kg})(0.600\text{ m})^2$

$= 108\text{ kg}\cdot\text{m}^2$

$r = 0.100\text{ m}$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad T_2 r - T_1 r = \bar{I}\alpha$

$(T_2 - T_1)(0.100\text{ m}) = 108\,\alpha$

$\alpha = (925.93\times 10^{-6})(T_2 - T_1)$ (2)

Belt Friction

Using μ_k instead of μ_s since the brake band is slipping:

$$\frac{T_2}{T_1} = e^{\mu_k \beta} \quad \text{or} \quad T_2 = T_1 e^{\mu_k \beta} \qquad (3)$$

Making $\mu_k = 0.30$ and $\beta = 180° = \pi$ rad in (3):

$$T_2 = T_1 e^{0.30\pi} \qquad T_2 = 2.5663 T_1 \qquad (4)$$

Substituting for T_2 from (4) into (1):

$$4T_1 - 2.5663T_1 = 250 \text{ N} \qquad T_1 = 174.38 \text{ N}$$

From (1): $\quad T_2 = 4(174.38) - 250 \qquad T_2 = 447.51 \text{ N}$

Substituting for T_1 and T_2 into (2):

$$\alpha = (925.93 \times 10^{-6})(447.51 - 174.38), \quad \boldsymbol{\alpha} = 0.2529 \text{ rad/s}^2$$

Kinematics

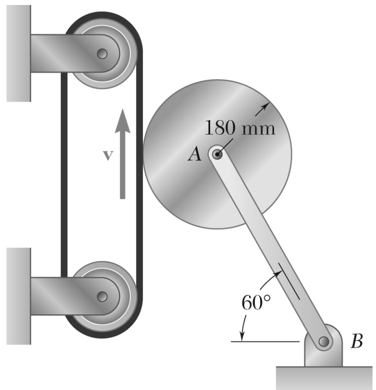
$$\boldsymbol{\omega}_0 = 180 \text{ rpm} \qquad \omega_0 = +18.850 \text{ rad/s}$$

$$\boldsymbol{\alpha} = 0.2529 \text{ rad/s}^2 \qquad \alpha = -0.2529 \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t: \quad 0 = 18.850 - 0.2529t$$

$t = 74.5$ s ◀

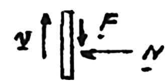
PROBLEM 16.30



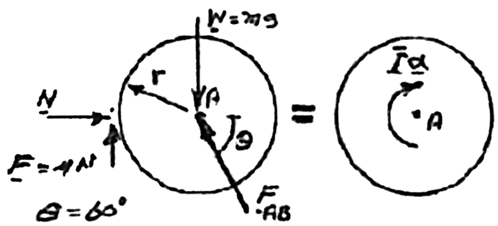
The 180-mm radius disk is at rest when it is placed in contact with a belt moving at a constant speed. Neglecting the weight of the link *AB* and knowing that the coefficient of kinetic friction between the disk and the belt is 0.40, determine the angular acceleration of the disk while slipping occurs.

SOLUTION

Belt: $F = \mu_k N$



Disk:



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \qquad N - F_{AB}\cos\theta = 0$$

$$F_{AB}\cos\theta = N \tag{1}$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad \mu_k N + F_{AB}\sin\theta - mg = 0$$

$$F_{AB}\sin\theta = mg - \mu_k N \tag{2}$$

$$\frac{\text{Eq. (2)}}{\text{Eq. (1)}}: \quad \tan\theta = \frac{mg - \mu_k N}{N}$$

$$N\tan\theta = mg - \mu_k N$$

$$N = \frac{mg}{\tan\theta + \mu_k}$$

$$F = \mu_k N = \frac{mg\mu_k}{\tan\theta + \mu_k}$$

PROBLEM 16.30 (Continued)

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$\alpha = \frac{r}{I}F$$

$$= \frac{r}{\frac{1}{2}mr^2} \cdot \frac{mg\mu_k}{\tan\theta + \mu_k}$$

$$= \frac{2g}{r} \cdot \frac{\mu_k}{\tan\theta + \mu_k}$$

Data:

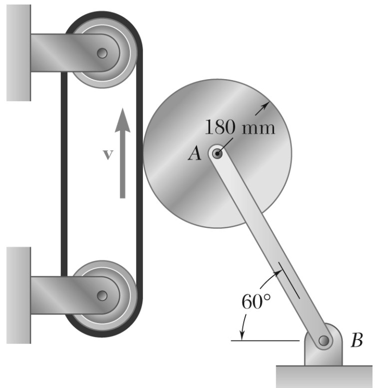
$$r = 0.18 \text{ m}$$

$$\theta = 60°$$

$$\mu_k = 0.40$$

$$\alpha = \frac{2(9.81 \text{ m/s}^2)}{0.18 \text{ m}} \cdot \frac{0.40}{\tan 60° + 0.40}$$

$\boldsymbol{\alpha} = 20.4 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 16.31

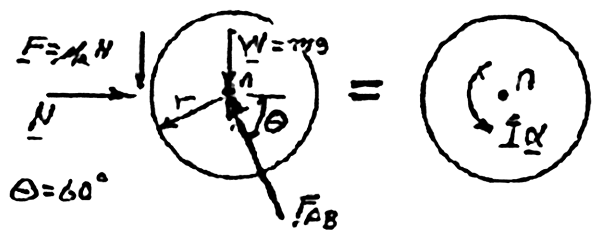
Solve Problem 16.30, assuming that the direction of motion of the belt is reversed.

PROBLEM 16.30 The 180-mm disk is at rest when it is placed in contact with a belt moving at a constant speed. Neglecting the weight of the link AB and knowing that the coefficient of kinetic friction between the disk and the belt is 0.40, determine the angular acceleration of the disk while slipping occurs.

SOLUTION

Belt: $F = \mu_k N$

Disk:



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad N - F_{AB}\cos\theta; \quad F_{AB}\cos\theta = N \qquad (1)$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad F_{AB}\sin\theta - mg - \mu_k N = 0$$

$$F_{AB}\sin\theta = mg + \mu_k N \qquad (2)$$

$$\frac{\text{Eq. (2)}}{\text{Eq. (1)}}: \quad \tan\theta = \frac{mg + \mu_k N}{N}$$

$$N\tan\theta = mg + \mu_k N; \quad N = \frac{mg}{\tan\theta - \mu_k}$$

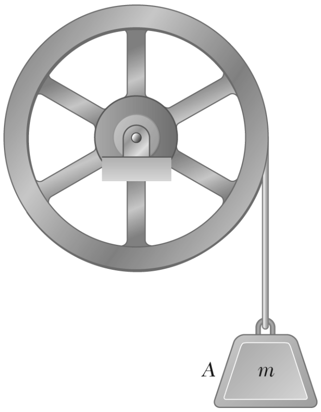
$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$\alpha = \frac{Fr}{r} = \frac{\mu_k N r}{I} = \frac{\mu_k r}{\frac{1}{2}mr^2} \cdot \frac{mg}{\tan\theta - \mu_k} = \frac{2g}{r} \cdot \frac{\mu_k}{\tan\theta - \mu_k}$$

Data: $r = 0.18 \text{ m}, \quad \theta = 60°, \quad \mu_k = 0.40$

$$\alpha = \frac{2(9.81 \text{ m/s}^2)}{0.18 \text{ m}} \cdot \frac{0.40}{\tan 60° - 0.40}$$

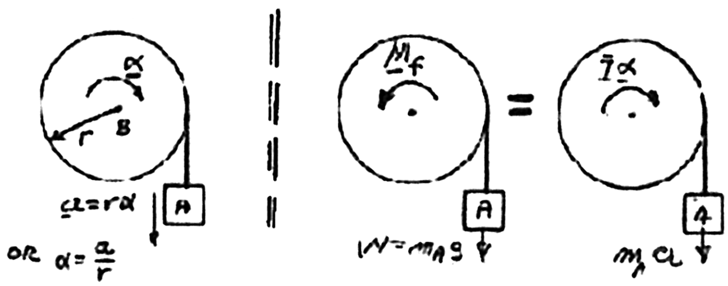
$\boldsymbol{\alpha} = 32.7 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 16.32

In order to determine the mass moment of inertia of a flywheel of radius 600 mm, a 12-kg block is attached to a wire that is wrapped around the flywheel. The block is released and is observed to fall 3 m in 4.6 s. To eliminate bearing friction from the computation, a second block of mass 24 kg is used and is observed to fall 3 m in 3.1 s. Assuming that the moment of the couple due to friction remains constant, determine the mass moment of inertia of the flywheel.

SOLUTION



$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad (m_A y)r - M_f = \bar{I}\alpha + (m_A a)r$$

$$m_A gr - M_f = \bar{I}\frac{a}{r} + m_A ar \qquad (1)$$

Case 1:

$$y = 3\text{ m}$$
$$t = 4.6\text{ s}$$
$$y = \frac{1}{2}at^2$$
$$3\text{ m} = \frac{1}{2}a(4.6\text{ s})^2$$
$$a = 0.2836\text{ m/s}^2$$
$$m_A = 12\text{ kg}$$

Substitute into Eq. (1)

$$(12\text{ kg})(9.81\text{ m/s}^2)(0.6\text{ m}) - M_f = \bar{I}\left(\frac{0.2836\text{ m/s}^2}{0.6\text{ m}}\right) + (12\text{ kg})(0.2836\text{ m/s}^2)(0.6\text{ m})$$

$$70.632 - M_f = \bar{I}(0.4727) + 2.0419 \qquad (2)$$

Case 2:

$$y = 3 \text{ m}$$
$$t = 3.1 \text{ s}$$
$$y = \frac{1}{2}at^2$$
$$3 \text{ m} = \frac{1}{2}a(3.1 \text{ s})^2$$
$$a = 0.6243 \text{ m/s}^2$$
$$m_A = 24 \text{ kg}$$

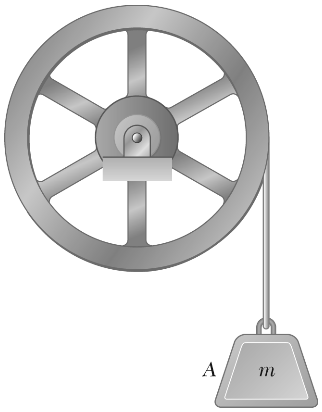
Substitute into Eq. (1):

$$(24 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m}) - M_f = \bar{I}\left(\frac{0.6243 \text{ m/s}^2}{0.6 \text{ m}}\right) + (24 \text{ kg})(0.6243 \text{ m/s}^2)(0.6 \text{ m})$$

$$141.264 - M_f = \bar{I}(1.0406) + 8.9899 \qquad (3)$$

Subtract Eq. (1) from Eq. (2) to eliminate M_f

$$70.632 = \bar{I}(1.0406 - 0.4727) + 6.948$$
$$63.684 = \bar{I}(0.5679)$$
$$\bar{I} = 112.14 \text{ kg}\cdot\text{m}^2 \qquad \bar{I} = 112.1 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

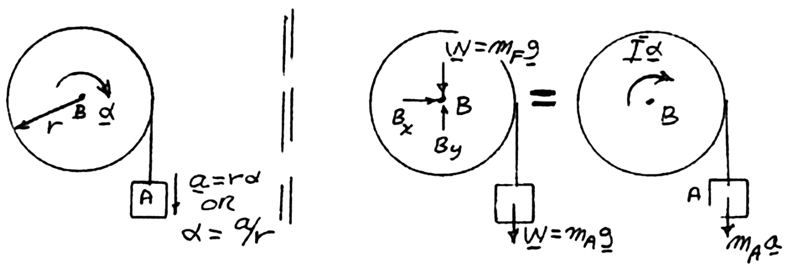


PROBLEM 16.33

The flywheel shown has a radius of 20 in. a weight of 250 lbs, and a radius of gyration of 15 in. A 30-lb block *A* is attached to a wire that is wrapped around the flywheel, and the system is released from rest. Neglecting the effect of friction, determine (*a*) the acceleration of block *A*, (*b*) the speed of block *A* after it has moved 5 ft.

SOLUTION

Kinematics: Kinetics:



$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad (m_A g)r = \bar{I}\alpha + (m_A a)r$$

$$m_A gr = m_F k^2\left(\frac{a}{r}\right) + m_A ar$$

$$a = \frac{W_A}{m_A + m_F\left(\frac{k}{r}\right)^2}$$

(*a*) Acceleration of *A*

$$a = \frac{(30 \text{ lb})}{\left(\frac{30 \text{ lb}}{32.2 \text{ ft/s}^2}\right) + \left(\frac{250 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(\frac{15 \text{ in.}}{20 \text{ in.}}\right)^2} = 5.6615 \text{ ft/s}^2$$

or $\mathbf{a}_A = 5.66 \text{ ft/s}^2 \downarrow$ ◀

(*b*) Velocity of *A*

$$v_A^2 = (v_A)_0^2 + 2a_A s$$

For $s = 5$ ft

$$v_A^2 = 0 + 2(5.6615 \text{ in./s}^2)(5 \text{ ft})$$

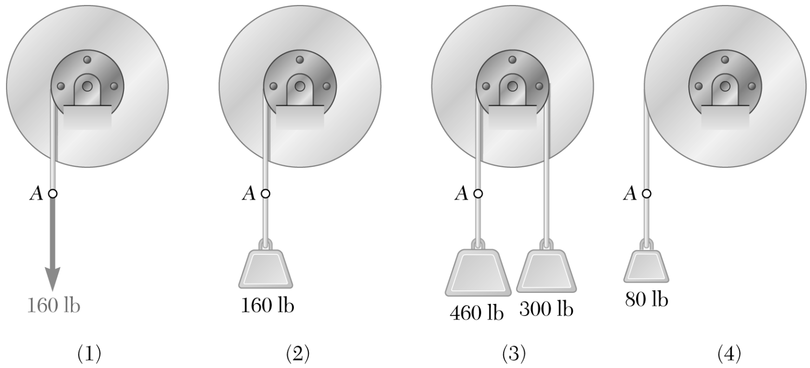
$$= 56.6154 \text{ ft}^2/\text{s}^2$$

$$v_A = 7.5243 \text{ ft/s}$$

or $v_A = 7.52$ ft/s ◀

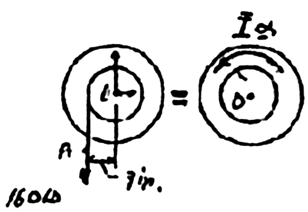
PROBLEM 16.34

Each of the double pulleys shown has a mass moment of inertia of $15\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ and is initially at rest. The outside radius is 18 in., and the inner radius is 9 in. Determine (*a*) the angular acceleration of each pulley, (*b*) the angular velocity of each pulley after Point *A* on the cord has moved 10 ft.



SOLUTION

Case 1:



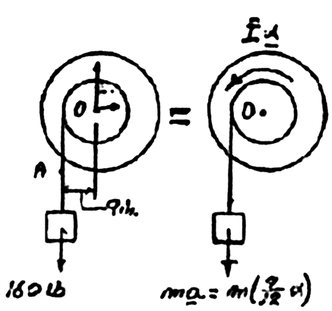
(*a*) $\stackrel{+}{\circlearrowleft}\Sigma M_0 = \Sigma(M_0)_{\text{eff}}:\quad (160\ \text{lb})\left(\dfrac{9}{12}\text{ft}\right) = (15\ \text{lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$

$$\boldsymbol{\alpha} = 8\ \text{rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \dfrac{10\ \text{ft}}{\left(\frac{9}{12}\text{ft}\right)} = 13.333\ \text{rad}$

$\omega^2 = 2\alpha\theta = 2(8\ \text{rad/s}^2)(13.33\ \text{rad})$ $\qquad \boldsymbol{\omega} = 14.61\ \text{rad/s} \circlearrowleft \blacktriangleleft$

Case 2:



(*a*) $\stackrel{+}{\circlearrowleft}\Sigma M_0 = \Sigma(M_0)_{\text{eff}}:\quad (160)\left(\dfrac{9}{12}\right) = 15\alpha + ma\left(\dfrac{9}{12}\right)$

$$120 = 15\alpha + \frac{160}{32.2}\left(\frac{9}{12}\alpha\right)\left(\frac{9}{12}\right)$$

$$120 = (15 + 2.795)\alpha$$

$$\alpha = 6.7435\ \text{rad/s}^2$$

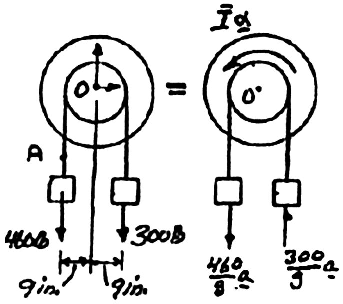
$$\boldsymbol{\alpha} = 6.74\ \text{rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \dfrac{10\ \text{ft}}{\left(\frac{9}{12}\text{ft}\right)} = 13.333\ \text{rad}$

$\omega^2 = 2\alpha\theta$

$= 2(6.7435\ \text{rad/s}^2)(13.333\ \text{rad})$ $\qquad \boldsymbol{\omega} = 13.41\ \text{rad/s} \circlearrowleft \blacktriangleleft$

Case 3:



(*a*) $+\circlearrowleft \Sigma M_0 = \Sigma (M_0)_{\text{eff}}:$

$$(460)\left(\frac{9}{12}\right) - (300)\left(\frac{9}{12}\right) = 15\alpha + \frac{460}{32.2}a\left(\frac{9}{12}\right) + \frac{300}{32.2}a\left(\frac{9}{12}\right)$$

$$120 = 15\alpha + \frac{460}{32.2}\left(\frac{9}{12}\right)^2 \alpha + \frac{300}{32.2}\left(\frac{9}{12}\right)^2 \alpha$$

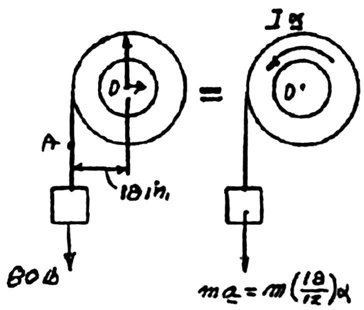
$$\alpha = 4.2437 \text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 4.24 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \dfrac{10 \text{ ft}}{\frac{9}{12}} = 13.333 \text{ rad}$

$$\omega^2 = 2\alpha\theta = 2(4.2437)(13.333) \qquad \boldsymbol{\omega} = 10.64 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

Case 4:



(*a*) $+\circlearrowleft \Sigma M_0 = \Sigma (M_0)_{\text{eff}}: \quad (80)\left(\frac{18}{12}\right) = 15\alpha + \frac{80}{32.2}a\left(\frac{18}{12}\right)$

$$120 = 15\alpha + \frac{80}{32.2}\left(\frac{18}{12}\right)^2 \alpha$$

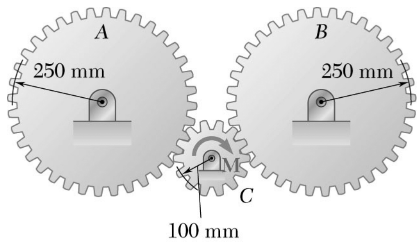
$$120 = (15 + 5.5901)\alpha$$

$$\alpha = 5.828 \text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 5.83 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \dfrac{10 \text{ ft}}{\left(\frac{18}{12} \text{ ft}\right)} = 6.6667 \text{ rad}$

$$\omega^2 = 2\alpha\theta = 2(5.828 \text{ rad/s}^2)(6.6667 \text{ rad}) \qquad \boldsymbol{\omega} = 8.82 \text{ rad/s} \circlearrowleft \blacktriangleleft$$



PROBLEM 16.35

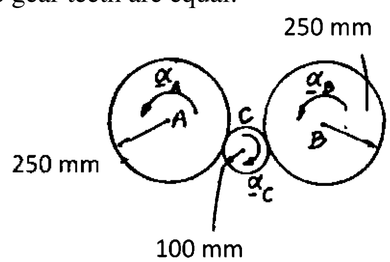
Each of the gears A and B has a mass of 9 kg and has a radius of gyration of 200 mm; gear C has a mass of 3 kg and has a radius of gyration of 75 mm. If a couple **M** of constant magnitude 5 N-m is applied to gear C, determine (*a*) the angular acceleration of gear A, (*b*) the tangential force which gear C exerts on gear A.

SOLUTION

Kinematics:

We express that the tangential components of the accelerations of the gear teeth are equal:

$$a_t = 0.25\alpha_A$$
$$= 0.25\alpha_B$$
$$= 0.1\alpha_C$$
$$\alpha_B = \alpha_A$$
$$\alpha_C = 2.5\alpha_A \qquad (1)$$



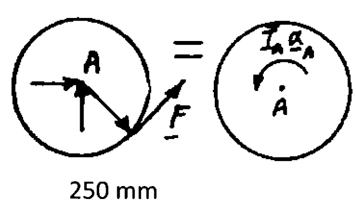
Kinetics:

Gear A: $\qquad \bar{I}_A = m_A \bar{k}_A^2 = 9(0.20)^2$

$$= 0.36 \text{ kg}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad F(0.25) = 0.36\alpha_A$$

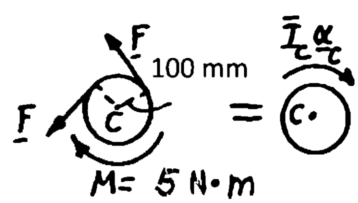
$$F = 1.44\alpha_A \qquad (2)$$



Because of symmetry, gear C exerts an equal force F on gear B.

Gear C: $\qquad \bar{I}_C = m_C \bar{k}_C^2$

$$= 3(0.075)^2$$
$$= 0.016875 \text{ kg}\cdot\text{m}^2$$



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad M - 2Fr_C = \bar{I}_C\alpha_C$$

$$5 \text{ N}\cdot\text{m} - 2F(0.1 \text{ m}) = 0.016875\alpha_C$$

(*a*) Angular acceleration of gear A.

Substituting for α_C from (1) and for F from (2):

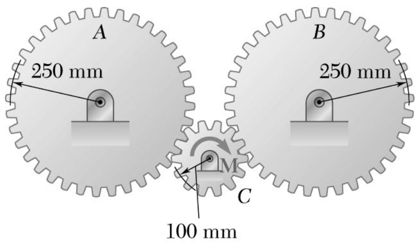
$$5 - 2(1.44\alpha_A)(0.1) = 0.016875(2.5\alpha_A)$$
$$5 - 0.288\alpha_A = 0.04219\alpha_A$$
$$5 = 0.3302\alpha_A$$
$$\alpha_A = 15.143 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_A = 15.14 \text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) Tangential force F.

Substituting for α_A into (2): $\qquad F = 1.44(15.143)$

$\mathbf{F}_A = 21.8 \text{ N} \nearrow$ ◀



PROBLEM 16.36

Solve Problem 16.35, assuming that the couple **M** is applied to disk *A*.

PROBLEM 16.35 Each of the gears *A* and *B* has a mass of 9 kg and has a radius of gyration of 200 mm; gear *C* has a mass of 3 kg and has a radius of gyration of 75 mm. If a couple **M** of constant magnitude 5 N-m is applied to gear *C*, determine (*a*) the angular acceleration of gear *A*, (*b*) the tangential force which gear *C* exerts on gear *A*.

SOLUTION

Kinematics: $\boldsymbol{\alpha}_A = \alpha_A \circlearrowright \qquad \boldsymbol{\alpha}_A = \alpha_A \circlearrowleft \qquad \boldsymbol{\alpha}_B = \alpha_B \circlearrowright$

At the contact point between gears *A* and *C*,

$$a_t = r_A\alpha_A = r_C\alpha_C$$

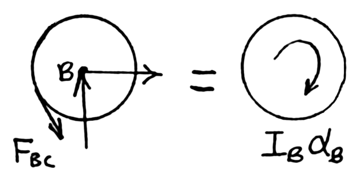
$$\alpha_C = \frac{r_A}{r_C}\alpha_A = \frac{0.25}{0.1}\alpha_A$$

At the contact point between gears *B* and *C*,

$$a_t = r_B\alpha_B = r_C\alpha_C$$

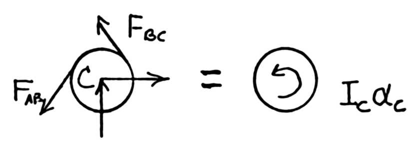
$$\alpha_B = \frac{r_C}{r_A}\alpha_C = \frac{0.1}{0.25}\cdot\frac{0.25}{0.1}\alpha_A = \alpha_A$$

Kinetics:



Gear *B*: $+\circlearrowright\Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad F_{BC}r_B = I_B\alpha_B = I_B\alpha_A$

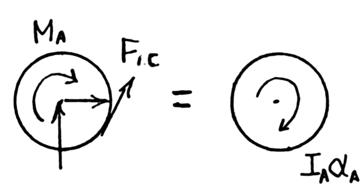
$$F_{BC} = \frac{I_B}{r_B}\alpha_A$$



Gear *C*: $+\circlearrowleft\Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad F_{BC}r_C + F_{AB}r_C = I_C\alpha_C$

$$\frac{r_C}{r_B}I_B\alpha_A + F_{AB}r_C = \frac{0.25}{0.1}I_C\alpha_A$$

$$F_{AC} = \frac{1}{r_C}\left(\frac{0.1}{0.25}I_B + \frac{0.25}{0.1}I_C\right)\alpha_A \qquad (1)$$



Gear *A*: $\circlearrowright + M_A = \Sigma(M_A)_{\text{eff}}: \qquad M_A - r_AF_{AB} = I_A\alpha_A$

$$M_A - \frac{r_A}{r_C}\left(\frac{0.1}{0.25}I_B + \frac{0.25}{0.1}I_C\right)\alpha_A = I_A\alpha_A$$

$$M_A = \left(I_A + I_B + \left(\frac{0.25}{0.1}\right)^2 I_C\right)\alpha_A \qquad (2)$$

Data: From Eq. (2)

$$m_A = m_B = 9 \text{ kg}$$
$$k_A = k_B = 0.2 \text{ m}$$
$$I_A = I_B = m_A k_A^2 = 9(0.2)^2$$
$$= 0.36 \text{ kg} \cdot \text{m}^2$$

$$m_C = 3 \text{ kg}$$
$$k_C = 0.075 \text{ m}$$
$$I_C = 3(0.075)^2 = 0.016875 \text{ kg} \cdot \text{m}^2$$
$$M_A = 5 \text{ N} \cdot \text{m}$$
$$5 = \left[0.36 + 0.36 + \left(\frac{0.25}{0.1}\right)^2 (0.016875)\right]\alpha_A$$

(*a*) Angular acceleration. $\alpha_A = 6.0572 \text{ rad/s}^2$

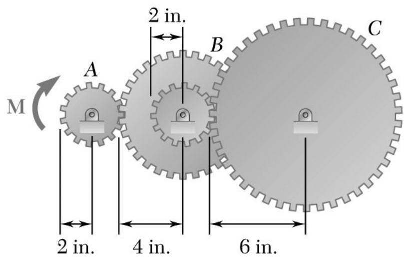
$\boldsymbol{\alpha}_A = 6.06 \text{ rad/s}^2$ ◀

(*b*) Tangential gear force.

From Eq. (1)

$$F_{AC} = \frac{1}{0.1}\left[\left(\frac{0.1}{0.25}\right)(0.36) + \left(\frac{0.25}{0.1}\right)(0.016875)\right](6.0572)$$
$$= 11.278$$

$\mathbf{F}_{AC} = 11.28 \text{ N}$ ◀



PROBLEM 16.37

Gear A weighs 1 lb and has a radius of gyration of 1.3 in; Gear B weighs 6 lb and has a radius of gyration of 3 in.; gear C weighs 9 lb and has a radius of gyration of 4.3 in. Knowing a couple **M** of constant magnitude of 40 lb · in is applied to gear A, determine (a) the angular acceleration of gear C, (b) the tangential force which gear B exerts on gear C.

SOLUTION

Masses and moments of inertia.

$$m_A = \frac{1\text{ lb}}{32.2\text{ ft/s}^2} = 0.031056\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_B = \frac{6\text{ lb}}{32.2\text{ ft/s}^2} = 0.18634\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_C = \frac{9\text{ lb}}{32.2\text{ ft/s}^2} = 0.27950\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\bar{I}_A = m_A\bar{k}_A^2 = (0.031056\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{1.3\text{ in.}}{12\text{ in./ft}}\right)^2$$

$$= 0.36448\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_B = m_B\bar{k}_B^2 = (0.18634\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{3\text{ in.}}{12\text{ in./ft}}\right)^2$$

$$= 11.646\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_C = m_C\bar{k}_C^2 = (0.27950\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{4.3\text{ in.}}{12\text{ in./ft}}\right)^2$$

$$= 35.889\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinematics. Gear A: $r_A = 2$ in.

Gear B: $r_1 = 4$ in., $r_2 = 2$ in.

Gear C: $r_C = 6$ in.

Point of contact between A and B.

$$a_t = r_A\alpha_A = r_1\alpha_B$$

$$\alpha_A = \frac{r_1}{r_A}\alpha_B = \frac{4\text{ in.}}{2\text{ in.}}\alpha_B$$

Point of contact between B and C.

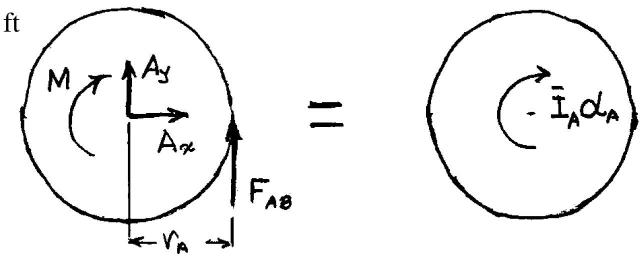
$$a_t = r_2\alpha_B = r_C\alpha_C$$

$$\alpha_B = \frac{r_C}{r_2}\alpha_C = \frac{6 \text{ in.}}{2 \text{ in.}}\alpha_C$$

Summary. $$\alpha_B = 3\alpha_C \qquad (1)$$

$$\alpha_A = 2\alpha_B = 6\alpha_C \qquad (2)$$

Kinetics. Applied couple: $M = 40 \text{ lb}\cdot\text{in.} = 3.3333 \text{ lb}\cdot\text{ft}$



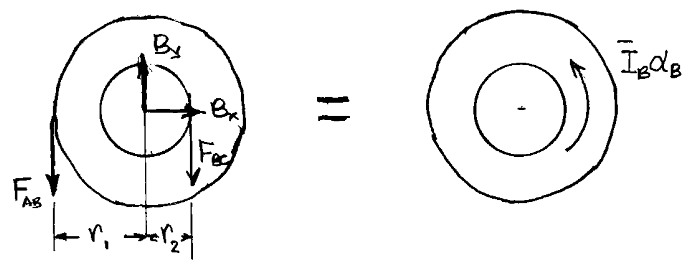
Gear A:

$$\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad M - F_{AB}r_A = \bar{I}_A\alpha_A$$

$$F_{AB} = \frac{M}{r_A} - \frac{\bar{I}_A(6\alpha_C)}{r_A}$$

$$= \frac{3.3333 \text{ lb}\cdot\text{ft}}{(2/12) \text{ ft}} - \frac{(0.36448\times10^{-3})(6)}{(2/12)}\alpha_C$$

$$= 20 \text{ lb} - 0.013121\alpha_C \qquad (3)$$



Gear B:

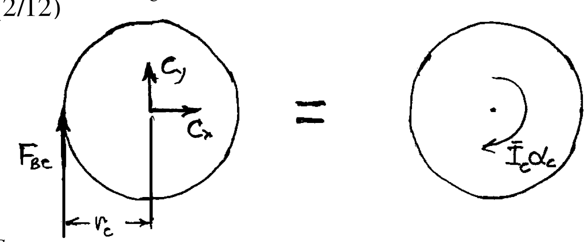
$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad F_{AB}r_1 - F_{BC}r_2 = \bar{I}_B\alpha_C$$

$$F_{BC} = F_{AB} = \frac{r_1}{r_2} - \frac{\bar{I}_B\alpha_B}{r_2}$$

$$= 2F_{AB} - \frac{3\bar{I}_B}{r_2}\alpha_C$$

$$= 2[20 \text{ lb} - 0.013121\alpha_C] - \frac{(3)(11.646\times10^{-3})}{(2/12)}\alpha_C$$

$$= 40 \text{ lb} - 0.23587\alpha_C$$



Gear C:

$$\circlearrowleft + \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad F_{BC}r_C = \bar{I}_C\alpha_C$$

$$(40 - 0.23587\alpha_C)\left(\frac{6}{12}\right) = (35.889\times10^{-3})\alpha_C$$

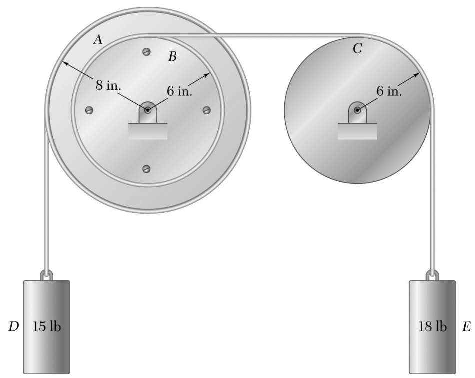
$$20 = 0.153824\alpha_C$$

(*a*) *Angular acceleration of gear C.* $\qquad \alpha_C = 130.0 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) *Tangential force which gear B exerts on gear C.*

$$F_{BC} = 40 \text{ lb} - (0.23587)(130.0) = 9.33 \text{ lb}$$

9.33 lb ↑ ◀



PROBLEM 16.38

Disks A and B are bolted together, and cylinders D and E are attached to separate cords wrapped on the disks. A single cord passes over disks B and C. Disk A weighs 20 lb and disks B and C each weigh 12 lb. Knowing that the system is released from rest and that no slipping occurs between the cords and the disks, determine the acceleration (*a*) of cylinder D, (*b*) of cylinder E.

SOLUTION

Masses and moments of inertia.

$$m_D = \frac{W_D}{g} = \frac{15 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.46584 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_E = \frac{W_E}{g} \frac{18 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.55901 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Assume disks have uniform thickness.

$$I_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2} \frac{20 \text{ lb}}{32.2 \text{ ft/s}^2} \left(\frac{8}{12} \text{ft}\right)^2 = 0.138026 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$I_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2} \frac{12 \text{ lb}}{32.2 \text{ ft/s}^2} \left(\frac{6}{12} \text{ft}\right)^2 = 0.046584 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$I_C = I_B = 0.046584 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$I_{AB} = I_A + I_B = 0.184610 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

Kinematics: $\mathbf{a}_D = a_D \downarrow, \ \mathbf{a}_E = a_E \uparrow, \ \boldsymbol{\alpha}_{AB} = \alpha_{AB} \circlearrowright, \ \boldsymbol{\alpha}_C = \alpha_C \circlearrowright$

For inextensible cord between disk A and cylinder D,

$$a_D = (a_t)_A = r_1 \alpha_{AB} = \left(\frac{8}{12} \text{ft}\right) \alpha_{AB} = 0.66667 \alpha_{AB} \qquad (1)$$

For inextensible cord between disks B and C,

$$r_2 \alpha_{AB} = r_3 \alpha_C$$

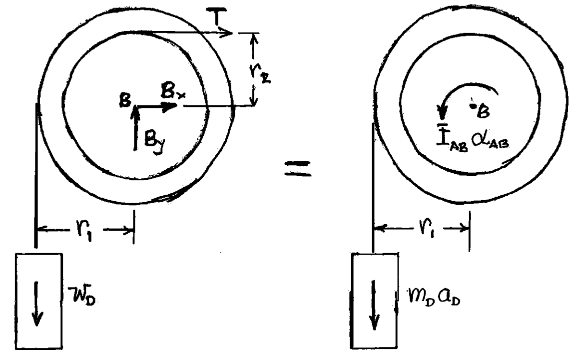
$$\alpha_C = \frac{r_2}{r_3} \alpha_{AB} = \left(\frac{6 \text{ in.}}{6 \text{ in.}}\right) \alpha_{AB} = \alpha_{AB}$$

For inextensible cord between disk C and cylinder E,

$$a_E = (a_t)_D = \left(\frac{6}{12} \text{ft}\right) \alpha_C = 0.5 \alpha_{AB} \qquad (2)$$

Kinetics.

Let T be the tension in the cord between disks B and C.



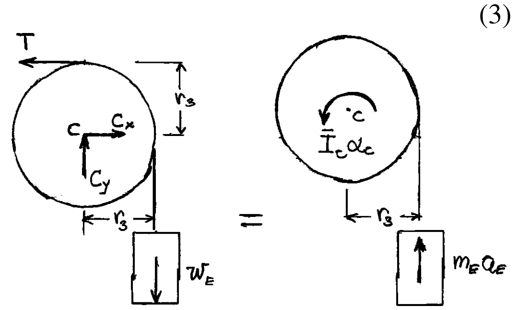
$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad r_1 W_D - r_2 T = I_{AB}\alpha_{AB} + r_1 m_D a_D$$

$$T = \frac{r_1}{r_2}W_D - \frac{I_{AB}\alpha_{AB}}{r_2} - \frac{r_1 m_D a_D}{r_2}$$

$$T = \frac{r_1}{r_2}W_D - \left[\frac{I_{AB}}{r_2} + \frac{m_D r_1^2}{r_2}\right]\alpha_{AB}$$

$$= \frac{8 \text{ in.}}{6 \text{ in.}}(15 \text{ lb}) - \left[\frac{0.184610}{6/12} + \frac{(0.46584)(8/12)^2}{6/12}\right]\alpha_{AB}$$

$$T = 20 \text{ lb} - 0.78330\alpha_{AB} \tag{3}$$



$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad r_3 T - r_3 W_E = I_C\alpha_C + r_3 m_E a_E$$

$$T = W_E + \left[\frac{I_C}{r_3} + m_E r_3\right]\alpha_{AB}$$

$$T = 18 \text{ lb} + \left[\frac{0.046584}{6/12} + (0.55901)(6/12)\right]\alpha_{AB}$$

$$T = 18 \text{ lb} + 0.37267\alpha_{AB} \tag{4}$$

Subtracting Eq. (4) from Eq. (3) to eliminate T,

$$0 = 2 \text{ lb} - 1.15597\alpha_{AB} \qquad \alpha_{AB} = 1.7301 \text{ rad/s}^2$$

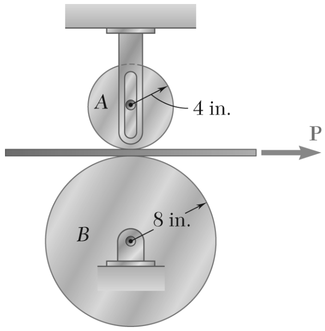
(*a*) *Acceleration of cylinder D.*

From Eq. (1), $\quad a_D = (0.66667)(7.7301) \qquad \mathbf{a}_D = 1.153 \text{ ft/s}^2 \downarrow$ ◀

(*b*) *Acceleration of cylinder E.*

From Eq. (2), $\quad a_E = (0.5)(1.7301) \qquad \mathbf{a}_E = 0.865 \text{ ft/s}^2 \uparrow$ ◀

PROBLEM 16.39



A belt of negligible mass passes between cylinders A and B and is pulled to the right with a force **P**. Cylinders A and B weigh, respectively, 5 and 20 lb. The shaft of cylinder A is free to slide in a vertical slot and the coefficients of friction between the belt and each of the cylinders are $\mu_s = 0.50$ and $\mu_k = 0.40$. For $P = 3.6$ lb, determine (*a*) whether slipping occurs between the belt and either cylinder, (*b*) the angular acceleration of each cylinder.

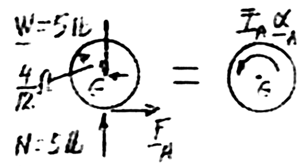
SOLUTION

<u>Assume that no slipping occurs</u>.

Then: $$a_{\text{belt}} = (4 \text{ in.})\alpha_A = (8 \text{ in.})\alpha_B \qquad \alpha_B = \frac{1}{2}\alpha_A \tag{1}$$

<u>Cylinder A</u>.

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F_A\left(\frac{4}{12}\text{ft}\right) = \bar{I}_A\alpha_A$$

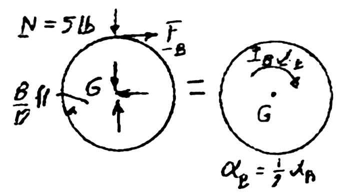


$$F_A\left(\frac{4}{12}\right) = \frac{1}{2}\frac{5}{g}\left(\frac{4}{12}\right)^2 \alpha_A$$

$$F_A = \frac{5}{6}\frac{\alpha_A}{g} \tag{2}$$

<u>Cylinder B</u>.

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F_B\left(\frac{8}{12}\text{ft}\right) = \bar{I}_B\alpha_B$$



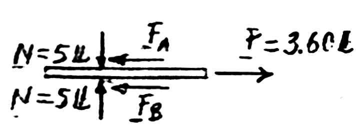
$$F_B\left(\frac{8}{12}\right) = \frac{1}{2}\frac{20}{g}\left(\frac{8}{12}\right)^2\left(\frac{1}{2}\alpha_A\right)$$

$$F_B = \frac{20}{6}\frac{\alpha_A}{g} \tag{3}$$

<u>Belt</u>

$$\xrightarrow{+} \Sigma F_A = 0: \quad P - F_A - F_B = 0 \tag{4}$$

$$3.60 - \frac{5}{6}\frac{\alpha_A}{g} - \frac{20}{6}\frac{\alpha_A}{g} = 0$$



$$\alpha_A = \frac{(3.60)6}{25}g$$

$$= 0.864g$$

$$\boldsymbol{\alpha}_A = 27.82 \text{ rad/s}^2 \circlearrowleft$$

Check that belt does not slip.

From (2): $$F_A = \frac{5}{6}(0.864) = 0.720 \text{ lb}$$

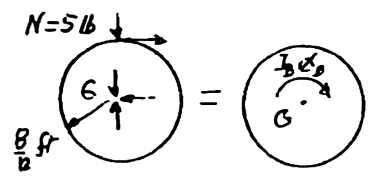
From (4): $$F_e = P - F_A = 3.60 - 0.720 = 2.88 \text{ lb}$$

But $$F_m = \mu_s N = 0.50(5 \text{ lb}) = 2.50 \text{ lb}$$

Since $F_e > F_m$, assumption is wrong.

<u>Slipping occurs between disk B and the belt</u>. ◀

<u>We redo analysis of B, assuming slipping</u>. $\left(\alpha_B \neq \frac{1}{2}\alpha_A\right)$



$$F_B = \mu_k N = 0.40(5 \text{ lb}) = 2 \text{ lb}$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad (2 \text{ lb})\left(\frac{8}{12} \text{ ft}\right) = \frac{1}{2}\frac{20}{g}\left(\frac{8}{12}\right)^2 \alpha_B$$

$$\alpha_B = 0.3g, \qquad \boldsymbol{\alpha}_B = 9.66 \text{ rad/s}^2 \circlearrowright \; ◀$$

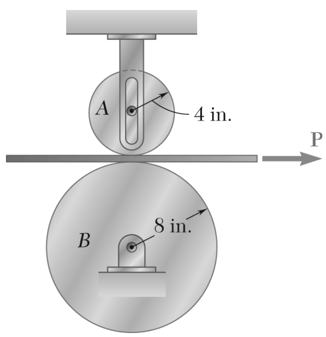
<u>Belt</u> Eq. (4):
$$P - F_A - F_B = 0$$
$$F_A = P - F_B$$
$$= 3.60 - 2$$
$$= 1.60 \text{ lb}$$

Since $F_A < F_m$, <u>There is no slipping between A and the belt</u>. ◀

Our analysis of disk A, therefore is valid. Using Eq. (2),

We have $$1.60 \text{ lb} = \frac{5}{6}\frac{\alpha_A}{g}$$

$$\alpha_A = 1.92g \qquad \boldsymbol{\alpha}_A = 61.8 \text{ rad/s}^2 \circlearrowright \; ◀$$



PROBLEM 16.40

Solve Problem 16.39 for $P = 2.00$ lb.

PROBLEM 16.39 A belt of negligible mass passes between cylinders A and B and is pulled to the right with a force **P**. Cylinders A and B weigh, respectively, 5 and 20 lb. The shaft of cylinder A is free to slide in a vertical slot and the coefficients of friction between the belt and each of the cylinders are $\mu_s = 0.50$ and $\mu_k = 0.40$. For $P = 3.60$ lb, determine (a) whether slipping occurs between the belt and either of the cylinders, (b) the angular acceleration of each cylinder.

SOLUTION

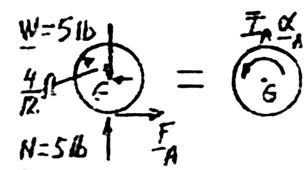
Assume that no slipping occurs.

Then:

$$a_{\text{belt}} = (4\text{ in.})\alpha_A = (8\text{ in.})\alpha_B \qquad \alpha_B = \frac{1}{2}\alpha_A \qquad (1)$$

Cylinder A

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F_A\left(\frac{4}{12}\text{ft}\right) = \bar{I}_A\alpha_A$$

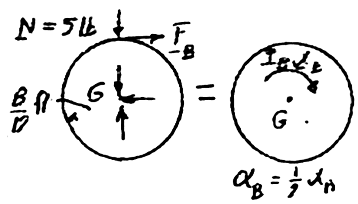


$$F_A\left(\frac{4}{12}\right) = \frac{1}{2}\frac{5}{g}\left(\frac{4}{12}\right)^2\alpha_A$$

$$F_A = \frac{5}{6}\frac{\alpha_A}{g} \qquad (2)$$

Cylinder B

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F_B\left(\frac{4}{12}\text{ft}\right) = \bar{I}_B\alpha_B$$



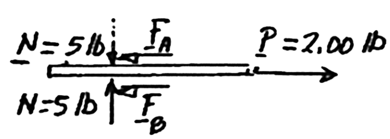
$$F_B\left(\frac{8}{12}\right) = \frac{1}{2}\frac{20}{g}\left(\frac{8}{12}\right)^2\left(\frac{1}{2}\alpha_A\right)$$

$$F_B = \frac{20}{6}\frac{\alpha_A}{g} \qquad (3)$$

Belt

$$\xrightarrow{+} \Sigma F_A = 0: \quad P - F_A - F_B = 0$$

$$2.00 - \frac{5}{6}\frac{\alpha_A}{g} - \frac{20}{6}\frac{\alpha_A}{g} = 0$$



$$\alpha_A = \frac{(2.00)6}{25}g$$

$$= 0.480g$$

$$\alpha_A = 15.46\text{ rad/s}^2 \circlearrowleft$$

From (2): $$F_A = \frac{5}{6}(0.480) = 0.400 \text{ lb}$$

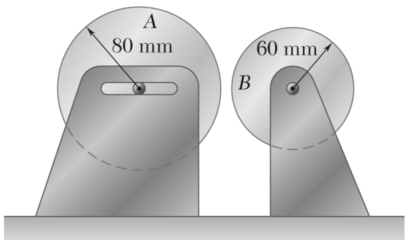
From (4): $$F_B = \frac{20}{6}(0.480) = 1.600 \text{ lb}$$

But $$F_M = \mu_s N = 0.50(5 \text{ lb}) = 2.50 \text{ lb}$$

Thus, F_A and F_B are both $< F_m$. Our assumption was right:

There is no slipping between cylinders and belt $\qquad \boldsymbol{\alpha}_A = 15.46 \text{ rad/s}^2 \circlearrowleft$ ◀

From (1): $\qquad \boldsymbol{\alpha}_B = 7.73 \text{ rad/s}^2 \circlearrowright$ ◀



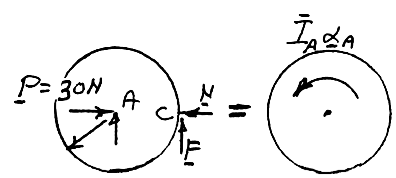
PROBLEM 16.41

Disk A has a mass of 6 kg and an initial angular velocity of 360 rpm clockwise; disk B has a mass of 3 kg and is initially at rest. The disks are brought together by applying a horizontal force of magnitude 20 N to the axle of disk A. Knowing that $\mu_k = 0.15$ between the disks and neglecting bearing friction, determine (a) the angular acceleration of each disk, (b) the final angular velocity of each disk.

SOLUTION

While slipping occurs, a friction force $F\uparrow$ is applied to disk A, and $F\downarrow$ to disk B.

Disk A:



$$I_A = \frac{1}{2} m_A r_A^2$$

$$= \frac{1}{2}(6 \text{ kg})(0.08 \text{ m})^2$$

$$= 0.0192 \text{ kg} \cdot \text{m}^2$$

$$\Sigma F: \quad N = P = 20 \text{ N}$$

$$F = \mu N = 0.15(20) = 3 \text{ N}$$

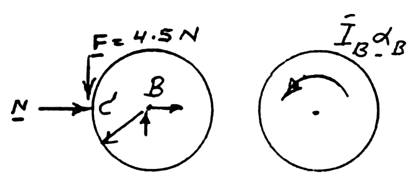
$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad F r_A = \bar{I}_A \alpha_A$$

$$(3 \text{ N})(0.08 \text{ m}) = (0.0192 \text{ kg} \cdot \text{m}^2)\alpha_A$$

$$\alpha_A = 10.227$$

$$\boldsymbol{\alpha}_A = 12.50 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

Disk B:



$$\bar{I}_B = \frac{1}{2} m_B r_B^2$$

$$= \frac{1}{2}(3 \text{ kg})(0.06 \text{ m})^2$$

$$= 0.0054 \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad F r_B = \bar{I}_B \alpha_B$$

$$(3 \text{ N})(0.06 \text{ m}) = (0.0054 \text{ kg} \cdot \text{m}^2)\alpha_B$$

$$\alpha_B = 33.333 \text{ rad/s}^2$$

$$\boldsymbol{\alpha}_B = 33.3 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

$$(\omega_A)_0 = 360 \text{ rpm} = 12\pi \text{ rad/s} \circlearrowright$$

PROBLEM 16.41 (Continued)

Sliding stops when $\mathbf{v}_C = \mathbf{v}_{C'}$ or $\omega_A r_A = \omega_B r_B$

$$r_A[(\omega_A)_0 - \alpha_A t] = r_B \alpha_B t$$

$$(0.08\text{ m})[12\pi\text{ rad/s} - (12.5\text{ rad/s}^2)t] = (0.06\text{ m})(33.333\text{ rad/s}^2)t$$

$$t = 1.00531\text{ s}$$

$$+\circlearrowleft\ \omega_A = (\omega_A)_0 - \alpha_A t$$

$$= 12\pi\text{ rad/s} - (12.5\text{ rad/s}^2)(1.00531\text{ s})$$

$$= 25.132\text{ rad/s}$$

$$\omega_A = (25.132\text{ rad/s})$$

$$= 240.00\text{ rpm}$$

or $\boldsymbol{\omega}_A = 240\text{ rpm} \circlearrowright$ ◀

$$+\circlearrowleft\ \omega_B = \alpha_B t$$

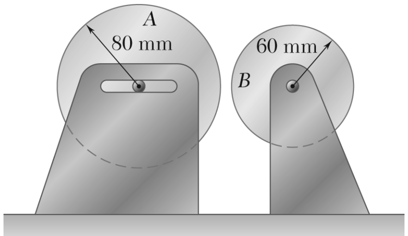
$$= 33.333\text{ rad/s}^2(1.00531\text{ s})$$

$$= 33.510\text{ rad/s}$$

$$\omega_B = (33.510\text{ rad/s})$$

$$= 320.00\text{ rpm}$$

or $\boldsymbol{\omega}_B = 320\text{ rpm} \circlearrowleft$ ◀



PROBLEM 16.42

Solve Problem 16.41, assuming that initially disk A is at rest and disk B has an angular velocity of 360 rpm clockwise.

PROBLEM 16.41 Disk A has a mass of 6 kg and an initial angular velocity of 360 rpm clockwise; disk B has a mass of 3 kg and is initially at rest. The disks are brought together by applying a horizontal force of magnitude 20 N to the axle of disk A. Knowing that $\mu_k = 0.15$ between the disks and neglecting bearing friction, determine (*a*) the angular acceleration of each disk, (*b*) the final angular velocity of each disk.

SOLUTION

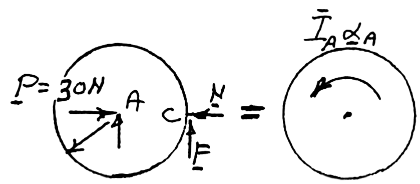
While slipping occurs, a friction force $F\uparrow$ is applied to disk A, and $F\downarrow$ to disk B.

Disk A:

$$I_A = \frac{1}{2} m_A r_A^2$$

$$= \frac{1}{2}(6 \text{ kg})(0.08 \text{ m})^2$$

$$= 0.0192 \text{ kg} \cdot \text{m}^2$$



$$\Sigma F: \quad N = P = 20 \text{ N}$$

$$F = \mu N = 0.15(20) = 3 \text{ N}$$

$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad F r_A = \bar{I}_A \alpha_A$$

$$(3 \text{ N})(0.08 \text{ m}) = (0.0192 \text{ kg} \cdot \text{m}^2)\alpha_A$$

$$\alpha_A = 10.227$$

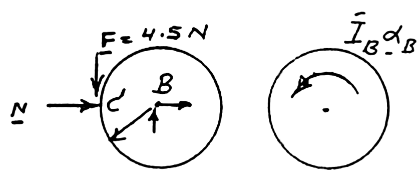
$$\boldsymbol{\alpha}_A = 12.50 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

Disk B:

$$\bar{I}_B = \frac{1}{2} m_B r_B^2$$

$$= \frac{1}{2}(3 \text{ kg})(0.06 \text{ m})^2$$

$$= 0.0054 \text{ kg} \cdot \text{m}^2$$



$$+\circlearrowright \Sigma M_A = \Sigma (M_B)_{\text{eff}}: \quad F r_B = \bar{I}_B \alpha_B$$

$$(3 \text{ N})(0.06 \text{ m}) = (0.0054 \text{ kg} \cdot \text{m}^2)\alpha_B$$

$$\alpha_B = 33.333 \text{ rad/s}^2$$

$$\boldsymbol{\alpha}_B = 33.3 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

Sliding starts when $\mathbf{v}_C = \mathbf{v}_{C'}$. That is when

$$\omega_A r_A = \omega_B r_B$$
$$(\alpha_A t) r_A = [(\omega_B)_0 - \alpha_B t] r_B$$
$$[(12.5 \text{ rad/s}^2)t](0.08 \text{ m}) = [12\pi \text{ rad/s} - (33.333 \text{ rad/s}^2)(t)](0.06 \text{ m})$$
$$t = 4.02124 \qquad t = 0.75398 \text{ s}$$

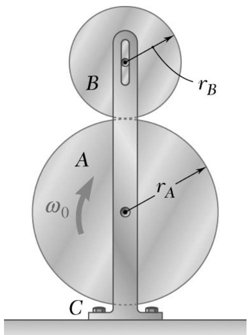
$$\omega_A = \alpha_A t = (12.5 \text{ rad/s}^2)(0.75398 \text{ s}) = 9.4248 \text{ rad/s}$$

$$+\ \omega_A = (9.4248 \text{ rad/s}) = 90.00 \text{ rpm}$$

or $\qquad \boldsymbol{\omega}_A = 90.0 \text{ rpm}$ ◀

$$+\ \omega_B = (\omega_B)_0 - \alpha_B t$$
$$= 12\pi \text{ rad/s} - (33.333 \text{ rad/s}^2)(0.75398 \text{ s})$$
$$= 12.566 \text{ rad/s}$$
$$\omega_B = (12.566 \text{ rad/s})$$
$$= 120.00 \text{ rpm}$$

or $\qquad \boldsymbol{\omega}_B = 120.0 \text{ rpm}$ ◀

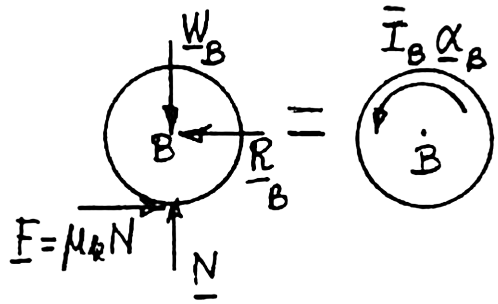


PROBLEM 16.43

Disk A has a mass m_A = 3 kg, a radius r_A = 300 mm, and an initial angular velocity $\omega_0 = 300$ rpm clockwise. Disk B has a mass m_B = 1.6 kg, a radius r_B = 180 mm, and is at rest when it is brought into contact with disk A. Knowing that $\mu_k = 0.35$ between the disks and neglecting bearing friction, determine (*a*) the angular acceleration of each disk, (*b*) the reaction at the support C.

SOLUTION

(*a*) Disk B.



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - W_B = 0$$

$$N = W_B = m_B g$$

Thus, $\qquad F = \mu_k N = \mu_k m_B g$

$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad F r_B = \bar{I}_B \alpha_B$$

$$\mu_k m_B g r_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

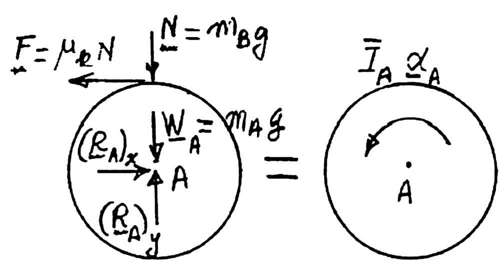
$$\boldsymbol{\alpha}_B = \frac{2\mu_k g}{r_B} \circlearrowright \qquad (1)$$

For the given data: $\alpha_B = \dfrac{2(0.35)(9.81\ \text{m/s}^2)}{0.180\ \text{m}} = 38.15\ \text{rad/s}^2$ $\qquad \boldsymbol{\alpha}_B = 38.2\ \text{rad/s}^2 \circlearrowright$ ◀

$$W_B = m_B g = (1.6\ \text{kg})(9.81\ \text{m/s}^2) = 15.696\ \text{N}$$
$$F = \mu_k m_B g = (0.35)(15.696) = 5.4936\ \text{N}$$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F - R_B = 0$ $\qquad \mathbf{R}_B = 5.4936\ \text{N} \longleftarrow$ ◀

Disk A:



$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad F r_A = \bar{I}_A \alpha_A$$

$$\mu_k m_B g r_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$\boldsymbol{\alpha}_A = \frac{2\mu_k g}{r_A} \frac{m_B}{m_A} \circlearrowright \qquad (2)$$

For the given data: $\alpha_A = \dfrac{2(0.35)(9.81\text{ m/s}^2)}{0.300\text{ m}}\dfrac{1.6\text{ kg}}{4\text{ kg}} = 9.156\text{ rad/s}^2$ $\qquad$ $\boldsymbol{\alpha}_A = 9.16\text{ rad/s}^2 \circlearrowright$ ◀

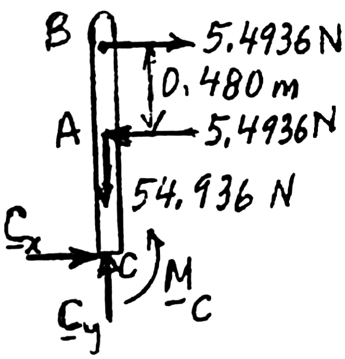
$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}:\quad (R_A)_x - F = 0$ $\qquad$ $(\mathbf{R}_A)_x = 5.4936\text{ N} \longrightarrow$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}:\quad (R_A)_y - m_A g - m_B g = 0$

$$(R_A)_y = (m_A + m_B)g = (4\text{ kg} + 1.6\text{ kg})(9.81\text{ m/s}^2)$$
$$= 54.936\text{ N}$$

$(\mathbf{R}_A)_y = 54.936\text{ N}\uparrow$

(*b*) *Reaction at C.*

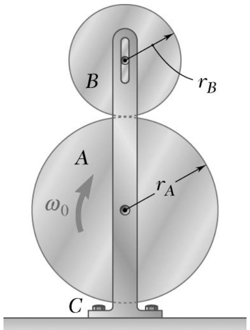


$\xrightarrow{+} \Sigma F_x = 0:\quad C_x = 0$

$+\uparrow \Sigma F_y = 0:\quad C_y - 54.936\text{ N} = 0$

$C_y = 54.936\text{ N}$ $\qquad$ $\mathbf{C} = 54.9\text{ N}\uparrow$ ◀

$+\circlearrowright \Sigma M_C = 0:\quad M_C - (5.4936\text{ N})(0.480\text{ m}) = 0$ $\qquad$ $\mathbf{M}_C = 2.64\text{ N}\cdot\text{m} \circlearrowright$ ◀



PROBLEM 16.44

Disk B is at rest when it is brought into contact with disk A, which has an initial angular velocity ω_0. (*a*) Show that the final angular velocities of the disks are independent of the coefficient of friction μ_k between the disks as long as $\mu_k \neq 0$. (*b*) Express the final angular velocity of disk A in terms of ω_0 and the ratio m_A/m_B of the masses of the two disks.

SOLUTION

(*a*) Disk B.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - w_B = 0$$

$$N = w_B = m_B g$$

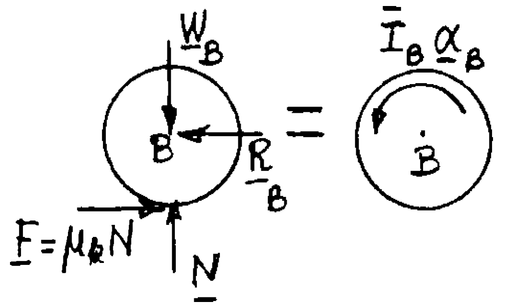
Thus,

$$F = \mu_k N = \mu_k m_B g$$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad F r_B = \bar{I}_B \alpha_B$$

$$\mu_k m_B g r_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$\boldsymbol{\alpha}_B = \frac{2\mu_k g}{r_B} \circlearrowright \qquad (1)$$

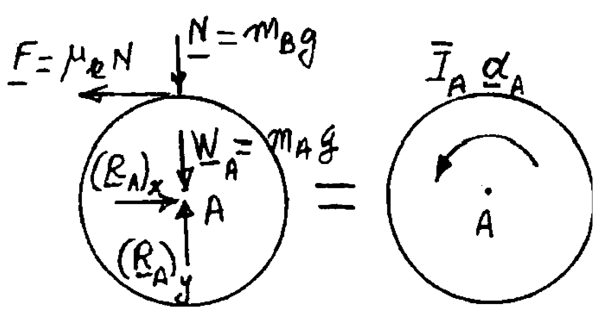


Disk A.

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad F r_A = \bar{I}_A \alpha_A$$

$$\mu_k m_B g r_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$\boldsymbol{\alpha}_A = \frac{2\mu_k g}{r_A} \frac{m_B}{m_A} \circlearrowright \qquad (2)$$



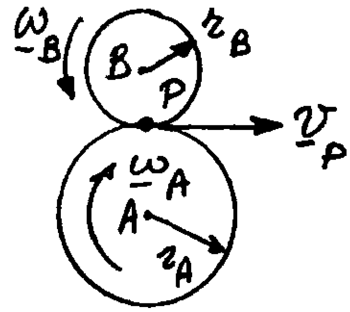
Disk A.

$$\omega_A = \omega_0 - \alpha_A t = \omega_0 - \frac{2\mu_k g}{r_A} \frac{m_B}{m_A} t \circlearrowright \qquad (3)$$

Disk B.

$$\omega_B = \alpha_B t = \frac{2\mu_k g}{r_B} t \circlearrowright \qquad (4)$$

When disks have stopped slipping:



$$v_P = \omega_A r_A = \omega_B r_B$$

$$\omega_0 r_A - (2\mu_k g)\frac{m_B}{m_A}t = 2\mu_k g t$$

$$t = \frac{\omega_0 r_A}{2\mu_k g}\frac{1}{1+\frac{m_B}{m_A}}$$

$$t = \frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A+m_B} \quad (5)$$

Substituting for t from (5) into (3) and (4):

$$\omega_A = \omega_0 - \frac{2\mu_k g}{r_A}\frac{m_B}{m_A}\frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A+m_B} = \omega_0 - \frac{\omega_0 m_B}{m_A+m_B}$$

$$\omega_A = \omega_0 \frac{m_A+m_B-m_B}{m_A+m_B} \qquad \omega_A = \omega_0 \frac{m_A}{m_A+m_B} \circlearrowright \quad (6)$$

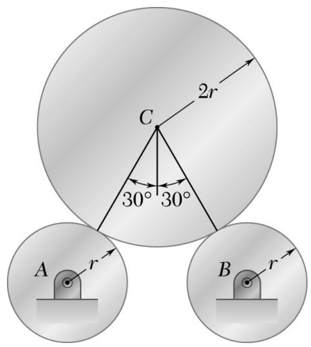
$$\omega_B = \frac{2\mu_k g}{r_A}\frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A+m_B} \qquad \omega_B = \omega_0 \frac{r_A}{r_B}\frac{m_A}{m_A+m_B} \circlearrowleft \quad (7)$$

(*a*) From Eqs. (6) and (7), it is apparent that ω_A and ω_B are independent of μ_k. However, if $\mu_k = 0$, we have from Eqs. (3) and (4) $\omega_A = \omega_0 \circlearrowright$ and $\omega_B = 0$.

(*b*) We can write (6) in the form $\omega_A = \omega_0/(1+m_B/m_A) \circlearrowright$ ◀

which shows that ω_A depends only upon ω_0 and m_B/m_A.

PROBLEM 16.45



Cylinder A has an initial angular velocity of 720 rpm clockwise, and cylinders B and C are initially at rest. Disks A and B each weigh 5 lb and have a radius $r = 4$ in. Disk C weighs 20 lb and has a radius of 8 in. The disks are brought together when C is placed gently onto A and B. Knowing that $\mu_k = 0.25$ between A and C and no slipping occurs between B and C, determine (a) the angular acceleration of each disk, (b) the final angular velocity of each disk.

SOLUTION

Assume Point C, the center of cylinder C, does not move. This is true provided the cylinders remain in contact as shown. Slipping occurs initially between disks A and C and ceases when the tangential velocities at their contact point are equal. We first determine the angular accelerations of each disk while slipping occurs.

Masses and moments of inertia:

$$m_A = m_B = \frac{W_A}{g} = \frac{5\text{ lb}}{32.2\text{ ft/s}^2} = 0.15528\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_C = \frac{W_C}{g} = \frac{20\text{ lb}}{32.2\text{ ft/s}^2} = 0.62112\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$I_A = I_B = \frac{1}{2}m_A r^2 = \frac{1}{2}(0.15528)\left(\frac{4}{12}\right)^2 = 0.0086266\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$I_C = \frac{1}{2}m_C(2r)^2 = \frac{1}{2}(0.62112)\left(\frac{8}{12}\right)^2 = 0.138027\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinematics: No slipping at contact BC.

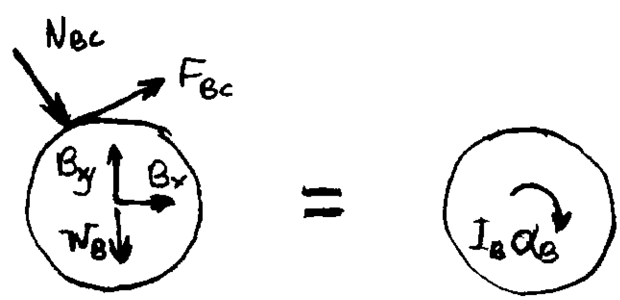
$$(\mathbf{a}_t)_{BC} = (a_t)_{BC} \measuredangle 30°$$

$$(a_t)_{BC} = r\alpha_B = 2r\alpha_C \qquad \alpha_B = 2\alpha_C \tag{1}$$

Friction condition: $$F_{AC} = \mu_k N_{AC} \tag{2}$$

Kinetics:

Disk B: $$\circlearrowright \Sigma F_B = \Sigma (F_B)_{\text{eff}}:$$

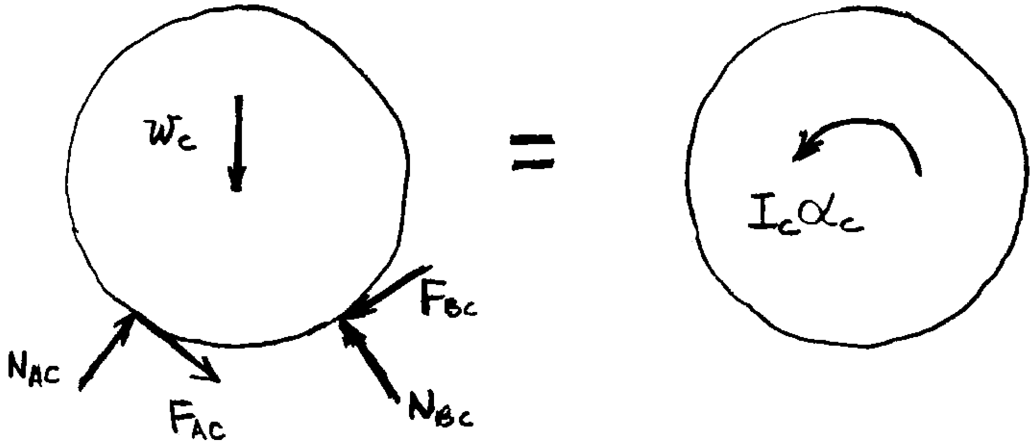


$$F_{BC}r = I_B\alpha_B$$

$$F_{BC} = \frac{I_B}{r}\alpha_B = \frac{2I_B}{r}\alpha_C$$

$$F_{BC} = \frac{(2)(0.0086266)}{4/12}\alpha_C$$

$$= 0.051760\alpha_C \tag{3}$$



Disk C:

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad F_{AC}(2r) - F_{BC}(2r) = I_C\alpha_C$$

$$F_{AC} = F_{BC} + \frac{I_C}{2r}\alpha_C$$

$$= 0.051760\alpha_C + \frac{0.138027}{8/12}\alpha_C$$

$$F_{AC} = 0.25880\alpha_C \qquad (4)$$

From Eq. (2), $\quad N_{AC} = \dfrac{F_{AC}}{0.25} = 1.03520\alpha_C \qquad (5)$

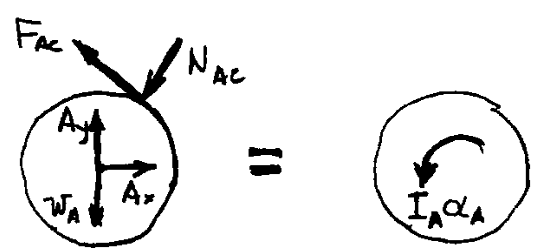
$$+\nearrow 30°\Sigma F = \Sigma F_{\text{eff}}: \quad W_B \sin 30° + F_{BC} - F_{AC}\cos 60° - N_{AC}\sin 60° = 0$$

$$(20)\sin 30° + (0.051760 - 0.25880\cos 60° - 1.03520\sin 60°)\alpha_C = 0$$

$$10 - 0.97415\alpha_C = 0$$

$$\alpha_C = 10.2654 \text{ rad/s}^2$$

$$F_{BC} = (0.051760)(10.2654) = 0.53134 \text{ lb.}$$
$$F_{AC} = (0.25880)(10.2654) = 2.6567 \text{ lb.}$$
$$N_{AC} = (1.03520)(10.2654) = 10.6267 \text{ lb.}$$



Check that $N_{BC} > 0$.

$$+\searrow 60°\Sigma F = \Sigma F_{\text{eff}}: \quad N_{BC} - W_C\cos 30° + N_{AC}\cos 60° - F_{AC}\sin 60° = 0$$

$$N_{BC} = (20 \text{ lb})\cos 30° - (10.6267 \text{ lb})\cos 60° + (2.6567 \text{ lb})\sin 60° = 0$$

$$N_{BC} = 14.3079 \text{ lb.}$$

Disk A:

$$+\Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad F_{AC}r = I_A\alpha_A$$

$$\alpha_A = \frac{F_{AC}r}{I_A} = \frac{(2.6567)(4/12)}{0.0086266} = 102.69 \text{ rad/s}^2$$

(*a*) *Angular accelerations of disks.* $\boldsymbol{\alpha}_A = 102.7 \text{ rad/s}^2$ ◀

From Eq. (1), $\boldsymbol{\alpha}_B = 20.5 \text{ rad/s}^2$ ◀

$\boldsymbol{\alpha}_C = 10.27 \text{ rad/s}^2$ ◀

(*b*) *Final angular velocities.*

Disk A: $\omega_0 = 720 \text{ rpm} = 24\pi \text{ rad/s}$

$$\omega_A = \omega_0 - \alpha_A t$$
$$= 24\pi - 102.69t$$

$$(v_t)_{AC} = r\omega_A = \frac{4}{12}(24\pi - 102.69t)$$

$$(\mathbf{v}_t)_{AC} = (8\pi - 34.23) \text{ ft/s} \quad 30°$$

Disk C: $\omega_C = \alpha_C t = 10.2654t$

$$(\mathbf{v}_t)_{CA} = 2r\omega_C = \left(\frac{8}{12}\right)(10.2654)t$$

$$(\mathbf{v}_t)_{CA} = 6.8436t \quad 30°$$

Time when tangential velocities are equal.

$$8\pi - 34.23t = 6.8436t \qquad t = 0.6119 \text{ s}$$

$\omega_A = 24\pi - (102.69)(0.6119) = 12.562 \text{ rad/s}$ $\boldsymbol{\omega}_A = 120.0 \text{ rpm}$ ◀

$\omega_C = (10.2654)(0.6119) = 6.2813 \text{ rad/s}$ $\boldsymbol{\omega}_C = 60.0 \text{ rpm}$ ◀

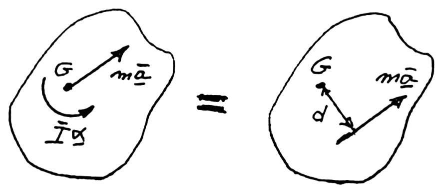
$\boldsymbol{\omega}_B = 120.0 \text{ rpm}$ ◀

PROBLEM 16.46

Show that the system of the effective forces for a rigid slab in plane motion reduces to a single vector, and express the distance from the mass center G of the slab to the line of action of this vector in terms of the centroidal radius of gyration $\bar{k}$ of the slab, the magnitude $\bar{a}$ of the acceleration of G, and the angular acceleration α.

SOLUTION

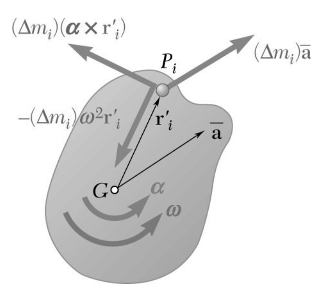
We know that the system of effective forces can be reduced to the vector $m\bar{\mathbf{a}}$ at G and the couple $\bar{I}\boldsymbol{\alpha}$. We further know from Chapter 3 on statics that a force-couple system in a plane can be further reduced to a single force.



The perpendicular distance d from G to the line of action of the single vector $m\bar{\mathbf{a}}$ is expressed by writing

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad \bar{I}\alpha = (m\bar{a})d$$

$$d = \frac{\bar{I}\alpha}{m\bar{a}} = \frac{m\bar{k}^2\alpha}{m\bar{a}} \qquad d = \frac{\bar{k}^2\alpha}{\bar{a}} \blacktriangleleft$$

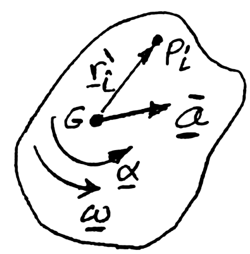


PROBLEM 16.47

For a rigid slab in plane motion, show that the system of the effective forces consists of vectors $(\Delta m_i)\,\bar{\mathbf{a}}$, $-(\Delta m_i)\omega^2\mathbf{r}_i'$, and $(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i')$ attached to the various particles P_i of the slab, where $\bar{\mathbf{a}}$ is the acceleration of the mass center G of the slab, $\boldsymbol{\omega}$ is the angular velocity of the slab, $\boldsymbol{\alpha}$ is its angular acceleration, and $\mathbf{r}_i'$ denotes the position vector of the particle P_i, relative to G. Further show, by computing their sum and the sum of their moments about G, that the effective forces reduce to a vector $m\bar{\mathbf{a}}$ attached at G and a couple $\bar{I}\boldsymbol{\alpha}$.

SOLUTION

Kinematics: The acceleration of P_L is



$$\mathbf{a}_i = \bar{\mathbf{a}} + \mathbf{a}_{P_i/G}$$

$$\mathbf{a}_i = \bar{\mathbf{a}} + \boldsymbol{\alpha}\times\mathbf{r}_i' + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}_i')$$

$$= \bar{\mathbf{a}} + \boldsymbol{\alpha}\times\mathbf{r}_i' - \omega^2\mathbf{r}_i'$$

Note: that $\boldsymbol{\alpha}\times\mathbf{r}_i'$ is $\perp$ to $\mathbf{r}_i'$

Thus, the effective forces are as shown in Figure P16.47 (also shown above). We write

$$(\Delta m_i)\mathbf{a}_i = (\Delta m_i)\bar{\mathbf{a}} + (\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i') - (\Delta m_i)\omega^2 r_i'$$

The sum of the effective forces is

$$\Sigma(\Delta m_i)\mathbf{a}_i = \Sigma(\Delta m_i)\bar{\mathbf{a}} + \Sigma(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i') - \Sigma(\Delta m_i)\omega^2\mathbf{r}_i'$$

$$\Sigma(\Delta m_i)\mathbf{a}_i = \bar{\mathbf{a}}\,\Sigma(\Delta m_i) + \boldsymbol{\alpha}\times\Sigma(\Delta m_i)\,\mathbf{r}_i' - \omega^2\Sigma(\Delta m_i)\mathbf{r}_i'$$

We note that $\Sigma(\Delta m_i) = m.$ And since G is the mass center,

$$\Sigma(\Delta m_i)\mathbf{r}_i' = m\,\bar{r}_i' = 0$$

Thus, $\qquad \Sigma(\Delta m_i)\mathbf{a}_i = m\bar{\mathbf{a}} \qquad (1)$

The sum of the moments about G of the effective forces is:

$$\Sigma(\mathbf{r}_i'\times\Delta m_i\mathbf{a}_i) = \Sigma\mathbf{r}_i'\times\Delta m_i\bar{\mathbf{a}} + \Sigma\mathbf{r}_i'\times(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i') - \Sigma\mathbf{r}_i'\times(\Delta m_i)\omega^2\mathbf{r}_i'$$

$$\Sigma(\mathbf{r}_i'\times\Delta m_i\mathbf{a}_i) = (\Sigma\mathbf{r}_i'\Delta m_i)\,\bar{\mathbf{a}} + \Sigma[\mathbf{r}_i'\times(\boldsymbol{\alpha}\times\mathbf{r}_i')\Delta m_i] - \omega^2\Sigma(\mathbf{r}_i'\times\mathbf{r}_i')\Delta m_i$$

Since G is the mass center, $\qquad \Sigma\mathbf{r}_i'\Delta m_i = 0$

Also, for each particle, $\qquad \mathbf{r}_i'\times\mathbf{r}_i' = 0$

Thus, $\qquad \Sigma(\mathbf{r}_i'\times\Delta m_i\mathbf{a}_i) = \Sigma[\mathbf{r}_i'\times(\boldsymbol{\alpha}_i\times\mathbf{r}_i')\Delta m_i]$

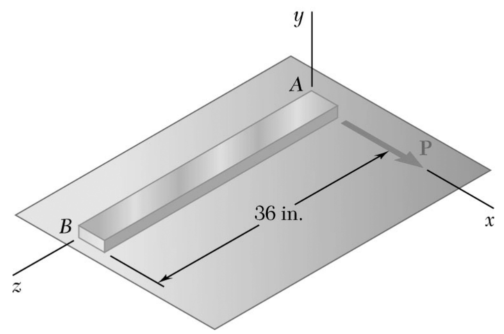
PROBLEM 16.47 (Continued)

Since $\alpha \perp \mathbf{r}_i'$, we have $\mathbf{r}_i' \times (\boldsymbol{\alpha} \times \mathbf{r}_i') = r_i^2 \alpha$ and

$$\Sigma(\mathbf{r}_{ii}' \times \Delta m_i \mathbf{a}_i) = \Sigma r_i'^2 (\Delta m_i)\alpha$$
$$= (\Sigma r_i'^2 \Delta m_i)\boldsymbol{\alpha}$$

Since $\Sigma r_i'^2 \Delta m_i = \bar{I}$ $\qquad \Sigma(\mathbf{r}_i' \times \Delta m_i \mathbf{a}_i) = \bar{I}\alpha$ (2)

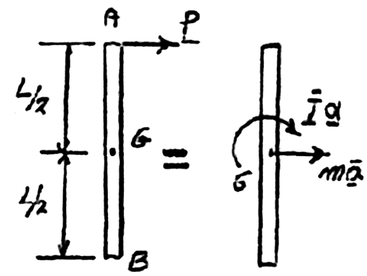
From Eqs. (1) and (2) we conclude that system of effective forces reduce to $m\bar{\mathbf{a}}$ attached at G and a couple $\bar{I}\boldsymbol{\alpha}$.



PROBLEM 16.48

A uniform slender rod AB rests on a frictionless horizontal surface, and a force $\mathbf{P}$ of magnitude 0.25 lb is applied at A in a direction perpendicular to the rod. Knowing that the rod weighs 1.75 lb, determine (a) the acceleration of Point A, (b) the acceleration of Point B, (c) the location of the point on the bar that has zero acceleration.

SOLUTION



$$m = \frac{W}{g}$$

$$I = \frac{1}{12}\frac{W}{g}L^2$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P = m\bar{a} = \frac{W}{g}\bar{a}$$

$$\bar{a} = \frac{P}{W}g = \frac{0.25 \text{ lb}}{1.75 \text{ lb}}g = \frac{1}{7}g \qquad \bar{\mathbf{a}} = \frac{1}{7}g \longrightarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad P\frac{L}{2} = \bar{I}\alpha = \frac{1}{12}\frac{W}{g}L^2\alpha$$

$$\alpha = 6\frac{P}{W}\frac{5}{L} = 6\frac{0.25 \text{ lb}}{1.75 \text{ lb}}\cdot\frac{g}{L} = \frac{6}{7}\frac{g}{L} \qquad \boldsymbol{\alpha} = \frac{6}{7}\frac{g}{L}\circlearrowright$$

We calculate the accelerations immediately after the force is applied. After the rod acquires angular velocity, there will be additional normal accelerations.

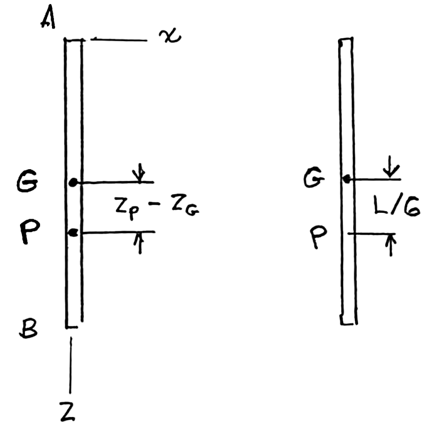
(a) <u>Acceleration of Point A</u>.

$$\xrightarrow{+} \mathbf{a}_A = \bar{\mathbf{a}} + \frac{L}{2}\alpha = \frac{1}{7}g + \frac{L}{2}\cdot\frac{6}{7}g = \frac{4}{7}g = \frac{4}{7}(32.2 \text{ ft/s}^2) \qquad \mathbf{a}_A = 18.40 \text{ ft/s}^2 \longrightarrow \blacktriangleleft$$

(b) <u>Acceleration of Point B</u>.

$$\xrightarrow{+} \mathbf{a}_B = \bar{\mathbf{a}} - \frac{L}{2}\alpha = \frac{1}{7}g - \frac{L}{2}\cdot\frac{6}{7}g = -\frac{2}{7}g = -\frac{2}{7}(32.2 \text{ ft/s}^2) \qquad \mathbf{a}_B = 9.20 \text{ ft/s}^2 \longleftarrow \blacktriangleleft$$

(*c*) Point of zero acceleration.



$$a_P = 0$$

$$\bar{a} - (z_P - z_G)\alpha = 0$$

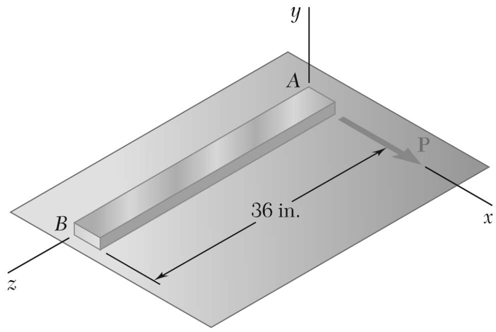
$$z_P - z_G = \frac{\bar{a}}{\alpha} = \frac{\frac{1}{7}g}{\frac{6}{7}\cdot\frac{g}{L}} = \frac{1}{6}L$$

Since $z_G = \dfrac{1}{2}L$

$$z_P = \frac{1}{2}L + \frac{1}{6}L = \frac{2}{3}L$$

$$z_P = \frac{2}{3}(36 \text{ in.})$$

$z_P = 24.0$ in. ◀



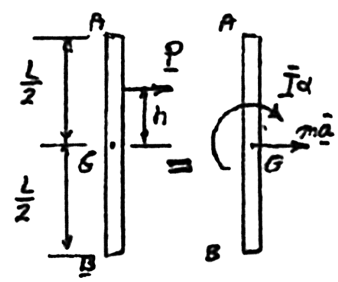
PROBLEM 16.49

(*a*) In Problem 16.48, determine the point of the rod *AB* at which the force **P** should be applied if the acceleration of Point *B* is to be zero. (*b*) Knowing that $P = 0.25$ lb, determine the corresponding acceleration of Point *A*.

SOLUTION

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_X)_{\text{eff}}: \quad P = m\bar{a} = \frac{W}{g}\bar{a}$$

$$\mathbf{a} = \frac{P}{W} g \longrightarrow$$



$$\circlearrowleft^{+} \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Ph = \bar{I}\alpha = \frac{1}{12}\frac{W}{g}L^2\alpha$$

$$\boldsymbol{\alpha} = \frac{12Ph}{WL^2} g \circlearrowright$$

(*a*) Position of force *P*.

$$\xrightarrow{+} \mathbf{a}_B = \bar{\mathbf{a}} - \frac{L}{2}\alpha$$

$$0 = \frac{P}{W}g - \frac{L}{2}\cdot\frac{12Ph}{WL^2}g$$

$$h = \frac{L}{6} = \frac{36 \text{ in.}}{6} = 6 \text{ in.}$$

Thus, *P* is located 12 in. from end *A*. ◀

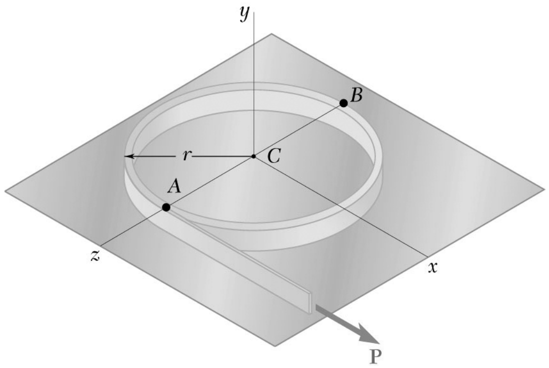
For $\quad h = \dfrac{L}{6}: \quad \alpha = \dfrac{12P\left(\frac{L}{6}\right)}{WL^2} g = 2\dfrac{P}{W}\cdot\dfrac{g}{L} \circlearrowright$

(*b*) Acceleration of Point *A*.

$$\xrightarrow{+} a_A = \bar{a} + \frac{L}{2}\alpha = \frac{P}{W}g + \frac{L}{2}\cdot 2\frac{P}{W}\frac{g}{L} = 2\frac{P}{W}g$$

$$a_A = 2\frac{0.25 \text{ lb}}{1.75 \text{ lb}}(32.2 \text{ ft/s}^2)$$

$\mathbf{a}_A = 9.20 \text{ ft/s}^2 \longrightarrow$ ◀

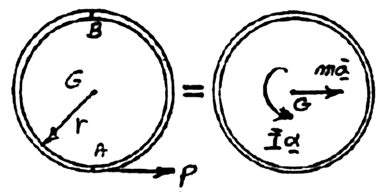


PROBLEM 16.50

A force **P** of magnitude 3 N is applied to a tape wrapped around a thin hoop of mass 2.4 kg. Knowing that the body rests on a frictionless horizontal surface, determine the acceleration of (*a*) Point *A*, (*b*) Point *B*.

SOLUTION

Hoop: $\bar{I} = mr^2$



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P = m\bar{a}$$

$$\bar{\mathbf{a}} = \frac{P}{m} \longrightarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Pr = \bar{I}\alpha = mr^2\alpha$$

$$\boldsymbol{\alpha} = \frac{P}{mr} \circlearrowright$$

(*a*) <u>Acceleration of Point *A*</u>.

$$\xrightarrow{+} a_A = \bar{a} + r\alpha = \frac{P}{m} + r\left(\frac{P}{mr}\right) = 2\frac{P}{m}$$

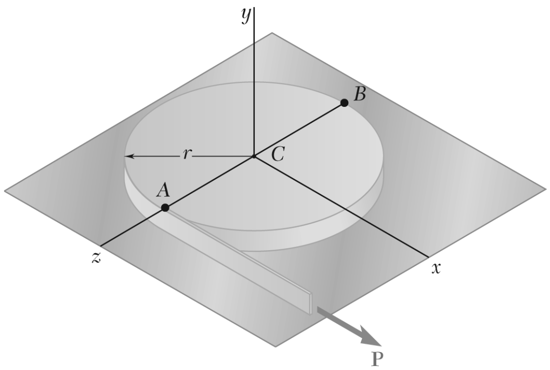
$$a_A = 2\frac{3\text{ N}}{2.4\text{ kg}} = 2.5\text{ m/s}^2$$

$$\mathbf{a}_A = 2.50\text{ m/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) <u>Acceleration of Point *B*</u>.

$$\xrightarrow{+} a_B = \bar{a} - r\alpha = \frac{P}{m} - r\left(\frac{P}{mr}\right) = 0$$

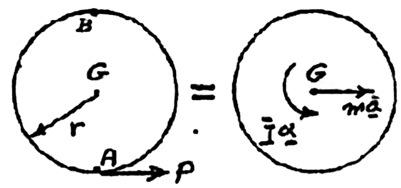
$$\mathbf{a}_B = 0 \blacktriangleleft$$



PROBLEM 16.51

A force **P** is applied to a tape wrapped around a uniform disk that rests on a frictionless horizontal surface. Show that for each 360° rotation of the disk the center of the disk will move a distance πr.

SOLUTION



Disk:
$$\bar{I} = \frac{1}{2}mr^2$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P = m\bar{a}$$

$$\bar{\mathbf{a}} = \frac{P}{m} \longrightarrow$$

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad Pr = \bar{I}\alpha$$

$$Pr = \frac{1}{2}mr^2\alpha$$

$$\alpha = \frac{2P}{mr} \circlearrowleft$$

Let t_1 be time required for 360° rotation.

$$\theta = \frac{1}{2}\alpha t_1^2$$

$$2\pi \text{ rad} = \frac{1}{2}\left(\frac{2P}{mr}\right)t_1^2$$

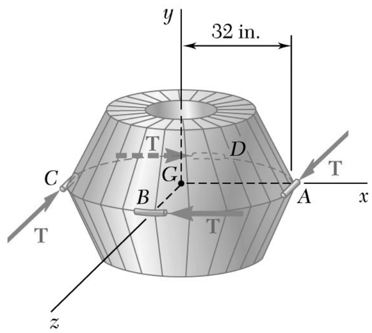
$$t_1^2 = \frac{2\pi mr}{P}$$

Let x_1 = distance G moves during 360° rotation.

$$x_1 = \frac{1}{2}\bar{a}t_1^2 = \frac{1}{2}\frac{P}{m}\left(\frac{2\pi mr}{P}\right)$$

$$x_1 = \pi r$$

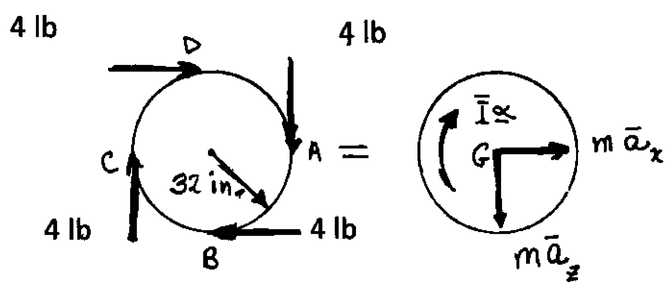
Q.E.D. ◀



PROBLEM 16.52

A 250-lb satellite has a radius of gyration of 24 in. with respect to the y axis and is symmetrical with respect to the zx plane. Its orientation is changed by firing four small rockets A, B, C, and D, each of which produces a 4-lb thrust $\mathbf{T}$ directed as shown. Determine the angular acceleration of the satellite and the acceleration of its mass center G (*a*) when all four rockets are fired, (*b*) when all rockets except D are fired.

SOLUTION



$$W = 250 \text{ lb}, \qquad m = \frac{W}{g} \qquad \bar{I} = mk_y^2 = \left(\frac{250 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(\frac{24 \text{ in.}}{12}\right)^2 = 31.056 \text{ slug}\cdot\text{ft}^2$$

(*a*) With all four rockets fired:

$$\Sigma F = \Sigma F_{\text{eff}}: \qquad 0 = m\bar{a} \qquad\qquad \bar{\mathbf{a}} = 0 \blacktriangleleft$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 4Tr = \bar{I}\alpha$$

$$-4(4 \text{ lb})\left(\frac{32 \text{ in.}}{12}\right) = 31.056\alpha \qquad\qquad \boldsymbol{\alpha} = -1.3739 \text{ rad/s}^2 \circlearrowright$$

$$\boldsymbol{\alpha} = -(1.374 \text{ rad/s}^2)\mathbf{j} \blacktriangleleft$$

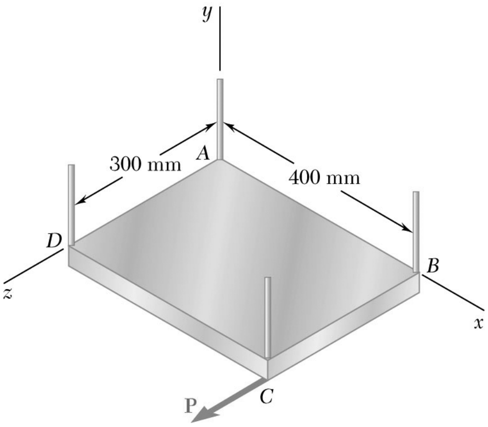
(*b*) With all rockets except D:

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad -4 \text{ lb} = \frac{250}{32.2}\bar{a}_x \qquad \bar{a}_x = -0.51520 \text{ ft/s}^2$$

$$+\downarrow \Sigma F_z = \Sigma(F_z)_{\text{eff}}: \qquad 0 = \frac{250}{32.2}\bar{a} \qquad \bar{a}_z = 0 \qquad\qquad \bar{\mathbf{a}} = -(0.515 \text{ ft/s}^2)\mathbf{i} \blacktriangleleft$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 3Tr = \bar{I}\alpha$$

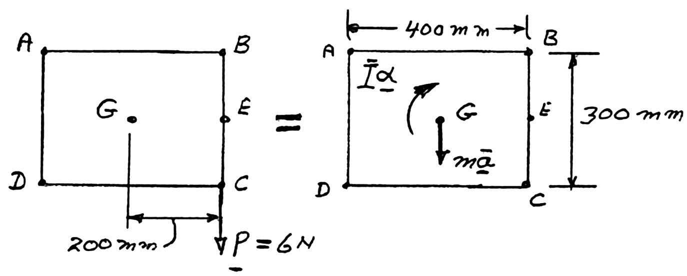
$$-3(4 \text{ lb})\left(\frac{32 \text{ in.}}{12}\right) = 31.056\alpha \qquad\qquad \boldsymbol{\alpha} = -(1.030 \text{ rad/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 16.53

A rectangular plate of mass 5 kg is suspended from four vertical wires, and a force **P** of magnitude 6 N is applied to corner *C* as shown. Immediately after **P** is applied, determine the acceleration of (*a*) the midpoint of edge *BC*, (*b*) corner *B*.

SOLUTION



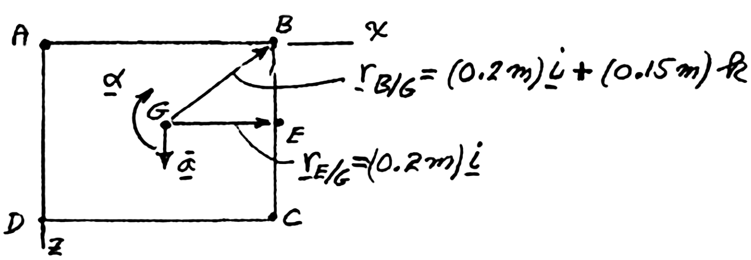
$$\bar{I} = \frac{1}{12}m(b^2 + h^2) = \frac{1}{12}(5\text{ kg})[(0.4\text{ m})^2 + (0.3\text{ m})^2] = 0.10417\text{ kg}\cdot\text{m}^2$$

$$+\downarrow \Sigma F = \Sigma F_{\text{eff}}: \qquad P = m\bar{a}$$

$$6N = (5\text{ kg})\bar{a} \qquad \bar{a} = +(1.2\text{ m/s}^2)\mathbf{k}$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad P(0.2\text{ m}) = \bar{I}\alpha$$

$$(6\text{ N})(0.2\text{ m}) = (0.10417\text{ kg}\cdot\text{m}^2)\alpha \qquad \boldsymbol{\alpha} = -(11.52\text{ rad/s}^2)\mathbf{j}$$



(*a*)

$$\mathbf{a}_E = \bar{\mathbf{a}} + \boldsymbol{\alpha} \times \mathbf{r}_{E/G}$$

$$= +(1.2\text{ m/s}^2)\mathbf{k} - (11.52\text{ rad/s}^2)\mathbf{j} \times (0.2\text{ m})\mathbf{i}$$

$$= +(1.2\text{ m/s}^2)\mathbf{k} + (2.304\text{ m/s}^2)\mathbf{k} \qquad \mathbf{a}_E = (3.50\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 16.53 (Continued)

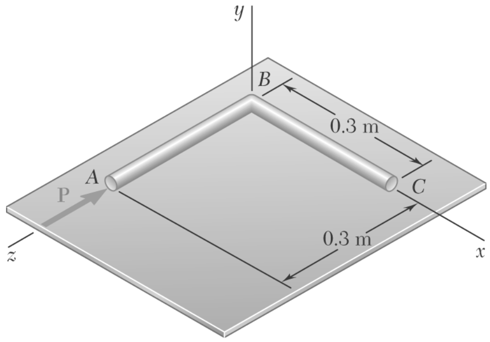
(*b*)

$$\mathbf{a}_B = \overline{\mathbf{a}} + \boldsymbol{\alpha} \times \mathbf{r}_{B/G}$$

$$= +(1.2 \text{ m/s}^2)\mathbf{k} - (11.52 \text{ rad/s})\mathbf{j} \times [(0.2 \text{ m})\mathbf{i} + (0.15 \text{ m})\mathbf{k}]$$

$$= +(1.2 \text{ m/s}^2)\mathbf{k} + (2.304 \text{ m/s}^2)\mathbf{k} + (1.728 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_B = (1.728 \text{ m/s}^2)\mathbf{i} + (3.5 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



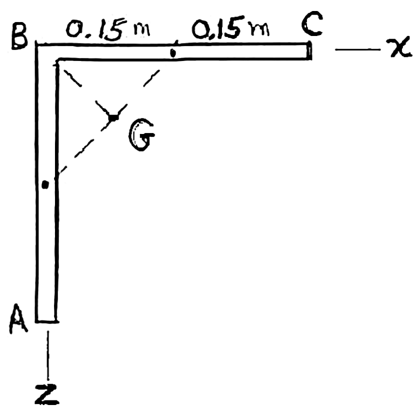
PROBLEM 16.54

A uniform slender L-shaped bar *ABC* is at rest on a horizontal surface when a force **P** of magnitude 4 N is applied at Point *A*. Neglecting friction between the bar and the surface and knowing that the mass of the bar is 2 kg, determine (*a*) the initial angular acceleration of the bar, (*b*) the initial acceleration of Point *B*.

SOLUTION

(*a*)

Mass center at G



$$\bar{x} = \frac{(m/2)\bar{x}_1 + (m/2)\bar{x}_2}{m}$$

$$\bar{x} = \frac{1(0.15) + 1(0)}{2} = 0.075 \text{ m}$$

$$\bar{x} = \bar{z}$$

$$\bar{I} = 2\left[\frac{1}{12}\left(\frac{m}{2}\right)(0.3)^2 + \frac{m}{2}\left((0.075)^2 + (0.075)^2\right)\right] = 0.0375 \text{ kg}\cdot\text{m}^2$$

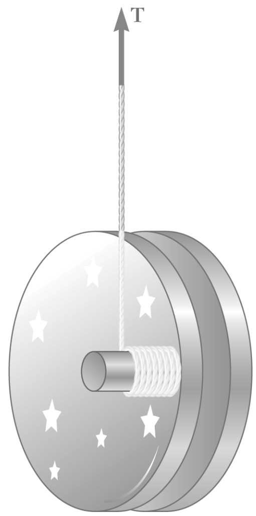
$$\Sigma F_x = 0 = ma_{Gx}, \qquad a_{Gx} = 0$$

$$\Sigma F_z = -4 = ma_{Gz}, \qquad a_{Gz} = -2 \text{ m/s}^2$$

$$\Sigma M_G = -4(0.075) = 0.0375\alpha, \qquad \boldsymbol{\alpha} = -(8 \text{ rad/s}^2)\mathbf{j} \blacktriangleleft$$

(*b*) $\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = -2\mathbf{k} - 8\mathbf{j}\times(-0.075\mathbf{i} - 0.075\mathbf{k})$

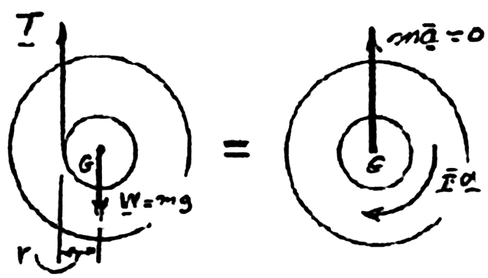
$$\mathbf{a}_B = (0.6 \text{ m/s}^2)\mathbf{i} - (2.6 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 16.55

By pulling on the string of a yo-yo, a person manages to make the yo-yo spin, while remaining at the same elevation above the floor. Denoting the mass of the yo-yo by m, the radius of the inner drum on which the string is wound by r, and the centroidal radius of gyration of the yo-yo by $\bar{k}$, determine the angular acceleration of the yo-yo.

SOLUTION



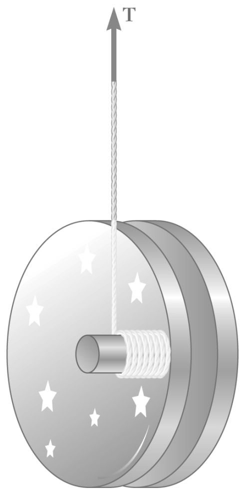
$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad T - mg = 0; \quad T = mg$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \qquad Tr = \bar{I}\alpha$$

$$mgr = m\bar{k}^2\alpha$$

$$\alpha = \frac{rg}{\bar{k}^2}$$

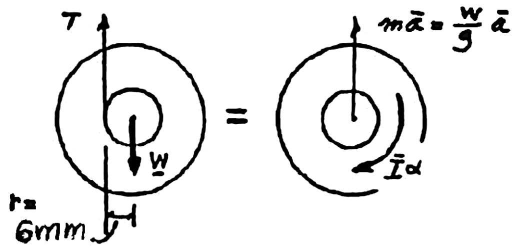
$$\boldsymbol{\alpha} = \frac{rg}{\bar{k}^2} \circlearrowright \blacktriangleleft$$



PROBLEM 16.56

The 80-g yo-yo shown has a centroidal radius of gyration of 30 mm. The radius of the inner drum on which a string is wound is 6 mm. Knowing that at the instant shown the acceleration of the center of the yo-yo is 1 m/s^2 upward, determine (*a*) the required tension **T** in the string, (*b*) the corresponding angular acceleration of the yo-yo.

SOLUTION



$$W = mg$$

$$W = 0.080 \text{ kg } (9.81 \text{ m/s}^2) = 0.7848 \text{ N}$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad T - W = \frac{W}{g}\bar{a}$$

$$T - (0.08 \text{ kg})(9.81 \text{ m/s}^2) = (0.08 \text{ kg})(1 \text{ m/s}^2)$$

$$T = 0.8648 \text{ N}$$

(*a*) *Tension in the string.* $T = 0.865 \text{ N}$ ◀

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad Tr = \bar{I}\alpha$$

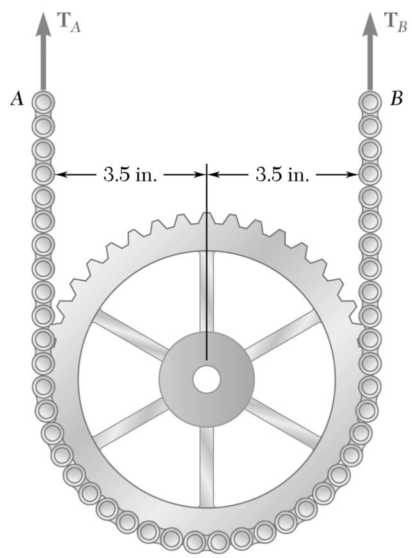
$$(0.8648 \text{ N})(0.006 \text{ m}) = m\bar{k}^2\alpha$$

$$5.1888 \times 10^{-3} \text{ N} \cdot \text{m} = (0.08 \text{ kg})(0.03 \text{ m})^2\alpha$$

(*b*) *Angular acceleration.*

$$\alpha = 72.067 \text{ rad/s}^2$$

$\alpha = 72.1 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 16.57

A 6-lb sprocket wheel has a centroidal radius of gyration of 2.75 in. and is suspended from a chain as shown. Determine the acceleration of Points A and B of the chain, knowing that $T_A = 3$ lb and $T_B = 4$ lb.

SOLUTION

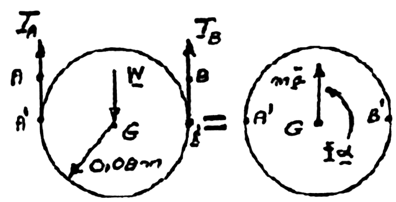
$$m = \frac{W}{g}$$

$$\bar{I} = m\bar{k}^2$$

$$= \left(\frac{6 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(\frac{2.75 \text{ in.}}{12 \text{ in./ft}}\right)^2$$

$$= 9.7858 \times 10^{-3} \text{ slug} \cdot \text{ft}^2$$

$$r = 3.5 \text{ in.}$$



$$+\!\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad T_A + T_B - W = m\bar{a}$$

$$T_A + T_B - 6 \text{ lb} = \left(\frac{(6 \text{ lb})}{32.2 \text{ ft/s}^2}\right)\bar{a}$$

$$+\!\uparrow \bar{a} = 5.3667(T_A + T_B - 6) \qquad (1)$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad T_B\left(\frac{3.5}{12}\text{ft}\right) - T_A\left(\frac{3.5}{12}\text{ft}\right) = \bar{I}\alpha$$

$$(T_B - T_A)\left(\frac{3.5}{12}\text{ft}\right) = (9.7858 \times 10^{-3} \text{ slug} \cdot \text{ft}^2)\alpha$$

$$+\circlearrowleft \alpha = 29.805\,(T_B - T_A) \qquad (2)$$

Given data: $T_A = 3$ lb, $T_B = 4$ lb

PROBLEM 16.57 (Continued)

Eq. (1): $\bar{a} = 5.3667(3+4-6) = 5.3667 \text{ ft/s}^2$

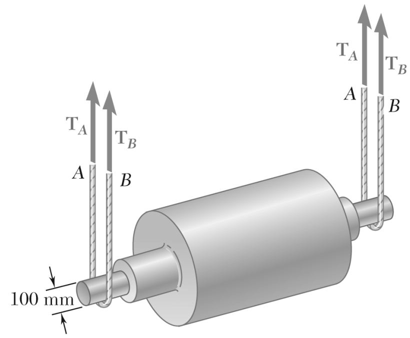
Eq. (2): $+\circlearrowleft \alpha = 29.805(4-3) = 29.805 \text{ rad/s}^2$

$$
\begin{aligned}
+\uparrow \mathbf{a}_A &= (a_A)_t \\
&= \bar{a} + r\alpha \\
&= 5.3667 - \left(\frac{3.5}{12}\right)(29.805) \\
&= -3.3264 \text{ ft/s}^2
\end{aligned}
$$

$\mathbf{a}_A = 3.33 \text{ ft/s}^2 \downarrow$ ◀

$$
\begin{aligned}
\mathbf{a}_A &= (a_A)_t \\
&= \bar{a} + r\alpha \\
&= 5.3667 + \left(\frac{3.5}{12}\right)(29.805) \\
&= +14.06 \text{ ft/s}^2
\end{aligned}
$$

$\mathbf{a}_B = 14.06 \text{ ft/s}^2 \uparrow$ ◀



PROBLEM 16.58

The steel roll shown has a mass of 1200 kg, a centroidal radius of gyration of 150 mm, and is lifted by two cables looped around its shaft. Knowing that for each cable $T_A = 3100$ N and $T_B = 3300$ N, determine (*a*) the angular acceleration of the roll, (*b*) the acceleration of its mass center.

SOLUTION

Data:

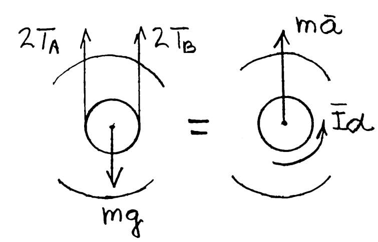
$$m = 1200 \text{ kg}$$

$$\bar{I} = mk^2 = (1200)(0.150)^2 = 27 \text{ kg}\cdot\text{m}^2$$

$$r = \frac{1}{2}d = \frac{1}{2}(0.100) = 0.050 \text{ m}$$

$$T_A = 3100 \text{ N}$$

$$T_B = 3300 \text{ N}$$



(*a*) <u>Angular acceleration</u>.

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 2T_B r - 2T_A r = \bar{I}\alpha$$

$$\alpha = \frac{2(T_B - T_A)r}{\bar{I}}$$

$$= \frac{(2)(3300 - 3100)(0.050)}{27}$$

$\boldsymbol{\alpha} = 0.741 \text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) <u>Acceleration of mass center</u>.

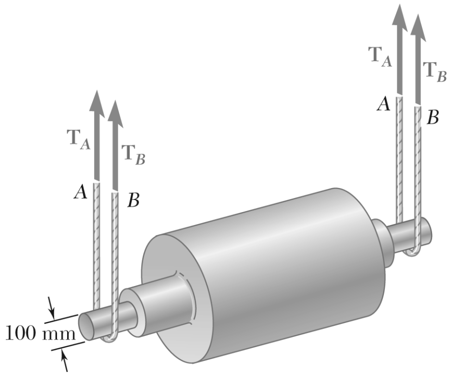
$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad 2T_A + 2T_B - mg = m\bar{a}$$

$$\bar{a} = \frac{2(T_A + T_B)}{m} - g$$

$$= \frac{2(3100 + 3300)}{1200} - 9.81$$

$\bar{\mathbf{a}} = 0.857 \text{ m/s}^2 \uparrow$ ◀

PROBLEM 16.59



The steel roll shown has a mass of 1200 kg, has a centroidal radius of gyration of 150 mm, and is lifted by two cables looped around its shaft. Knowing that at the instant shown the acceleration of the roll is 150 mm/s^2 downward and that for each cable $T_A = 3000$ N, determine (*a*) the corresponding tension T_B, (*b*) the angular acceleration of the roll.

SOLUTION

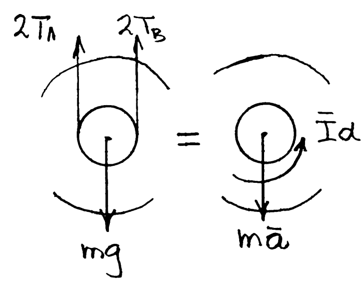
Data: $m = 1200$ kg

$$\bar{I} = mk^2 = (1200)(0.150)^2 = 27 \text{ kg} \cdot \text{m}^2$$

$$r = \frac{1}{2}d = \frac{1}{2}(0.100) = 0.050 \text{ m}$$

$$T_A = 3000 \text{ N}$$

$$\bar{\mathbf{a}} = 0.150 \text{ m/s}^2 \downarrow$$



(*a*) Tension in cable *B*.

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad 2T_A + 2T_B - mg = -m\bar{a}$$

$$T_B = \frac{mg - ma}{2} - T_A$$

$$= \frac{m(g - \bar{a})}{2} - T_A$$

$$= \frac{(1200)(9.81 - 0.150)}{2} - 3000$$

$$= 2796 \text{ N}$$

$T_B = 2800$ N ◀

(*b*) Angular acceleration.

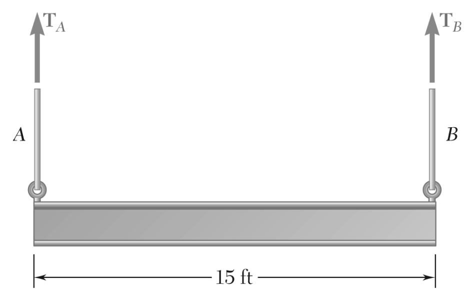
$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad 2T_B r - 2T_A r = \bar{I}\alpha$$

$$\alpha = \frac{2(T_B - T_A)r}{\bar{I}}$$

$$= \frac{(2)(2796 - 3000)}{27}$$

$$= -15.11 \text{ rad/s}^2$$

$\boldsymbol{\alpha} = 15.11 \text{ rad/s}^2 \circlearrowleft$ ◀



PROBLEM 16.60

A 15-ft beam weighing 500 lb is lowered by means of two cables unwinding from overhead cranes. As the beam approaches the ground, the crane operators apply brakes to slow the unwinding motion. Knowing that the deceleration of cable A is 20 ft/s^2 and the deceleration of cable B is 2 ft/s^2, determine the tension in each cable.

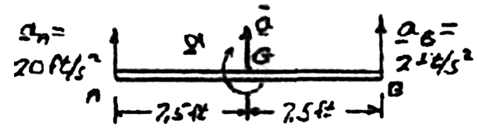
SOLUTION

Kinematics:

$$a_B = a_A + (15 \text{ ft})\alpha$$

$$2\uparrow = 20\uparrow + 15\alpha$$

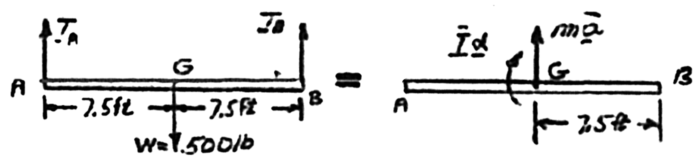
$$\alpha = 1.2 \text{ rad/s}^2 \circlearrowright$$



$$\bar{a} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(2 + 20)$$

$$\bar{\mathbf{a}} = 11 \text{ ft/s}^2 \uparrow$$

Kinetics:



$$\bar{I} = \frac{1}{12}mL^2$$

$$= \frac{1}{12}\frac{500}{32.2 \text{ ft/s}^2}(15 \text{ ft})^2$$

$$= 291.15 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad T_A(15 \text{ ft}) - W(2.5 \text{ ft}) = m\bar{a}(7.5 \text{ ft}) + \bar{I}\alpha$$

$$T_A(15 \text{ ft}) - (500 \text{ lb})(7.5 \text{ ft}) = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2}(11 \text{ ft/s}^2)(7.5 \text{ ft})$$

$$+ (291.15 \text{ lb} \cdot \text{ft/s})(1.2 \text{ rad/s}^2)$$

$$15T_A - 3750 = 1281 + 349.3$$

$$T_A = 358.7 \text{ lb}$$

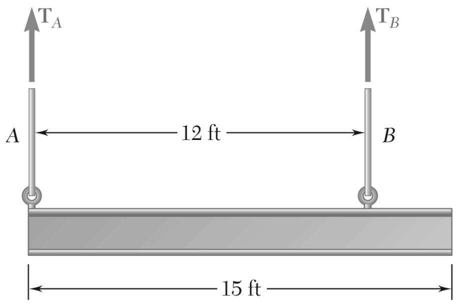
$T_A = 359$ lb ◀

$$+\uparrow \Sigma F = \Sigma F_{\text{eff}}: \quad T_A + T_B - W = m\bar{a}$$

$$358.7 \text{ lb} + T_B - 500 \text{ lb} = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2}(11 \text{ ft/s}^2)$$

$$T_B = 312.2 \text{ lb}$$

$T_B = 312$ lb ◀



PROBLEM 16.61

A 15-ft beam weighing 500 lb is lowered by means of two cables unwinding from overhead cranes. As the beam approaches the ground, the crane operators apply brakes to slow the unwinding motion. Knowing that the deceleration of cable A is 20 ft/s^2 and the deceleration of cable B is 2 ft/s^2, determine the tension in each cable.

SOLUTION

Kinematics:

$$a_B = a_A + 12\alpha$$

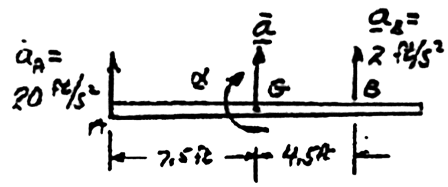
$$2\uparrow = 20\uparrow + 12\alpha$$

$$\alpha = 1.5 \text{ rad/s}^2 \circlearrowright$$

$$\bar{\mathbf{a}} = a_A + 7.5\alpha$$

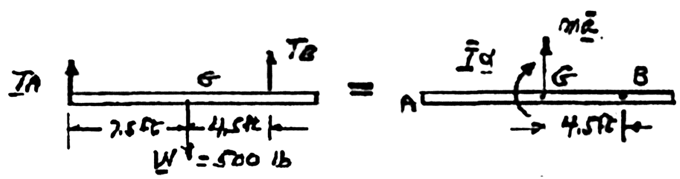
$$= 20\uparrow + (7.5)(1.5)\downarrow$$

$$\bar{\mathbf{a}} = 8.75 \text{ ft/s}^2\uparrow$$



$$\bar{I} = \frac{1}{2}mL^2 = \frac{1}{12}\frac{500}{32.2}(15)^2 = 291.15 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Kinetics:



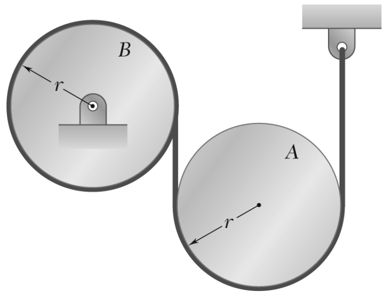
$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad T_A(12 \text{ ft}) + W(4.5 \text{ ft}) = m\bar{a}(4.5 \text{ ft}) + \bar{I}\alpha$$

$$T_A(12 \text{ ft}) - (500 \text{ lb})(4.5 \text{ ft}) = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2}(8.75 \text{ ft/s}^2)(4.5 \text{ ft})$$
$$+ (291.15)(1.5 \text{ rad/s}^2)$$

$$12T_A - 2250 = 611.4 + 436.7 \qquad T_A = 275 \text{ lb} \blacktriangleleft$$

$$+\uparrow \Sigma F = \Sigma F_{\text{eff}}: \quad T_A + T_B - W = m\bar{a}$$

$$275 \text{ lb} + T_B - 500 = \frac{500}{32.2}(8.75) \qquad T_B = 361 \text{ lb} \blacktriangleleft$$



PROBLEM 16.62

Two uniform cylinders, each of weight $W = 14$ lb and radius $r = 5$ in., are connected by a belt as shown. If the system is released from rest, determine (*a*) the angular acceleration of each cylinder, (*b*) the tension in the portion of belt connecting the two cylinders, (*c*) the velocity of the center of the cylinder A after it has moved through 3 ft.

SOLUTION

Kinematics

Let $\mathbf{a}_A = a_A \downarrow$ be the acceleration of the center of cylinder A, $\mathbf{a}_{AB} = a_{AB} \downarrow$ be acceleration of the cord between the disks, $\boldsymbol{\alpha}_A = \alpha_A \circlearrowright$ be the angular acceleration of disk A, and $\boldsymbol{\alpha}_B = \alpha_B \circlearrowright$ be the angular acceleration of disk B.

$$a_A = r\alpha_A \tag{1}$$

$$a_{AB} = a_A + r\alpha_A = 2r\alpha_A = r\alpha_B \tag{2}$$

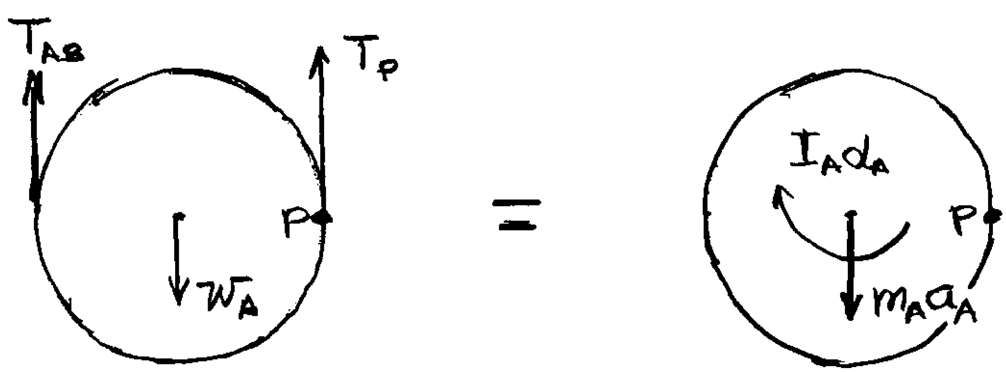
Masses and moments of inertia

$$m_A = m_B = m \tag{3}$$

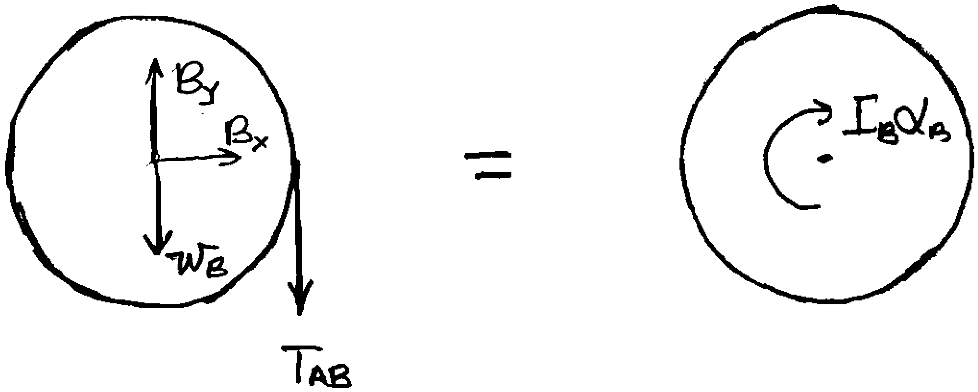
$$\bar{I}_A = \bar{I}_B = \frac{1}{2}mr^2 \tag{4}$$

Kinetics: Let T_{AB} be the tension in the portion of the cable between disks A and B.

Disk A: $\circlearrowright \Sigma M_P = \Sigma(M_P)_{\text{eff}}: \quad rW_A - 2rT_{AB} = rm_A a_A + \bar{I}_A \alpha_A$ (5)



Disk B: $\quad \circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad rT_{AB} = \bar{I}_B \alpha_B$ (6)



Add $2 \times \text{Eq.}(6)$ to Eq. (5) to eliminate T_{AB}.

$$rW_A = rm_A a_A + \bar{I}_A \alpha_A + 2\bar{I}_B \alpha_B \tag{7}$$

Use Eqs. (1) and (2) to eliminate a_A and α_B

$$\begin{aligned} rW_A &= rm_A(r\alpha_A) + \bar{I}_A\alpha_A + 2\bar{I}_B \cdot (2\alpha_A) \\ &= (m_A r^2 + \bar{I}_A + 4\bar{I}_B)\alpha_A \\ &= \left[mr^2 + \frac{1}{2}mr^2 + 4\left(\frac{1}{2}mr^2\right)\right]\alpha_A \\ &= 3.5\, mr^2 \alpha_A \end{aligned}$$

$$\alpha_A = \frac{rW_A}{3.5\, mr^2} = \frac{1g}{3.5r}$$

$$\alpha_B = 2\alpha_A = \frac{2g}{3.5r}$$

From Eq. (2), $$T_{AB} = \frac{\bar{I}_B \alpha_B}{r} = \frac{\left(\frac{1}{2}mr^2\right)(2g)}{3.5r^2}$$

$$= \frac{mg}{3.5} = \frac{W}{3.5}$$

Data: $\quad W = 14 \text{ lb}, \quad g = 32.2 \text{ ft/s}^2,$

$$r = 5 \text{ in.} = \frac{5}{12} \text{ ft}$$

(*a*) *Angular accelerations.*

$$\alpha_A = \frac{32.2 \text{ ft/s}^2}{(3.5)\left(\frac{5}{12} \text{ ft}\right)} = 22.08 \text{ rad/s} \qquad \boldsymbol{\alpha}_A = 22.1 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

$$\alpha_B = 2\alpha_A \qquad \boldsymbol{\alpha}_B = 44.2 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(*b*) *Tension T_{AB}.*

$$T_{AB} = \frac{W}{3.5} = \frac{14 \text{ lb}}{3.5} \qquad T_{AB} = 4.00 \text{ lb} \blacktriangleleft$$

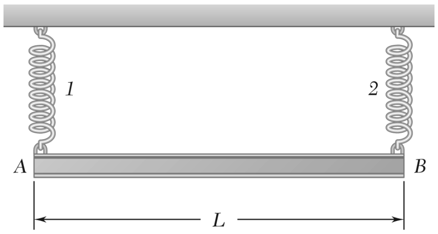
(*c*) *Velocity of the center of A.*

$$a_A = \frac{5}{12}\alpha_A = \frac{5}{12}(22.08) = 9.20 \text{ ft/s}^2$$

$$v_A^2 = [(v_A)_0]^2 + 2a_A d_A$$

$$= 0 + (2)(9.20 \text{ ft/s})(3 \text{ ft}) = 55.2 \text{ ft}^2/\text{s}^2$$

$$v_A = 7.43 \text{ ft/s} \qquad \mathbf{v}_A = 7.43 \text{ ft/s} \downarrow \blacktriangleleft$$



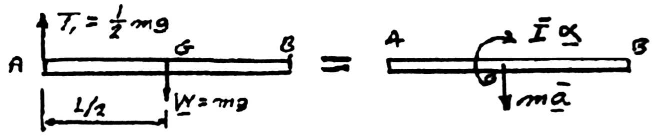
PROBLEM 16.63

A beam AB of mass m and of uniform cross section is suspended from two springs as shown. If spring 2 breaks, determine at that instant (a) the angular acceleration of the bar, (b) the acceleration of Point A, (c) the acceleration of Point B.

SOLUTION

Statics: $$T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$$

(a) Angular acceleration:



$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad T\left(\frac{L}{2}\right) = \bar{I}\alpha$$

$$\frac{1}{2}mg\left(\frac{L}{2}\right) = \frac{1}{12}mL^2\alpha$$

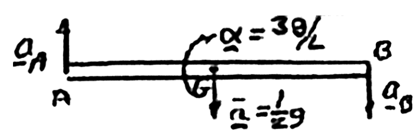
$$\alpha = \frac{3g}{L} \qquad\qquad \boldsymbol{\alpha} = \frac{3g}{L} \circlearrowright \blacktriangleleft$$

$$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad W - T_1 = m\bar{a}$$

$$mg - \frac{1}{2}mg = m\bar{a}$$

$$\bar{a} = \frac{1}{2}g \qquad \bar{\mathbf{a}} = \frac{1}{2}g\downarrow$$

(b) Acceleration of A:



$$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G}$$

$$+\downarrow \mathbf{a}_A = \frac{1}{2}g - \frac{L}{2}\alpha$$

$$= \frac{1}{2}g - \frac{L}{2}\left(\frac{3g}{L}\right)$$

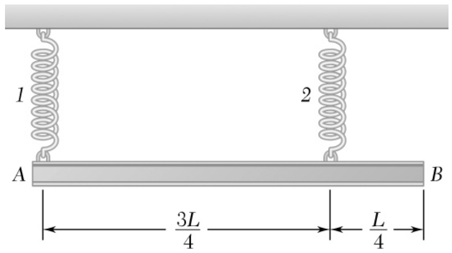
$$a_A = -g \qquad\qquad \mathbf{a}_A = g\uparrow \blacktriangleleft$$

(c) Acceleration of B:

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$

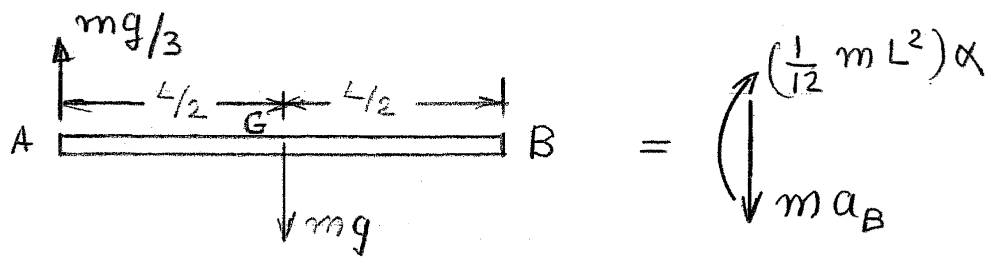
$$+\downarrow a_B = \bar{a} + \frac{L}{2}\alpha = \frac{1}{2}g + \frac{L}{2}\left(\frac{3g}{L}\right) = +2g \qquad\qquad \mathbf{a}_B = 2g\downarrow \blacktriangleleft$$

PROBLEM 16.64



A beam AB of mass m and of uniform cross section is suspended from two springs as shown. If spring 2 breaks, determine at that instant (*a*) the angular acceleration of the beam, (*b*) the acceleration of Point A, (*c*) the acceleration of Point B.

SOLUTION



$$+\downarrow \Sigma F_y = \frac{2mg}{3} = ma_G$$

$$+\circlearrowright \Sigma M_G = \left(\frac{mg}{3}\right)\frac{L}{2} = \frac{1}{12}mL^2\alpha$$

(*a*) $\mathbf{a}_G = \frac{2g}{3}\downarrow, \qquad \boldsymbol{\alpha} = \frac{2g}{L}\circlearrowright$ ◀

(*b*) $\mathbf{a}_A = \downarrow\frac{2g}{3} + \frac{2g}{L}\frac{L}{2}\uparrow = \frac{g}{3}\uparrow,$

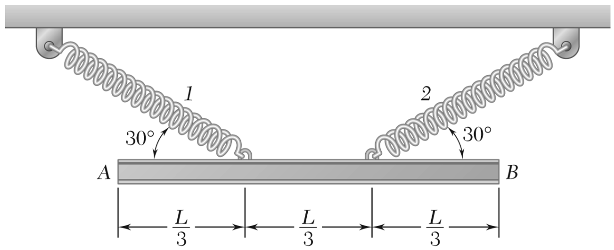
$\mathbf{a}_A = \frac{g}{3}\uparrow$ ◀

(*c*) $\mathbf{a}_B = \downarrow\frac{2g}{3} + \frac{2g}{L}\frac{L}{2}\downarrow = \frac{5g}{3}\downarrow$

$\mathbf{a}_B = \frac{5g}{3}\downarrow$ ◀

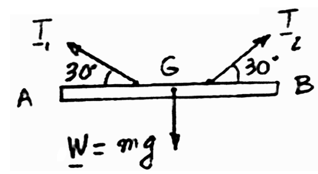
PROBLEM 16.65

A beam AB of mass m and of uniform cross section is suspended from two springs as shown. If spring 2 breaks, determine at that instant (a) the angular acceleration of the bar, (b) the acceleration of Point A, (c) the acceleration of Point B.



SOLUTION

Before spring 1 breaks:



$$+\uparrow \Sigma F_y = 0: \quad T_1 \sin 30° + T_2 \sin 30° - W = 0$$

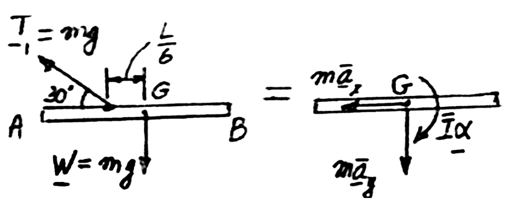
Since $T_1 = T_2$ by symmetry,

$$2T_1 \sin 30° = W = mg$$

$$\mathbf{T}_1 = mg \measuredangle 30°$$

Immediately after spring 2 breaks, elongation of spring 1 is unchanged. Thus, we still have

$$\mathbf{T}_1 = mg \measuredangle 30°$$



(a) Angular acceleration:

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad (T_1 \sin 30°)\frac{L}{6} = \bar{I}\alpha$$

$$(mg \sin 30°)\frac{L}{6} = \frac{1}{12}mL^2\alpha \qquad \boldsymbol{\alpha} = \frac{g}{L} \circlearrowright \blacktriangleleft$$

$$\overset{+}{\leftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad T_1 \cos 30° = m\bar{a}_x$$

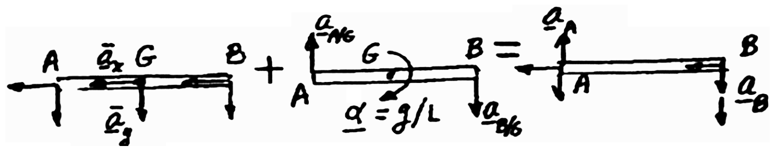
$$mg \cos 30° = m\bar{a}, \qquad \bar{\mathbf{a}}_x = 0.866g \leftarrow$$

$$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad W - T_1 \sin 30° = m\bar{a}_y$$

$$mg - mg \sin 30° = m\bar{a}_y \qquad \bar{\mathbf{a}}_y = 0.5g \downarrow$$

PROBLEM 16.65 (Continued)

Accelerations of A and B



Translation + Rotation about G

(b) Acceleration of A:

$$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G} = [0.866g \leftarrow] + [0.5g \downarrow] + \left(\frac{g}{L}\right)\left(\frac{L}{2}\right)\uparrow$$

$$= [0.866g \leftarrow] + 0$$

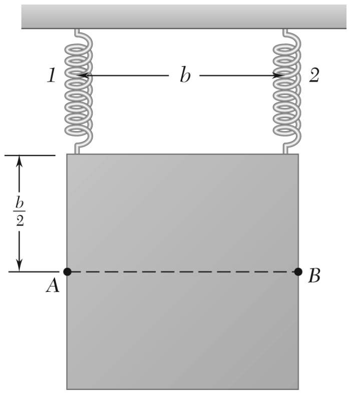
$\mathbf{a}_A = 0.866g \leftarrow$ ◀

(c) Acceleration of B:

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = [0.866g \leftarrow] + [0.5g \downarrow] + \left(\frac{g}{L}\right)\left(\frac{L}{2}\right)\downarrow$$

$$= [0.366g \leftarrow] + [g \downarrow]$$

$\mathbf{a}_B = 1.323g \swarrow 49.1°$ ◀



PROBLEM 16.66

A thin plate of the shape indicated and of mass m is suspended from two springs as shown. If spring 2 breaks, determine the acceleration at that instant (*a*) of Point A, (*b*) of Point B.

A square plate of side b.

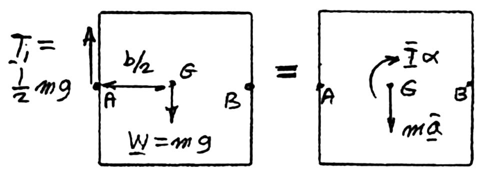
SOLUTION

$$\bar{I} = \frac{1}{12}m(b^2 + b^2)$$

$$\bar{I} = \frac{1}{6}mb^2$$

Statics: $$T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$$

Kinetics: $$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad T\frac{b}{2} = \bar{I}\alpha$$



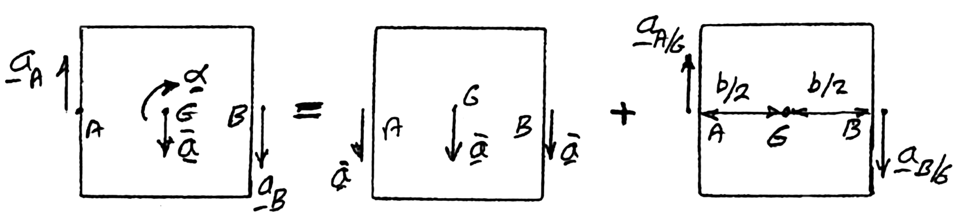
$$\frac{1}{2}mg\left(\frac{b}{2}\right) = \frac{1}{6}mb^2\alpha$$

$$\boldsymbol{\alpha} = \frac{3g}{2b} \circlearrowright$$

$$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad W - T_1 = m\bar{a}$$

$$mg - \frac{1}{2}mg = m\bar{a} \qquad \bar{\mathbf{a}} = \frac{1}{2}g \downarrow$$

Kinematics:



Plane motion = Translation + Rotation

PROBLEM 16.66 (Continued)

(*a*) $$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G} = \bar{\mathbf{a}}\downarrow + \frac{b}{2}\alpha\uparrow$$

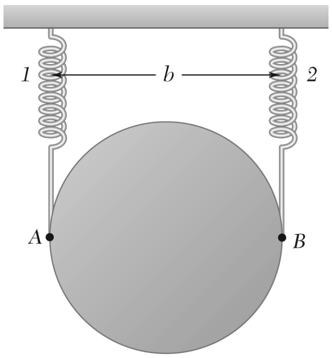
$$\mathbf{a}_A = \frac{g}{2}\downarrow + \frac{b}{2}\left(\frac{3g}{2b}\right)\uparrow = \frac{g}{4}\uparrow$$

$$\mathbf{a}_A = \frac{1}{4}g\uparrow \blacktriangleleft$$

(*b*) $$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = \bar{\mathbf{a}}\downarrow + \frac{b}{2}\alpha\downarrow$$

$$\mathbf{a}_B = \frac{1}{2}g\downarrow + \frac{b}{2}\left(\frac{3g}{2b}\right)\downarrow = \frac{5}{4}g\downarrow$$

$$\mathbf{a}_B = \frac{5}{4}g\downarrow \blacktriangleleft$$



PROBLEM 16.67

A thin plate of the shape indicated and of mass m is suspended from two springs as shown. If spring 2 breaks, determine the acceleration at that instant (a) of Point A, (b) of Point B.

A circular plate of diameter b.

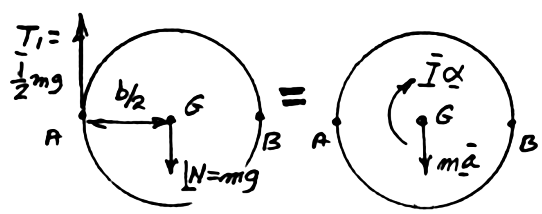
SOLUTION

$$\bar{I} = \frac{1}{2}m\left(\frac{b}{2}\right)^2 = \frac{1}{8}mb^2$$

Statics:

$$T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$$

Kinetics: $+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad T_1\frac{b}{2} = \bar{I}\alpha$



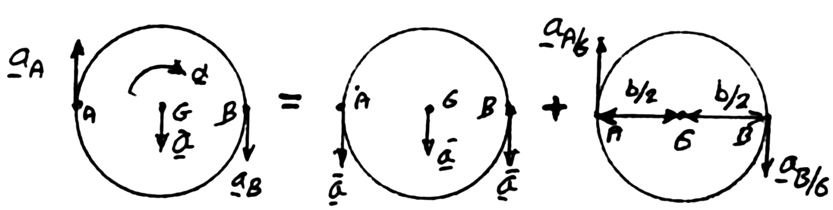
$$\frac{1}{2}mg\left(\frac{b}{2}\right) = \frac{1}{8}mb^2\alpha$$

$$\boldsymbol{\alpha} = 2\frac{g}{b} \circlearrowright$$

$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad W - T_1 = m\bar{a}$

$$mg - \frac{1}{2}mg = m\bar{a} \qquad \bar{\mathbf{a}} = \frac{1}{2}g\downarrow$$

Kinematics:

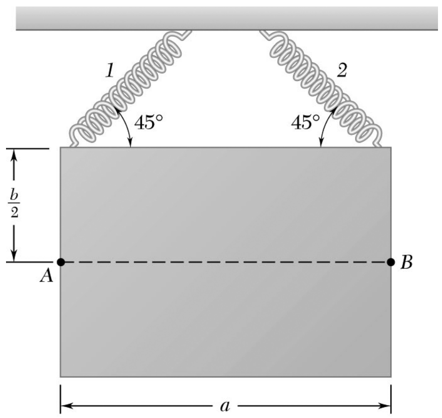


Plane motion = Translation + Rotation

(a) $\quad \mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G} = \bar{\mathbf{a}}\downarrow + \frac{b}{2}\alpha\uparrow = \frac{1}{2}g\downarrow + \frac{b}{2}\left(2\frac{g}{b}\right)\uparrow \qquad \mathbf{a}_A = \frac{1}{2}g\uparrow$ ◀

(b) $\quad \mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = \bar{\mathbf{a}}\downarrow + \frac{b}{2}\alpha\downarrow = \frac{1}{2}g\downarrow + \frac{b}{2}\left(2\frac{g}{b}\right)\downarrow \qquad \mathbf{a}_B = \frac{3}{2}g\downarrow$ ◀

PROBLEM 16.68



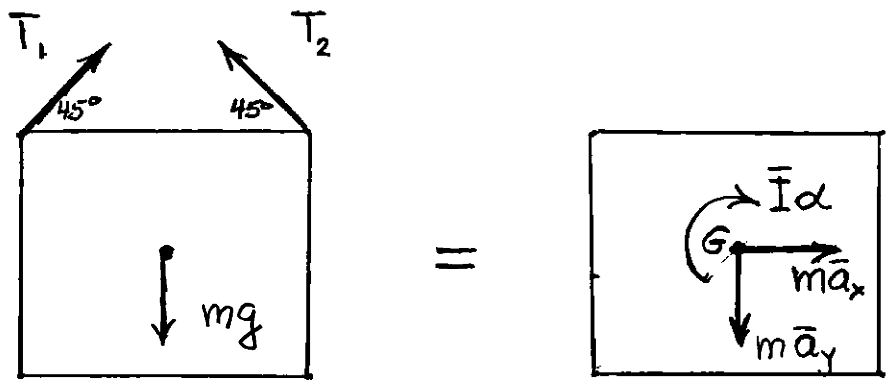
A thin plate of the shape indicated and of mass m is suspended from two springs as shown. If spring 2 breaks, determine the acceleration at that instant (*a*) of Point A, (*b*) of Point B.

A rectangular plate of height b and width a.

SOLUTION

Moment of inertia.

$$I = \frac{1}{12}m(a^2 + b^2)$$

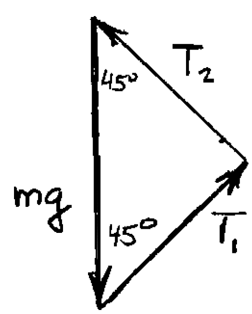


Statics:

$$a_x = 0, \quad a_y = 0, \quad \alpha = 0$$

Draw the force triangle showing equilibrium.

$$T_1 = T_2 = mg\sin 45°$$



Kinetics: $T_2 = 0$

Since there is no time for displacements to occur, the tension in spring 1 remains equal to

$$T_1 = mg\sin 45°$$

Then

$$\mathbf{T}_1 = \left[\frac{1}{2}mg \longrightarrow\right] + \left[\frac{1}{2}mg \uparrow\right]$$

PROBLEM 16.68 (Continued)

$$+\Sigma\mathbf{F} = m\bar{\mathbf{a}}: \quad [mg \downarrow] + \left[\frac{1}{2}mg \longrightarrow\right] + \left[\frac{1}{2}mg \uparrow = m\bar{\mathbf{a}}\right]$$

$$\bar{\mathbf{a}} = \left[\frac{1}{2}g \longrightarrow\right] + \left[\frac{1}{2}g \downarrow\right]$$

$$\circlearrowleft + \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad \frac{1}{2}mg\left(\frac{a}{2}\right) + \frac{1}{2}mg\left(\frac{b}{2}\right) = \bar{I}\alpha$$

$$\frac{1}{4}mg(a+b) = \frac{1}{12}m(a^2+b^2)\alpha \qquad \boldsymbol{\alpha} = \frac{3g(a+b)}{a^2+b^2} \circlearrowright$$

In vector notation, $\bar{\mathbf{a}} = \frac{1}{2}g(\mathbf{i}-\mathbf{j})$

$$\boldsymbol{\alpha} = -\frac{3g(a+b)}{a^2+b^2}\mathbf{k}$$

Kinematics. $\mathbf{a}_P = \bar{\mathbf{a}} + \boldsymbol{\alpha}\times\mathbf{r}_{P/G} - \omega^2\mathbf{r}_{P/G}$

Since there is no time to acquire angular velocity, $\omega^2 = 0$

(*a*) *Acceleration at A.* $\mathbf{r}_{A/G} = -\frac{1}{2}a\mathbf{i}$

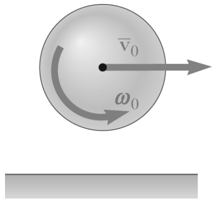
$$\mathbf{a}_A = \frac{1}{2}g(\mathbf{i}-\mathbf{j}) + \left[-\frac{3g(a+b)}{a^2+b^2}\mathbf{k}\right]\times\left(-\frac{1}{2}a\mathbf{i}\right)$$

$$\mathbf{a}_A = \frac{1}{2}g(\mathbf{i}-\mathbf{j}) + \frac{3g(a+b)a}{a^2+b^2}\mathbf{j} \blacktriangleleft$$

(*b*) *Acceleration at B.* $\mathbf{r}_{B/G} = \frac{1}{2}a\mathbf{i}$

$$\mathbf{a}_B = \frac{1}{2}g(\mathbf{i}-\mathbf{j}) + \left[-\frac{3g(a+b)}{a^2+b^2}\mathbf{k}\right]\times\left(\frac{1}{2}a\mathbf{i}\right)$$

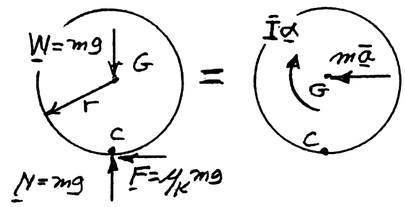
$$\mathbf{a}_B = \frac{1}{2}g(\mathbf{i}-\mathbf{j}) - \frac{3g(a+b)a}{a^2+b^2}\mathbf{j} \blacktriangleleft$$



PROBLEM 16.69

A sphere of radius r and mass m is projected along a rough horizontal surface with the initial velocities indicated. If the final velocity of the sphere is to be zero, express, in terms of v_0, r, and μ_k, (*a*) the required magnitude of $\boldsymbol{\omega}_0$, (*b*) the time t_1 required for the sphere to come to rest, (*c*) the distance the sphere will move before coming to rest.

SOLUTION



Kinetics:

$$I = m\bar{k}^2$$

$$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\bar{a}$$

$$\mu_k mg = m\bar{a} \qquad \bar{\mathbf{a}} = \mu_k g \longleftarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = m\bar{k}^2\alpha$$

$$\boldsymbol{\alpha} = \frac{\mu_k gr}{\bar{k}^2} \circlearrowright$$

Kinematics:

$$\xrightarrow{+} v = v_0 - \bar{a}t$$

$$v = v_0 - \mu_k gt$$

For $v = 0$ when $t = t_1$

$$0 = v_0 - \mu_k gt_1; \qquad t_1 = \frac{v_0}{\mu_k g} \tag{1}$$

$$+\circlearrowright \omega = \omega_0 - \alpha t$$

$$\omega = \omega_0 - \frac{\mu_k gr}{\bar{k}^2}t$$

For $\omega = 0$ when $t = t_1$

$$0 = \omega_0 - \frac{\mu_k gr}{\bar{k}^2}t_1; \qquad t_1 = \frac{\bar{k}^2}{\mu_k gr}\omega_0 \tag{2}$$

Set Eq. (1) = Eq. (2)

$$\frac{v_0}{\mu_k g} = \frac{\bar{k}^2}{\mu_k gr}\omega_0; \qquad \omega_0 = \frac{r}{\bar{k}^2}v_0 \tag{3}$$

Distance traveled:

$$s_1 = v_0 t_1 - \frac{1}{2}\bar{a}t_1^2$$

$$s_1 = v_0\left(\frac{v_0}{\mu_k g}\right) - \frac{1}{2}(\mu_k g)\left(\frac{v_0}{\mu_k g}\right)^2; \qquad s_1 = \frac{v_0^2}{2\mu_k g} \tag{4}$$

PROBLEM 16.69 (Continued)

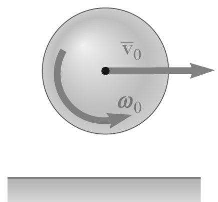
For a solid sphere $\bar{k}^2 = \frac{2}{5}r^2$

(*a*) Eq. (3): $\omega_0 = \frac{r}{\frac{2}{5}r^2}v_0 = \frac{5}{2}\frac{v_0}{r}$ $\boldsymbol{\omega}_0 = \frac{5}{2}\frac{v_0}{r}$ ◀

(*b*) Eq. (1) $t_1 = \frac{v_0}{\mu_k g}$ ◀

(*c*) Eq. (4) $s_1 = \frac{v_0^2}{2\mu_k g}$ ◀

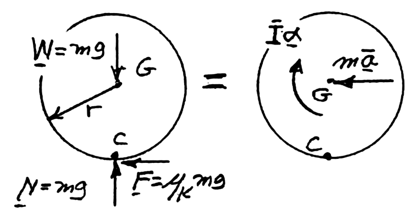
PROBLEM 16.70



Solve Problem 16.69, assuming that the sphere is replaced by a uniform thin hoop of radius r and mass m.

PROBLEM 16.69 A sphere of radius r and mass m is projected along a rough horizontal surface with the initial velocities indicated. If the final velocity of the sphere is to be zero, express, in terms of v_0, r, and μ_k, (*a*) the required magnitude of $\boldsymbol{\omega}_0$, (*b*) the time t_1 required for the sphere to come to rest, (*c*) the distance the sphere will move before coming to rest.

SOLUTION



Kinetics: $\qquad I = m\overline{k}^2$

$$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\overline{a}$$

$$\mu_k mg = m\overline{a} \qquad \overline{\mathbf{a}} = \mu_k g \longleftarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Fr = \overline{I}\alpha$$

$$(\mu_k mg)r = m\overline{k}^2\alpha$$

$$\boldsymbol{\alpha} = \frac{\mu_k gr}{\overline{k}^2} \circlearrowright$$

Kinematics: $\qquad \xrightarrow{+} v = v_0 - \overline{a}t$

$$v = v_0 - \mu_k gt$$

For $v = 0$ when $t = t_1$

$$0 = v_0 - \mu_k gt_1; \qquad t_1 = \frac{v_0}{\mu_k g} \tag{1}$$

$$+\circlearrowleft \omega = \omega_0 - \alpha t$$

$$\omega = \omega_0 - \frac{\mu_k gr}{\overline{k}^2}t$$

For $\omega = 0$ when $t = t_1$

$$0 = \omega_0 - \frac{\mu_k gr}{\overline{k}^2}t_1; \qquad t_1 = \frac{\overline{k}^2}{\mu_k gr}\omega_0 \tag{2}$$

Set Eq. (1) = Eq. (2)

$$\frac{v_0}{\mu_k g} = \frac{\overline{k}^2}{\mu_k gr}\omega_0; \qquad \omega_0 = \frac{r}{\overline{k}^2}v_0 \tag{3}$$

Distance traveled:

$$s_1 = v_0 t_1 - \frac{1}{2}\overline{a}t_1^2$$

$$s_1 = v_0\left(\frac{v_0}{\mu_k g}\right) - \frac{1}{2}(\mu_k g)\left(\frac{v_0}{\mu_k g}\right)^2; \qquad s_1 = \frac{v_0^2}{2\mu_k g} \tag{4}$$

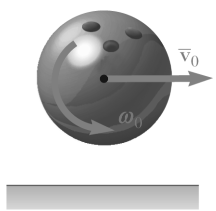
For a hoop, $\bar{k} = r$

(*a*) Eq. (3): $\omega_0 = \dfrac{r}{r^2} r_0 = \dfrac{v_0}{r}$ $\qquad \boldsymbol{\omega}_0 = \dfrac{v_0}{r}$ ◀

(*b*) Eq. (1): $\qquad t_1 = \dfrac{v_0}{\mu_k g}$ ◀

(*c*) Eq. (4): $\qquad s_1 = \dfrac{v_0^2}{2\mu_k g}$ ◀

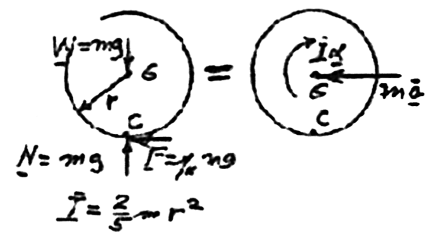
PROBLEM 16.71



A bowler projects an 8-in.-diameter ball weighing 12 lb along an alley with a forward velocity $\mathbf{v}_0$ of 15 ft/s and a backspin $\boldsymbol{\omega}_0$ of 9 rad/s. Knowing that the coefficient of kinetic friction between the ball and the alley is 0.10, determine (*a*) the time t_1 at which the ball will start rolling without sliding, (*b*) the speed of the ball at time t_1, (*c*) the distance the ball will have traveled at time t_1.

SOLUTION

<u>Kinetics</u>:



$$\overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad \mu_k mg = m\bar{a}$$

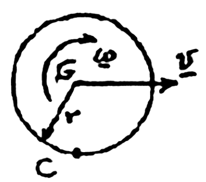
$$\bar{\mathbf{a}} = \mu_k g \longleftarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \qquad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = \frac{2}{5}mr^2\alpha$$

$$\boldsymbol{\alpha} = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowright$$

<u>Kinematics</u>: When the ball rolls, the instant center of rotation is at C, and when



$$t = t_1 \quad v = r\omega \tag{1}$$

$$\bar{v} = \bar{v}_0 - \bar{a}t = \bar{v}_0 - \mu_k gt \tag{2}$$

$$\omega = -\omega_0 + \alpha t = -\omega_0 + \frac{5}{2}\frac{\mu_k g}{r}t$$

When $t = t_1$:

Eq. (1): $v = r\omega$:

$$\bar{v}_0 - \mu_k gt_1 = \left(-\omega_0 + \frac{5}{2}\frac{\mu_k g}{r}t_1\right)r$$

$$\bar{v}_0 - \mu_k gt_1 = -\omega_0 r + \frac{5}{2}\mu_k gt_1$$

$$t_1 = \frac{2}{g}\frac{(\bar{v}_0 + r\omega_0)}{\mu_k g} \tag{3}$$

$$\bar{v}_0 = 15 \text{ ft/s}, \quad \omega_0 = 9 \text{ rad/s}, \quad r = 4 \text{ in.} = \frac{1}{3}\text{ft}$$

(*a*)

$$t_1 = \frac{2}{7}\frac{\left(15 + \frac{1}{3}(9)\right)}{0.1(32.2)} = 1.5972 \text{ s}$$

$t_1 = 1.597$ s ◀

PROBLEM 16.71 (Continued)

(*b*) Eq. (2):

$$\bar{v}_1 = v_0 - \mu_k g t_1$$
$$= 15 - 0.1(32.2)(1.5972)$$
$$\bar{v}_1 = 15 - 5.1429$$
$$= 9.857 \text{ ft/s}$$

$$\bar{v}_1 = 9.86 \text{ ft/s} \blacktriangleleft$$

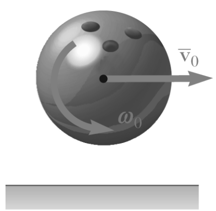
(*c*)

$$\bar{a} = \mu_k g = 0.1(32.2 \text{ ft/s}^2) = 3.22 \text{ ft/s}^2 \longleftarrow$$

$$\xrightarrow{+} s_1 = \bar{v}_0 t_1 - \frac{1}{2}\bar{a}t_1^2$$
$$= (15 \text{ ft/s})(1.597 \text{ s}) - \frac{1}{2}(3.22 \text{ ft/s}^2)(1.597 \text{ s})^2$$
$$= 23.96 - 4.11 = 19.85 \text{ ft}$$

$$s_1 = 19.85 \text{ ft} \blacktriangleleft$$

PROBLEM 16.72

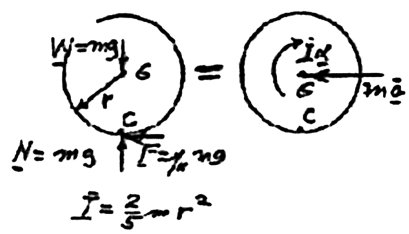


Solve Problem 16.71, assuming that the bowler projects the ball with the same forward velocity but with a backspin of 18 rad/s.

PROBLEM 16.71 A bowler projects an 8-in.-diameter ball weighing 12 lb along an alley with a forward velocity $\mathbf{v}_0$ of 15 ft/s and a backspin $\boldsymbol{\omega}_0$ of 9 rad/s. Knowing that the coefficient of kinetic friction between the ball and the alley is 0.10, determine (*a*) the time t_1 at which the ball will start rolling without sliding, (*b*) the speed of the ball at time t_1, (*c*) the distance the ball will have traveled at time t_1.

SOLUTION

<u>Kinetics</u>:



$$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad \mu_k mg = m\bar{a}$$

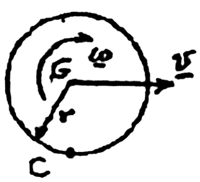
$$\bar{\mathbf{a}} = \mu_k g \longleftarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = \frac{2}{5} mr^2 \alpha$$

$$\alpha = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowright$$

<u>Kinematics</u>: When the ball rolls, the instant center of rotation is at C, and when



$$t = t_1 \quad v = r\omega \tag{1}$$

$$\bar{v} = \bar{v}_0 - \bar{a}t = \bar{v}_0 - \mu_k g^t \tag{2}$$

$$\omega = -\omega_0 + \alpha t = -\omega_0 + \frac{5}{2}\frac{\mu_k g}{r} t$$

When $t = t_1$:

Eq. (1) $\quad v = r\omega$:

$$\bar{v}_0 - \mu_k g t_1 = \left(-\omega_0 + \frac{5}{2}\frac{\mu_k g}{r} t_1 \right) r$$

$$\bar{v}_0 - \mu_k g t_1 = -\omega_0 r + \frac{5}{2}\mu_k g t_1$$

$$t_1 = \frac{2}{7}\frac{(\bar{v}_0 + r\omega_0)}{\mu_k g} \tag{3}$$

$$\bar{v}_0 = 15 \text{ ft/s}, \quad \omega_0 = 18 \text{ rad/s}, \quad r = \frac{1}{3} \text{ ft}$$

PROBLEM 16.72 (Continued)

(*a*) Eq. (3): $t_1 = \frac{2}{7}\frac{\left(15+\frac{1}{3}(18)\right)}{0.1(32.2)} = 1.8634 \text{ s}$ $t_1 = 1.863 \text{ s}$ ◀

(*b*) Eq. (2): $v_1 = v_0 - \mu_k g^t$

$= 15 - 0.1(32.2)(1.8634)$

$v_1 = 15 - 6.000$

$= 9 \text{ ft/s}$ $\bar{v}_1 = 9 \text{ ft/s}$ ◀

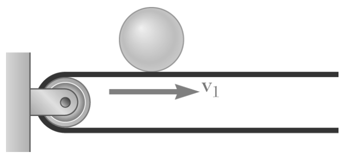
(*c*) $\bar{a} = \mu_k g = 0.1(32.2 \text{ ft/s}^2) = 3.22 \text{ ft/s}^2 \leftarrow$

$\xrightarrow{+} s_1 = \bar{v}_0 t_1 - \frac{1}{2}\bar{a}t_1^2$

$= (15 \text{ ft/s})(1.8634 \text{ s}) - \frac{1}{2}(3.22 \text{ ft/s}^2)(1.8634 \text{ s})^2$

$= 27.95 - 5.59$

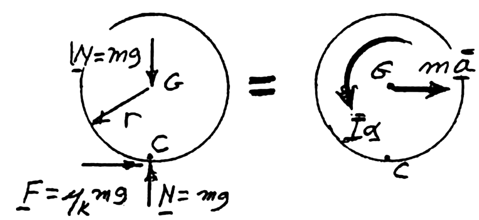
$= 22.36 \text{ ft}$ $s_1 = 22.4 \text{ ft}$ ◀



PROBLEM 16.73

A uniform sphere of radius r and mass m is placed with no initial velocity on a belt that moves to the right with a constant velocity $\mathbf{v}_1$. Denoting by μ_k the coefficient of kinetic friction between the sphere and the belt, determine (*a*) the time t_1 at which the sphere will start rolling without sliding, (*b*) the linear and angular velocities of the sphere at time t_1.

SOLUTION



Kinetics: $\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\bar{a}$

$$\mu_k mg = m\bar{a}$$

$$\bar{\mathbf{a}} = \mu_k g \longrightarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = \frac{2}{5} mr^2 \alpha$$

$$\boldsymbol{\alpha} = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowright$$

Kinematics: $\xrightarrow{+} \bar{v} = \bar{a}t = \mu_k gt \qquad (1)$

$$+\circlearrowright \omega = \alpha t = \frac{5}{2}\frac{\mu_k g}{r} t \qquad (2)$$

Point C is the point of contact with belt.

$$\xrightarrow{+} v_C = \bar{v} + \omega r = \mu_k gt + \left(\frac{5}{2}\frac{\mu_k g}{r} t\right) r$$

$$v_C = \frac{7}{2}\mu_k gt$$

(*a*) When sphere starts rolling $(t = t_1)$, we have

$$v_C = v_1$$

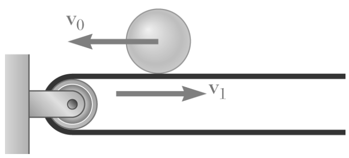
$$v_1 = \frac{7}{2}\mu_k g t_1 \qquad t_1 = \frac{2}{7}\frac{v_1}{\mu_k g} \blacktriangleleft$$

(*b*) Velocities when $t = t_1$

Eq. (1): $\bar{v} = \mu g \left(\frac{2}{7}\frac{v_1}{\mu_k g}\right) \qquad \bar{\mathbf{v}} = \frac{2}{7} v_1 \longrightarrow \blacktriangleleft$

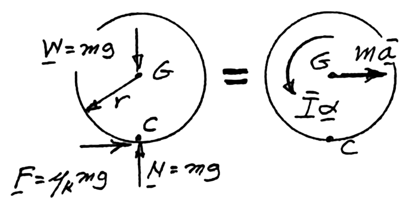
Eq. (2): $\omega = \left(\frac{5}{2}\frac{\mu_k g}{r}\right)\left(\frac{2}{7}\frac{v_1}{\mu_k g}\right) \qquad \boldsymbol{\omega} = \frac{5}{7}\frac{v_1}{r} \circlearrowright \blacktriangleleft$

PROBLEM 16.74



A sphere of radius r and m has a linear velocity $\mathbf{v}_0$ directed to the left and no angular velocity as it is placed on a belt moving to the right with a constant velocity $\mathbf{v}_1$. If after first sliding on the belt the sphere is to have no linear velocity relative to the ground as it starts rolling on the belt without sliding, determine in terms of v_1 and the coefficient of kinetic friction μ_k between the sphere and the belt (*a*) the required value of v_0, (*b*) time t_1 at which the sphere will start rolling on the belt, (*c*) the distance the sphere will have moved relative to the ground at time t_1.

SOLUTION



Kinetics: $\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\bar{a}$

$$\mu_k mg = m\bar{a}$$

$$\bar{\mathbf{a}} = \mu_k g \longrightarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad F_r = \bar{I}\alpha$$

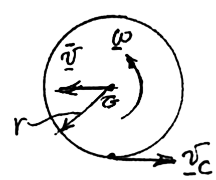
$$(\mu_k mg) r = \frac{5}{2} mr^2 \alpha$$

$$\boldsymbol{\alpha} = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowright$$

Kinematics: $\xleftarrow{+} \bar{v} = v_0 - \bar{a}t = v_0 - \mu_k g t$ (1)

$$+\circlearrowright \omega = \alpha t = \frac{5}{2}\frac{\mu_k g}{r} t \qquad (2)$$

Point C is the point of contact with belt.



$$\xrightarrow{+} v_C = -\bar{v} + r\omega$$

$$v_C = -\bar{v} + r\frac{5}{2}\frac{\mu_k g}{r} t$$

$$v_C = -\bar{v} + \frac{5\mu_k g}{2} t \qquad (3)$$

But, when $t = t_1$, $\bar{v} = 0$ and $v_c = v_1$

Eq. (3): $v_1 = \dfrac{5\mu_k g}{2} t_1$ $\qquad t_1 = \dfrac{2v_1}{5\mu_k g}$ ◀

Eq. (1): $\bar{v} = v_0 - \mu_k g t$

When $t = t_1$, $\bar{v} = 0$, $\quad 0 = v_0 - \mu_k g\left(\dfrac{2v_1}{5\mu_k g}\right)$ $\qquad v_0 = \dfrac{2}{5} v_1$ ◀

PROBLEM 16.74 (Continued)

Distance when $t = t_1$:

$$\overset{+}{\longleftarrow} s = v_0 t_1 - \frac{1}{2} a t_1^2$$

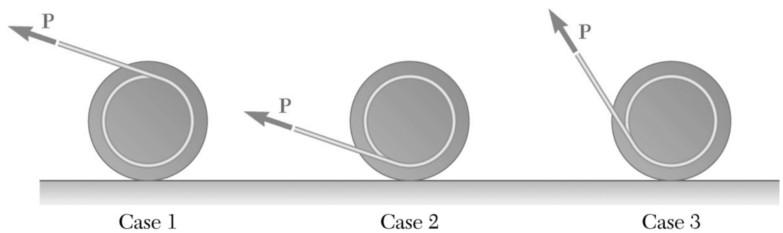
$$s = \left(\frac{2}{5} v_1\right)\left(\frac{2v_1}{5\mu_k g}\right) - \frac{1}{2}(\mu_k g)\left(\frac{2v_1}{5\mu_k g}\right)^2$$

$$s = \frac{v_1^2}{\mu_k g}\left(\frac{4}{25} - \frac{2}{25}\right); \qquad \mathbf{s} = \frac{2}{25}\frac{v_1^2}{\mu_k g} \longleftarrow \blacktriangleleft$$

PROBLEM 16.CQ4

A cord is attached to a spool when a force **P** is applied to the cord as shown. Assuming the spool rolls without slipping, what direction does the spool move for each case?

Case 1:	(*a*) left	(*b*) right	(*c*) It would not move.
Case 2:	(*a*) left	(*b*) right	(*c*) It would not move.
Case 3:	(*a*) left	(*b*) right	(*c*) It would not move.



SOLUTION

Answer:

Case 1: (*a*)

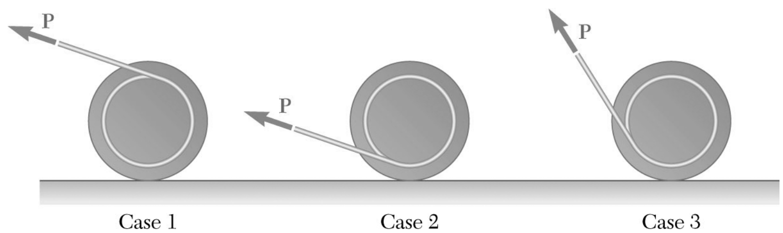
Case 2: (*a*)

Case 3: (*b*)

PROBLEM 16.CQ5

A cord is attached to a spool when a force **P** is applied to the cord as shown. Assuming the spool rolls without slipping, in what direction does the friction force act for each case?

Case 1:	(*a*) left	(*b*) right	(*c*) The friction force would be zero.
Case 2:	(*a*) left	(*b*) right	(*c*) The friction force would be zero.
Case 3:	(*a*) left	(*b*) right	(*c*) The friction force would be zero.



SOLUTION

Answer:

Case 1: (*b*)

Case 2: (*b*)

Case 3: (*b*)

PROBLEM 16.CQ6

A front wheel drive car starts from rest and accelerates to the right. Knowing that the tires do not slip on the road, what is the direction of the friction force the road applies to the front tires?

(*a*) left

(*b*) right

(*c*) The friction force is zero.

SOLUTION

Answer: (*b*)

PROBLEM 16.CQ7

A front wheel drive car starts from rest and accelerates to the right. Knowing that the tires do not slip on the road, what is the direction of the friction force the road applies to the rear tires?

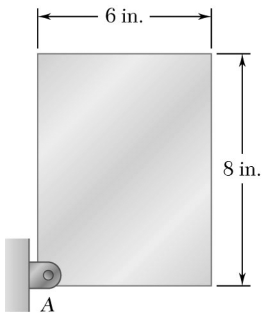
(*a*) left

(*b*) right

(*c*) The friction force is zero.

SOLUTION

Answer: (*a*)

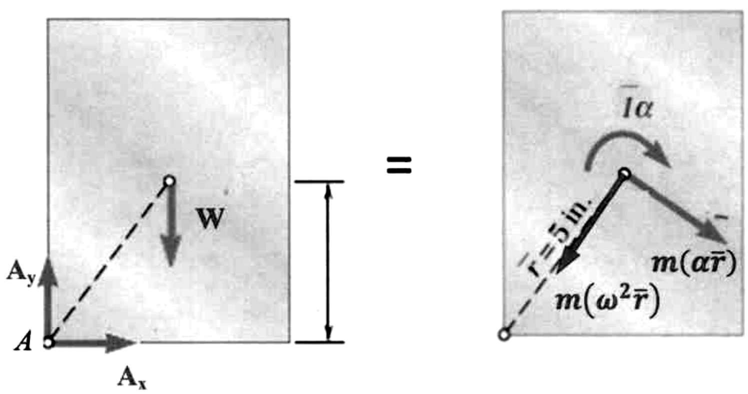


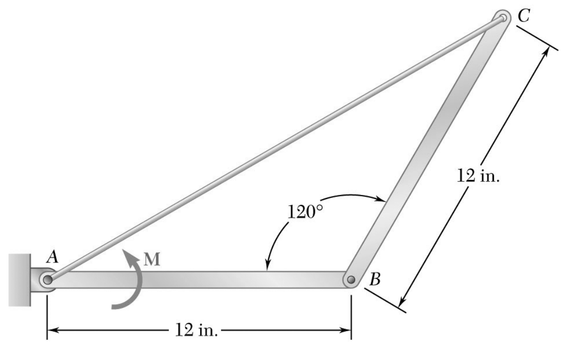
PROBLEM 16.F5

A uniform 6 × 8-in. rectangular plate of mass m is pinned at A. Knowing the angular velocity of the plate at the instant shown is ω, draw the FBD and KD.

SOLUTION

Answer:



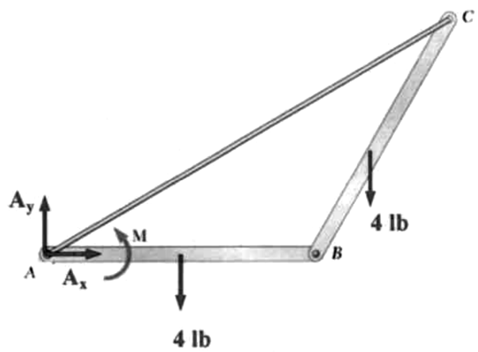


PROBLEM 16.F6

Two identical 4-lb slender rods *AB* and *BC* are connected by a pin at *B* and by the cord *AC*. The assembly rotates in a vertical plane under the combined effect of gravity and a couple **M** applied to rod *AB*. Knowing that in the position shown the angular velocity of the assembly is ω, draw the FBD and KD that can be used to determine the angular acceleration of the assembly and the tension in cord *AC*.

SOLUTION

Answer:



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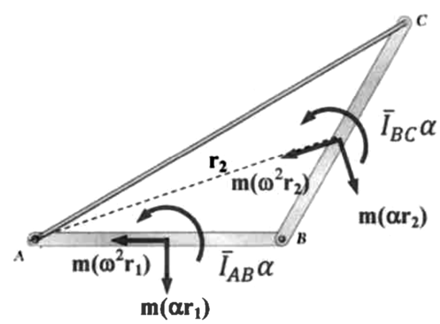
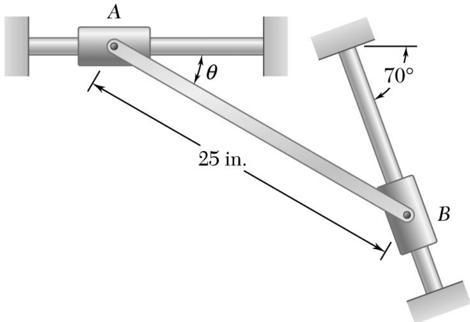


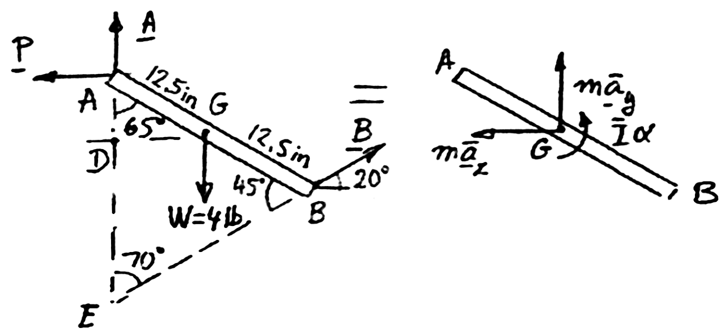


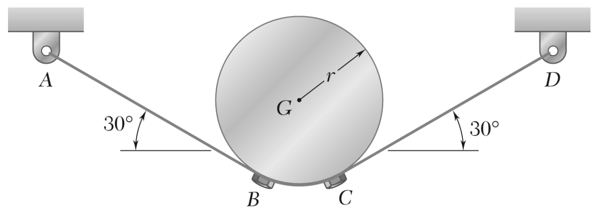
PROBLEM 16.F7

The 4-lb uniform rod *AB* is attached to collars of negligible mass which may slide without friction along the fixed rods shown. Rod *AB* is at rest in the position $\theta = 25°$ when an horizontal force **P** is applied to collar *A* causing it to start moving to the left. Draw the FBD and KD for the rod.

SOLUTION

Answer:



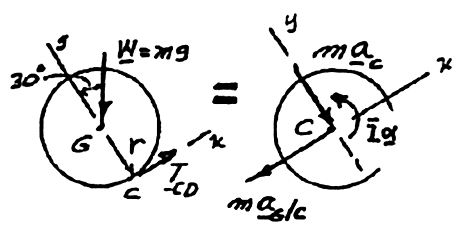


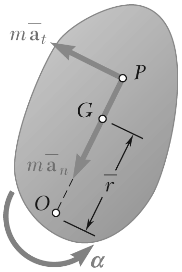
PROBLEM 16.F8

A uniform disk of mass $m = 4$ kg and radius $r = 150$ mm is supported by a belt *ABCD* that is bolted to the disk at *B* and *C*. If the belt suddenly breaks at a point located between *A* and *B*, draw the FBD and KD for the disk immediately after the break.

SOLUTION

Answer:

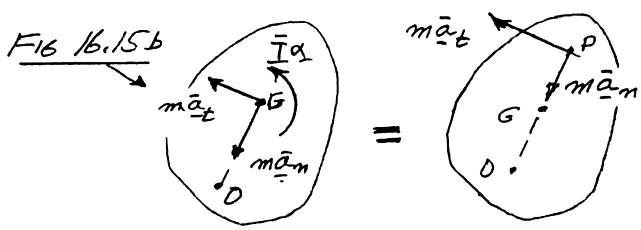




PROBLEM 16.75

Show that the couple $\bar{I}\boldsymbol{\alpha}$ of Figure 16.15 can be eliminated by attaching the vectors $m\bar{\mathbf{a}}_t$ and $m\bar{\mathbf{a}}_n$ at a Point P called the *center of percussion,* located on line OG at a distance $GP = \bar{k}^2/\bar{r}$ from the mass center of the body.

SOLUTION



$$OG = \bar{r} \qquad \bar{\mathbf{a}}_t = \bar{r}\alpha$$

We first observe that the sum of the vectors is the same in both figures. To have the same sum of moments about G, we must have

$$+\circlearrowleft \Sigma M_G = \Sigma M_G: \quad \bar{I}\alpha = (m\bar{a}_t)(GP)$$

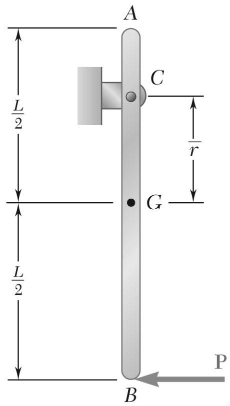
$$m\bar{k}^2\alpha = m\bar{r}\alpha(GP) \qquad\qquad GP = \frac{\bar{k}^2}{\bar{r}} \quad \text{(Q.E.D.)} \blacktriangleleft$$

Note: The center of rotation and the center of percussion are interchangeable. Indeed, since $OG = \bar{r}$, we may write

$$GP = \frac{\bar{k}^2}{GO} \quad \text{or} \quad GO = \frac{\bar{k}^2}{GP}$$

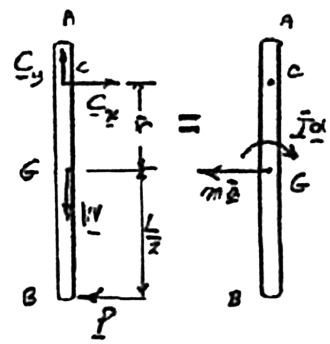
Thus, if Point P is selected as center of rotation, then Point O is the center of percussion.

PROBLEM 16.76



A uniform slender rod of length L = 900 mm and mass m = 4 kg is suspended from a hinge at C. A horizontal force **P** of magnitude 75 N is applied at end B. Knowing that $\bar{r} = 225$ mm, determine (a) the angular acceleration of the rod, (b) the components of the reaction at C.

SOLUTION



(a) Angular acceleration.

$$\bar{a} = \bar{r}\alpha \qquad \bar{I} = \frac{1}{12}mL^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad P\left(\bar{r} + \frac{L}{2}\right) = (m\bar{a})\bar{r} + \bar{I}\alpha$$

$$= (m\bar{r}\alpha)\bar{r} + \frac{1}{12}mL^2\alpha$$

$$P\left(\bar{r} + \frac{L}{2}\right) = m\left(\bar{r}^2 + \frac{1}{12}L^2\right)\alpha$$

Substitute data: $$(75\text{ N})\left[0.225\text{ m} + \frac{0.9\text{ m}}{2}\right] = (4\text{ kg})\left[(0.225\text{ m})^2 + \frac{1}{12}(0.9\text{ m})^2\right]\alpha$$

$$50.625 = 0.4725\alpha$$

$$\alpha = 107.14\text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 107.1\text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(b) Components of reaction at C.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad C_y - W = 0$$

$$C_y = W = mg = (4\text{ kg})(9.81\text{ m/s}^2)$$

$$\mathbf{C}_y = 39.2\text{ N}\uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad C_x - P = -m\bar{a}$$

$$C_x = P - m\bar{a} = P - m(\bar{r}\alpha)$$

$$= 75\text{ N} - (4\text{ kg})(0.225\text{ m})(107.14\text{ rad/s}^2)$$

$$C_x = 75\text{ N} - 96.4\text{ N}$$

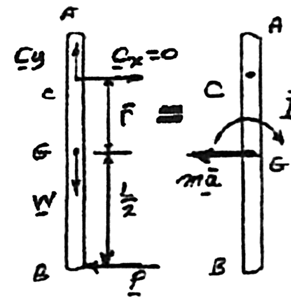
$$= -21.4\text{ N}$$

$$\mathbf{C}_x = 21.4\text{ N} \longleftarrow \blacktriangleleft$$

PROBLEM 16.77

In Problem 16.76, determine (*a*) the distance $\bar{r}$ for which the horizontal component of the reaction at *C* is zero, (*b*) the corresponding angular acceleration of the rod.

SOLUTION



(*a*) Distance $\bar{r}$.

$$\bar{a} = \bar{r}\alpha$$

$$\bar{I} = \frac{1}{12}mL^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P = m\bar{a}$$

$$P = m(\bar{r}\alpha)$$

$$\alpha = \frac{P}{m\bar{r}} \qquad (1)$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad P\frac{L}{2} = \bar{I}\alpha$$

$$P\frac{L}{2} = \frac{1}{12}mL^2\alpha$$

$$P\frac{L}{2} = \frac{1}{12}mL^2\left(\frac{P}{m\bar{r}}\right)$$

$$\frac{L}{2} = \frac{L^2}{12\bar{r}}$$

$$\bar{r} = \frac{1}{6}L \quad \bar{r} = \frac{900 \text{ mm}}{6}$$

$\bar{r} = 150 \text{ mm}$ ◀

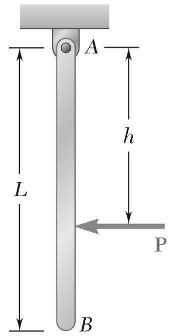
(*b*) Angular acceleration.

Eq. (1):

$$\alpha = \frac{P}{m\bar{r}} = \frac{P}{m\left(\frac{L}{6}\right)} = \frac{6P}{mL}$$

$$\alpha = \frac{6(75 \text{ N})}{(4 \text{ kg})(0.9 \text{ m})} = 125 \text{ rad/s}^2$$

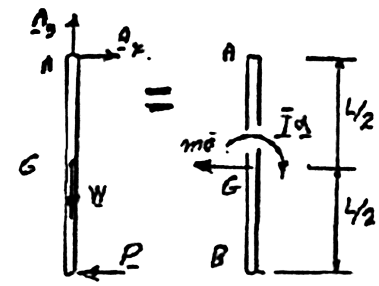
$\alpha = 125 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 16.78

A uniform slender rod of length $L = 36$ in. and weight $W = 4$ lb hangs freely from a hinge at A. If a force **P** of magnitude 1.5 lb is applied at B horizontally to the left ($h = L$), determine (a) the angular acceleration of the rod, (b) the components of the reaction at A.

SOLUTION



$$\bar{a} = \frac{1}{2}\alpha \qquad \bar{I} = \frac{1}{12}mL^2$$

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$$

$$= \left(m\frac{L}{2}\alpha\right)\frac{L}{2} + \frac{1}{12}mL^2\alpha$$

$$PL = \frac{1}{3}mL^2\alpha$$

(a) Angular acceleration.

$$\alpha = \frac{3P}{mL}$$

$$= \frac{3(1.5 \text{ lb})}{\left(\frac{4 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(3 \text{ ft})}$$

$$= 12.08 \text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 12.08 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(b) Components of the reaction at A.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A_y - W = 0$$

$$A_y = W = 4 \text{ lb}$$

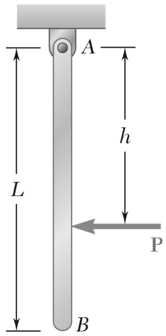
$$\mathbf{A}_y = 4.00 \text{ lb} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A_x - P = -m\bar{a}$$

$$A_x = P - m\left(\frac{L}{2}\alpha\right) = P - m\frac{L}{2}\left(\frac{3P}{mL}\right) = -\frac{P}{2}$$

$$A_x = -\frac{P}{2} = -\frac{1.5 \text{ lb}}{2} = -0.75 \text{ lb}$$

$$\mathbf{A}_x = 0.750 \text{ lb} \longleftarrow \blacktriangleleft$$

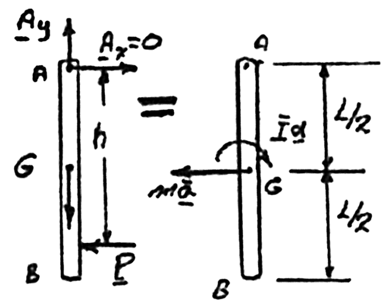


PROBLEM 16.79

In Problem 16.78, determine (*a*) the distance h for which the horizontal component of the reaction at A is zero, (*b*) the corresponding angular acceleration of the rod.

PROBLEM 16.78 A uniform slender rod of length $L = 36$ in. and weight $W = 4$ lb hangs freely from a hinge at A. If a force **P** of magnitude 1.5 lb is applied at B horizontally to the left ($h = L$), determine (*a*) the angular acceleration of the rod, (*b*) the components of the reaction at A.

SOLUTION



$$\bar{a} = \frac{L}{2}\alpha$$

$$\bar{I} = \frac{1}{12}mL^2$$

$\overset{+}{\leftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}:$

$$P = m\bar{a}$$

$$P = m\left(\frac{L}{2}\alpha\right)$$

(*b*) Angular acceleration.

$$\alpha = \frac{2P}{mL}$$

$$\alpha = \frac{2(1.5\text{ lb})}{\left(\frac{4\text{ lb}}{32.2\text{ ft/s}^2}\right)(3\text{ ft})}$$

$\boldsymbol{\alpha} = 8.05\text{ rad/s}^2 \circlearrowright$ ◀

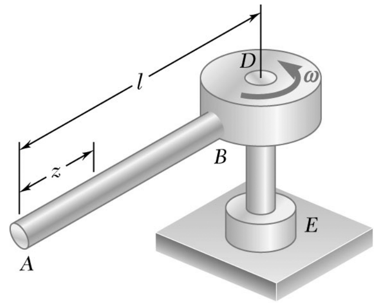
$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}:$ $\quad P\left(h - \frac{L}{2}\right) = \bar{I}\alpha: \quad P(h - L)$

$$P\left(h - \frac{L}{2}\right) = \frac{1}{12}mL^2\left(\frac{2P}{mL}\right) = \frac{PL}{6}$$

$$\left(h - \frac{L}{2}\right) = \frac{L}{6}; \quad h = \frac{L}{2} + \frac{L}{6} = \frac{2}{3}L$$

(*a*) Distance h. $\qquad h = 24$ in. ◀

PROBLEM 16.80



The uniform slender rod AB is welded to the hub D, and the system rotates about the vertical axis DE with a constant angular velocity $\boldsymbol{\omega}$, (*a*) Denoting by ω the mass per unit length of the rod, express the tension in the rod at a distance z from end A in terms of w, l, z, and $\boldsymbol{\omega}$, (*b*) Determine the tension in the rod for $w = 0.3$ kg/m, $l = 400$ mm, $z = 250$ mm, and $\boldsymbol{\omega} = 150$ rpm.

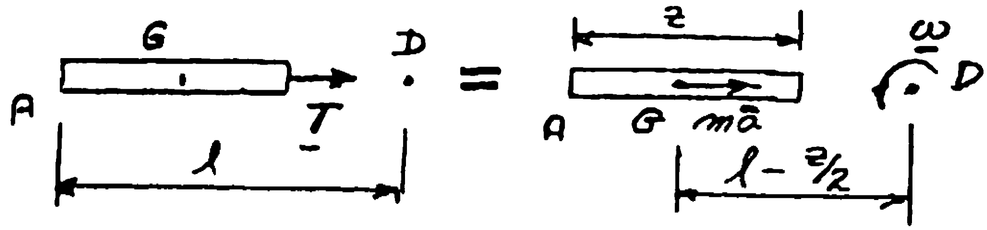
SOLUTION

Consider motion in the horizontal plane. Since ω is constant, the angular acceleration is zero. Only normal acceleration, i.e., along the rod occurs. For a section defined by the coordinate z, the acceleration of the mass center of the portion extending from z to the section is

$$\bar{a} = r\omega^2 = (l - z/2)\omega^2$$

Kinetics. The mass of the section is

$$m = wz$$



(*a*) $\xrightarrow{+} \Sigma F = \Sigma F_{\text{eff}}: \quad T = m\bar{a}$

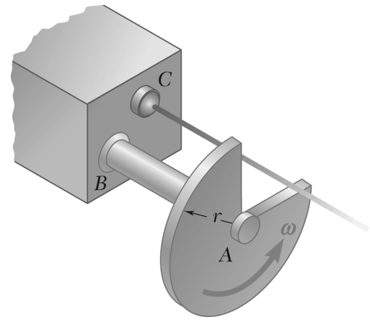
$$= (wz)\left(l - \frac{z}{2}\right)\omega^2 \qquad\qquad T = w\left(lz - \frac{z^2}{2}\right)\omega^2 \blacktriangleleft$$

(*b*) Data: $\omega = 150 \text{ rpm} = \dfrac{(150)(2\pi)}{60} = 5\pi \text{ rad/s}$

$z = 0.250 \text{ m}, \quad l = 0.400 \text{ m}, \quad w = 0.3 \text{ kg/m}$

$$T = (0.3 \text{ kg/m})\left[(0.4 \text{ m})(0.25 \text{ m}) - \frac{(0.25 \text{ m})^2}{2}\right](5\pi \text{ rad/s})^2 = 5.09 \text{ N}$$

$$T = 5.09 \text{ N} \blacktriangleleft$$

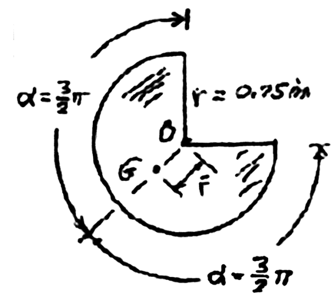


PROBLEM 16.81

The shutter shown was formed by removing one quarter of a disk of 0.75-in. radius and is used to interrupt a beam of light emanating from a lens at C. Knowing that the shutter weighs 0.125 lb and rotates at the constant rate of 24 cycles per second, determine the magnitude of the force exerted by the shutter on the shaft at A.

SOLUTION

See inside front cover for centroid of a circular sector.



$$\bar{r} = \frac{2r\sin\alpha}{3\alpha}$$

$$\bar{r} = \frac{2(0.75\text{ in.})\sin(\frac{3}{4}\pi)}{3(\frac{3}{4}\pi)}$$

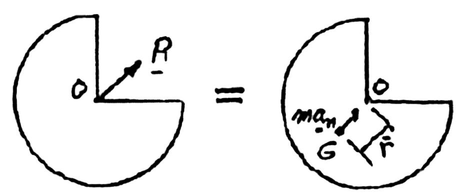
$$\bar{r} = 0.15005\text{ in.}$$

$$a_n = r\omega^2$$

$$\omega = 24\text{ rad/s}$$

$$= 24(2\pi)\text{ rad/s}$$

$$\omega = 150.8\text{ rad/s}$$



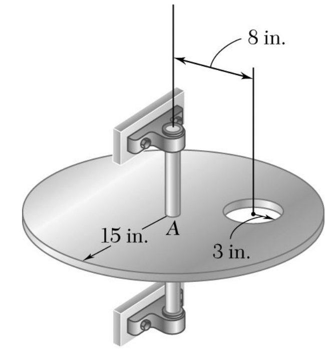
$+\nearrow\ \Sigma F = \Sigma F_{\text{eff}}$: $\quad R = m\,a_n = m\bar{r}\omega^2$

$$= \frac{(0.125\text{ lb})}{32.2\text{ ft/s}^2}\left(\frac{0.15005}{12}\text{ft}\right)(150.8\text{ rad/s})^2$$

$$R = 1.1038\text{ lb} \nearrow$$

Force on shaft is $\quad R = 1.104\text{ lb} \swarrow$

Magnitude: $\quad R = 1.104\text{ lb}$ ◀



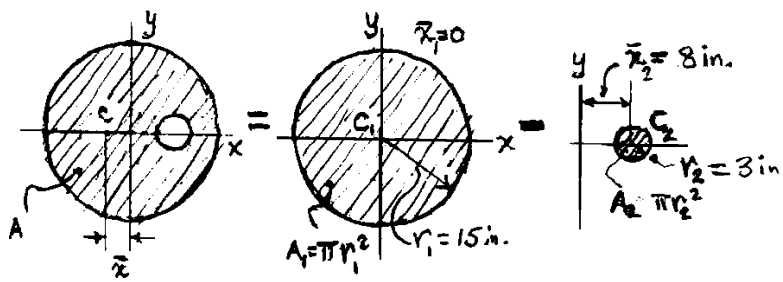
PROBLEM 16.82

A 6-in.-diameter hole is cut as shown in a thin disk of 15-in.-diameter. The disk rotates in a horizontal plane about its geometric center A at the constant rate of 480 rpm. Knowing that the disk has a mass of 60 lb after the hole has been cut, determine the horizontal component of the force exerted by the shaft on the disk at A.

SOLUTION

Determination of mass center of disk

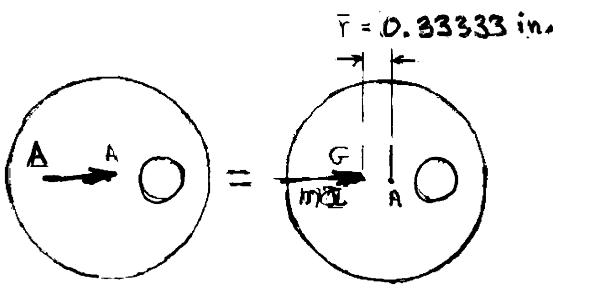
We determine the centroid of the composite area:



$$\bar{x}A = \bar{x}_1 A_1 - \bar{x}_2 A_2 \quad \text{or} \quad \bar{x}(A_1 - A_2) = \bar{x}_1 A_1 - \bar{x}_2 A_2$$

$$\bar{x} = \frac{\bar{x}A_1 - \bar{x}_2 A_2}{A_1 - A_2} = \frac{0 - (8)\pi(3)^2}{\pi(15)^2 - \pi(3)^2} = -\frac{(8)(3)^2}{(15)^2 - (3)^2} = -0.33333 \text{ in.}$$

Kinetics



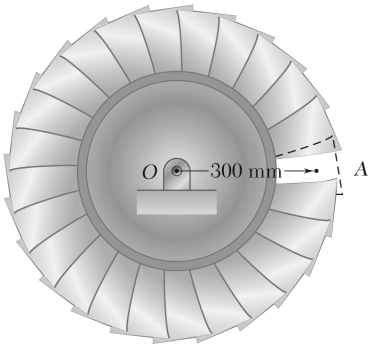
Mass center G coincides with centroid C

$\omega = 480 \text{ rpm} = 50.265 \text{ rad/s}$

$$\bar{a} = \bar{r}\omega^2 = \left(\frac{0.33333}{12}\right)(50.265 \text{ rad/s})^2 = 70.183 \text{ ft/s}^2$$

$$\Sigma F = \Sigma(F)_{\text{eff}}: \ \mathbf{A} = m\bar{\mathbf{a}} = \left(\frac{60 \text{ lb}}{32.2}\right)(70.183 \text{ ft/s}^2)$$

$\mathbf{A} = 130.8 \text{ N} \longrightarrow$ ◀



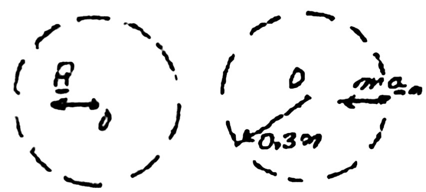
PROBLEM 16.83

A turbine disk of mass 26 kg rotates at a constant rate of 9600 rpm. Knowing that the mass center of the disk coincides with the center of rotation O, determine the reaction at O immediately after a single blade at A, of mass 45 g, becomes loose and is thrown off.

SOLUTION

$$\omega = 9600 \text{ rpm}\left(\frac{2\pi}{60}\right)$$

$$\omega = 320\pi \text{ rad/s}$$



Consider before it is thrown off.

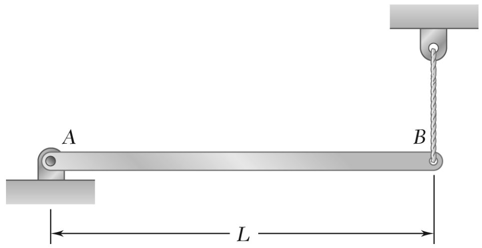
$$\xleftarrow{+} \Sigma F = \Sigma F_{\text{eff}}: \quad R = ma_n = mF\omega^2$$

$$= (45\times 10^{-3}\text{ kg})(0.3\text{ m})(320\,\pi)^2$$

$$R = 13.64 \text{ kN}$$

Before blade was thrown off, the disk was balanced $(R = 0)$. Removing vane at A also removes its reaction, so disk is unbalanced and reaction is

$$\mathbf{R} = 13.64 \text{ kN} \longrightarrow \blacktriangleleft$$

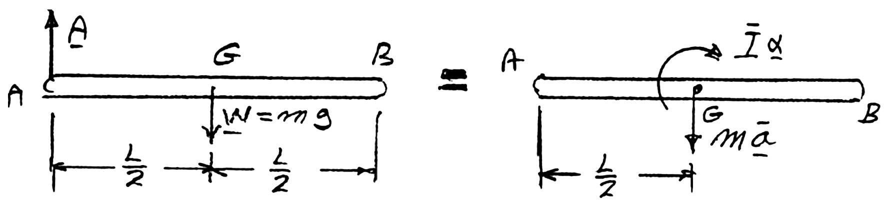


PROBLEM 16.84

A uniform rod of length L and mass m is supported as shown. If the cable attached at end B suddenly breaks, determine (*a*) the acceleration of end B, (*b*) the reaction at the pin support.

SOLUTION

$$w = 0 \quad \bar{a} = \frac{L}{2}\alpha$$



$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad W\frac{L}{2} = \bar{I}\alpha + m\bar{a}\frac{L}{2}$$

$$mg\frac{L}{2} = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{2}\alpha\right)\frac{L}{2}$$

$$mg\frac{L}{2} = \frac{1}{3}mL^2\alpha \quad \alpha = \frac{3}{2}\frac{g}{L} \circlearrowright$$

(*b*) Reaction at *A*.

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A - mg = -m\bar{a} = -m\frac{L}{2}\alpha$$

$$A - mg = -m\left(\frac{L}{2}\right)\left(\frac{3}{2}\frac{g}{L}\right)$$

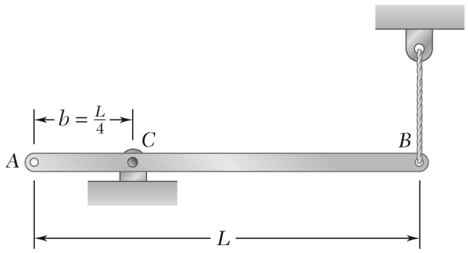
$$A - mg = -\frac{3}{4}mg$$

$$A = \frac{1}{4}mg \qquad \mathbf{A} = \frac{1}{4}mg \uparrow \blacktriangleleft$$

(*a*) Acceleration of *B*.

$$\mathbf{a}_B = \mathbf{a}_n + \mathbf{a}_{B/A} = 0 + L\alpha \downarrow$$

$$\mathbf{a}_B = L\left(\frac{3}{2}\frac{g}{L}\right) = \frac{3}{2}g \downarrow \qquad \mathbf{a}_B = \frac{3}{2}g \downarrow \blacktriangleleft$$

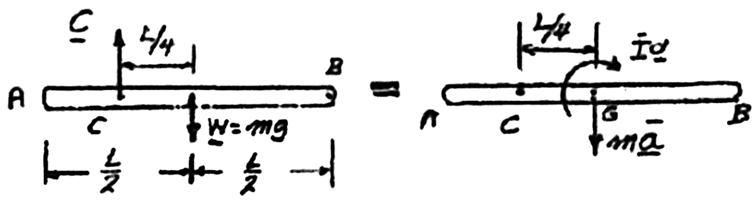


PROBLEM 16.85

A uniform rod of length L and mass m is supported as shown. If the cable attached at end B suddenly breaks, determine (*a*) the acceleration of end B, (*b*) the reaction at the pin support.

SOLUTION

$$\omega = 0 \qquad \bar{a} = \frac{L}{4}\alpha$$



$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W\frac{L}{4} = \bar{I}\alpha + m\bar{a}\frac{L}{4}$$

$$mg\frac{L}{4} = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{4}\alpha\right)\frac{L}{4}$$

$$mg\frac{L}{4} = \frac{7}{48}mL^2\alpha \qquad \boldsymbol{\alpha} = \frac{12g}{7L} \circlearrowright$$

(*b*) Reaction at C.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad C - mg = -m\bar{a} = -m\frac{L}{4}\alpha$$

$$C - mg = -m\left(\frac{L}{4}\right)\left(\frac{12g}{7L}\right)$$

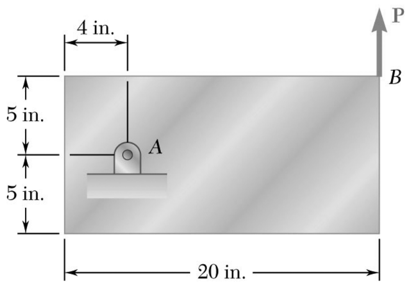
$$C - mg = -\frac{3}{7}mg$$

$$C = \frac{4}{7}mg \qquad \mathbf{C} = \frac{4}{7}mg\uparrow \blacktriangleleft$$

(*a*) Acceleration of B.

$$a_B = a_C + a_{C/B} = 0 + \frac{3L}{4}\alpha$$

$$a_B = \frac{3L}{4}\left(\frac{12g}{7L}\right) = \frac{9}{7}g \qquad \mathbf{a}_B = \frac{9}{7}g\downarrow \blacktriangleleft$$



PROBLEM 16.86

A 12-lb uniform plate rotates about A in a vertical plane under the combined effect of gravity and of the vertical force **P**. Knowing that at the instant shown the plate has an angular velocity of 20 rad/s and an angular acceleration of 30 rad/s^2 both counterclockwise, determine (a) the force **P**, (b) the components of the reaction at A.

SOLUTION

Kinematics.

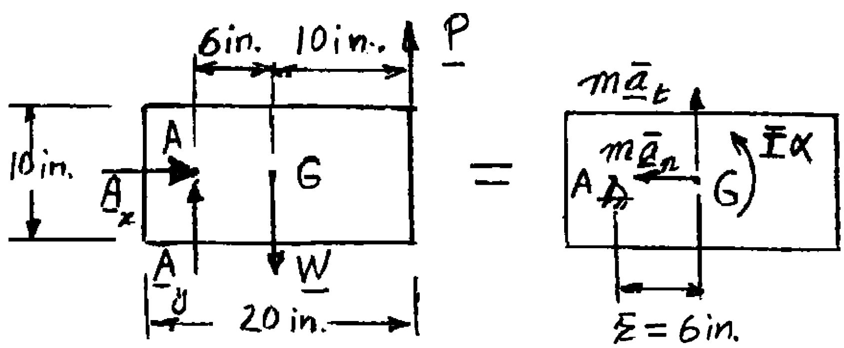
$$\bar{a}_t = r\alpha = \left(\frac{6}{12}\text{ ft}\right)(30\text{ rad/s}^2) = 15\text{ ft/s}^2$$

$$\bar{a}_n = r\omega^2 = \left(\frac{6}{12}\text{ ft}\right)(20\text{ rad/s})^2 = 200\text{ ft/s}^2$$

Mass and moment of inertia. $$m = \frac{W}{g} = \frac{12\text{ lb}}{32.2\text{ ft/s}^2} = 0.37267\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\bar{I} = \frac{m}{12}\left[\left(\frac{10}{12}\right)^2 + \left(\frac{20}{12}\right)^2\right] = (0.37267\text{ lb}\cdot\text{s}^2/\text{ft})(0.28935\text{ ft}^2) = 0.10783\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinetics.



(a) Force **P**.

$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad P\left(\frac{16}{12}\text{ ft}\right) - W\left(\frac{6}{12}\text{ ft}\right) = m\bar{a}_t\left(\frac{6}{12}\text{ ft}\right) + \bar{I}\alpha$$

$$\frac{4}{3}P = (12)\left(\frac{1}{2}\right) + (0.37267)(15)\left(\frac{1}{2}\right) + (0.10783)(30)$$

$$P = 9.0224\text{ lb.}$$

$\mathbf{P} = 9.02\text{ lb}\uparrow$ ◀

PROBLEM 16.86 (Continued)

(*b*) Reaction at *A*.

$$\xrightarrow{+} \Sigma F_y = \Sigma (F_x)_{\text{eff}}: \quad A_x = -m\bar{a}_n = -\frac{12}{32.2}(200)$$

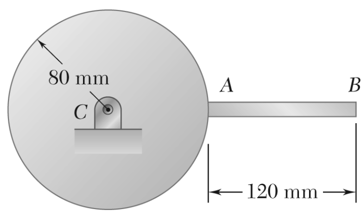
$$A_x = -74.53 \text{ lb}$$

$$\mathbf{A}_x = 74.5 \text{ lb} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A_y + P - W = m\bar{a}_t$$

$$A_y = W + m\bar{a}_t - P = 12 + \frac{12}{32.2}(15) - 9.02 = 8.57 \text{ lb}$$

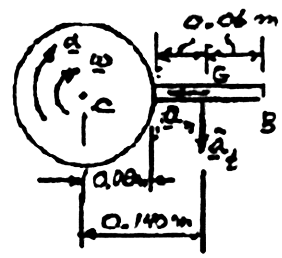
$$\mathbf{A}_y = 8.57 \text{ lb}\uparrow \blacktriangleleft$$



PROBLEM 16.87

A 1.5-kg slender rod is welded to a 5-kg uniform disk as shown. The assembly swings freely about C in a vertical plane. Knowing that in the position shown the assembly has an angular velocity of 10 rad/s clockwise, determine (*a*) the angular acceleration of the assembly, (*b*) the components of the reaction at C.

SOLUTION



Kinematics:

$$\bar{a}_n = (CG)\omega^2 = (0.14\text{ m})(10\text{ rad/s}^2)$$

$$\bar{\mathbf{a}}_n = 14\text{ m/s}^2 \longleftarrow$$

$$\bar{\mathbf{a}}_t = (CG)\alpha = (0.14\text{ m})\alpha \downarrow$$

Kinetics:

$$\bar{I}_{\text{disk}} = \frac{1}{2}m_{\text{disk}}(CG)^2$$

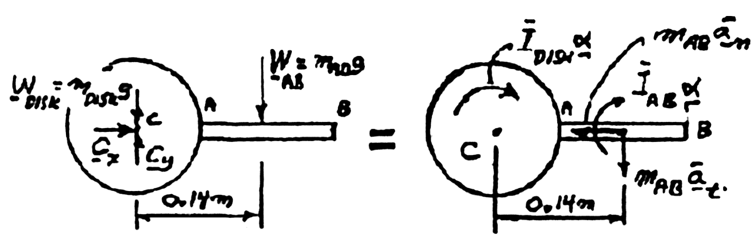
$$= \frac{1}{2}(5\text{ kg})(0.08\text{ m})^2$$

$$= 16\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{AB} = \frac{1}{12}m_{AB}(AB)^2$$

$$= \frac{1}{12}(1.5\text{ kg})(0.12\text{ m})^2$$

$$= 1.8\times10^{-3}\text{ kg}\cdot\text{m}^2$$



(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}:$$

$$W_{AB}(0.14\text{ m}) = \bar{I}_{\text{disk}}\alpha + m_{AB}\bar{a}_t(0.14\text{ m}) + \bar{I}_{AB}\alpha$$

$$(1.5\text{ kg})(9.81\text{ m/s}^2)(0.14\text{ m}) = \bar{I}_{\text{disk}}\alpha + (1.5\text{ kg})(0.14\text{ m})^2\alpha + \bar{I}_{AB}\alpha$$

$$2.060\text{ N}\cdot\text{m} = (16\times10^{-3} + 29.4\times10^{-3} + 1.8\times10^{-3})\alpha$$

$$2.060\text{ N}\cdot\text{m} = (47.2\times10^{-3}\text{ kg}\cdot\text{m}^2)\alpha$$

$$\alpha = 43.64\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 43.6\text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

PROBLEM 16.87 (Continued)

(*b*) Components of reaction of *C*.

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad C_x = -m_{AB}\bar{a}_n = -(1.5\text{ kg})(14\text{ m/s}^2)$$

$$C_x = -21.0\text{ N} \qquad\qquad \mathbf{C}_x = 21.0\text{ N} \longleftarrow \blacktriangleleft$$

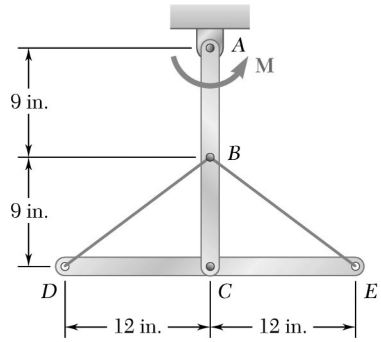
$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad a_t = (0.14\text{ m})(\alpha)$$

$$C_y - m_{\text{disk}}g - m_{AB}g = -m_{AB}\bar{a}_t$$

$$C_y - (5\text{ kg})9.81 - (1.5\text{ kg})\,9.81 = -(1.5\text{ kg})(0.14\text{ m})(43.64\text{ rad/s}^2)$$

$$C_y - 49.05\text{ N} - 14.715\text{ N} = -9.164\text{ N}$$

$$C_y = +54.6\text{ N} \qquad\qquad \mathbf{C}_y = 54.6\text{ N} \uparrow \blacktriangleleft$$



PROBLEM 16.88

Two uniform rods, *ABC* of weight 6-lb and *DCE* of weight 8-lb, are connected by a pin at *C* and by two cords *BD* and *BE*. The T-shaped assembly rotates in a vertical plane under the combined effect of gravity and of a couple **M** which is applied to rod *ABC*. Knowing that at the instant shown the tension in cord *BE* is 2 lb and the tension in cord *BD* is 0.5 lb, determine (*a*) the angular acceleration of the assembly, (*b*) the couple **M**.

SOLUTION

We first consider the *entire system* and express that the *external* forces are equivalent to the *effective* forces of both rods.

$$(\bar{a}_{AC})_t = 0.75\alpha$$

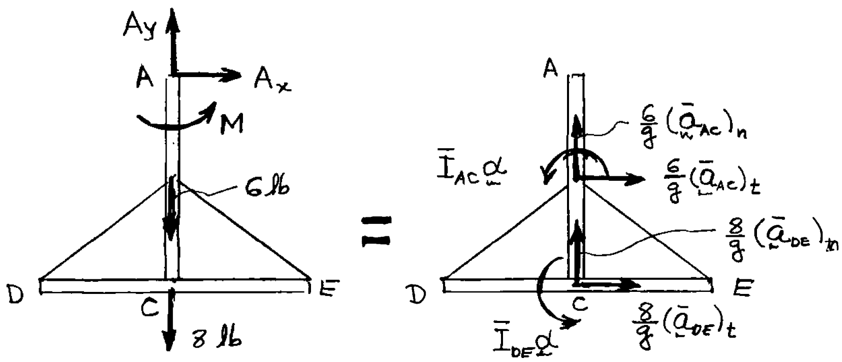
$$(\bar{a}_{DE})_t = 1.5\alpha$$

$$\bar{I}_{AC} = \frac{1}{12}\left(\frac{6\text{ lb}}{32.2}\right)(1.5\text{ ft})^2$$

$$= 34.938\times 10^{-3}\,\text{slug}\cdot\text{ft}^2$$

$$\bar{I}_{DE} = \frac{1}{12}\left(\frac{8\text{ lb}}{32.2}\right)(2\text{ ft})^2$$

$$= 82.816\times 10^{-3}\,\text{slug}\cdot\text{ft}^2$$

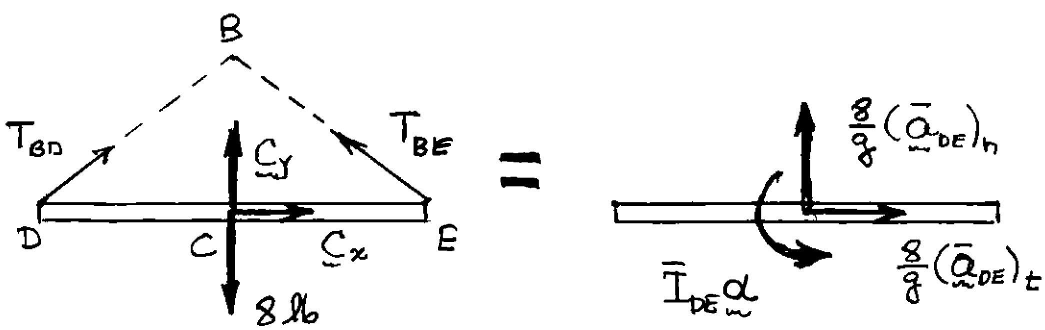


$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad M = \bar{I}_{AC}\alpha + \frac{6}{32.2}(\bar{a}_{AC})_t(0.75) + \bar{I}_{DE}\alpha + \frac{8}{32.2}(\bar{a}_{DE})_t(1.5)$$

$$M = 34.938\times 10^{-3}\alpha + \frac{6}{32.2}(0.75\alpha)(0.75) + 82.816\times 10^{-3}\alpha + \frac{8}{32.2}(1.5\alpha)(1.5)$$

$$M = 0.78157\alpha \qquad (1)$$

We now consider rod *DE* alone:



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad \left(\frac{3}{5}T_{BE}\right)(1\text{ ft}) - \left(\frac{3}{5}T_{BD}\right)(1\text{ ft}) = \bar{I}_{DE}\alpha$$

$$0.6(T_{BE} - T_{BD}) = 82.816\times 10^{-3}\alpha$$

$$T_{BE} - T_{BD} = 0.13803\,\alpha \qquad (2)$$

Given data:

$$T_{BD} = 0.5\text{ lb}$$

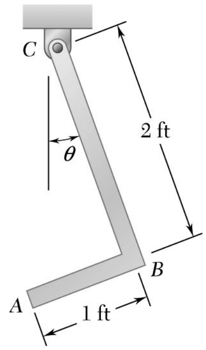
$$T_{BE} = 2\text{ lb}$$

(*a*) Angular acceleration.

Substitute into (2): $\quad 2 - 0.5 = 0.13803\alpha \qquad \boldsymbol{\alpha} = 10.87\text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) Couple **M**.

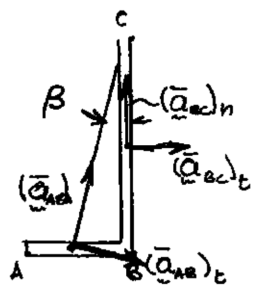
Carry value of α into (1): $\quad M = 0.78157(10.868) = 8.4938\text{ ft}\cdot\text{lb} \qquad \mathbf{M} = 8.49\text{ ft}\cdot\text{lb} \circlearrowleft$ ◀



PROBLEM 16.89

The object ABC consists of two slender rods welded together at Point B. Rod AB has a weight of 2 lb and bar BC has a weight of 4 lb. Knowing the magnitude of the angular velocity of ABC is 10 rad/s when $\theta = 0$, determine the components of the reaction at Point C when $\theta = 0$.

SOLUTION



Masses and lengths: $W_{AB} = 2\text{ lb}, \quad L_{AB} = 1\text{ ft}, \quad W_{BC} = 4\text{ lb}, \quad L_{BC} = 2\text{ ft}$

Moments of inertia: $\bar{I}_{AB} = \dfrac{1}{12} m_{AB} L_{AB}^2 = \dfrac{1}{12}\left(\dfrac{2}{32.2}\right)(1)^2 = 5.1760 \times 10^{-3}\text{ slug}\cdot\text{ft}^2$

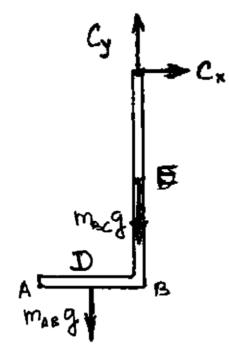
$$I_{BC} = \frac{1}{12} m_{BC} L_{BC}^2 = \frac{1}{12}\left(\frac{4}{32.2}\right)(2)^2 = 41.408 \times 10^{-3}\text{ slug}\cdot\text{ft}^2$$

Geometry: $r_{CD} = \sqrt{2^2 + 0.5^2} = 2.0616\text{ ft}$

$$r_{CE} = \frac{1}{2} L_{AB} = 1\text{ ft}$$

$$\tan\beta = \frac{0.5}{2}$$

$$\beta = 14.036°$$



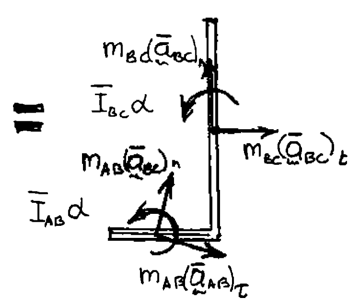
Kinematics: Let $\boldsymbol{\alpha} = \alpha \circlearrowright$ be the angular acceleration of object ABC.

$$(\bar{\mathbf{a}}_{AB})_t = r_{CD}\alpha \searrow \beta$$

$$(\bar{\mathbf{a}}_{AB})_n = r_{CD}\omega^2 \nearrow \beta$$

$$(\bar{\mathbf{a}}_{BC})_t = r_{CE}\alpha \longrightarrow$$

$$(\bar{\mathbf{a}}_{BC})_n = r_{CE}\omega^2 \uparrow$$



Kinetics: $+\circlearrowright \Sigma M_C = +\circlearrowright \Sigma(M_C)_{\text{eff}}$: $\quad W_{AB}\dfrac{L_{AB}}{2} = \bar{I}_{AB}\alpha + r_{CD} m_{AB}(\bar{a}_{AB})_t$

$$+ \bar{I}_{BC}\alpha + r_{CE} m_{BC}(\bar{a}_{BC})_t$$

$$= \left(\bar{I}_{AB} + m_{AB} r_{CD}^2 + \bar{I}_{BC} + m_{BC} r_{CE}^2\right)\alpha$$

$$(2)(0.5) = \left[(5.1760 \times 10^{-3}) + \left(\frac{2}{32.2}\right)(2.0616)^2 + (41.408 \times 10^{-3}) + \left(\frac{4}{32.2}\right)(1)^2\right]\alpha \qquad \boldsymbol{\alpha} = 2.3\text{ rad/s}^2 \circlearrowright$$

PROBLEM 16.89 (Continued)

$\xrightarrow{+} \Sigma F_x = \xrightarrow{+} \Sigma (F_x)_{\text{eff}}$: $\quad C_x = m_{AB}(\bar{a}_{AB})_t \cos\beta + m_{AB}(\bar{a}_{AB})_n \sin\beta + m_{BC}(a_{BC})_t$

$$= m_{AB} r_{CD}(\alpha\cos\beta + \omega^2 \sin\beta) + m_{BC} r_{CE}\alpha$$

$$= \left(\frac{2}{32.2}\right)(2.0616)(2.3\cos 14.035° + 10^2 \sin 14.035°) + \left(\frac{4}{32.2}\right)(1)(2.3)$$

$$= 3.6770 \text{ lb}$$

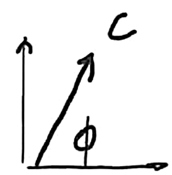
$+\!\uparrow \Sigma F_y = +\!\uparrow \Sigma (F_y)_{\text{eff}}$: $\quad C_y - m_{AB}g - m_{BC}g = -m_{AB}(\bar{a}_{AB})_t \sin\beta + m_{AB}(\bar{a}_{AB})_n \cos\beta + m_{BC}(\bar{a}_{BC})_n$

$$C_y = (W_{AB} + W_{BC}) + m_{AB} r_{CD}(\omega^2 \cos\beta - \alpha\sin\beta) + m_{BC} r_{CE}\,\omega$$

$$C_y = 6 + \left(\frac{2}{32.2}\right)(2.0616)(10^2 \cos 14.035° - 2.3\sin 14.035°)$$

$$+ \left(\frac{4}{32.2}\right)(1)(10)^2$$

$$C_y = 30.773 \text{ lb}$$



Reaction at C.

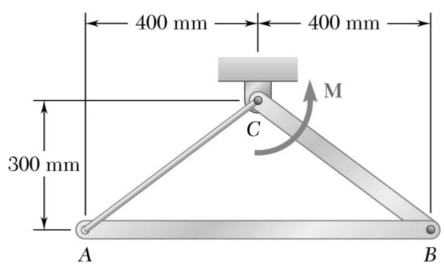
$$C = \sqrt{3.6770^2 + 30.773^2}$$

$$= 30.992 \text{ lb}$$

$$\tan\phi = \frac{30.773}{3.6770}$$

$$\phi = 83.186°$$

$\mathbf{C} = 31.0 \text{ lb} \angle 83.2°$ ◀



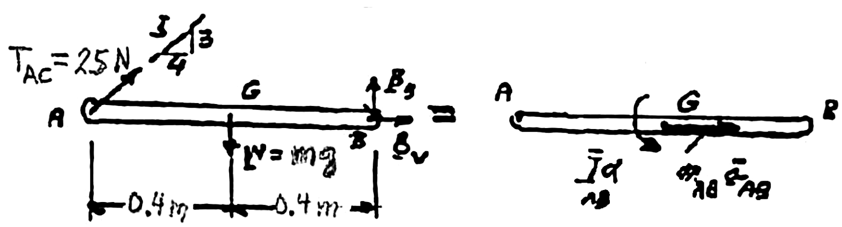
PROBLEM 16.90

A 3.5-kg slender rod AB and a 2-kg slender rod BC are connected by a pin at B and by the cord AC. The assembly can rotate in a vertical plane under the combined effect of gravity and a couple $\mathbf{M}$ applied to rod BC. Knowing that in the position shown the angular velocity of the assembly is zero and the tension in cord AC is equal to 25 N, determine (a) the angular acceleration of the assembly, (b) the magnitude of the couple $\mathbf{M}$.

SOLUTION

(a) Angular acceleration.

Rod AB:
$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2$$
$$= \frac{1}{12} 3.5\ \text{kg}(0.8\ \text{m})^2$$
$$= 0.18667\ \text{kg}\cdot\text{m}^2$$



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad \frac{3}{5}(25\ \text{N})(0.8\ \text{m}) - (3.5\ \text{kg})(9.81\ \text{m/s}^2)(0.4\ \text{m}) = \bar{I}_{AB}\alpha$$

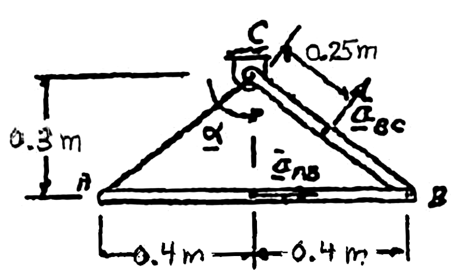
$$\bar{I}_{AB}\alpha = -1.7340\ \text{N}\cdot\text{m} \qquad (1)$$

$$(0.18667\ \text{kg}\cdot\text{m}^2)\alpha = -1.7340\ \text{N}\cdot\text{m}$$

$$\boldsymbol{\alpha} = -9.2893\ \text{rad/s}^2 \qquad \boldsymbol{\alpha} = 9.29\ \text{rad/s}^2 \circlearrowright \blacktriangleleft$$

Entire assembly: Since AC is taut, assembly rotates about C as a rigid body.

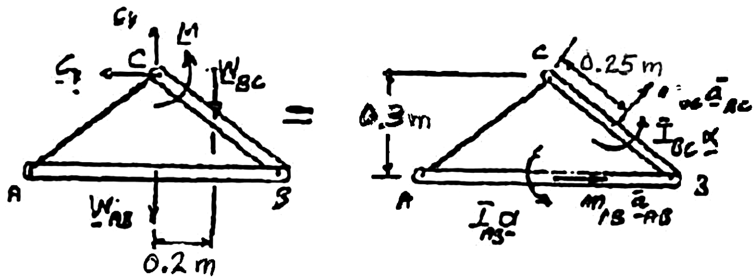
Kinematics:



$$CB = \sqrt{(0.3)^2 + (0.4)^2} = 0.5\ \text{m}$$
$$CG_{BC} = \frac{1}{2} CB = 0.25\ \text{m}$$
$$\bar{\mathbf{a}}_{BC} = (0.25\ \text{m})\alpha \nearrow$$
$$\bar{\mathbf{a}}_{AB} = (0.3\ \text{m})\alpha \longrightarrow$$

Kinetics:

$$\bar{I}_{BC} = \frac{1}{12} m_{BC}(CB)^2$$
$$= \frac{1}{12}(2\text{ kg})(0.5\text{ m})^2$$
$$= 0.041667\text{ kg}\cdot\text{m}^2$$



(*b*) Couple **M**.

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}:$$

$$M - m_{BC}g(0.2\text{ m}) = m_{BC}\bar{a}_{BC}(0.25\text{ m}) + \bar{I}_{BC}\alpha + m_{AB}\bar{a}_{AB}(0.3\text{ m}) + \bar{I}_{AB}\alpha$$
$$M - (2\text{ kg})(9.81\text{ m/s}^2)(0.2\text{ m}) = 2\text{ kg}(0.25\text{ m})^2\alpha + (0.041667\text{ kg}\cdot\text{m}^2)\alpha + 3.5\text{ kg}(0.3\text{ m})^2\alpha + \bar{I}_{AB}\alpha$$

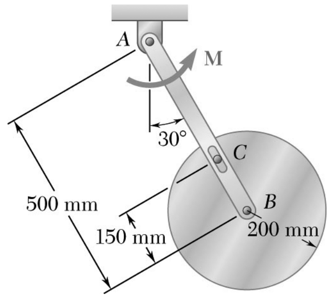
Substitute $\alpha = 9.29\text{ rad/s}^2$ $\quad \bar{I}_{AB}\alpha = -1.7340\text{ N}\cdot\text{m}$

$$M - 3.9240 = (0.125)(9.2893) + (0.041667)(9.2893) + (0.315)(9.2893) + (0.18667)(9.2893)$$
$$M - 3.9240 = 1.1612 + 0.3871 + 2.9261 + 1.7340$$
$$M - 3.9240 = 6.2084$$
$$M = +10.132\text{ N}\cdot\text{m}$$

$$\mathbf{M} = 10.13\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

$$M - m_{BC}g(0.2\text{ m}) = I_C\alpha$$

Since C is fixed, we could also use: $M - (3.5\text{ kg})(9.81\text{ m/s}^2)(0.2\text{ m}) = \left(\frac{1}{3}m_{CB}\overline{CB}^2 + \bar{I}_{AB} + 3.5\text{ kg}(0.3\text{ m})^2\right)\alpha$



PROBLEM 16.91

A 9-kg uniform disk is attached to the 5-kg slender rod AB by means of frictionless pins at B and C. The assembly rotates in a vertical plane under the combined effect of gravity and of a couple **M** which is applied to rod AB. Knowing that at the instant shown the assembly has an angular velocity of 6 rad/s and an angular acceleration of 25 rad/s^2, both counterclockwise, determine (a) the couple **M**, (b) the force exerted by pin C on member AB.

SOLUTION

We first consider the entire system and express that the external forces are equivalent to the effective forces of the disk and the rod.

$$m_R = 5\text{ kg}, \qquad m_D = 9\text{ kg}$$

$$W_R = m_D g = (5\text{ kg})(9.81\text{ m/s}^2) = 49.05\text{ N}$$

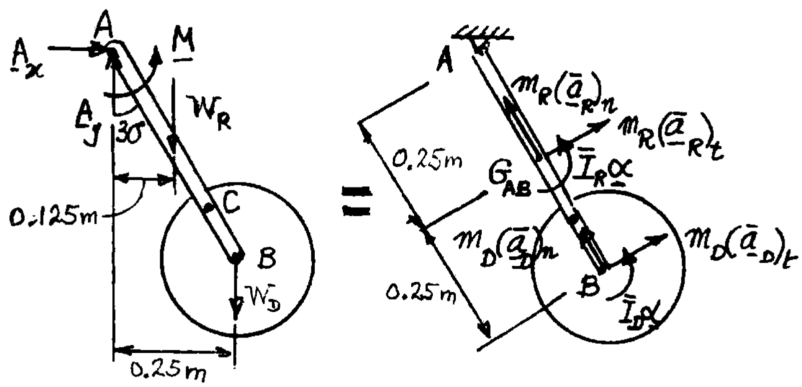
$$W_D = m_D g = (9\text{ kg})(9.81\text{ m/s}^2) = 88.29\text{ N}$$

$$I_R = \frac{1}{12} m_R L_{AB}^2 = \frac{1}{12}(5\text{ kg})(0.5\text{ m})^2 = 0.104167\text{ kg}\cdot\text{m}^2$$

$$I_D = \frac{1}{2} m_D r_D^2 = \frac{1}{2}(9\text{ kg})(0.2\text{ m})^2 = 0.18\text{ kg}\cdot\text{m}^2$$

$$\bar{a}_R = (0.25\text{ m})\alpha$$

$$\bar{a}_D = (0.5\text{ m})\alpha$$



(a) $\quad \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad M - W_R(0.125) - W_D(0.25) = \bar{I}_R\alpha + m_R(a_R)_t(0.25) + \bar{I}_D\alpha + m_D(\bar{a}_D)_t(0.5)$

$$M - (49.05)(0.125) - (88.29)(0.25) = 0.104167\alpha + (5)(0.25\alpha)(0.25) + 0.18\alpha + (9)(0.5\alpha)(0.5)$$

$$M - 6.1312 - 22.073 = (0.10417 + 0.3125 + 0.18 + 2.25)\alpha$$

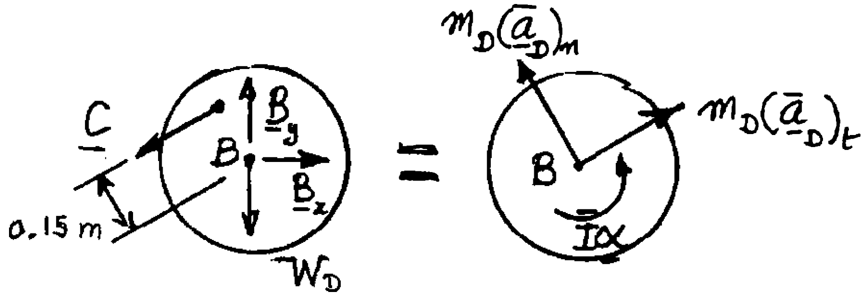
$$M - 28.204 = (2.8467)(25)$$

$$M = 99.370\text{ N-m}$$

$M = 99.4$ N-m ◀

PROBLEM 16.91 (Continued)

(*b*) Consider now the <u>disk alone</u>:



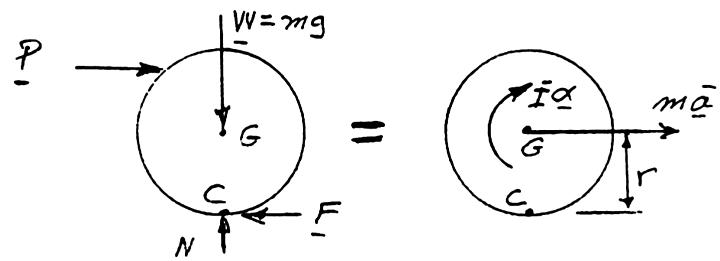
$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad C(0.15) = \bar{I}_D \alpha = (0.18)(25)$$

C = 30.0 N ∡ 30° ◀

PROBLEM 16.92

Derive the equation $\Sigma M_C = I_C \alpha$ for the rolling disk of Figure 16.17, where ΣM_C represents the sum of the moments of the external forces about the instantaneous center C, and I_C is the moment of inertia of the disk about C.

SOLUTION



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad \Sigma M_C = (m\bar{a})r + \bar{I}\alpha = (mr\alpha)\bar{r} + \bar{I}\alpha$$

$$= (mr\alpha)\bar{r} + \bar{I}\alpha$$

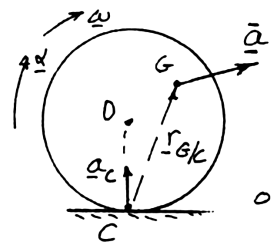
But, we know that $\qquad I_C = mr^2 + \bar{I}$

Thus: $\qquad \Sigma M_C = I_C \alpha \quad$ (Q.E.D.) ◀

PROBLEM 16.93

Show that in the case of an unbalanced disk, the equation derived in Problem 16.92 is valid only when the mass center G, the geometric center O, and the instantaneous center C happen to lie in a straight line.

SOLUTION



Kinematics:

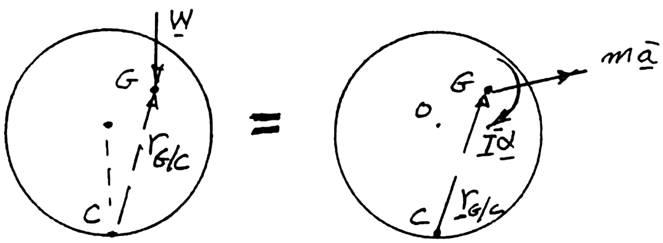
$$\bar{\mathbf{a}} = \mathbf{a}_C + \mathbf{a}_{G/C}$$

$$= \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{G/C} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{G/C})$$

or, since $\boldsymbol{\omega} \perp \mathbf{r}_{G/C}$

$$\bar{\mathbf{a}} = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{G/C} - \omega^2 \mathbf{r}_{G/C} \qquad (1)$$

Kinetics:



$$\Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad \Sigma \mathrm{M}_C = \bar{I}\boldsymbol{\alpha} + \mathbf{r}_{G/C} \times m\bar{\mathbf{a}}$$

Recall Eq. (1):
$$\Sigma \mathbf{M}_C = \bar{I}\alpha + \mathbf{r}_{G/C} \times m(\mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{G/C} - \omega^2 r_{G/C})$$

$$\Sigma \mathbf{M}_C = \bar{I}\alpha + \mathbf{r}_{G/C} \times m\mathbf{a}_C + m\mathbf{r}_{G/C} \times (\boldsymbol{\alpha} \times \mathbf{r}_{G/C}) - m\omega^2 \mathbf{r}_{G/C} \times \mathbf{r}_{G/C}$$

But: $\quad \mathbf{r}_{G/C} \times \mathbf{r}_{G/C} = 0 \quad$ and $\quad \boldsymbol{\alpha} \perp \mathbf{r}_{G/C}$

$$\mathbf{r}_{G/C} \times m(\boldsymbol{\alpha} \times \mathbf{r}_{G/C}) = m r_{G/C}^2 \alpha$$

Thus:
$$\Sigma \mathbf{M}_C = \left(\bar{I} + m r_{G/C}^2\right)\alpha + \mathbf{r}_{G/C} \times m\mathbf{a}_C$$

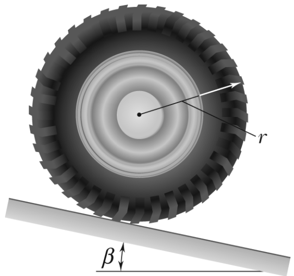
Since
$$I_C = \bar{I} + m r_{G/C}^2$$

$$\Sigma \mathbf{M}_C = I_C \alpha + \mathbf{r}_{G/C} \times m\mathbf{a}_C \qquad (2)$$

Eq. (2) reduces to $\Sigma M_C = I_C \alpha$ when $\mathbf{r}_{G/C} \times m\mathbf{a}_C = 0$; that is, when $\mathbf{r}_{G/C}$ and $\mathbf{a}_C$ are collinear.

Referring to the first diagram, we note that this will occur only when Points G, O, and C lie in a straight line.

(Q.E.D.) ◀



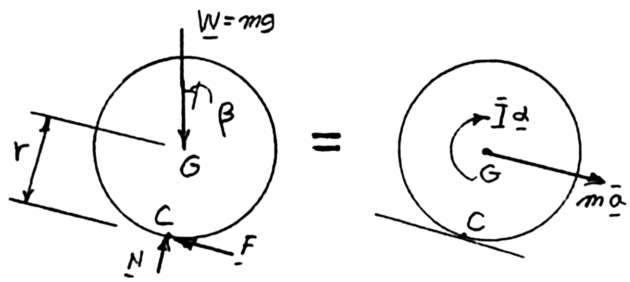
PROBLEM 16.94

A wheel of radius r and centroidal radius of gyration $\bar{k}$ is released from rest on the incline and rolls without sliding. Derive an expression for the acceleration of the center of the wheel in terms of r, $\bar{k}$, β, and g.

SOLUTION

$$\bar{I}\alpha = m\bar{k}^2\alpha$$

$$m\bar{a} = mr\alpha$$



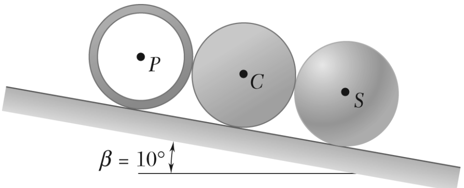
$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (W \sin\beta)r = (m\bar{a})r + \bar{I}\alpha$$

$$(mg \sin\beta)r = (mr\alpha)r + m\bar{k}^2\alpha$$

$$rg \sin\beta = (r^2 + \bar{k}^2)\alpha$$

$$\alpha = \frac{rg \sin\beta}{r^2 + \bar{k}^2}$$

$$\bar{a} = r\alpha = r\frac{rg \sin\beta}{r^2 + k^2} \qquad \bar{a} = \frac{r^2}{r^2 + \bar{k}^2} g \sin\beta \blacktriangleleft$$



PROBLEM 16.95

A homogeneous sphere S, a uniform cylinder C, and a thin pipe P are in contact when they are released from rest on the incline shown. Knowing that all three objects roll without slipping, determine, after 4 s of motion, the clear distance between (*a*) the pipe and the cylinder, (*b*) the cylinder and the sphere.

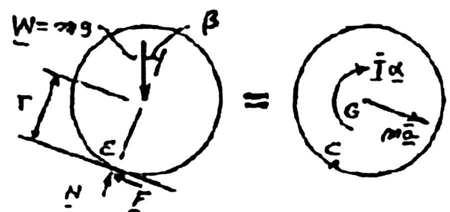
SOLUTION

General case: $\quad \bar{I} = m\bar{k}^2 \qquad \bar{a} = r\alpha$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad (w \sin\beta) r = \bar{I}\alpha + m\bar{a}r$$

$$mg \sin\beta r = m\bar{k}^2\alpha + mr^2\alpha$$

$$\alpha = \frac{r\theta \sin\beta}{r^2 + \bar{k}^2}$$



$$\bar{a} = r\alpha = r\frac{rg\sin\beta}{r^2+\bar{k}^2} \qquad \bar{a} = \frac{r^2}{r^2+\bar{k}^2} g\sin\beta \tag{1}$$

For pipe: $\quad \bar{k} = r \qquad \bar{a}_P = \dfrac{r^2}{r^2+r^2} g\sin\beta = \dfrac{1}{2} g\sin\beta$

For cylinder: $\quad \bar{k}^2 = \dfrac{1}{2} \qquad a_C = \dfrac{r^2}{r^2+\dfrac{r^2}{2}} g\sin\beta = \dfrac{2}{3} g\sin\beta$

For sphere: $\quad \bar{k}^2 = \dfrac{2}{5} \qquad a_S = \dfrac{r^2}{r^2+\dfrac{2}{5}r^2} g\sin\beta = \dfrac{5}{7} g\sin\beta$

(*a*) <u>Between pipe and cylinder</u>.

$$a_{C/P} = a_C - a_P = \left(\frac{2}{3} - \frac{1}{2}\right) g\sin\beta = \frac{1}{6} g\sin\beta$$

$$x_{C/P} = \frac{1}{2} a_{C/P} t^2 = \frac{1}{2}\left(\frac{1}{6} g\sin\beta\right) t^2$$

SI units: $\quad x_{C/P} = \dfrac{1}{2}\left(\dfrac{1}{6} 9.81 \text{ m/s}^2\right)\sin 10°(4 \text{ s})^2 = 2.27 \text{ m}$ ◀

US units: $\quad x_{C/P} = \dfrac{1}{2}\left(\dfrac{1}{6} 32.2 \text{ ft/s}^2\right)\sin 10°(4 \text{ s})^2 = 7.46 \text{ ft}$ ◀

PROBLEM 16.95 (Continued)

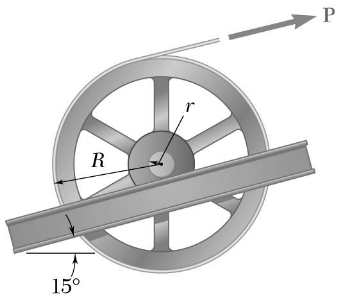
(*b*) Between sphere and cylinder.

$$a_{S/C} = a_S - a_C = \left(\frac{5}{7} - \frac{2}{3}\right) g \sin\beta = \frac{1}{21} g \sin\beta$$

$$x_{S/C} = \frac{1}{2} a_{S/C} t^2 = \frac{1}{2}\left(\frac{1}{21} g \sin\beta\right) t^2$$

SI units: $$x_{S/C} = \frac{1}{2}\left(\frac{1}{21} 9.81 \text{ m/s}^2\right) \sin 10°(4 \text{ s})^2 = 0.649 \text{ m} \blacktriangleleft$$

US units: $$x_{S/C} = \frac{1}{2}\left(\frac{1}{21} 32.2 \text{ ft/s}^2\right) \sin 10°(4 \text{ s})^2 = 2.13 \text{ ft} \blacktriangleleft$$



PROBLEM 16.96

A 40-kg flywheel of radius $R = 0.5$ m is rigidly attached to a shaft of radius $r = 0.05$ m that can roll along parallel rails. A cord is attached as shown and pulled with a force **P** of magnitude 150 N. Knowing the centroidal radius of gyration is $\bar{k} = 0.4$ m, determine (*a*) the angular acceleration of the flywheel, (*b*) the velocity of the center of gravity after 5 s.

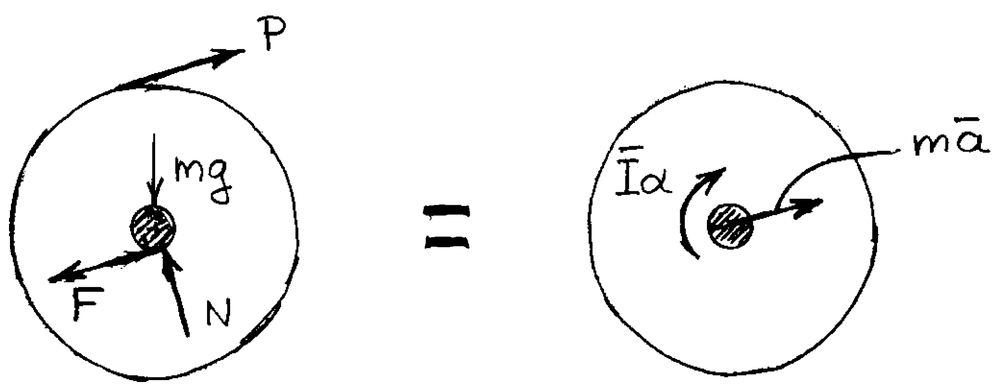
SOLUTION

Mass and moment of inertia: $m = 40$ kg

$$\bar{I} = m\bar{k}^2 = (40\text{ kg})(0.4\text{ m})^2 = 6.4\text{ kg}\cdot\text{m}^2$$

Kinematics: (no slipping) $\bar{\mathbf{a}} = r\alpha \measuredangle 15°$

Kinetics:



Let Point C be the contact point between the flywheel and the rails.

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad P(R+r) - mgr\sin 15° = \bar{I}\alpha + (m\bar{a})r$$

$$P(R+r) - mgr\sin 15° = (\bar{I} + mr^2)\alpha$$

(*a*) *Angular acceleration of the flywheel.*

$$\alpha = \frac{P(R+r) - mgr\sin 15°}{\bar{I} + mr^2}$$

$$= \frac{(150\text{ N})(0.55\text{ m}) - (40\text{ kg})(9.81\text{ m/s}^2)(0.05\text{ m})\sin 15°}{(6.4\text{ kg}\cdot\text{m}^2) + (40\text{ kg})(0.05\text{ m})^2}$$

$$= 11.911\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 11.91\text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

PROBLEM 16.96 (Continued)

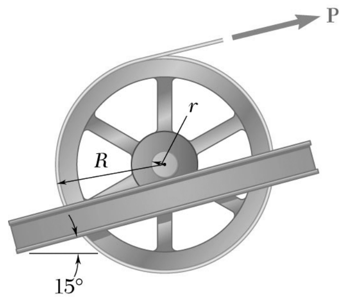
(*b*) *Velocity of center of gravity after* 5 s.

$$\bar{a} = r\alpha = (0.05\text{ m})(11.911\text{ rad/s}^2) = 0.59555\text{ m/s}^2$$

$$\bar{\mathbf{a}} = 0.59555\text{ m/s}^2 \measuredangle 15°$$

$$\bar{\mathbf{v}} = \bar{\mathbf{v}}_0 + \bar{\mathbf{a}}t = 0 + (0.59555\text{ m/s}^2)(5\text{ s})$$

$$\bar{\mathbf{v}} = 2.98\text{ m/s} \measuredangle 15° \blacktriangleleft$$



PROBLEM 16.97

A 40-kg flywheel of radius R = 0.5 m is rigidly attached to a shaft of radius r = 0.05 m that can roll along parallel rails. A cord is attached as shown and pulled with a force **P**. Knowing the centroidal radius of gyration is $\bar{k} = 0.4$ m and the coefficient of static friction is $\mu_s = 0.4$, determine the largest magnitude of force **P** for which no slipping will occur.

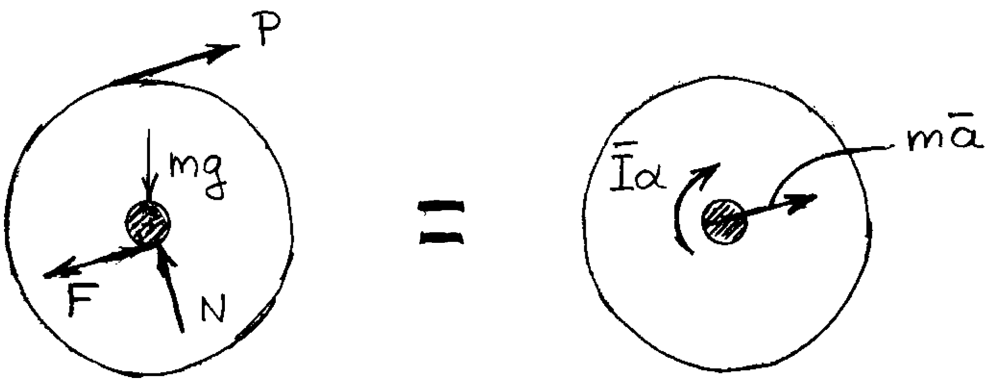
SOLUTION

Mass and moment of inertia: $m = 40$ kg

$$\bar{I} = m\bar{k}^2 = (40\text{ kg})(0.4\text{ m})^2 = 6.4\text{ kg}\cdot\text{m}^2$$

Kinematics: (no slipping) $\bar{\mathbf{a}} = r\alpha \measuredangle 15°$

Kinetics:



$$+\Sigma F_n = \Sigma(F_n)_{\text{eff}}: \quad N - mg\cos 15° = 0$$

$$N = (40\text{ kg})(9.81\text{ m/s}^2)\cos 15° = 379.03\text{ N}$$

For impending slipping, $F = \mu_s N = (0.4)(379.03)$

$$= 151.61\text{ N}$$

$$+\measuredangle 15°\ \Sigma F = \Sigma(F)_{\text{eff}}: \quad P - F - mg\sin 15° = ma$$

$$P - 151.61\text{ N} = (40\text{ kg})(9.81\text{ m/s}^2)\sin 15° = (40\text{ kg})(0.05\text{ m})\alpha$$

$$P - 253.17\text{ N} = (2\text{ kg}\cdot\text{m})\alpha \qquad (1)$$

PROBLEM 16.97 (Continued)

Let C be the contact point between the flywheel and the rails.

$$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad P(R+r) - mgr\sin 15^\circ = \bar{I}a + (m\bar{a})r$$

$$P(R+r) - mgr\sin 15^\circ = (\bar{I} + mr^2)\alpha$$

$$(0.55 \text{ m})P - (40 \text{ kg})(9.81 \text{ m/s}^2)(0.05 \text{ m})\sin 15^\circ$$

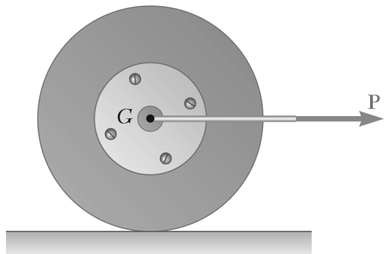
$$= [6.4 \text{ kg}\cdot\text{m}^2 + (40 \text{ kg})(0.05 \text{ m})^2]\alpha \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$P = 303 \text{ N} \qquad \alpha = 24.8 \text{ rad/s}^2$$

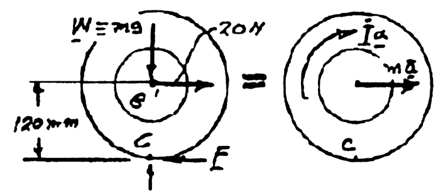
$P = 303 \text{ N}$ ◀

PROBLEM 16.98



A drum of 60-mm radius is attached to a disk of 120-mm radius. The disk and drum have a total mass of 6 kg and a combined radius of gyration of 90 mm. A cord is attached as shown and pulled with a force **P** of magnitude 20 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{k}^2$$

$$= (6 \text{ kg})(0.09 \text{ m})^2$$

$$\bar{I} = 48.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (20 \text{ N})(0.12 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$2.4 \text{ N} \cdot \text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + 48.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$2.4 = 135.0 \times 10^{-3}\alpha$$

(*a*) $\alpha = 17.778 \text{ rad/s}^2$ $\qquad \boldsymbol{\alpha} = 17.78 \text{ rad/s}^2 \circlearrowright$ ◀

$$\bar{a} = r\alpha = (0.12 \text{ m})(17.778 \text{ rad/s}^2)$$

$$= 2.133 \text{ m/s}^2 \qquad \bar{\mathbf{a}} = 2.13 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

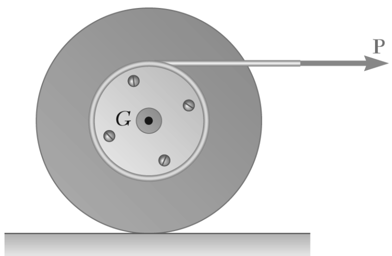
(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - mg = 0$

$$N = (6 \text{ kg})(9.81 \text{ m/s}^2) \qquad \mathbf{N} = 58.86 \text{ N}\uparrow$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad 20 \text{ N} - F = m\bar{a}$$

$$20 \text{ N} - F = (6 \text{ kg})(2.133 \text{ m/s}^2) \qquad \mathbf{F} = 7.20 \text{ N} \longleftarrow$$

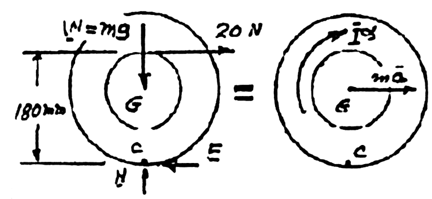
$$(\mu_s)_{\min} = \frac{F}{N} = \frac{7.20 \text{ N}}{58.86 \text{ N}} \qquad (\mu_s)_{\min} = 0.122 \blacktriangleleft$$



PROBLEM 16.99

A drum of 60-mm radius is attached to a disk of 120-mm radius. The disk and drum have a total mass of 6 kg and a combined radius of gyration of 90 mm. A cord is attached as shown and pulled with a force **P** of magnitude 20 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{k}^2$$

$$= (6 \text{ kg})(0.09 \text{ m})^2$$

$$\bar{I} = 48.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (20 \text{ N})(0.18 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$3.6 \text{ N} \cdot \text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + 48.6 \text{ kg} \cdot \text{m}^2$$

$$3.6 = 135 \times 10^{-3}\alpha$$

(*a*) $\alpha = 26.667 \text{ rad/s}^2$ $\qquad$ $\boldsymbol{\alpha} = 26.7 \text{ rad/s}^2 \circlearrowright$ ◀

$$\bar{a} = r\alpha = (0.12 \text{ m})(26.667 \text{ rad/s}^2)$$

$= 3.2 \text{ m/s}^2$ $\qquad$ $\bar{\mathbf{a}} = 3.20 \text{ m/s}^2 \longrightarrow$ ◀

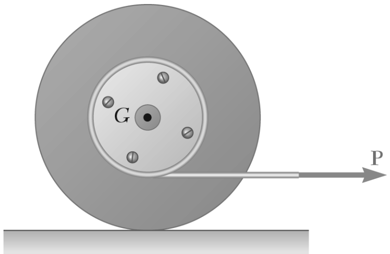
(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - mg = 0$

$N = (6 \text{ kg})(9.81 \text{ m/s}^2)\uparrow$ $\qquad$ $\mathbf{N} = 58.86 \text{ N}\uparrow$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad 20 \text{ N} - F = m\bar{a}$$

$20 \text{ N} - F = (6 \text{ kg})(3.2 \text{ m/s}^2)$ $\qquad$ $\mathbf{F} = 0.8 \text{ N} \longleftarrow$

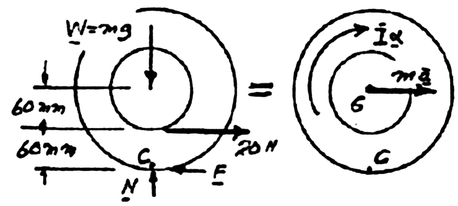
$(\mu_s)_{\min} = \dfrac{F}{N} = \dfrac{0.8 \text{ N}}{58.86 \text{ N}}$ $\qquad$ $(\mu_s)_{\min} = 0.0136$ ◀



PROBLEM 16.100

A drum of 60-mm radius is attached to a disk of 120-mm radius. The disk and drum have a total mass of 6 kg and a combined radius of gyration of 90 mm. A cord is attached as shown and pulled with a force **P** of magnitude 20 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = (0.12\text{ m})\alpha$$

$$\bar{I} = m\bar{k}^2 = (6\text{ kg})(0.09\text{ m})^2$$

$$\bar{I} = 48.6\text{ kg}\cdot\text{m}^2$$

$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}$: $(20\text{ N})(0.06\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$$1.2\text{ N}\cdot\text{m} = (6\text{ kg})(0.12\text{ m})^2\alpha + 48.6\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$1.2 = 135\times10^{-3}\alpha$$

(*a*) $\alpha = 8.889\text{ rad/s}^2$ $\qquad\qquad$ $\boldsymbol{\alpha} = 8.89\text{ rad/s}^2 \circlearrowright$ ◀

$$\bar{a} = r\alpha = (0.12\text{ m})(8.889\text{ rad/s}^2) = 1.0667\text{ m/s}^2$$

$\bar{\mathbf{a}} = 1.067\text{ m/s}^2 \longrightarrow$ ◀

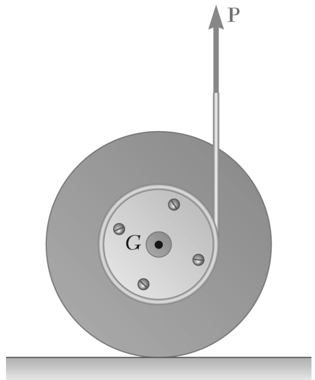
(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $N - mg = 0$

$N = (6\text{ kg})(9.81\text{ m/s}^2)$ $\qquad$ $\mathbf{N} = 58.86\text{ N}\uparrow$

$\xrightarrow{+} \Sigma F_k = \Sigma(F_x)_{\text{eff}}$: $20\text{ N} - F = m\bar{a}$

$(20\text{ N}) - F = (6\text{ kg})(1.0667\text{ m/s}^2)$ $\qquad$ $\mathbf{F} = 13.6\text{ N} \longleftarrow$

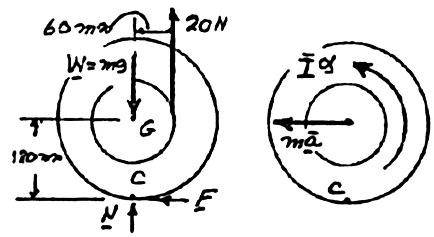
$(\mu_s)_{\min} = \dfrac{F}{N} = \dfrac{13.6\text{ N}}{58.86\text{ N}}$ $\qquad$ $(\mu_s)_{\min} = 0.231$ ◀



PROBLEM 16.101

A drum of 60-mm radius is attached to a disk of 120-mm radius. The disk and drum have a total mass of 6 kg and a combined radius of gyration of 90 mm. A cord is attached as shown and pulled with a force **P** of magnitude 20 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{k}^2$$

$$= (6 \text{ kg})(0.09 \text{ m})^2$$

$$\bar{I} = 48.6 \text{ kg}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (20 \text{ N})(0.06 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$1.2 \text{ N}\cdot\text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + (48.6\times10^{-3} \text{kg}\cdot\text{m}^2)\alpha$$

$$1.2 = 135\times10^{-3}\alpha$$

(*a*) $$\alpha = 8.889 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 8.89 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

$$\bar{a} = r\alpha = (0.12 \text{ m})(8.889 \text{ rad/s}^2)$$

$$= 1.0667 \text{ m/s}^2 \qquad \bar{\mathbf{a}} = 1.067 \text{ m/s}^2 \longleftarrow \blacktriangleleft$$

(*b*) $$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N + 20 \text{ N} - mg = 0$$

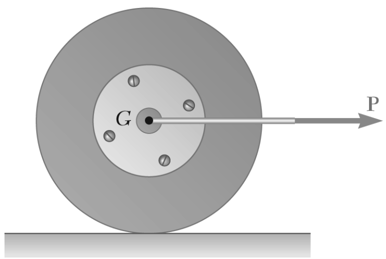
$$N + 20 \text{ N} - (6 \text{ kg})(9.81 \text{ m/s}^2) \qquad \mathbf{N} = 38.86 \text{ N}\uparrow$$

$$\xleftarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F = m\bar{a}$$

$$F = (6 \text{ kg})(1.0667 \text{ m/s}^2) \qquad \mathbf{F} = 6.4 \text{ N} \longleftarrow$$

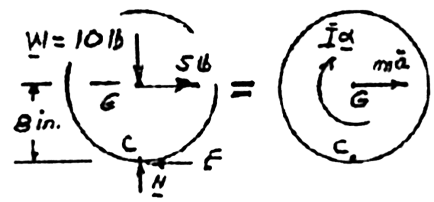
$$(\mu_s)_{\min} = \frac{F}{N} = \frac{6.4 \text{ N}}{38.86 \text{ N}} \qquad (\mu_s)_{\min} = 0.165 \blacktriangleleft$$

PROBLEM 16.102



A drum of 4-in. radius is attached to a disk of 8-in. radius. The disk and drum have a combined weight of 10 lb and a combined radius of gyration of 6 in. A cord is attached as shown and pulled with a force **P** of magnitude 5 lb. Knowing that the coefficients of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively, determine (*a*) whether or not the disk slides, (*b*) the angular acceleration of the disk and the acceleration of *G*.

SOLUTION



Assume disk rolls:

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$$

$$\bar{I} = m\bar{k}^2 = \frac{10 \text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$$

$$\bar{I} = 0.07764 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (5 \text{ lb})\left(\frac{8}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$$

$$3.333 \text{ lb}\cdot\text{ft} = \frac{10 \text{ lb}}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2 \alpha + 0.07764\alpha$$

$$3.333 = 0.21566\alpha$$

$$\alpha = 15.456 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 15.46 \text{ rad/s}^2 \circlearrowright$$

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)(15.456 \text{ rad/s}^2) \qquad \bar{\mathbf{a}} = 10.30 \text{ ft/s}^2 \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad -F + 5 \text{ lb} = m\bar{a}$$

$$-F + 5 \text{ lb} = \frac{10 \text{ lb}}{32.2}(10.30 \text{ ft/s}^2)$$

$$F = 1.80 \text{ lb}$$

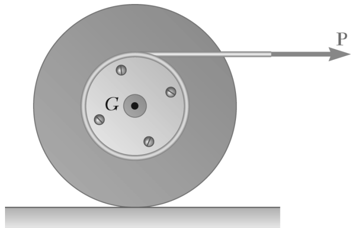
$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - 10 \text{ lb} = 0 \qquad N = 10 \text{ lb}$$

$$F_m = \mu_s N = 0.25(10 \text{ lb}) = 2.5 \text{ lb}$$

(*a*) Since $F < F_m$, Disk rolls without sliding ◀

(*b*) Angular acceleration of the disk. $\boldsymbol{\alpha} = 15.46 \text{ rad/s}^2 \circlearrowright$ ◀

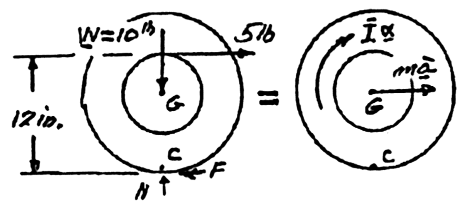
Acceleration of *G*. $\bar{\mathbf{a}} = 10.30 \text{ ft/s}^2 \longrightarrow$ ◀



PROBLEM 16.103

A drum of 4-in. radius is attached to a disk of 8-in. radius. The disk and drum have a combined weight of 10 lb and a combined radius of gyration of 6 in. A cord is attached as shown and pulled with a force **P** of magnitude 5 lb. Knowing that the coefficients of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively, determine (*a*) whether or not the disk slides, (*b*) the angular acceleration of the disk and the acceleration of *G*.

SOLUTION



Assume disk rolls: $\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$

$$\bar{I} = m\bar{k}^2$$

$$= \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$$

$$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (5\text{ lb})(1\text{ ft}) = (m\bar{a})r + \bar{I}\alpha$$

$$5\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2 \alpha + 0.07764\alpha$$

$$5 = 0.21566\alpha$$

$$\alpha = 23.184\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 23.2\text{ rad/s}^2 \circlearrowright$$

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)(23.184\text{ rad/s}^2) \qquad \bar{\mathbf{a}} = 15.46\text{ ft/s}^2 \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad -F + 5\text{ lb} = m\bar{a}$$

$$-F + 5\text{ lb} = \frac{10\text{ lb}}{32.2}(15.46\text{ ft/s}^2); \quad F = 0.20\text{ lb}$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - 10\text{ lb} = 0 \qquad N = 10\text{ lb}$$

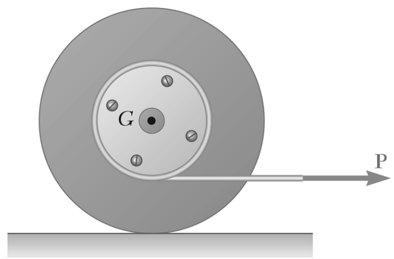
$$F_m = \mu_s N = 0.25(10\text{ lb}) = 2.5\text{ lb}$$

(*a*) Since $F < F_m$, <u>Disk rolls without sliding</u> ◀

(*b*) <u>Angular acceleration of the disk</u>. $\boldsymbol{\alpha} = 23.2\text{ rad/s}^2 \circlearrowright$ ◀

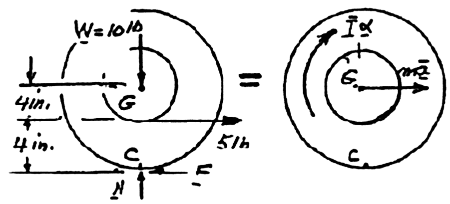
<u>Acceleration of *G*</u>. $\bar{\mathbf{a}} = 15.46\text{ ft/s}^2 \longrightarrow$ ◀

PROBLEM 16.104



A drum of 4-in. radius is attached to a disk of 8-in. radius. The disk and drum have a combined weight of 10 lb and a combined radius of gyration of 6 in. A cord is attached as shown and pulled with a force **P** of magnitude 5 lb. Knowing that the coefficients of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively, determine (*a*) whether or not the disk slides, (*b*) the angular acceleration of the disk and the acceleration of *G*.

SOLUTION



Assume disk rolls: $\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$

$$\bar{I} = m\bar{k}^2 = \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$$

$$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$+\circlearrowright\Sigma M_C = \Sigma(M_C)_{\text{eff}}:\quad (5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$$

$$1.6667\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ft}\right)^2\alpha + 0.07764\alpha$$

$$1.6667 = 0.21566\alpha$$

$$\alpha = 7.728\text{ rad/s}^2 \qquad\qquad \boldsymbol{\alpha} = 7.73\text{ rad/s}^2 \circlearrowright$$

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)7.728\text{ rad/s}^2 \qquad\qquad \bar{\mathbf{a}} = 5.153\text{ ft/s}^2 \longrightarrow$$

$$\xrightarrow{+}\Sigma F_x = \Sigma(F_x)_{\text{eff}}:\qquad -F + 5\text{ lb} = m\bar{a}$$

$$-F + 5\text{ lb} = \frac{10\text{ lb}}{32.2}(5.153\text{ ft/s}^2)$$

$$F = 3.40\text{ lb}$$

$$+\uparrow\Sigma F_y = \Sigma(F_y)_{\text{eff}}:\qquad N - 10\text{ lb} = 0 \qquad N = 10\text{ lb}$$

$$F_m = \mu_s N = 0.25(10\text{ lb}) = 2.5\text{ lb}$$

(*a*) Since $F < F_m$, <u>Disk slides</u> ◀

Knowing that disk slides

$$F = \mu_k N = 0.20(10\text{ lb}) = 2\text{ lb}$$

(*b*) Angular acceleration.

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F\left(\frac{8}{12}\text{ft}\right) - (5\text{ lb})\left(\frac{4}{12}\text{ft}\right) = \bar{I}\alpha$$

$$(2\text{ lb})\left(\frac{8}{12}\text{ft}\right) - 1.6667\text{ lb}\cdot\text{ft} = (0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$$

$$-0.3333 = 0.07764\alpha$$

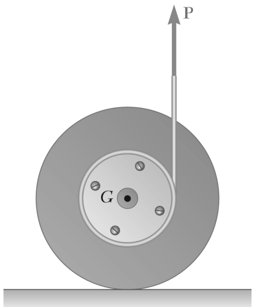
$$\alpha = -4.29\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 4.29\text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(*c*) Acceleration of *G*.

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad -F + 5\text{ lb} = m\bar{a}$$

$$-2\text{ lb} + 5\text{ lb} = \frac{10\text{ lb}}{32.2}\bar{a}$$

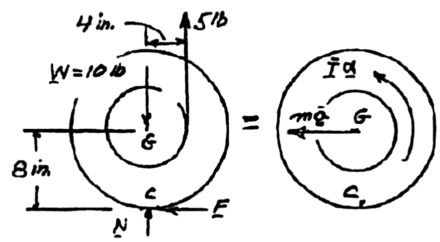
$$\bar{a} = 9.66\text{ ft/s}^2 \qquad \bar{\mathbf{a}} = 9.66\text{ ft/s}^2 \longrightarrow \blacktriangleleft$$



PROBLEM 16.105

A drum of 4-in. radius is attached to a disk of 8-in. radius. The disk and drum have a combined weight of 10 lb and a combined radius of gyration of 6 in. A cord is attached as shown and pulled with a force **P** of magnitude 5 lb. Knowing that the coefficients of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively, determine (*a*) whether or not the disk slides, (*b*) the angular acceleration of the disk and the acceleration of *G*.

SOLUTION



Assume disk rolls: $\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$

$$\bar{I} = m\bar{k}^2 = \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$$

$$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}$: $\quad (5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$$1.6667\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2\alpha + 0.07764\alpha$$

$$1.6667 = 0.21566\alpha$$

$$\alpha = 7.728\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 7.73\text{ rad/s}^2 \circlearrowleft$$

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)(7.728\text{ rad/s}^2) \qquad \bar{\mathbf{a}} = 5.153\text{ ft/s}^2 \longleftarrow$$

$\xleftarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\quad F = m\bar{a}$

$$F = \frac{10\text{ lb}}{32.2}(5.153\text{ ft/s}^2); \quad F = 1.60\text{ lb}$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $\quad N - 10\text{ lb} + 5\text{ lb} = 0 \qquad N = 5\text{ lb}$

$$F_m = \mu_s N = 0.25(5\text{ lb}) = 1.25\text{ lb}$$

(*a*) Since $F > F_m$, Disk slides ◀

Knowing that disk slides

$$F = \mu_k N = 0.2(5)$$

$$F = 1.00\text{ lb}$$

PROBLEM 16.105 (Continued)

(*b*) Angular acceleration.

$$+\circlearrowleft \Sigma M_S = \Sigma (M_S)_{\text{eff}}: \quad (5 \text{ lb})\left(\frac{4}{12}\text{ ft}\right) - F\left(\frac{8}{12}\text{ ft}\right) = \bar{I}\alpha$$

$$(5 \text{ lb})\left(\frac{4}{12}\text{ ft}\right) - (1.00 \text{ lb})\left(\frac{8}{12}\text{ ft}\right) = 0.07764\alpha$$

$$1.000 = 0.07764\alpha$$

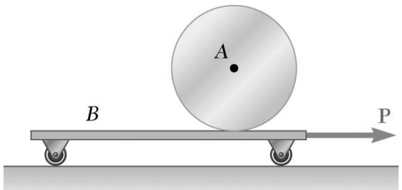
$$\alpha = 12.88 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 12.88 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*c*) Acceleration of *G*.

$$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\bar{a}$$

$$1.00 \text{ lb} = \frac{10 \text{ lb}}{32.2}\bar{a}$$

$$\bar{a} = 3.22 \text{ ft/s}^2 \qquad \bar{\mathbf{a}} = 3.22 \text{ ft/s}^2 \longleftarrow \blacktriangleleft$$



PROBLEM 16.106

A 12-in.-radius cylinder of weight 16 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 4 lb is applied. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine (*a*) the acceleration of the carriage, (*b*) the acceleration of Point *A*, (*c*) the distance the cylinder has rolled with respect to the carriage after 0.5 s.

SOLUTION

Masses and moments of inertia.

$$m_A = \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.49689 \text{ lb}\cdot\text{s}^2/\text{ft} \qquad m_B = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.18634 \text{ lb}\cdot\text{s}^2/\text{ft}$$

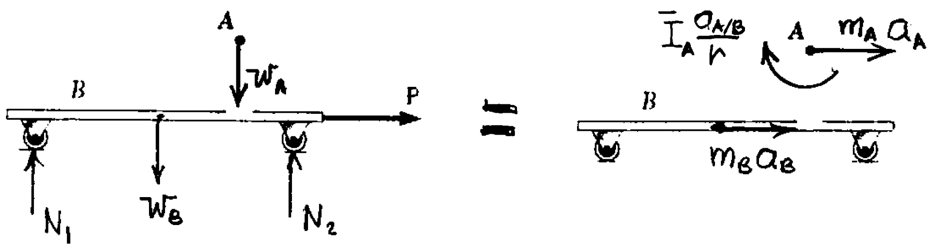
$$\bar{I}_A = \frac{1}{2}m_A r^2 = \frac{1}{2}(0.49689 \text{ lb}\cdot\text{s}^2/\text{ft})(1 \text{ ft})^2 = 0.24895 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinematics: Let $\mathbf{a}_A = a_A \longrightarrow$, $\mathbf{a}_B = a_B \longrightarrow$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{A/B} = (a_A - a_B) \longrightarrow \qquad \boldsymbol{\alpha} = \frac{a_{A/B}}{r} \circlearrowright \tag{1}$$

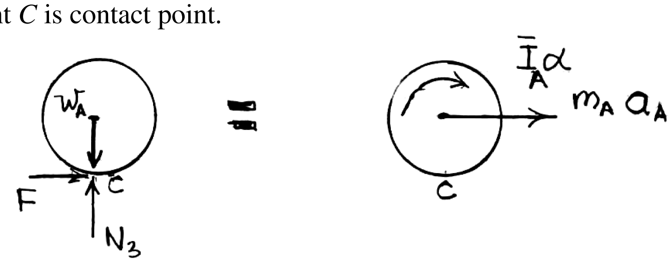
Kinetics: Carriage and cylinder



$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P = m_A a_A + m_B a_B$$

$$4 \text{ lb} = (0.49689 \text{ lb}\cdot\text{s}^2/\text{ft})\ a_A + (0.18634 \text{ lb}\cdot\text{s}^2/\text{ft})\ a_B \tag{2}$$

Cylinder alone. Point *C* is contact point.



PROBLEM 16.106 (Continued)

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad 0 = \bar{I}_A \alpha + m_A a_A r$$

Substituting from Eqs. (1) and (2),

$$0 = \bar{I}_A \frac{a_A - a_B}{r} + m_A a_A r$$

$$0 = \left(\frac{\bar{I}_A}{r} + m_A r \right) a_A - \frac{\bar{I}_A}{r} a_B$$

Data: $$\frac{I_A}{r} = \frac{0.24895 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}}{1 \text{ ft}} = 0.24895 \text{ lb} \cdot \text{s}^2$$

$$m_A r = (0.49689 \text{ lb} \cdot \text{s}^2/\text{ft})(1 \text{ ft}) = 0.49689 \text{ lb} \cdot \text{s}^2$$

$$0 = 0.74584\, a_A - 0.24895\, a_B \qquad (3)$$

Solving Eqs. (2) and (3) simultaneously,

$$a_A = 3.7909 \text{ ft/s}^2 \quad a_B = 11.3574 \text{ ft/s}^2$$

(*a*) *Acceleration of the carriage.* $\mathbf{a}_B = 11.36 \text{ ft/s}^2 \longrightarrow$ ◀

(*b*) *Acceleration of Point A.* $\mathbf{a}_A = 3.79 \text{ ft/s}^2 \longrightarrow$ ◀

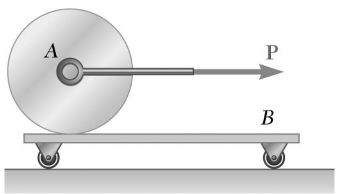
(*c*) *Relative displacement after* 0.5 s.

$$a_{A/B} = 11.3574 \text{ ft/s}^2 - 3.7909 \text{ ft/s}^2 = 7.5665 \text{ ft/s}^2$$

$$x_{A/B} = \frac{1}{2}(a_{A/B})t^2$$

$$= \frac{1}{2}(7.5665 \text{ ft/s})^2 (0.5 \text{ s})^2$$

$\mathbf{x}_{B/A} = 0.946 \text{ ft} \longrightarrow$ ◀



PROBLEM 16.107

A 12-in.-radius cylinder of weight 16 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 4 lb is applied. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine (*a*) the acceleration of the carriage, (*b*) the acceleration of Point *A*, (*c*) the distance the cylinder has rolled with respect to the carriage after 0.5 s.

SOLUTION

Masses and moments of inertia.

$$m_A = \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.49689 \text{ lb} \cdot \text{s}^2/\text{ft} \qquad m_B = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.18634 \text{ lb} \cdot \text{s}^2/\text{ft}$$

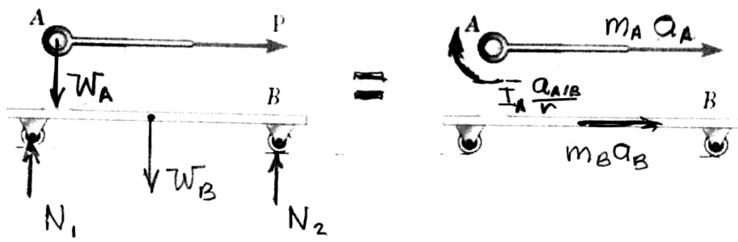
$$\bar{I}_A = \frac{1}{2} m_A r^2 = \frac{1}{2}(0.49689 \text{ lb} \cdot \text{s}^2/\text{ft})(1 \text{ ft})^2 = 0.24895 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

Kinematics: Let $\mathbf{a}_A = a_A \longrightarrow$, $\mathbf{a}_B = a_B \longrightarrow$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{A/B} = (a_B - a_B) \longrightarrow \qquad \alpha = \frac{a_{A/B}}{r} \circlearrowright \qquad (1)$$

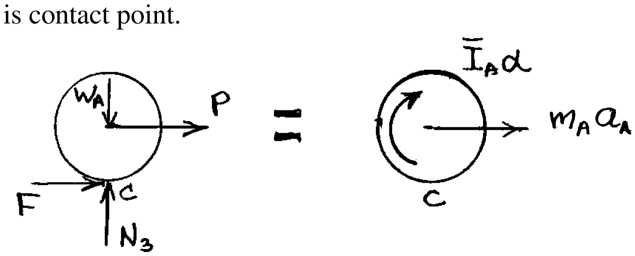
Kinetics: Carriage and cylinder



$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P = m_A a_A + m_B a_B$$

$$4 \text{ lb} = (0.49689 \text{ lb} \cdot \text{s}^2/\text{ft})\, a_A + (0.18634 \text{ lb} \cdot \text{s}^2/\text{ft}) a_B \qquad (2)$$

Cylinder alone. Point *C* is contact point.



PROBLEM 16.107 (Continued)

$$+\circlearrowleft \Sigma M_C = \Sigma(M_B)_{\text{eff}}: \quad Pr = \bar{I}_A \alpha + m_A a_A r$$

Substituting from Eqs. (1) and (2),

$$Pr = \bar{I}_A \frac{a_A - a_B}{r} + m_A a_A r$$

$$Pr = \left(\frac{\bar{I}_A}{r} + m_A r\right) a_A - \frac{\bar{I}_A}{r} a_B$$

Data:

$$\frac{\bar{I}_A}{r} = \frac{0.24895 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}}{1 \text{ ft}} = 0.24895 \text{ lb} \cdot \text{s}^2$$

$$m_A r = (0.49689 \text{ lb} \cdot \text{s}^2/\text{ft})(1 \text{ ft}) = 0.49689 \text{ lb} \cdot \text{s}^2$$

$$(4 \text{ lb})(1 \text{ ft}) = 0.74584\, a_A - 0.24895\, a_B \tag{3}$$

Solving Eqs. (2) and (3) simultaneously,

$$a_A = 6.6284 \text{ ft/s}^2 \qquad a_B = 3.7909 \text{ ft/s}^2$$

(*a*) *Acceleration of the carriage.* $\mathbf{a}_B = 3.79 \text{ ft/s}^2 \longrightarrow$ ◀

(*b*) *Acceleration of Point A.* $\mathbf{a}_A = 6.63 \text{ ft/s}^2 \longrightarrow$ ◀

(*c*) *Relative displacement after* 0.5 s.

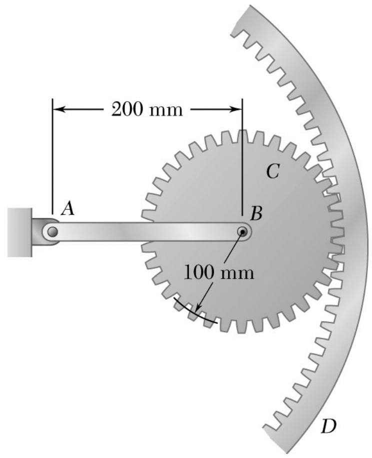
$$a_{A/B} = 6.6284 \text{ ft/s}^2 - 3.7909 \text{ ft/s}^2 = 2.8375 \text{ ft/s}^2$$

$$x_{A/B} = \frac{1}{2}(a_{A/B})t^2$$

$$= \frac{1}{2}(2.8375 \text{ ft/s}^2)(0.5 \text{ s})^2$$

$\mathbf{x}_{A/B} = 0.355 \text{ ft} \longrightarrow$ ◀

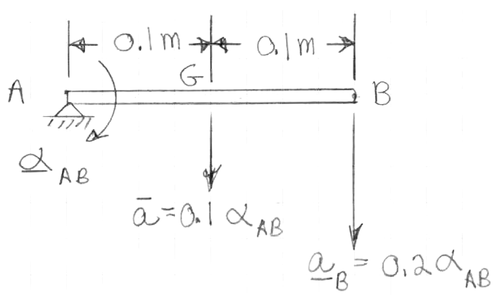
PROBLEM 16.108

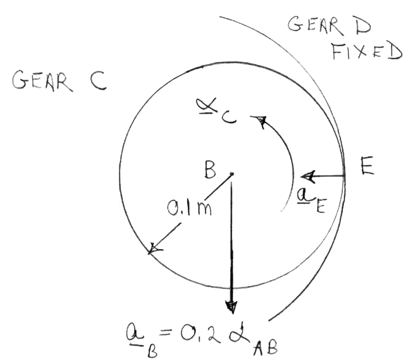


Gear C has a mass of 5 kg and a centroidal radius of gyration of 75 mm. The uniform bar AB has a mass of 3 kg and gear D is stationary. If the system is released from rest in the position shown, determine (a) the angular acceleration of gear C, (b) the acceleration of Point B.

SOLUTION

Kinematics:





Since gear D is fixed, we have for Point E of gear C: $\downarrow (a_E)_t = 0$

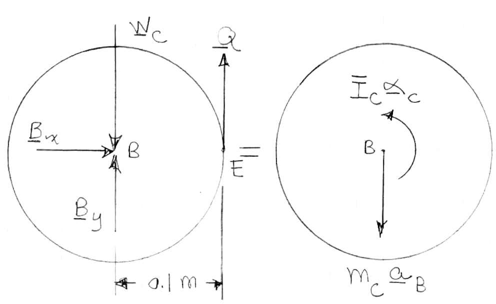
But $\quad \mathbf{a}_E = \mathbf{a}_B + \mathbf{a}_{E/B}$

$+\downarrow (a_E)_t = (a_B)_t + (\mathbf{a}_{E/B})_t$

$0 = 0.2\,\alpha_{AB} - 0.1\,\alpha_C$

$$\alpha_{AB} = \frac{1}{2}\alpha_C \qquad (1)$$

Gear C



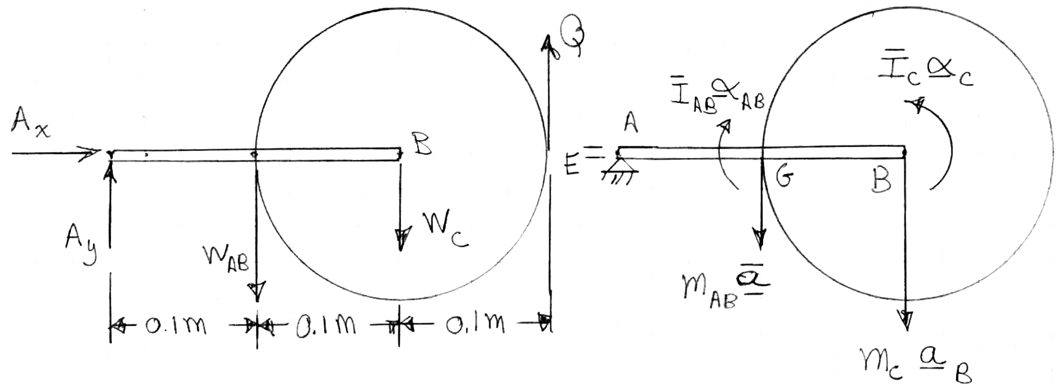
$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}:$

$Q\,(0.1\text{m}) = \bar{I}_C \alpha_C$

$= (5\text{ kg})(0.075\text{ m})^2 \alpha_C$

$$Q = 0.28125\,\alpha_C \qquad (2)$$

Bar AB and gear C



(*a*) $+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}$:

$$W_{AB}(0.1) + W_C(0.2) - Q(0.3) = (m_{AB}\,\bar{a})\,0.1 + \bar{I}_{AB}\alpha_{AB} + (m_C\,a_B)0.2 - \bar{I}_C\alpha_C$$

$$(3)g(0.1) + (5)g(0.2) - Q(0.3) = 3(0.1\alpha_{AB})0.1 + \frac{1}{12}(3)(0.2)^2\,\alpha_{AB}$$

$$+\,5(0.2\,\alpha_{AB})0.2 - 5(0.075)^2\,\alpha_C$$

$$(1.3)g - 0.3Q = 0.24\alpha_{AB} - 0.028125\alpha_C$$

Substituting for α_{AB} and Q from (2) and (1):

$$1.3\,g - 0.3(0.28125\alpha_C) = 0.24\left(\frac{1}{2}\,\alpha_C\right) - 0.028125\,\alpha_C$$

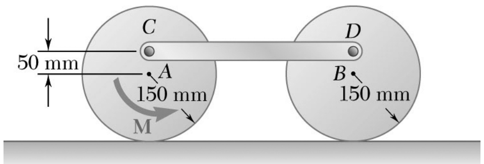
$$1.3g = 0.17625\,\alpha_C \qquad \alpha_C = 7.3759(9.81)$$

$$\alpha_C = 72.36$$

$\boldsymbol{\alpha}_C = 72.4\ \text{rad/s}^2 \circlearrowright$ ◀

(*b*) $a_B = 0.2\alpha_{AB} = 0.2\left(\frac{1}{2}\,\alpha_C\right) = 0.1\alpha_C = 0.1(72.36)$

$\mathbf{a}_B = 7.24\ \text{m/s}^2 \downarrow$ ◀



PROBLEM 16.109

Two uniform disks A and B, each of mass of 2 kg, are connected by a 1.5 kg rod CD as shown. A counterclockwise couple $\mathbf{M}$ of moment 2.5 N-m is applied to disk A. Knowing that the disks roll without sliding, determine (*a*) the acceleration of the center of each disk, (*b*) the horizontal component of the force exerted on disk B by pin D.

SOLUTION

Geometry: $r = 150 \text{ mm} = 0.15 \text{ m}$

$$b = \overline{AC} = \overline{BD} = 50 \text{ mm} = 0.05 \text{ m}$$

$$r + b = 0.20 \text{ m}$$

Masses: $m_A = m_B = m = 2 \text{ kg} \qquad m_{CD} = 1.5 \text{ kg}$

Moment of inertia: $I_A = I_B = I = \dfrac{1}{2}mr^2$

Kinematics: $\alpha_{CD} = 0$

$$\mathbf{a}_A = \mathbf{a}_B = \mathbf{a} = a \longleftarrow$$

Angular accelerations of disks: $\alpha = \dfrac{a}{r} \circlearrowleft$

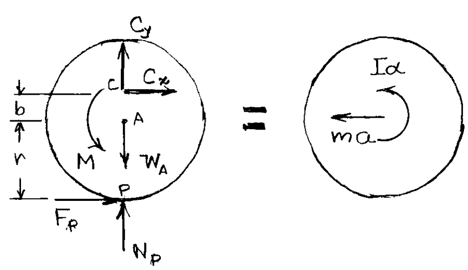
$$\mathbf{a}_C = \mathbf{a}_A + b\alpha \longleftarrow + b\omega^2 \downarrow$$

$$\mathbf{a}_D = \mathbf{a}_B + b\alpha \longleftarrow + b\omega^2 \downarrow$$

For rod CD,

$$\bar{a} = (a + b\alpha) \longleftarrow + b\omega^2 \downarrow$$

$$= \left(1 + \frac{b}{r}\right)a \longleftarrow + b\omega^2 \downarrow$$

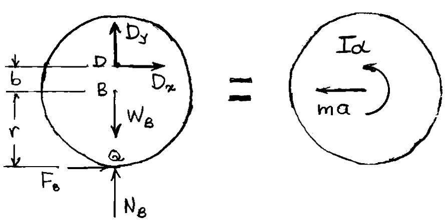


Kinetics:

Disk A: $+\circlearrowleft \Sigma M_P = \Sigma (M_P)_{\text{eff}}: \quad M - (r+b)C_x = mar + I\alpha$

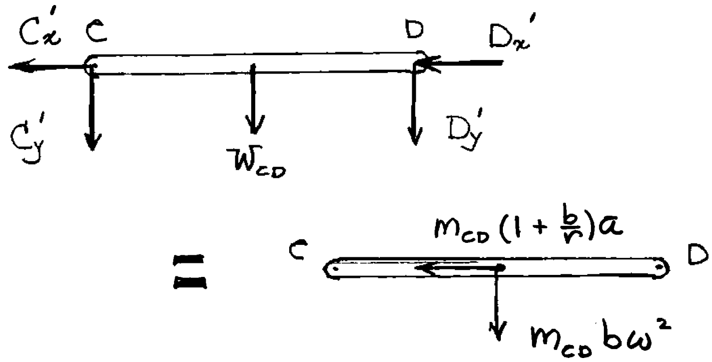
$$= \left(mr + \frac{I}{r}\right)(a) \qquad (1)$$

Disk B: $+\circlearrowleft \Sigma M_Q = \Sigma (M_Q)_{\text{eff}}: \quad -(r+b)D_x = mar + I\alpha$



$$= \left(mr + \frac{I}{r}\right)a \qquad (2)$$

Rod *CD*: $\quad \overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad C_x + D_x = m_{CD}\left(1 + \dfrac{b}{r}\right)a$



Multiply by $(r+b)$

$$(r+b)(C_x + D_x) = m_{CD} r \left(1 + \frac{b}{r}\right)^2 a \qquad (3)$$

Add Eqs. (1), (2), and (3) to eliminate C_x and D_x

$$M = 2\left(mr + \frac{I}{r}\right)a + m_{CD} r \left(1 + \frac{b}{r}\right)^2 a$$

Apply the numerical data. $\quad mr + \dfrac{I}{r} = (2 \text{ kg})(0.15 \text{ m}) + \dfrac{1}{2}(2 \text{ kg})\dfrac{(0.15 \text{ m})^2}{0.15 \text{ m}} = 0.45 \text{ kg} \cdot \text{m}$

$$m_{CD} r \left(1 + \frac{b}{r}\right)^2 = (1.5 \text{ kg})(0.15 \text{ m})\left(1 + \frac{0.05}{0.15}\right)^{-2} = 0.40 \text{ kg} \cdot \text{m}$$

$$2.5 \text{ N} \cdot \text{m} = 2(0.45 \text{ kg} \cdot \text{m})a + (0.40 \text{ kg} \cdot \text{m})a$$

$$a = 1.9231 \text{ m/s}^2$$

(*a*) *Acceleration of the center of each disk*:

$$\mathbf{a}_A = \mathbf{a}_B = 1.923 \text{ m/s}^2 \longleftarrow \blacktriangleleft$$

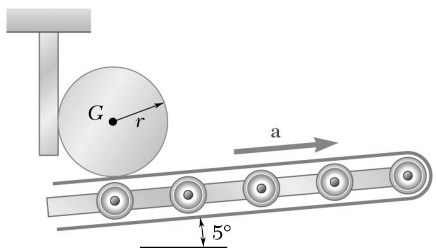
(*b*) *Horizontal component of the force exerted on disk B by pin D*.

From Eq. (2), $\quad D_x = -\dfrac{1}{r+b}\left(mr + \dfrac{I}{r}\right)a$

$$= -\frac{1}{0.20 \text{ m}}(0.45 \text{ kg} \cdot \text{m})(1.9231 \text{ m/s}^2) = -4.33 \text{ N}$$

$$\mathbf{D}_x = 4.33 \text{ N} \longleftarrow \blacktriangleleft$$

PROBLEM 16.110



A 10-lb cylinder of radius $r = 4$ in. is resting on a conveyor belt when the belt is suddenly turned on and it experiences an acceleration of magnitude $a = 6\text{ft/s}^2$. The smooth vertical bar holds the cylinder in place when the belt is not moving. Knowing the cylinder rolls without slipping and the friction between the vertical bar and the cylinder is negligible, determine (*a*) the angular acceleration of the cylinder, (*b*) the components of the force the conveyor belt applies to the cylinder.

SOLUTION

Mass and moment of inertia.

$$m = \frac{W}{g} = \frac{10\text{ lb}}{32.2\text{ ft/s}^2} = 0.31056\text{ lb}\cdot\text{s}^2/\text{ft}$$

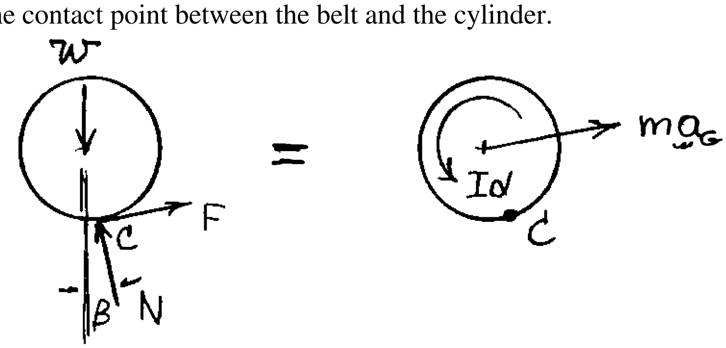
$$I = \frac{1}{2}mr^2 = \frac{1}{2}(0.31056\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{4}{12}\text{ ft}\right)^2 = 0.017253\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinematics: The cylinder rolls without slipping on the belt which is accelerating at $6\text{ ft/s}^2 \measuredangle 5°$.

$$\mathbf{a}_G = (6\text{ ft/s}^2 - r\alpha)\measuredangle 5° = \left[6\text{ ft/s}^2 - \left(\frac{4}{12}\text{ ft}\right)\alpha\right]\measuredangle 5°$$

where $\boldsymbol{\alpha} = \alpha \circlearrowright$ is the angular acceleration of the cylinder.

Kinetics: Let Point C be the contact point between the belt and the cylinder.



$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad Wr\sin\beta = -rma_G + I\alpha$$

$$(10\text{ lb})\left(\frac{4}{12}\text{ ft}\right)\sin 5° = -\left(\frac{4}{12}\text{ ft}\right)(0.31056\text{ lb}\cdot\text{s}^2/\text{ft})\left[6\text{ ft/s}^2 - \left(\frac{4}{12}\text{ft}\right)\alpha\right] + (0.017253\text{ lb}\cdot\text{s}^2\text{ft})\alpha$$

$$0.29052\text{ lb}\cdot\text{ft} = -0.62112\text{ lb}\cdot\text{ft} + (0.051760\text{ lb}\cdot\text{s}^2\cdot\text{ft})\alpha$$

PROBLEM 16.110 (Continued)

(*a*) *Angular acceleration.*

$$\alpha = 17.613 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 17.61 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

$$a_G = 6 \text{ ft/s}^2 - \left(\frac{4}{12} \text{ ft}\right)(17.613 \text{ rad/s}^2) = 0.129 \text{ ft/s}^2$$

(*b*) *Components of contact force:*

$$\measuredangle 5° \; \Sigma F = \Sigma(F_{\text{eff}}): \quad F - W \sin 5° = ma_G$$

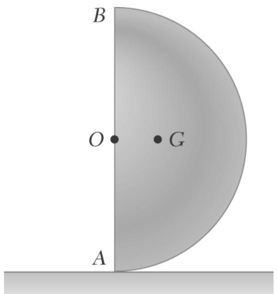
$$F = W \sin 5° + ma_G = (10 \text{ lb}) \sin 5° + (0.31056 \text{ lb} \cdot \text{s}^2/\text{ft})(0.129 \text{ ft/s}^2)$$

$$\mathbf{F} = 0.912 \text{ lb} \measuredangle 5° \blacktriangleleft$$

$$+\searrow 85° \; \Sigma F = \Sigma F = \Sigma F_{\text{eff}}: \quad N - W \cos 5° = 0$$

$$N = W \cos 5° = (10 \text{ lb}) \cos 5°$$

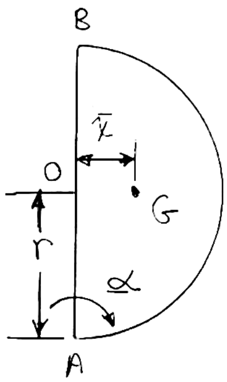
$$\mathbf{N} = 9.96 \text{ lb} \searrow 85° \blacktriangleleft$$



PROBLEM 16.111

A hemisphere of weight W and radius r is released from rest in the position shown. Determine (*a*) the minimum value of μ_s for which the hemisphere starts to roll without sliding, (*b*) the corresponding acceleration of Point B. [*Hint*: Note that $OG = \frac{3}{8}r$ and that, by the parallel-axis theorem, $\bar{I} = \frac{2}{5}mr^2 - m(OG)^2$.]

SOLUTION



Kinematics: $\omega = 0$

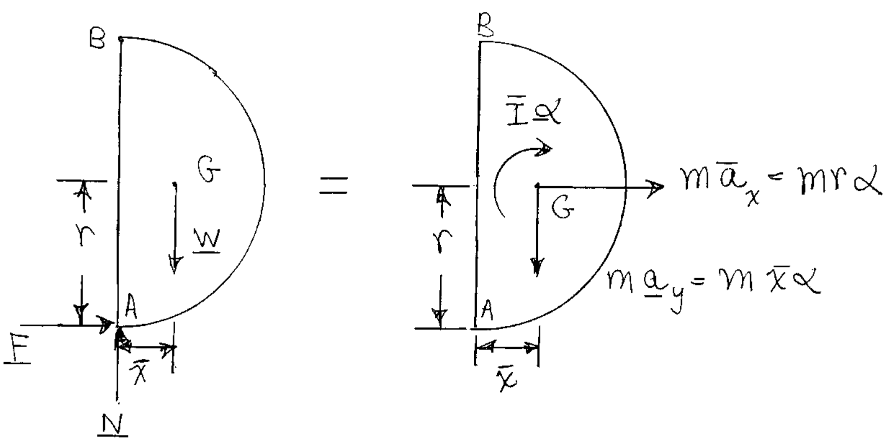
$$\mathbf{a}_O = \mathbf{a}_A + \mathbf{a}_{O/A} = 0 + [r\alpha \longrightarrow]$$

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$= [r\alpha \longrightarrow] + [\bar{x}\alpha \downarrow]$$

Thus, $\bar{\mathbf{a}}_x = r\alpha \longrightarrow, \qquad \bar{\mathbf{a}}_y = \bar{x}\alpha \downarrow$ (1)

Kinetics:



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad W\bar{x} = (m\bar{a}_x)r + (m\bar{a}_y)\,\bar{x} + \bar{I}\alpha$$

$$mg\bar{x} = (mr\alpha)\,r + (m\bar{x}\alpha)\,\bar{x} + m\bar{k}^2\alpha$$

$$\alpha = \frac{g\bar{x}}{r^2 + \bar{x}^2 + \bar{k}^2} \qquad (2)$$

PROBLEM 16.111 (Continued)

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \qquad F = ma_x \qquad F = mr\alpha$$

$$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \qquad N - W = -m\bar{a}_y \qquad N = mg - m\bar{x}\alpha$$

$$\mu_{\min} = \frac{F}{N} = \frac{mr\alpha}{mg - \bar{x}\alpha} \qquad \mu_{\min} = \frac{r\alpha}{g - \bar{x}\alpha} \tag{3}$$

For a hemisphere:

$$\bar{x} = OG = \frac{3}{8}r \qquad \bar{I} = I_O - m\bar{x}^2 = \frac{2}{5}mr^2 - m\left(\frac{3}{8}r\right)^2$$

$$\bar{I} = \frac{2}{5}mr^2 - \frac{9}{64}mr^2 \qquad \bar{k}^2 = \frac{\bar{I}}{m} = \left(\frac{2}{5} - \frac{9}{64}\right)r^2$$

Substituting into (2)

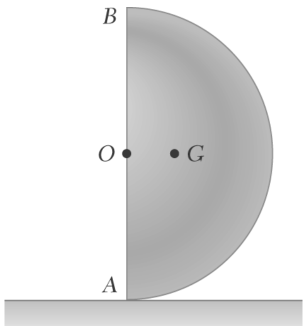
$$\alpha = \frac{g\left(\frac{3}{8}r\right)}{r^2 + \frac{9}{64}r^2 + \left(\frac{2}{5} - \frac{9}{64}\right)r^2} = \frac{\frac{3}{8}}{\frac{7}{5}}\frac{g}{r} \qquad \alpha = \frac{15}{56}\frac{g}{r}$$

(*a*) Substituting into (3)

$$\mu_{\min} = \frac{\frac{15}{56}}{1 - \left(\frac{3}{8}\right)\left(\frac{15}{56}\right)} = \frac{0.26786}{0.89955} \qquad \mu_{\min} = 0.298 \blacktriangleleft$$

(*b*) $a_B = (2r)\alpha = (2r)\left(\frac{15g}{56r}\right) = \frac{30g}{56}$ $\qquad \mathbf{a}_B = 0.536g \longrightarrow \blacktriangleleft$

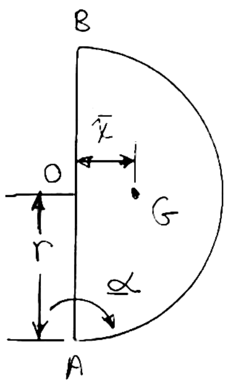
Note: In this problem we *cannot* use the equation $\Sigma M_A = I_A\alpha$, since Points *A*, *O*, and *G* are not aligned.



PROBLEM 16.112

Solve Problem 16.111, considering a half cylinder instead of a hemisphere. [*Hint*. Note that $OG = 4r/3\pi$ and that, by the parallel-axis theorem, $\bar{I} = \frac{1}{2}mr^2 - m(OG)^2$.]

SOLUTION



Kinematics: $\omega = 0$

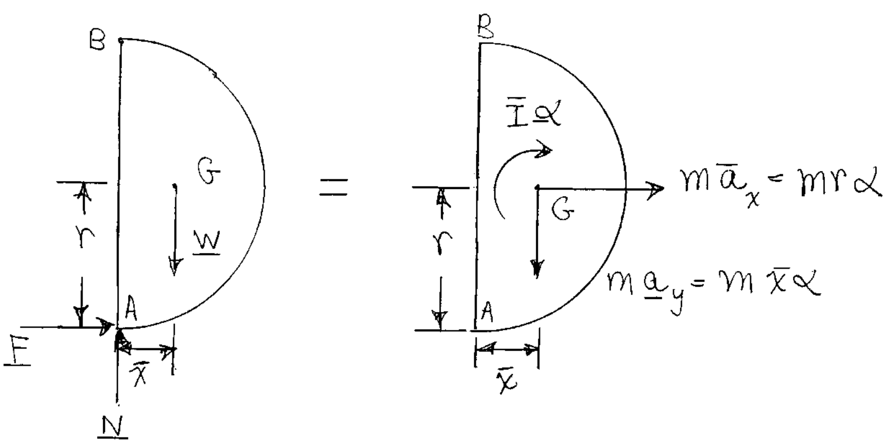
$$\mathbf{a}_O = \mathbf{a}_A + \mathbf{a}_{O/A} = 0 + [r\alpha \longrightarrow]$$

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$= [r\alpha \longrightarrow] + [\bar{x}\alpha \downarrow]$$

Thus, $\mathbf{a}_x = r\alpha \longrightarrow, \qquad \mathbf{a}_y = \bar{x}\alpha \downarrow$ (1)

Kinetics:



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad W\bar{x} = (m\bar{a}_x)r + (m\bar{a}_y)\,\bar{x} + \bar{I}\alpha$$

$$mg\bar{x} = (mr\alpha)\,r + (m\bar{x}\alpha)\,\bar{x} + m\bar{k}^2\alpha$$

$$\alpha = \frac{g\bar{x}}{r^2 + \bar{x}^2 + \bar{k}^2} \qquad (2)$$

PROBLEM 16.112 (Continued)

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \qquad F = ma_x \qquad F = mr\alpha$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \qquad N - W = -m\bar{a}_y \qquad N = mg - m\bar{x}\alpha$$

$$\mu_{\min} = \frac{F}{N} = \frac{mr\alpha}{mg - \bar{x}\alpha} \qquad \mu_{\min} = \frac{r\alpha}{g - \bar{x}\alpha} \tag{3}$$

For a half cylinder:

$$\bar{x} = OG = \frac{4r}{3\pi} \qquad \bar{I} = I_O - m\bar{x}^2 = \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2$$

$$\bar{k}^2 = \frac{\bar{I}}{m} = \frac{1}{2}r^2 - \left(\frac{4r}{3\pi}\right)^2$$

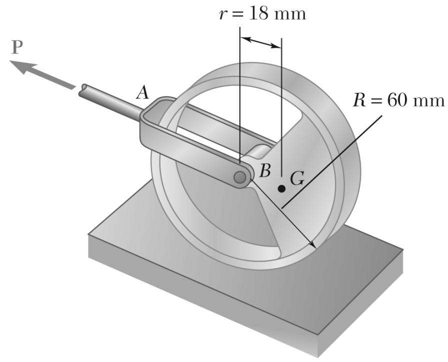
Substituting into (2):

$$\alpha = \frac{g\left(\frac{4r}{3\pi}\right)}{r^2 + \left(\frac{4r}{3\pi}\right)^2 + \frac{1}{2}r^2 - \left(\frac{4r}{3\pi}\right)^2} = \frac{\frac{4}{3\pi}}{\frac{3}{2}}\frac{g}{r} \qquad \alpha = \frac{8}{9\pi}\frac{g}{r}$$

(*a*) Substituting into (3):

$$\mu_{\min} = \frac{\frac{8}{9\pi}}{1 - \left(\frac{4}{3\pi}\right)\left(\frac{8}{9\pi}\right)} = \frac{0.28294}{0.87992} \qquad \mu_{\min} = 0.322 \blacktriangleleft$$

(*b*) $a_B = (2r)\,\alpha = (2r)\left(\dfrac{8g}{9\pi r}\right) = \dfrac{16g}{9\pi}$ $\qquad a_B = 0.566g \blacktriangleleft$

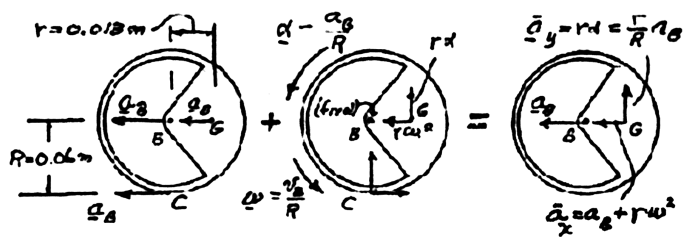


PROBLEM 16.113

The center of gravity G of a 1.5-kg unbalanced tracking wheel is located at a distance $r = 18$ mm from its geometric center B. The radius of the wheel is $R = 60$ mm and its centroidal radius of gyration is 44 mm. At the instant shown the center B of the wheel has a velocity of 0.35 m/s and an acceleration of 1.2 m/s^2, both directed to the left. Knowing that the wheel rolls without sliding and neglecting the mass of the driving yoke AB, determine the horizontal force **P** applied to the yoke.

SOLUTION

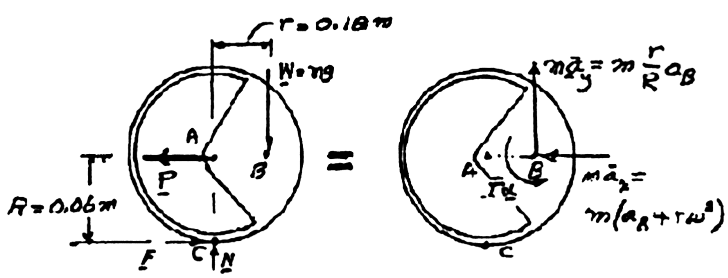
<u>Kinematics</u>: Choose positive $\mathbf{v}_B$ and $\mathbf{a}_B$ to left.



<u>Trans. with B</u> + <u>Rotation about B</u> = <u>Rolling motion</u>

$$\bar{\mathbf{a}} = [a_B + r\omega^2] \longleftarrow + \left[\frac{r}{R}a_B\right]\uparrow$$

<u>Kinetics</u>:



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad PR - Wr = (m\bar{a}_y)r + (m\bar{a}_x)R + \bar{I}\alpha$$

$$PR - mgr = m\left(\frac{r}{R}a_B\right)r + m(a_B + r\omega^2)R + m\bar{k}^2\frac{a_B}{R}$$

$$= ma_B\left(\frac{r^2}{R} + R + \frac{\bar{k}^2}{R}\right) + mr\left(\frac{v_B}{r}\right)^2 R$$

$$P = mg\left(\frac{r}{R}\right) + ma_B\left(1 + \frac{r^2 + \bar{k}^2}{R^2}\right) + m\frac{r}{R^2}v_B^2 \qquad (1)$$

PROBLEM 16.113 (Continued)

Substitute: $m = 1.5 \text{ kg}$

$r = 0.018 \text{ m}$

$R = 0.06 \text{ m}$

$\bar{k} = 0.044 \text{ mm}$ and $g = 9.81 \text{ m/s}^2$ in Eq. (1)

$$P = 1.5(9.81)\frac{0.018}{0.06} + 1.5(a_B)\left(1 + \frac{0.018^2 + 0.044^2}{0.06^2}\right) + 1.5\frac{0.018}{0.06^2}v_B^2$$

$$P = 4.4145 + 2.4417a_B + 7.5v_B^2 \tag{2}$$

Data: $\mathbf{v}_B = 0.35 \text{ m/s} \longleftarrow; \quad v_B = +0.35 \text{ m/s}$

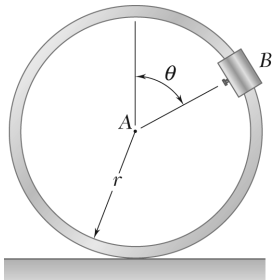
$\mathbf{a}_B = 1.2 \text{ m/s}^2 \longleftarrow; \quad a_B = +1.2 \text{ m/s}^2$

Substitute in Eq. (2): $P = 4.4145 + 2.4417(+1.2) + 7.5(+0.35)^2$

$= 4.4145 + 2.9300 + 0.9188$

$= +8.263 \text{ N}$

$\mathbf{P} = 8.26 \text{ N} \longleftarrow$ ◀

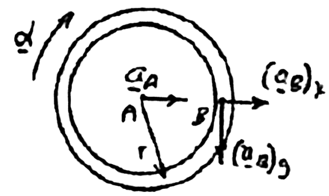


PROBLEM 16.114

A small clamp of mass m_B is attached at B to a hoop of mass m_h. The system is released from rest when $\theta = 90°$ and rolls without sliding. Knowing that $m_h = 3m_B$, determine (*a*) the angular acceleration of the hoop, (*b*) the horizontal and vertical components of the acceleration of B.

SOLUTION

<u>Kinematics</u>:



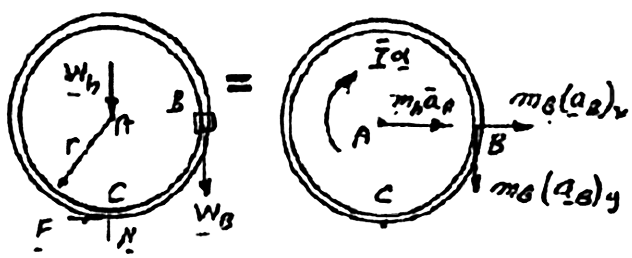
$$\mathbf{a}_A = r\alpha \longrightarrow, \quad a_{B/A} = r\alpha \downarrow$$

$$\mathbf{a}_B = \mathbf{a}_A \longrightarrow + a_{B/A} \downarrow$$

$$\mathbf{a}_B = r\alpha \longrightarrow + r\alpha \downarrow$$

$$(\mathbf{a}_B)_x = r\alpha \longrightarrow \quad (a_B)_y = r\alpha \downarrow$$

<u>Kinetics</u>:



$$m_h = 3m_B$$

$$\bar{I} = m_h r^2 = 3m_B r^2$$

(*a*) <u>Angular acceleration</u>.

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W_B r = \bar{I}\alpha + m_h \bar{a}_A r + m_B (a_B)_x r + m_B (a_B)_y r$$

$$m_B g r = 3m_B r^2 \alpha + (3m_B) r^2 \alpha + m_B r^2 \alpha + m_B r^2 \alpha$$

$$g r = 8 r^2 \alpha$$

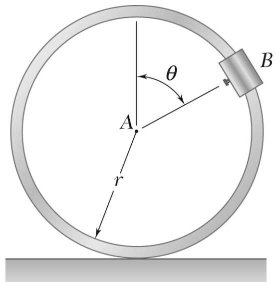
$$\alpha = \frac{1}{8}\frac{g}{r} \circlearrowright \blacktriangleleft$$

(*b*) <u>Components of acceleration of *B*</u>.

$$(\mathbf{a}_B)_x = r\alpha = \frac{1}{8} g \longrightarrow \quad (\mathbf{a}_B)_y = r\alpha = \frac{1}{8} g \downarrow$$

$$(\mathbf{a}_B)_x = \frac{1}{8} g \longrightarrow \blacktriangleleft$$

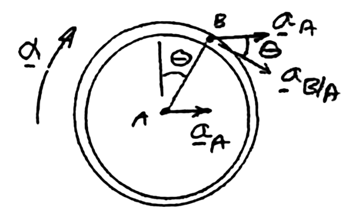
$$(\mathbf{a}_B)_y = \frac{1}{8} g \downarrow \blacktriangleleft$$



PROBLEM 16.115

A small clamp of mass m_B is attached at B to a hoop of mass m_h. Knowing that the system is released from rest and rolls without sliding, derive an expression for the angular acceleration of the hoop in terms of m_B, m_h, r, and θ.

SOLUTION



Kinematics:

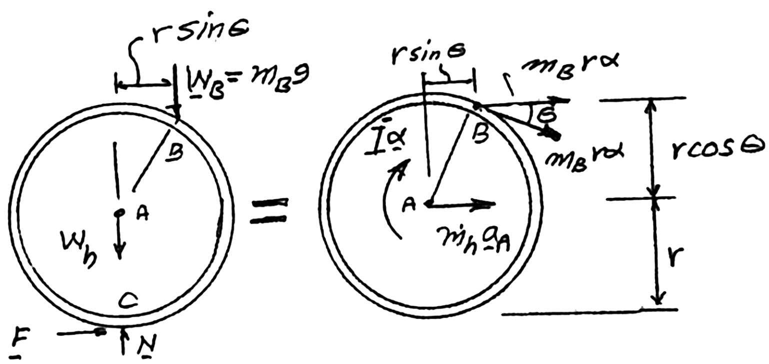
$$\mathbf{a}_A = r\alpha \longrightarrow \quad a_{B/A} = r\alpha \; ⦪\, \theta$$

$$\mathbf{a}_B = a_A \longrightarrow + \; a_{B/A} \; ⦪\, \theta$$

$$\mathbf{a}_B = r\alpha \longrightarrow + \; r\alpha \; ⦪\, \theta$$

Kinetics:

$$\bar{I} = m_h r^2$$



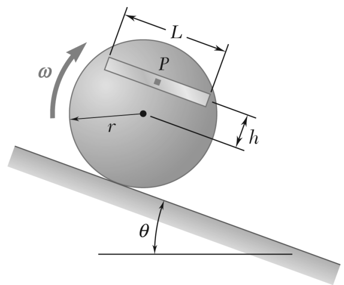
$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W_B r\sin\theta = \bar{I}\alpha + m_h a_A r + m_B r\alpha(r + r\cos\theta) + m_B r\alpha\sin\theta(r\sin\theta) + m_B r\alpha\cos\theta(r + r\cos\theta)$$

$$m_B g r\sin\theta = m_h r^2\alpha + m_h(r\alpha)r + m_B r\alpha(1+\cos\theta)(r + r\cos\theta) + m_B r\alpha\sin\theta(r\sin\theta)$$

$$m_B g r\sin\theta = 2m_h r^2\alpha + m_B r^2\alpha[(1+\cos\theta)^2 + \sin^2\theta]$$

$$= 2m_h r^2\alpha + m_B r^2\alpha[1 + 2\cos\theta + \cos^2\theta + \sin^2\theta]$$

$$m_B g r\sin\theta = r^2\alpha[2m_h + m_B(2 + 2\cos\theta)] \qquad \alpha = \frac{g}{2r}\,\frac{m_B\sin\theta}{m_h + m_B(1+\cos\theta)} \blacktriangleleft$$



PROBLEM 16.116

A 4-lb bar is attached to a 10-lb uniform cylinder by a square pin, P, as shown. Knowing that $r = 16$ in., $h = 8$ in., $\theta = 20°$, $L = 20$ in. and $\omega = 2$ rad/s at the instant shown, determine the reactions at P at this instant assuming that the cylinder rolls without sliding down the incline.

SOLUTION

Masses and moments of inertia.

Bar: $$m_B = \frac{W_B}{g} = \frac{4\text{lb}}{32.2} = 0.12422 \text{ slug}$$

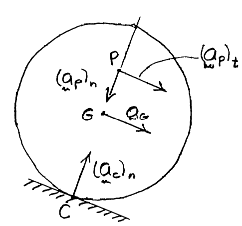
$$\bar{I}_B = \frac{1}{12} m_B L^2 = \frac{1}{12}\left(\frac{4 \text{ lb}}{32.2}\right)\left(\frac{20 \text{ in.}}{12}\right)^2 = 0.028755 \text{ slug}\cdot\text{ft}^2$$

Disk: $$m_D = \frac{W_D}{g} = \frac{10 \text{ lb}}{32.2} = 0.31056 \text{ slug}$$

$$\bar{I}_D = \frac{1}{2} m_C r^2 = \frac{1}{2}\left(\frac{10 \text{ lb}}{32.2}\right)\left(\frac{16 \text{ in}}{12}\right)^2 = 0.27605 \text{ slug}\cdot\text{ft}^2$$

Kinematics. Let Point C be the point of contact between the cylinder and the incline and Point G be the mass center of the cylinder without the bar. Assume that the mass center of the bar lies at Point P.

$$\boldsymbol{\alpha} = \alpha \circlearrowright. \quad \mathbf{a}_G = a_G \searrow 20°$$



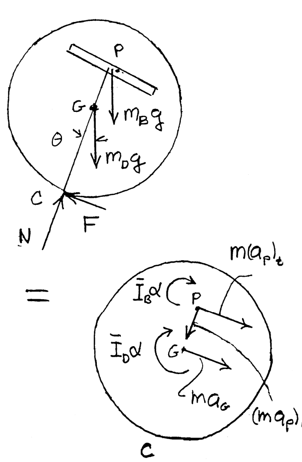
For rolling without slipping, $$(a_C)_t = a_G - r\alpha = 0 \quad a_G = r\alpha$$

$$(a_P)_t = a_G + h\alpha$$

$$(a_P)_t = (r+h)\alpha$$

$$(a_P)_h = 0 + h\omega^2 = h\omega^2$$

Kinetics: Using the cylinder plus the bar as a free body,



$$+\circlearrowright \Sigma M_C = +\Sigma(M_C)_{\text{eff}}: \quad m_D g r \sin\theta + m_B g (r+h)\sin\theta$$

$$= \bar{I}_D\alpha + m_D a_G r + \bar{I}_B\alpha + m_B (a_P)_t (r+h)$$

$$= [\bar{I}_D + m_D r^2 + \bar{I}_B + m_B (r+h)^2]\alpha$$

$$\alpha = \frac{[m_D g r + m_B g (r+h)]\sin\theta}{\bar{I}_D + m_D r^2 + \bar{I}_B + m_B (r+h)^2}$$

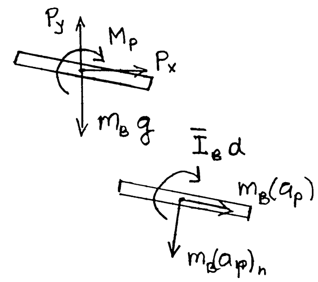
$$= \frac{\left[(10)\left(\frac{16}{12}\right) + (4)\left(\frac{24}{12}\right)\right]\sin 20°}{0.27605 + \left(\frac{10}{32.2}\right)\left(\frac{16}{12}\right)^2 + (0.028755) + \left(\frac{4}{32.2}\right)\left(\frac{24}{12}\right)^2}$$

$$= 5.3896 \text{ rad/s}^2$$

$$(a_P)_t = \left(\tfrac{24}{12}\right)(5.3896) = 10.7791 \text{ ft/s}^2$$

$$(a_P)_n = \left(\tfrac{8}{12}\right)(2)^2 = 2.6667 \text{ ft/s}^2$$

Using the bar alone as a free body,



$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P_x = m_B(a_P)_t \cos 20° - m_B(a_P)_n \sin 20°$$

$$= \left(\frac{4}{32.2}\right)(10.779)\cos 20° - \left(\frac{4}{32.2}\right)(2.6667)\sin 20°$$

$$P_x = 1.1450 \text{ lb}$$

$$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad P_y - m_B g = -m_B(a_P)_t \sin 20° - m_B(a_P)_n \cos 20°$$

$$P_y = (4) - \left(\frac{4}{32.2}\right)(10.779)\sin 20° - \left(\frac{4}{32.2}\right)(2.6667)\cos 20°$$

$$P_y = 3.2307 \text{ lb}$$

$$P = \sqrt{1.145^2 + 3.2307^2} = 3.4276 \text{ lb}$$

$$\tan\beta = \frac{3.2307}{1.145}$$

$$\beta = 70.5°$$

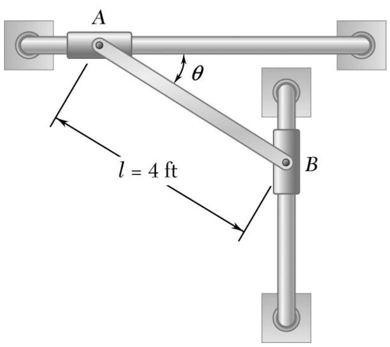
$$\mathbf{P} = 3.43 \text{ lb} \measuredangle 70.5° \blacktriangleleft$$

Recognizing that P is the CG of the bar.

$$+\circlearrowright \Sigma M_P = \Sigma(M_P)_{\text{eff}}: \quad M_P = \bar{I}_B \alpha$$

$$= (0.028755)(5.3896)$$

$$\mathbf{M}_P = 0.1550 \text{ ft} \cdot \text{lb} \circlearrowright \blacktriangleleft$$

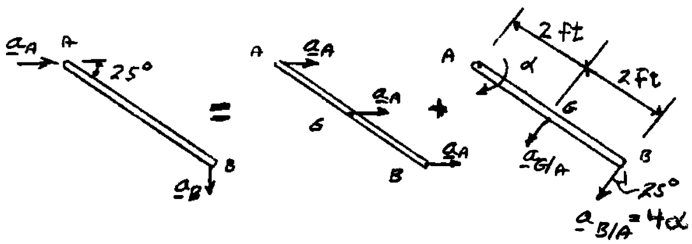


PROBLEM 16.117

The ends of the 20-lb uniform rod AB are attached to collars of negligible mass that slide without friction along fixed rods. If the rod is released from rest when $\theta = 25°$, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the reaction at A, (*c*) the reaction at B.

SOLUTION

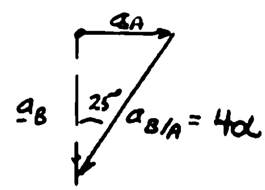
Kinematics: Assume α ↻ $\qquad\qquad \omega = 0$



$$\mathbf{a}_B \downarrow = \mathbf{a}_A + \mathbf{a}_{B/A} = [a_A \longrightarrow] + [4\alpha \measuredangle 25°]$$

$$a_B = (4\alpha)\cos 25° = 3.6252\alpha$$

$$a_A = (4\alpha)\sin 25° = 1.6905\alpha$$



$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A} = [a_A \longrightarrow] + [2\alpha \measuredangle 25°]$$

$$\mathbf{a}_G = [1.6905\alpha \longrightarrow] + [2\alpha \measuredangle 25°]$$

$$\bar{a}_x = (a_G)_x = [1.6905\alpha \longrightarrow] + [0.84524\alpha \longleftarrow]$$

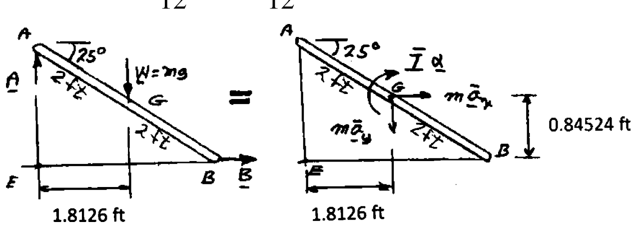
$$\bar{\mathbf{a}}_x = 0.84524\alpha \longrightarrow$$

$$\bar{a}_y = [2\alpha \cos 25° \downarrow] = 1.8126\alpha \downarrow$$

We have found for α ↻ $\qquad \bar{\mathbf{a}}_x = 0.84524\alpha \longrightarrow$

$$\bar{\mathbf{a}}_y = 1.8126\alpha \downarrow$$

Kinetics: $\qquad \bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}m(4\text{ ft})^2$



PROBLEM 16.117 (Continued)

(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_E = \Sigma (M_E)_{\text{eff}}: \quad mg(1.8126 \text{ ft}) = \bar{I}\alpha + m\bar{a}_x(0.84524 \text{ ft}) + m\bar{a}_y(1.8126 \text{ ft})$$

$$mg(1.8126) = \frac{1}{12}m(4)^2\alpha + m(0.84524)^2\alpha + m(1.8126)^2\alpha$$

$$g(1.8126) = 5.3333\alpha \qquad \alpha = 0.33988g \qquad \boldsymbol{\alpha} = 10.944 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(*b*) $+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A - mg = -m\bar{a}_y = m(1.8126\alpha)$

$$A - 20 = -\left(\frac{20}{32.2}\right)(1.8126)(10.944)$$

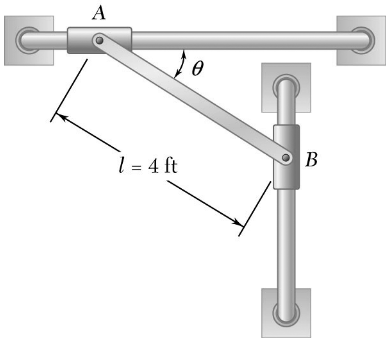
$$A = 20 - 12.321$$

$$= 7.6791 \text{ lb} \qquad \mathbf{A} = 7.68 \text{ lb} \uparrow \blacktriangleleft$$

(*c*) $\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad B = m\bar{a}_x = m(0.84524\alpha)$

$$B = \frac{20}{32.2}(0.84524)(10.944)$$

$$B = 5.7453 \text{ lb} \qquad \mathbf{B} = 5.75 \text{ lb} \longrightarrow \blacktriangleleft$$

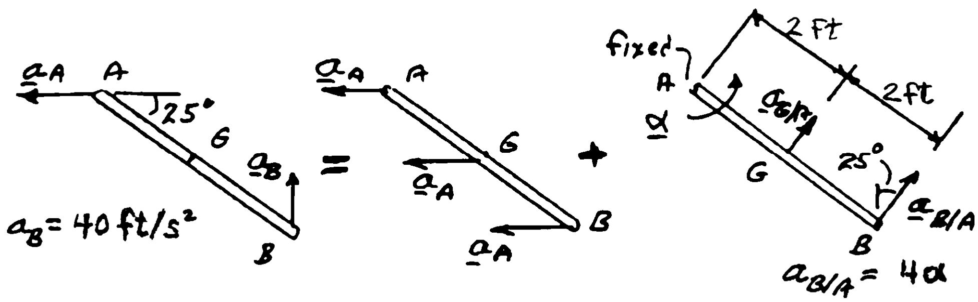


PROBLEM 16.118

The ends of the 20-lb uniform rod AB are attached to collars of negligible mass that slide without friction along fixed rods. A vertical force **P** is applied to collar B when $\theta = 25°$, causing the collar to start from rest with an upward acceleration of 40 ft/s^2. Determine (*a*) the force **P**, (*b*) the reaction at A.

SOLUTION

Kinematics: $\omega = 0$

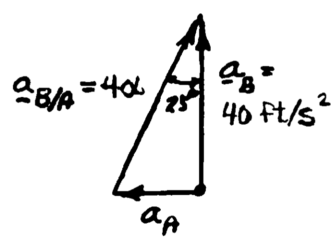


$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}; \quad [40 \text{ ft/s}^2 \uparrow] = [a_A \longleftarrow] + [4\alpha \measuredangle 25°]$$

$$a_B = a_{B/A} \cos 25°$$

$$40 \text{ ft/s}^2 = (4\alpha)\cos 25°$$

$$\boldsymbol{\alpha} = 11.034 \text{ rad/s}^2 \circlearrowleft$$



$$\mathbf{a}_A = 40 \tan 25° = 18.6523 \text{ ft/s}^2 \longleftarrow$$

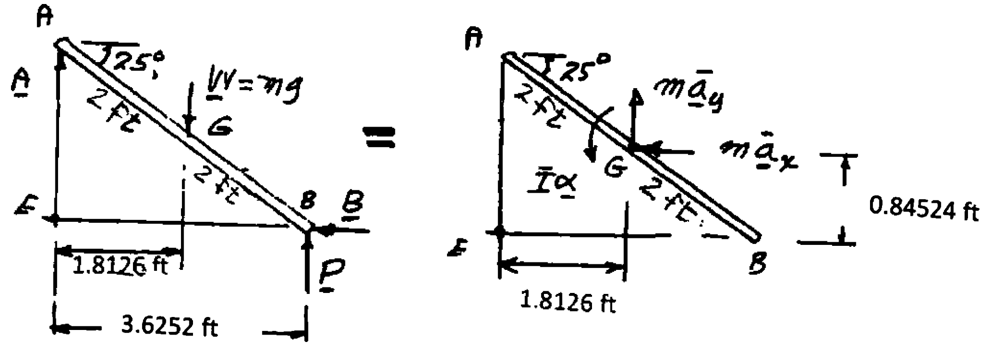
$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}: \quad \mathbf{a}_G = [18.652 \longleftarrow] + [2\alpha \measuredangle 25°]$$

$$\mathbf{a}_G = [18.652 \longleftarrow] + [2(11.034) \measuredangle 25°]$$

$$\bar{\mathbf{a}}_x = (\mathbf{a}_G)_x = 9.3262 \text{ ft/s}^2 \longleftarrow$$

$$\bar{\mathbf{a}}_y = (\mathbf{a}_G)_y = 20.00 \text{ ft/s}^2 \uparrow$$

Kinetics: $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}m(4)^2$



(*a*) $+\circlearrowright \Sigma M_E = \Sigma (M_E)_{\text{eff}}: \quad P(3.6252) - W(1.8126) = \bar{I}\alpha + m\bar{a}_x(0.84524) + m\bar{a}_y(1.8126)$

$$W = mg = 20 \text{ lb}$$

$$\bar{I}\alpha = \frac{1}{12}mL^2\alpha$$

$$= \frac{1}{12}\left(\frac{20}{32.2}\right)(4)^2(11.034)$$

$$= 9.1377 \text{ ft}\cdot\text{lb}$$

$$m\bar{a}_x = \left(\frac{20}{32.2}\right)(9.3262) = 5.7926 \text{ lb}$$

$$m\bar{a}_y = \left(\frac{20}{32.2}\right)(20) = 12.4224 \text{ lb}$$

$$P(3.6252) - (20)(1.8126) = 9.1377 + (5.7926)(0.84524) + (12.422)(1.8126)$$

$$P(3.6252) - 36.252 = 9.1377 + 4.8962 + 22.517$$

$$P(3.6252) = 72.8031$$

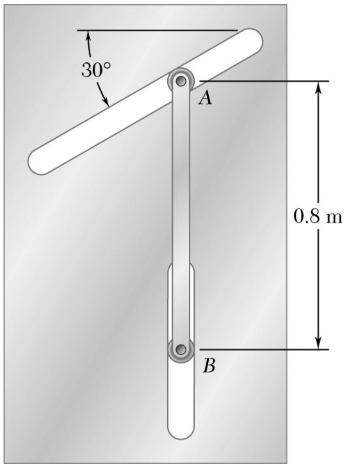
$$P = 20.082 \text{ lb}$$

$\mathbf{P} = 20.1 \text{ lb}\uparrow$ ◀

(*b*) $+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A - W + P = m\bar{a}_y$

$$A - 20 + 20.082 = 12.4224 \text{ lb}$$

$A = 12.34 \text{ lb}\uparrow$ ◀

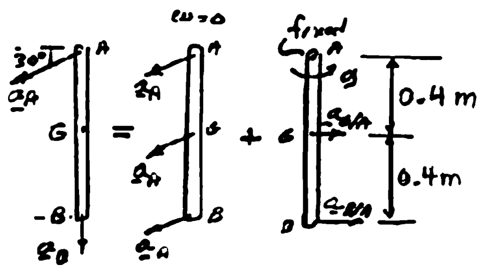


PROBLEM 16.119

The motion of the 3-kg uniform rod *AB* is guided by small wheels of negligible weight that roll along without friction in the slots shown. If the rod is released from rest in the position shown, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the reaction at *B*.

SOLUTION

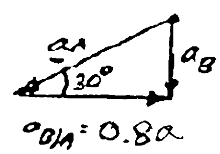
Kinematics:



$$\mathbf{a}_{G/A} = (0.4\text{ m})\alpha \longrightarrow$$

$$\mathbf{a}_{B/A} = (0.8\text{ m})\alpha \longrightarrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}: \quad [a_B \downarrow] = [a_A \swarrow 30°] + [0.8\alpha \longrightarrow]$$



$$\mathbf{a}_A = \frac{0.8\alpha}{\cos 30°} = 0.92376\alpha \swarrow 30°$$

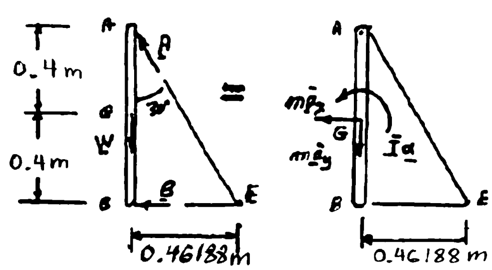
$$\mathbf{a}_B = (0.8\alpha)\tan 30° = 0.46188\alpha \downarrow$$

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}; \quad \bar{\mathbf{a}} = [0.92376\alpha \swarrow 30°] + [0.4\alpha + \longrightarrow]$$

$$\bar{\mathbf{a}}_x = [0.8\alpha \longleftarrow] + [0.4\alpha \longrightarrow] = 0.4\alpha \longleftarrow$$

$$\mathbf{a}_y = [0.46188\alpha \downarrow] = 0.46188\alpha \downarrow$$

We have:



$$\bar{\mathbf{a}}_x = 0.4\alpha \longleftarrow; \quad \mathbf{a}_y = 0.46188\alpha \downarrow$$

$$\bar{I} = \frac{1}{12} \cdot mL^2 = \frac{1}{12} \cdot (3 \text{ kg})(0.8 \text{ m})^2$$

$$\bar{I} = 0.16 \text{ kg} \cdot \text{m}^2$$

(*a*) Angular acceleration.

$$+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{\text{eff}}: \quad mg(0.46188 \text{ m}) = \bar{I}\alpha + m\bar{a}_x(0.4 \text{ m}) + m\bar{a}_y(0.46188 \text{ m})$$

$$3(9.81)(0.46188) = 0.16\alpha + 3(0.4)^2\alpha + 3(0.46188)^2\alpha$$

$$13.593 = 1.28\alpha$$

$$\alpha = 10.620 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 10.62 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

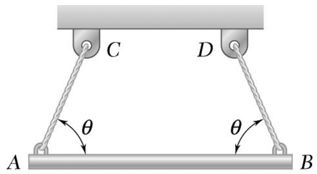
(*b*) Reaction at *B*.

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad B(0.8 \text{ m}) = -\bar{I}\alpha + m\bar{a}_x(0.4 \text{ m})$$

$$0.8B = -(0.16)(10.620) + 3(0.4)(10.620)(0.4)$$

$$0.8B = -1.6991 + 5.0974$$

$$B = 4.2479 \text{ N} \qquad \mathbf{B} = 4.25 \text{ N} \longleftarrow \blacktriangleleft$$



PROBLEM 16.120

A beam *AB* of length *L* and mass *m* is supported by two cables as shown. If cable *BD* breaks, determine at that instant the tension in the remaining cable as a function of its initial angular orientation θ.

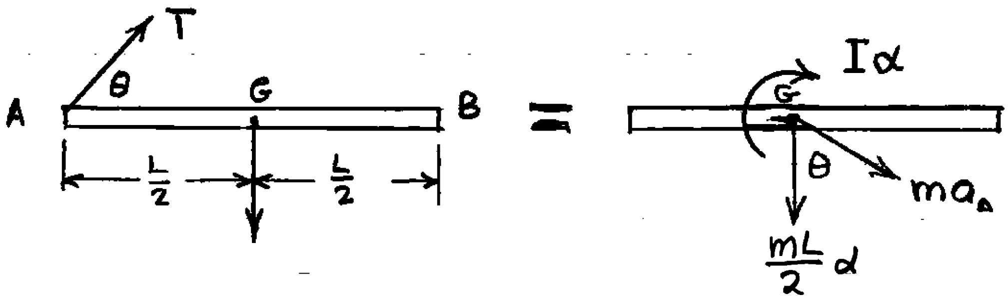
SOLUTION

Kinematics: At the instant just after the cable break,

$$\omega = 0 \qquad \boldsymbol{\alpha} = \alpha \circlearrowright$$

$$\mathbf{a}_G = \mathbf{a}_A + \frac{L}{2}\alpha \downarrow = \alpha_A \nwarrow \theta + \frac{L}{2}\alpha \downarrow$$

Kinetics: $I = \dfrac{1}{12} mL^2$



$$\circlearrowleft + \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad (T\sin\theta)\frac{L}{2} = I\alpha = \frac{1}{12} mL^2 \alpha$$

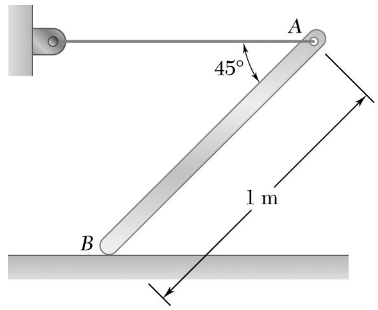
$$\alpha = \frac{6T\sin\theta}{mL}$$

$$\measuredangle \theta \; \Sigma F = \Sigma F_{\text{eff}}: \quad T - mg\sin\theta = -\frac{mL}{2}\alpha$$

$$= -\frac{mL}{2} \cdot \frac{6T\sin\theta}{mL}$$

$$= -3T\sin\theta$$

Solving for *T*, $$T = \frac{mg\sin\theta}{1 + 3\sin\theta} \blacktriangleleft$$



PROBLEM 16.121

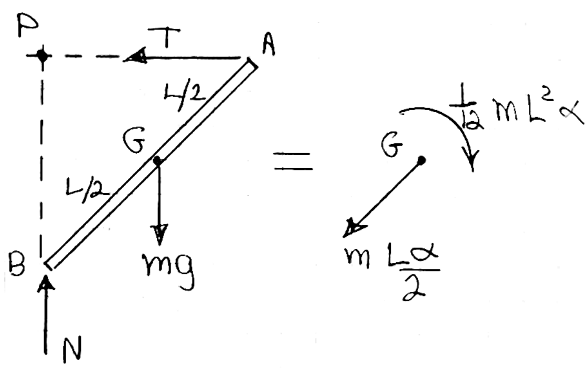
End A of a uniform 10-kg bar is attached to a horizontal rope and end B contacts a floor with negligible friction. Knowing that the bar is released from rest in the position shown, determine immediately after release (a) the angular acceleration of the bar, (b) the tension in the rope, (c) the reaction at B.

SOLUTION

Kinematics: $\quad \boldsymbol{\alpha} = \alpha \curvearrowright$

$$[a_B \longleftarrow] = [a_A \downarrow] + [L\alpha \searrow 45^\circ] \qquad \mathbf{a}_A = 0.7071\, L\alpha \downarrow$$

$$\mathbf{a}_G = \mathbf{a}_A + \left[\frac{L\alpha}{2} \searrow 45^\circ\right] = [0.7071\, \alpha \downarrow] + \left[\frac{L\alpha}{2} \searrow 45^\circ\right] = \frac{L\alpha}{2} \searrow 45^\circ$$



$$+\curvearrowright \Sigma M_P = \cancel{m} g\, \frac{L}{2}(0.7071)$$

$$= \frac{1}{12} \cancel{m}\, L^2 \alpha + \cancel{m}\, \frac{L}{2} \alpha \left(\frac{L}{2}\right)$$

$$\alpha = 0.7071 \left(\frac{3g}{2L}\right)$$

$$g = 9.81 \text{ m/s}^2, \qquad L = 1 \text{ m},$$

(a) $\boldsymbol{\alpha} = 10.405 \text{ rad/s}^2 \curvearrowright$ ◀

$$\overset{+}{\longleftarrow} \Sigma F_x = T = \frac{mL\alpha}{2}(0.7071) = \frac{m\cancel{L}\,(0.5)}{2}\left(\frac{3g}{\cancel{L}}\right) = \frac{3}{8}\, mg = 36.788 \text{ N}$$

(b) $T = 36.8 \text{ N}$ ◀

$$+\uparrow \Sigma F_y = N - mg = -\frac{3}{8} mg, \qquad N = \frac{5}{8}\, mg = 61.313 \text{ N}$$

(c) $N = 61.3 \text{ N} \uparrow$ ◀

Alternate solution:

Kinematics (realizing that the rope constrains Point A to the **j**-direction at the instant shown):

$$a_A\mathbf{j} = a_B\mathbf{i} + \alpha\mathbf{k}\, x(L\cos 45^\circ \mathbf{i} + L\sin 45^\circ \mathbf{j})$$

$$= a_B\mathbf{i} + (L\cos 45^\circ)\alpha\mathbf{j} - (L\sin 45^\circ)\alpha\mathbf{i}$$

x-components

$$0 = a_B - (L\sin 45°)\alpha$$
$$a_B = (L\sin 45°)\alpha$$

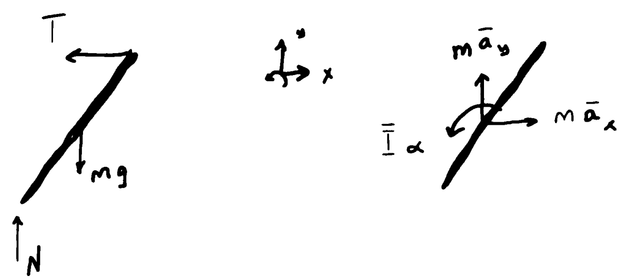
y-components

$$a_A = (L\cos 45°)\alpha$$

Acceleration of CG

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha}\times\mathbf{r}_{G/B} - \omega^2\mathbf{r}_{G/B} = \mathbf{a}_B - \boldsymbol{\alpha}\times\mathbf{r}_{G/B}$$
$$a_G = (L\sin 45°)\alpha\mathbf{i} + \alpha\mathbf{k}\times\left(\frac{L}{2}\cos(45°)\mathbf{i} + \frac{L}{2}\sin(45°)\mathbf{j}\right)$$
$$= (L\sin 45)\alpha\mathbf{i} + \left(\frac{L}{2}\cos(45°)\alpha\mathbf{j} - \frac{L}{2}\sin(45°)\alpha\mathbf{i}\right)$$
$$a_G = \frac{L}{2}\sin 45°\alpha\,\mathbf{i} + \frac{L}{2}\cos 45°\alpha\mathbf{j}$$

Kinetics:

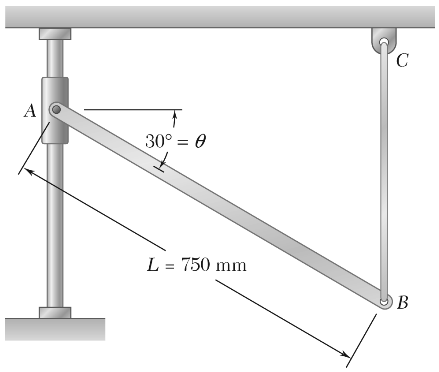


Equations of motion:

$$\Sigma F_x = m\bar{a}_x$$
$$-T = m\left(\frac{L}{2}\sin 45°\right)\alpha \qquad (1)$$
$$\Sigma F_y = m\bar{a}_y$$
$$N - mg = m\left(\frac{L}{2}\cos 45°\right)\alpha$$
$$N = mg + m\left(\frac{L}{2}\cos 45\right)\alpha \qquad (2)$$
$$\Sigma M_G = \bar{T}\alpha$$
$$T\left(\frac{L}{2}\right)\sin 45° - N\left(\frac{L}{2}\cos 45°\right) = \frac{1}{12}mL^2\alpha \qquad (3)$$

Solving the three simultaneous equations gives the same results as above.

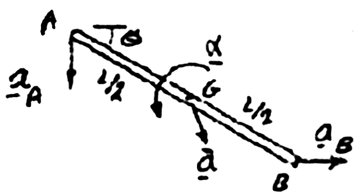
PROBLEM 16.122



End A of the 8-kg uniform rod AB is attached to a collar that can slide without friction on a vertical rod. End B of the rod is attached to a vertical cable BC. If the rod is released from rest in the position shown, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the reaction at A.

SOLUTION

Kinematics:



$$\omega = 0$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}; \quad [a_B \longrightarrow] = [a_A \downarrow] + [L\alpha \measuredangle \theta]$$

$$+\downarrow 0 = a_A - L\alpha\cos\theta$$

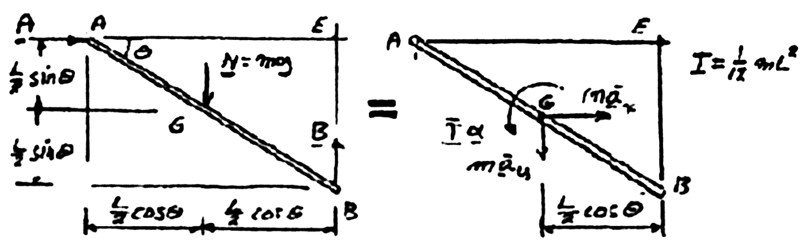
$$\mathbf{a}_A = L\alpha\cos\theta \downarrow$$

$$\bar{\mathbf{a}} = \mathbf{a}_A + \mathbf{a}_{G/A}$$

$$\bar{\mathbf{a}} = [L\alpha\cos\theta \downarrow] + \left[\frac{L}{2}\alpha \measuredangle \theta\right]$$

$$\bar{\mathbf{a}}_x = \frac{L}{2}\alpha\sin\theta \longrightarrow; \quad \mathbf{a}_y = \frac{L}{2}\alpha\cos\theta \uparrow$$

Kinetics:



$$+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{\text{eff}}: \quad mg\frac{L}{2}\cos\theta = I\alpha + m\bar{a}_x\left(\frac{L}{2}\sin\theta\right) + m\bar{a}_y\left(\frac{L}{2}\cos\theta\right)$$

$$mg\frac{L}{2}\cos\theta = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{2}\sin\theta\right)^2\alpha + m\left(\frac{L}{2}\cos\theta\right)^2\alpha$$

$$mg\frac{L}{2}\cos\theta = \frac{1}{3}mL^2\alpha \qquad \boldsymbol{\alpha} = \frac{3}{2}\frac{g}{L}\cos\theta \circlearrowleft$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad A = m\bar{a}_x = m\frac{L}{2}\alpha \sin\theta$$

$$A = m\frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\cos\theta\right)\sin\theta$$

$$\mathbf{A} = \frac{3}{4}mg\sin\theta\cos\theta \longrightarrow$$

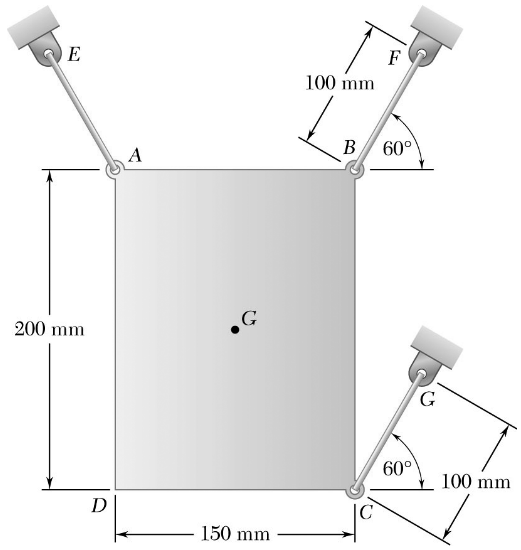
Data: $m = 8\text{ kg}, \quad \theta = 30°, \quad L = 0.75\text{ m}$

(*a*) Angular acceleration.

$$\alpha = \frac{3}{2}\frac{9.81\text{ m/s}^2}{0.75\text{ m}}\cos 30° \qquad \boldsymbol{\alpha} = 16.99\text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(*b*) Reaction at A.

$$A = \frac{3}{4}(8\text{ kg})(9.81\text{ m/s}^2)\sin 30° \cos 30° \qquad \mathbf{A} = 25.5\text{ N} \longrightarrow \blacktriangleleft$$



PROBLEM 16.123

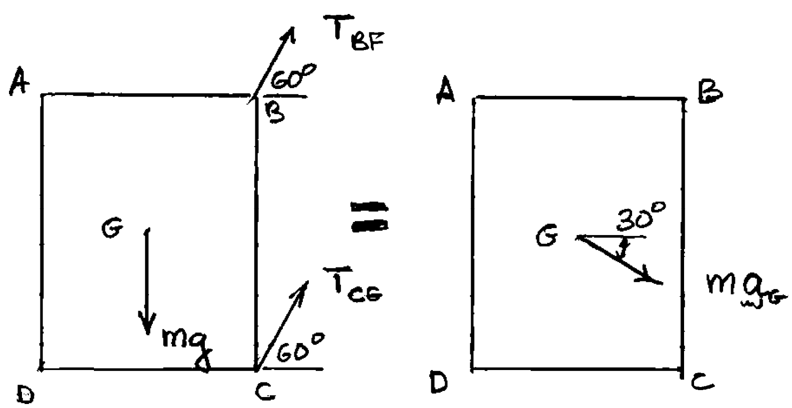
A uniform thin plate *ABCD* has a mass of 8 kg and is held in position by three inextensible cords *AE*, *BF*, and *CG*. If cord *AE* is cut, determine at that instant (*a*) if the plate is undergoing translation or general plane motion, (*b*) the tension in cords *BF* and *CG*.

SOLUTION

Immediately after cord *AE* breaks, $\omega = 0$.

(*a*) Assume that the cords *BF* and *CG* constrain the plate to undergo curvilinear translation, making $\boldsymbol{\alpha} = 0$.

$$\mathbf{a}_G = \bar{a} \searrow 30°$$



$$+\searrow 30° \Sigma F = \Sigma F_{\text{eff}}: \quad mg\sin 30° = ma_G$$

$$a_G = g\sin 30°$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad 0.075mg + 0.200T_{CG}\cos 60° = mg\sin 30°(0.100 + 0.075\sin 30°)$$

$$0.100T_{CG} = -0.00625mg \qquad T_{CG} = -0.0625mg$$

Since T_C is negative, the cord becomes slack so that the plate undergoes general plane motion with $T_{CG} = 0$.

general plane motion ◀

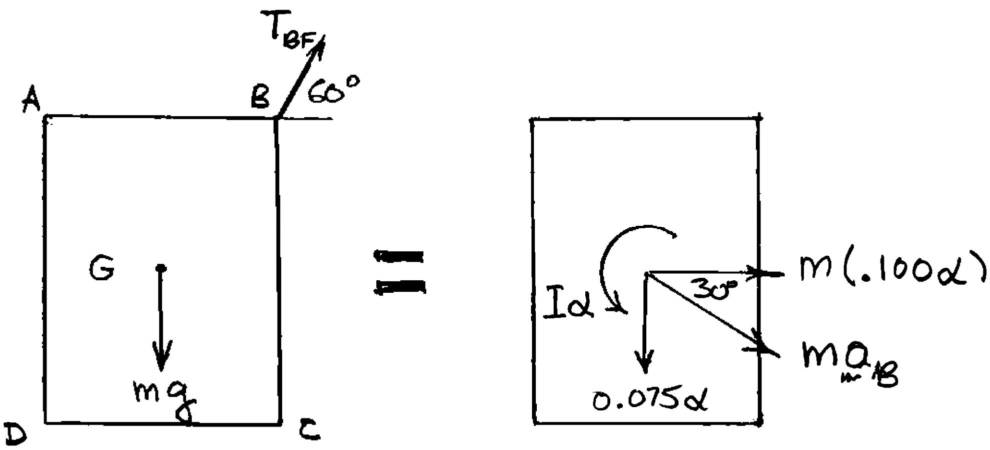
(*b*) *Kinematics*:

$$\mathbf{a}_B = a_A \searrow 30°$$

$$\boldsymbol{\alpha} = \alpha \circlearrowright$$

$$\mathbf{a}_G = \mathbf{a}_B + 0.100\alpha \longrightarrow + 0.075\alpha \downarrow$$

Kinetics:



$$m = 8 \text{ kg}$$

$$I = \frac{1}{12}(8 \text{ kg})[(0.150 \text{ m})^2 + (0.200 \text{ m})^2] = 0.041667 \text{ kg} \cdot \text{m}^2$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad T_{BF}\cos 60° = (8 \text{ kg})(0.100\alpha) + (8 \text{ kg})a_B \cos 30° \tag{1}$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad T_{BF}\sin 60° - (8 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= -(8 \text{ kg})(0.075\alpha) - (8 \text{ kg})a_B \sin 30° \tag{2}$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 0.075T_{BF}\sin 60° - 0.100T_{BF}\cos 60° = I\alpha$$

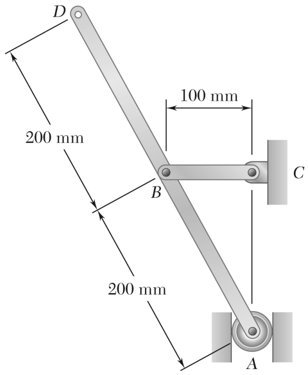
$$0.014952\, T_{BF} = 0.041667\alpha \tag{3}$$

Solving Eqs. (1), (2), and (3) simultaneously,

$$T_{BF} = 65.168 \text{ N}, \quad \alpha = 23.385 \text{ rad/s}^2, \quad a_B = 2.0028 \text{ m/s}^2$$

Tension in cords: $T_{BF} = 65.2 \text{ N}$ ◀

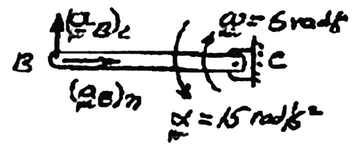
$T_{CG} = 0$ ◀



PROBLEM 16.124

The 4-kg uniform rod *ABD* is attached to the crank *BC* and is fitted with a small wheel that can roll without friction along a vertical slot. Knowing that at the instant shown crank *BC* rotates with an angular velocity of 6 rad/s clockwise and an angular acceleration of 15 rad/s^2 counterclockwise, determine the reaction at *A*.

SOLUTION

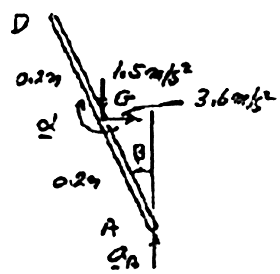


Crank *BC*:

$$BC = 0.1 \text{ m}$$

$$(a_B)_t = (BC)\alpha = (0.1 \text{ m})(15 \text{ rad/s}^2) = 1.5 \text{ m/s}^2 \qquad (\mathbf{a}_B)_t = 1.5 \text{ rad/s}^2 \downarrow$$

$$(a_B)_n = (BC)\omega^2 = (0.1 \text{ m})(6 \text{ rad/s})^2 = 3.6 \text{ m/s}^2 \qquad (\mathbf{a}_B)_n = 3.6 \text{ m/s}^2 \longrightarrow$$



Rod *ABD*:

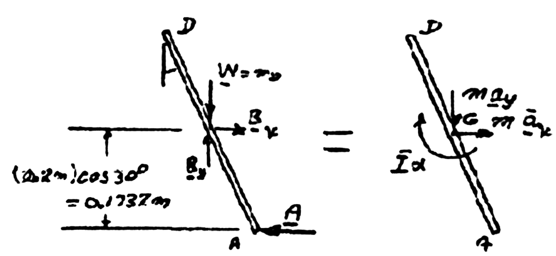
$$\theta = \sin^{-1}\frac{BC}{AB} = \sin^{-1}\frac{0.1 \text{ m}}{0.2 \text{ m}} = 30°$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/G}$$

$$[a_A \uparrow] = [1.5 \downarrow + 3.6 \longrightarrow] + [0.2\alpha \nearrow \beta]$$

$$\xrightarrow{+} \; 0 = 3.6 - (0.2\alpha)\cos\beta$$

$$\alpha = \frac{3.6}{0.2\cos\beta} = \frac{18}{\cos 30°} = 20.78 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 20.78 \text{ rad/s}^2 \circlearrowright$$

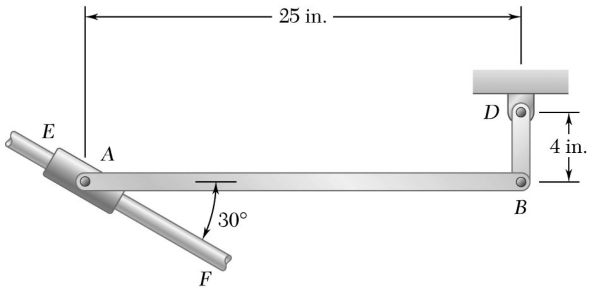


Kinetics:

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad A(0.1732 \text{ m}) = I\alpha = \frac{1}{12}mL^2\alpha$$

$$= \frac{1}{12}(4 \text{ kg})(0.4 \text{ m})^2(20.78 \text{ rad/s}^2)$$

$$A = 6.399 \text{ N} \qquad \mathbf{A} = 6.40 \text{ N} \longleftarrow \blacktriangleleft$$

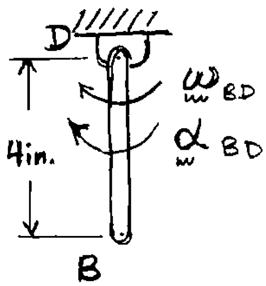


PROBLEM 16.125

The 7-lb uniform rod AB is connected to crank BD and to a collar of negligible weight, which can slide freely along rod EF. Knowing that in the position shown crank BD rotates with an angular velocity of 15 rad/s and an angular acceleration of 60 rad/s^2, both clockwise, determine the reaction at A.

SOLUTION

Crank BD:



$$\boldsymbol{\omega}_{BD} = 15 \text{ rad/s}, \quad \mathbf{v}_B = \left(\frac{4}{12}\text{ft}\right)(15 \text{ rad/s}) = 5 \text{ ft/s} \longleftarrow$$

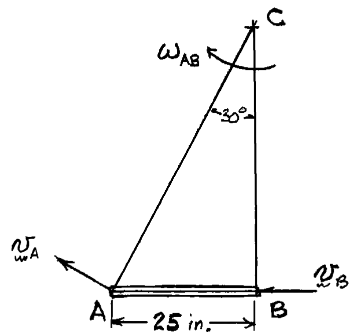
$$\boldsymbol{\alpha}_{BD} = 60 \text{ rad/s}^2$$

$$(\mathbf{a}_B)_x = \left(\frac{4}{12}\text{ ft}\right)(60 \text{ rad/s}^2) = 20 \text{ ft/s}^2 \longleftarrow$$

$$(\mathbf{a}_B)_y = \left(\frac{4}{12}\text{ ft}\right)(15 \text{ rad/s})^2 = 75 \text{ ft/s}^2 \uparrow$$

Rod AB:

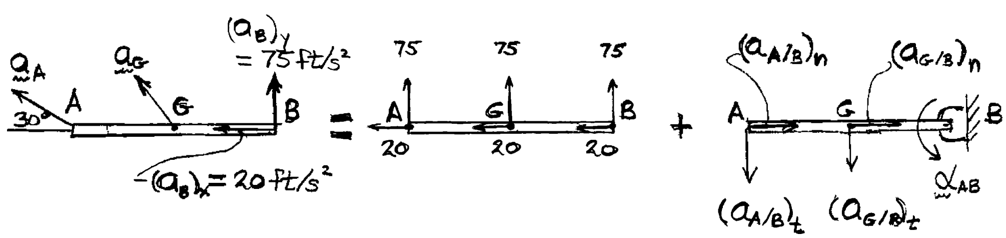
<u>Velocity</u>: Instantaneous center at C.



$$CB = \left(\frac{25}{12}\text{ ft}\right)/\tan 30° = 3.6084 \text{ ft}$$

$$\boldsymbol{\omega}_{AB} = \frac{v_B}{CB} = \frac{5 \text{ ft/s}}{3.6084 \text{ ft}} = 1.3856 \text{ rad/s}$$

<u>Acceleration</u>:



$$(\mathbf{a}_{A/B})_t = (AB)\alpha_{AB} = \frac{25}{12}\alpha_{AB} \downarrow$$

$$(\mathbf{a}_{A/B})_n = (AB)\omega_{AB}^2 = \left(\frac{25}{12}\right)(1.3856)^2 = 4 \text{ ft/s}^2 \longrightarrow$$

$$(\mathbf{a}_{G/B})_t = (GB)\alpha_{AB} \downarrow = \frac{12.5}{12}\alpha_{AB} \downarrow$$

PROBLEM 16.125 (Continued)

$$(\mathbf{a}_{G/B})_n \longrightarrow = (GB)\omega_{AB}^2 = \left(\frac{12.5}{12}\right)(1.3856)^2 = 2 \text{ ft/s}^2 \longrightarrow$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A} = \mathbf{a}_B + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{G/A})_n$$

$$[a_A \searrow 30°] = [20 \longleftarrow] + [75 \uparrow] + \left[\frac{25}{12}\alpha_{AB} \downarrow\right] + [4 \longrightarrow]$$

$$\overset{+}{\longleftarrow}\ a_A \cos 30° = 20 - 4; \qquad \mathbf{a}_A = 18.475 \text{ ft/s}^2 \searrow 30°$$

$$+\uparrow (18.475)\sin 30° = 75 - \frac{25}{12}\alpha_{AB}; \quad \boldsymbol{\alpha}_{AB} = 31.566 \text{ rad/s}^2 \circlearrowright$$

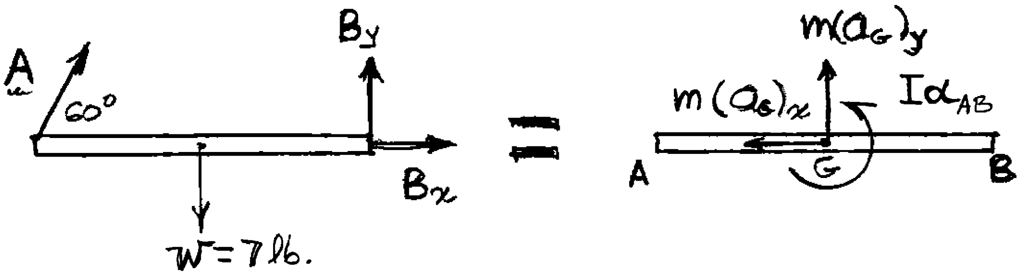
$$\bar{\mathbf{a}} = \mathbf{a}_B + \mathbf{a}_{G/B} = \mathbf{a}_B + (\mathbf{a}_{G/B})_t + (\mathbf{a}_{G/B})_n$$

$$\bar{\mathbf{a}} = [20 \longleftarrow] + [75 \uparrow] + [\tfrac{12.5}{12}(31.566) \downarrow] + [2 \longrightarrow]$$

$$\overset{+}{\longleftarrow}\ \bar{a}_x = 20 - 2 = 18; \qquad \bar{\mathbf{a}}_x = 18 \text{ ft/s}^2 \longleftarrow$$

$$+\uparrow\ a_y = 75 - 32.881 = 42.119; \quad \bar{\mathbf{a}}_y = 42.119 \text{ ft/s}^2 \uparrow$$

Kinetics: $\bar{I} = \frac{1}{12}m(AB)^2 = \frac{7 \text{ lb}}{12(32.2)}\left(\frac{25}{12} \text{ ft}\right)^2 = 0.078628 \text{ slug} \cdot \text{ft}^2$

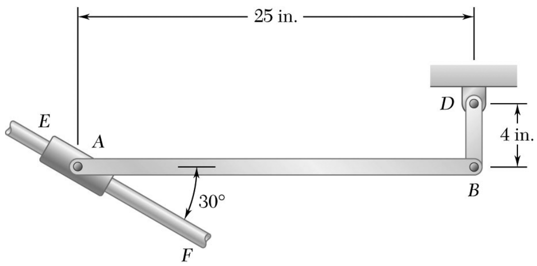


$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad (A\sin 60°)\left(\frac{25}{12} \text{ ft}\right) - mg\left(\frac{12.5}{12} \text{ ft}\right) = -\bar{I}\alpha_{AB} + m\bar{a}_y\left(\frac{12.5}{12} \text{ ft}\right)$$

$$1.8042A - (7 \text{ lb})\left(\frac{12.5}{12} \text{ ft}\right) = -(0.078628 \text{ slug} \cdot \text{ft}^2)(31.566 \text{ rad/s}^2) + \left(\frac{7}{32.2} \text{ slug}\right)(42.119 \text{ ft/s}^2)\left(\frac{12.5}{12} \text{ ft}\right)$$

$$1.8042A - 7.2917 = -2.4820 + 9.5378$$

$$A = 7.9522 \text{ lb} \qquad \mathbf{A} = 7.95 \text{ lb} \angle 60° \blacktriangleleft$$



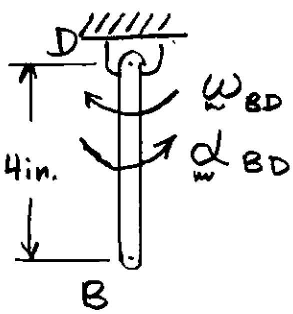
PROBLEM 16.126

In Problem 16.125, determine the reaction at A, knowing that in the position shown crank BD rotates with an angular velocity of 15 rad/s clockwise and an angular acceleration of 60 rad/s^2 counterclockwise.

PROBLEM 16.125 The 7-lb uniform rod AB is connected to crank BD and to a collar of negligible weight, which can slide freely along rod EF. Knowing that in the position shown crank BD rotates with an angular velocity of 15 rad/s and an angular acceleration of 60 rad/s^2, both clockwise, determine the reaction at A.

SOLUTION

Crank BD:



$$\boldsymbol{\omega}_{BD} = 15 \text{ rad/s},$$

$$\mathbf{v}_B = \left(\frac{4}{12}\text{ ft}\right)(15 \text{ rad/s}) = 5 \text{ ft/s} \longleftarrow$$

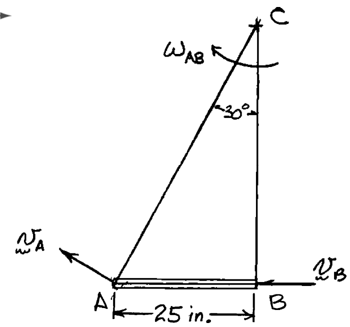
$$\boldsymbol{\alpha}_{BD} = 60 \text{ rad/s}^2$$

$$(\mathbf{a}_B)_x = \left(\frac{4}{12}\text{ ft}\right)(60 \text{ rad/s}^2) = 20 \text{ ft/s}^2 \longrightarrow$$

$$(\mathbf{a}_B)_y = \left(\frac{4}{12}\text{ ft}\right)(15 \text{ rad/s})^2 = 75 \text{ ft/s}^2 \uparrow$$

Rod AB:

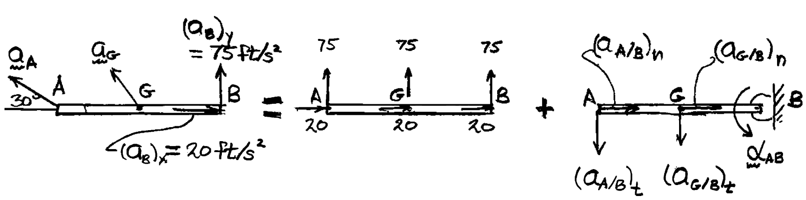
<u>Velocity</u>: Instantaneous center at C.



$$CB = \left(\frac{25}{12}\text{ ft}\right)/\tan 30° = 3.6084 \text{ ft}$$

$$\boldsymbol{\omega}_{AB} = \frac{v_B}{CB} = \frac{5 \text{ ft/s}}{3.6084 \text{ ft}} = 1.3856 \text{ rad/s}$$

<u>Acceleration</u>:



$$(\mathbf{a}_{A/B})_t = (AB)\alpha_{AB} = \frac{25}{12}\alpha_{AB} \downarrow$$

$$(\mathbf{a}_{A/B})_n = (AB)\omega_{AB}^2 = \left(\frac{25}{12}\right)(1.3856)^2 = 4 \text{ ft/s}^2 \longrightarrow$$

$$(\mathbf{a}_{G/B})_t = (GB)\alpha_{AB} = \frac{12.5}{12}\alpha_{AB}\downarrow$$

$$(\mathbf{a}_{G/B})_n = (GB)\omega_{AB}^2 = \left(\frac{12.5}{12}\right)(1.3856)^2 = 2 \text{ ft/s}^2 \longrightarrow$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A} = \mathbf{a}_B + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$[\mathbf{a}_A \measuredangle 30°] = [20 \longrightarrow] + [75\uparrow] + \left[\frac{25}{12}\alpha_{AB}\downarrow\right] + [4 \longrightarrow]$$

$\xrightarrow{+}$: $a_A \cos 30° = 20 + 4$; $\quad \mathbf{a}_A = 27.713 \text{ ft/s}^2 \measuredangle 30°$

$+\downarrow$: $(27.713)\sin 30° = -75 + \dfrac{25}{12}\alpha_{AB}$; $\quad \boldsymbol{\alpha}_{AB} = 42.651 \text{ rad/s}^2 \circlearrowleft$

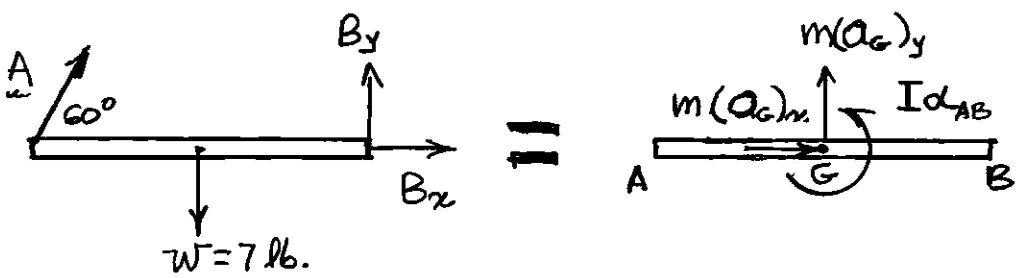
$$\bar{\mathbf{a}} = \mathbf{a}_B + \mathbf{a}_{G/B} = \mathbf{a}_B + (\mathbf{a}_{G/B})_t + (\mathbf{a}_{G/A})_n$$

$$\bar{\mathbf{a}} = [20 \longrightarrow] + [75\uparrow] + \left[\frac{12.5}{12}(42.651)\downarrow\right] + [2 \longrightarrow]$$

$\xrightarrow{+}$ $a_x = 20 + 2 = 22$; $\quad \mathbf{a}_x = 22 \text{ ft/s}^2 \longrightarrow$

$+\uparrow$ $a_y = 75 - 44.428 = 30.572$; $\quad \mathbf{a}_y = 30.6 \text{ ft/s}^2 \uparrow$

Kinetics: $\quad \bar{I} = \dfrac{7 \text{ lb}}{12(32.2)}\left(\dfrac{25}{12} \text{ ft}\right)^2 = 0.078628 \text{ slug}\cdot\text{ft}^2$



$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}$: $\quad (A\sin 60°)\left(\dfrac{25}{12} \text{ ft}\right) - mg\left(\dfrac{12.5}{12} \text{ ft}\right) = -\bar{I}\alpha_{AB} + m\bar{a}_y\left(\dfrac{12.5}{12} \text{ ft}\right)$

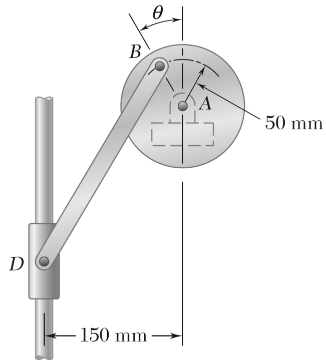
$$1.8042A - (7 \text{ lb})\left(\frac{12.5}{12} \text{ ft}\right) = -(0.078628 \text{ slug}\cdot\text{ft}^2)(42.651 \text{ rad/s}^2) + \left(\frac{7}{32.2} \text{ slug}\right)(30.572 \text{ ft/s}^2)\left(\frac{12.5}{12} \text{ ft}\right)$$

$$1.8042A - 7.2917 = -3.3536 + 6.9230$$

$$A = 6.0198 \text{ lb}$$

$\mathbf{A} = 6.02 \text{ lb} \measuredangle 60°$ ◀

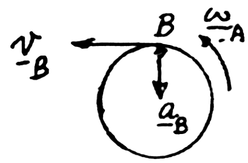
PROBLEM 16.127



The 250-mm uniform rod BD, of mass 5 kg, is connected as shown to disk A and to a collar of negligible mass, which may slide freely along a vertical rod. Knowing that disk A rotates counterclockwise at a constant rate of 500 rpm, determine the reactions at D when $\theta = 0$.

SOLUTION

Kinematics: For disk A: $(\alpha_A = 0)$

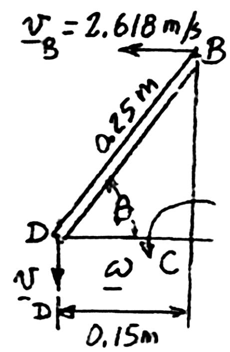


$$\omega_A = 500 \text{ rpm} = 52.36 \text{ rad/s}$$

$$v_B = r\omega_A = (0.05 \text{ m})(52.36 \text{ rad/s})$$
$$= 2.618 \text{ m/s}$$

$$a_B = r\omega_A^2 = (0.05 \text{ m})(52.36 \text{ rad/s}^2)$$
$$= 137.08 \text{ m/s}$$

<u>For rod (velocities)</u>



$$BC = \sqrt{(0.25)^2 - (0.15)^2}$$
$$= 0.20 \text{ m}$$

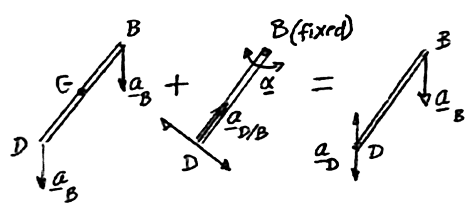
$$\cos\beta = \frac{0.15}{0.25} = \frac{3}{5}$$

$$\sin\beta = \frac{0.20}{0.25} = \frac{4}{5}$$

$$\omega = \frac{v_B}{BC} = \frac{2.618 \text{ m/s}}{0.20 \text{ m}}$$

$$\boldsymbol{\omega} = 13.09 \text{ rad/s} \circlearrowright$$

<u>Kinematics of rod (accelerations)</u>

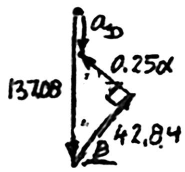


$$\mathbf{a}_B + \mathbf{a}_{D/B} = \mathbf{a}_D$$

$$[a_B\downarrow] + [(a_{D/B})_n \measuredangle \beta] + [(a_{D/B})_t \nwarrow \beta] = a_D\uparrow$$

$$[137.08\downarrow] + [0.25(13.09)^2 \measuredangle \beta] + [0.25\alpha \nwarrow \beta] = a_D\uparrow$$

x components:



$$\frac{3}{5}(42.84)-\frac{4}{5}(0.25\alpha)=0$$

$$25.704-0.2\alpha=0 \qquad \boldsymbol{\alpha}=128.52\text{ rad/s}^2$$

$$\bar{\mathbf{a}}=\mathbf{a}_G=\mathbf{a}_B+\mathbf{a}_{G/B}$$

$$=[137.08\downarrow]+[0.125(13.09)^2\nearrow]+[0.125(128.52)\nwarrow]$$

$$=[137.08\downarrow]+[21.42\measuredangle\beta]+[16.065\searrow\beta]$$

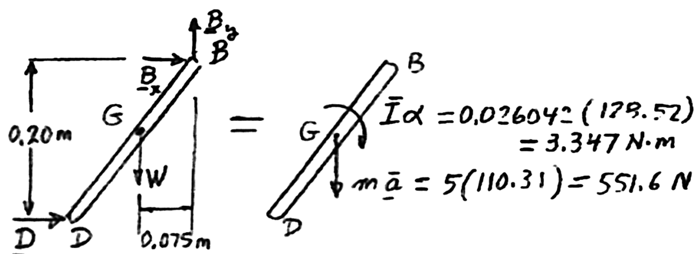
$$\xrightarrow{+}\bar{a}_x=\frac{3}{5}(21.42)-\frac{4}{5}(16.065)=0$$

$$+\downarrow\bar{a}_y=137.08-\frac{4}{5}(21.42)-\frac{3}{5}(16.065)=110.31\text{ m/s}^2$$

Kinetics

$$\bar{I}=\frac{1}{12}ml^2=\frac{1}{12}(5\text{ kg})(0.25\text{ m})^2$$

$$=0.026042\text{ kg}\cdot\text{m}^2$$



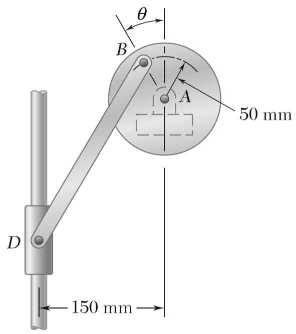
$$+\circlearrowright\Sigma M_B=\Sigma(M_B)_{\text{eff}}:\quad D(0.20\text{ m})+W(0.075\text{ m})=m\bar{a}(0.075)-\bar{I}\alpha$$

$$0.2D+5(9.81)(0.075)=551.6(0.075)-3.347$$

$$0.2D=41.370-3.347-3.679=34.344$$

$$D=171.7\text{ N}$$

$$\mathbf{D}=171.7\text{ N}\longrightarrow \blacktriangleleft$$



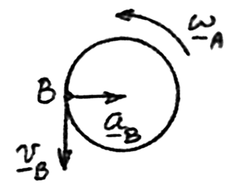
PROBLEM 16.128

Solve Problem 16.127 when $\theta = 90°$.

PROBLEM 16.127 The 250-mm uniform rod *BD*, of mass 5 kg, is connected as shown to disk *A* and to a collar of negligible mass, which may slide freely along a vertical rod. Knowing that disk *A* rotates counterclockwise at a constants rate of 500 rpm, determine the reactions at *D* when $\theta = 0$.

SOLUTION

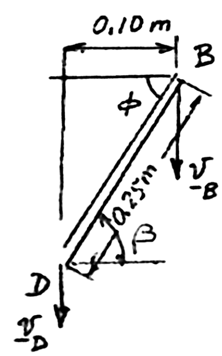
<u>Kinematics</u>: *For disk A*: $(\alpha_A = 0)$



$$\omega_A = 500 \text{ rpm} = 52.36 \text{ rad/s}$$

$$v_B = r\omega_A = (0.05 \text{ m})(52.36 \text{ rad/s})$$
$$= 2.618 \text{ m/s}$$

$$a_B = r\omega_A^2 = (0.05 \text{ m})(52.36 \text{ rad/s})^2$$
$$= 137.08 \text{ m/s}^2$$



<u>For rod (velocities)</u>

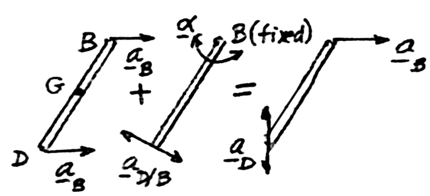
Since $\mathbf{v}_D$ is parallel to, $\mathbf{v}_B$

we have $\underline{\omega = 0}$

We also note that $\cos\phi = \dfrac{0.10}{0.25}$

$$\phi = 66.42°$$

<u>For rod (accelerations)</u>



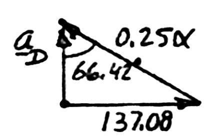
$$(a_{D/B})_n = 0$$

Since $\omega = 0$

$$\mathbf{a}_B + \mathbf{a}_{D/B} = \mathbf{a}_D$$

$$137.08 \rightarrow +0.25\alpha \searrow \phi = a_D \uparrow$$

$$(0.25\alpha)\sin 66.42° = 137.08$$



$$\boldsymbol{\alpha} = 598.3 \text{ rad/s}^2 \curvearrowright$$

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$

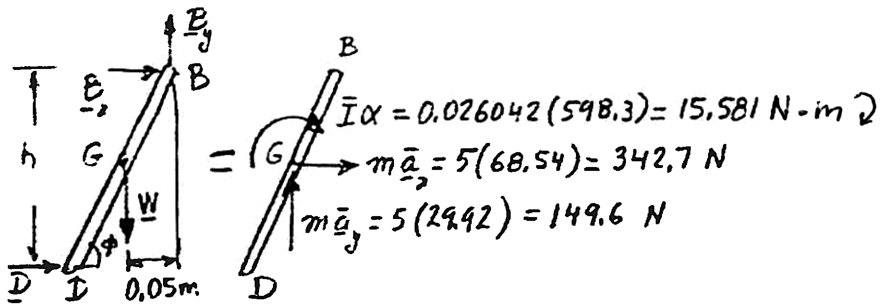
$$\xrightarrow{+} \bar{a}_x = 137.08 - 0.125(598.3)\sin 66.42° = +68.54 \text{ m/s}^2$$

$$+\uparrow \bar{a}_y = 0.125(598.3)\cos 66.42° = +29.92 \text{ m/s}^2$$

Summary of kinematics: $\boldsymbol{\alpha} = 598.3\ \text{rad/s}^2$ ↻, $\quad \bar{\mathbf{a}}_x = 68.54\ \text{m/s}^2 \longrightarrow, \quad \bar{\mathbf{a}}_y = 29.92\ \text{m/s}^2 \uparrow$

$$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}(5\ \text{kg})(0.25\ \text{m})^2$$
$$= 0.026042\ \text{kg}\cdot\text{m}^2$$

Kinetics:

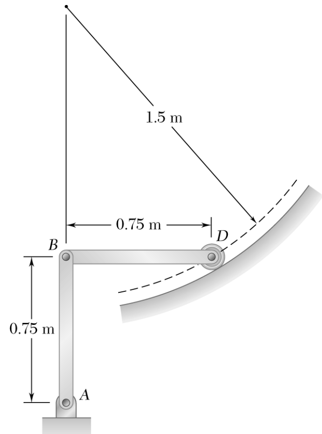


We recall that $\phi = 66.42°$. Thus: $h = (0.25\ \text{m})\sin\phi = 0.2291\ \text{m}$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad Dh + W(0.05) = m\bar{a}_x\left(\frac{h}{2}\right) - m\bar{a}_y(0.05) - \bar{I}\alpha$$

$$D(0.2291) + 5(9.81)(0.05) = (342.7)\frac{0.2291}{2} - (149.6)(0.05) - 15.581$$

$$0.2291D = 39.256 - 7.480 - 15.581 - 2.453 = 13.742 \qquad \mathbf{D} = 60.0\ \text{N} \longrightarrow ◀$$

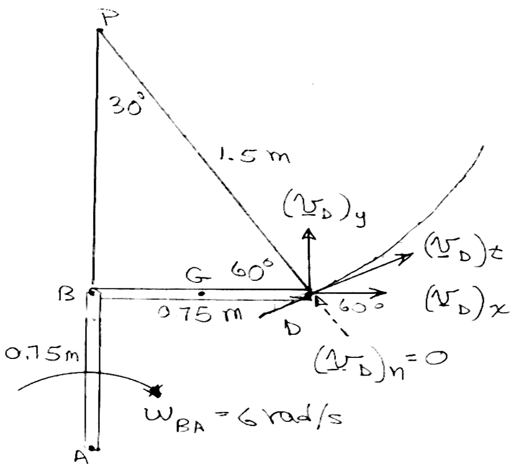


PROBLEM 16.129

The 4-kg uniform slender bar BD is attached to bar AB and a wheel of negligible mass which rolls on a circular surface. Knowing that at the instant shown bar AB has an angular velocity of 6 rad/s and no angular acceleration, determine the reaction at Point D.

SOLUTION

Kinematics:



Since $(\mathbf{v}_D)_n = 0$,

$$(v_D)_x \cos 60° = (v_D)_y \cos 30°$$

$$(v_D)_x = (v_B)_x = (AB)\,\omega_{BA} = (0.75 \text{ m})(6 \text{ rad/s})$$

$$(v_D)_y = (v_D)_x \frac{\cos 60°}{\cos 30°}$$

$$= (0.75)(6)\frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{0.75\,(6)}{\sqrt{3}}$$

But

$$(v_D)_y = \omega_{BD}\,(BD)$$

$$\omega_{BD} = \frac{(v_D)_y}{BD} = \frac{0.75\,(6)}{0.75\sqrt{3}} = \frac{6}{\sqrt{3}} \text{ rad/s}$$

$$\mathbf{a}_B = AB\,\omega_{BA}^2 = (0.75 \text{ m})(6 \text{ rad/s})^2 = 27 \text{ m/s}^2 \downarrow$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$$

$$[(\mathbf{a}_D)_t \measuredangle 30°] + [(1.5)(6/\sqrt{3})^2 \searrow 60°] = [27 \downarrow] + [0.75\,\alpha_{AB} \uparrow] + [(0.75)(6/\sqrt{3})^2 \longleftarrow]$$

$$60° \searrow: \qquad 0 + 18 = -27\sin 60° + (0.75\sin 60°)\alpha_{BD} + 9\sin 30°$$

$$\alpha_{BD} = \frac{18 + 27\sin 60° - 9\sin 30°}{0.75\sin 60°} = 56.7846 \text{ rad/s}$$

$$\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$

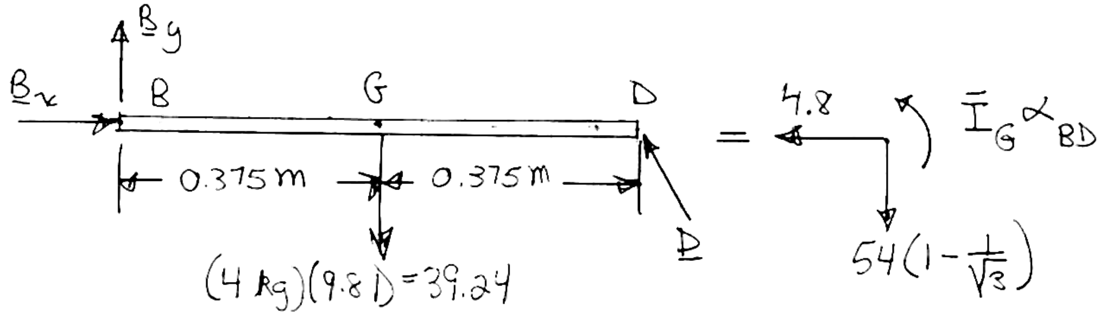
$$= [27 \downarrow] + [0.375\alpha_{BD} \uparrow] + [(0.375)(6/\sqrt{3})^2 \longleftarrow]$$

$$= [5.7058 \text{ m/s}^2 \downarrow] + [4.5 \text{ m/s}^2 \longleftarrow]$$

PROBLEM 16.129 (Continued)

Kinetics:

$$m = 4 \text{ kg} \qquad I_G = \frac{1}{12}mL^2 = \frac{1}{12}(4)(0.75)^2 = 0.1875 \text{ kg}\cdot\text{m}^2$$



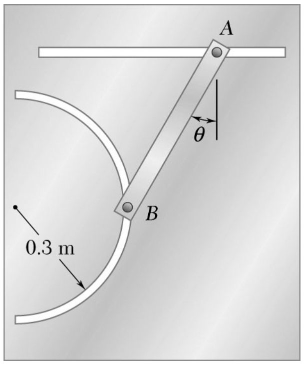
$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}$:

$$-(39.24 \text{ N})(0.375 \text{ m}) + D\cos 30°\,(0.75 \text{ m}) = \left[(0.1875)(56.7846) - (54)(0.375)\left(1 - \frac{1}{\sqrt{3}}\right)\right] \text{N}\cdot\text{m}$$

$$0.64952\,D = (14.715 + 10.6471 - 8.5587) \text{ N}\cdot\text{m}$$

$$\mathbf{D} = 25.87 \text{ N} \measuredangle 60°$$

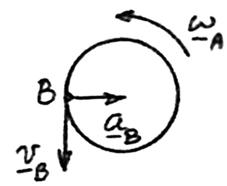
or $\mathbf{D} = 25.9 \text{ N} \measuredangle 60° \blacktriangleleft$



PROBLEM 16.130

The motion of the uniform slender rod of length $L = 0.5$ m and mass $m = 3$ kg is guided by pins at A and B that slide freely in frictionless slots, circular and horizontal, cut into a vertical plate as shown. Knowing that at the instant shown the rod has an angular velocity of 3 rad/s counter-clockwise and $\theta = 30°$, determine the reactions at Points A and B.

SOLUTION



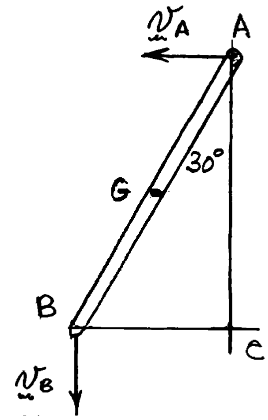
Mass and moment of inertia: $m = 3$ kg

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(3\text{ kg})(0.5\text{ m})^2 = 0.0625\text{ kg}\cdot\text{m}^2$$

Kinematics: $\boldsymbol{\omega} = 3\text{ rad/s} \circlearrowleft$

$$\mathbf{v}_A = v_A \longleftarrow \qquad \mathbf{v}_B = v_B \downarrow$$

Locate the instantaneous center C by drawing line $\overline{AC}$ perpendicular to $\mathbf{v}_A$ and line $\overline{BC}$ perpendicular to $\mathbf{v}_B$.



$$\begin{aligned} v_A &= (L\cos 30°)\omega \\ &= (0.5\text{ m}\cos 30°)(3\text{ rad/s}) \\ &= 1.29904\text{ m/s} \end{aligned}$$

$$\begin{aligned} v_B &= (L\sin 30°)\omega \\ &= (0.5\text{ m}\sin 30°)(3\text{ rad/s}) \\ &= 0.75\text{ m/s} \end{aligned}$$

$$\alpha = \alpha \circlearrowleft \qquad \mathbf{a}_A = a_A \longrightarrow$$

$$\mathbf{a}_B = \frac{v_B^2}{R} \longleftarrow + (a_B)_y \uparrow = \frac{(0.75\text{ m/s})^2}{0.3\text{ m}} \longleftarrow (a_B)_y \uparrow$$

$$= 1.875\text{ m/s}^2 \longleftarrow + (a_B)_y \uparrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

where $(\mathbf{a}_{B/A})_t = L\alpha \nwarrow 30° = 0.5\alpha \nwarrow 30°$

and $(\mathbf{a}_{B/A})_n = L\omega^2 \measuredangle 60° = (0.5)(3)^2 \measuredangle 60° = 4.5\text{ m/s}^2 \measuredangle 60°$

PROBLEM 16.130 (Continued)

Equating the two expressions for $\mathbf{a}_B$ gives

$$1.875\ \text{m/s}^2 \longleftarrow +(a_B)_y \uparrow = \mathbf{a}_A + 0.5\alpha \measuredangle 30^\circ + 4.5\ \text{m/s}^2 \measuredangle 60^\circ$$

$$\longrightarrow: \quad -1.875 = a_A + 0.5\alpha\cos 30^\circ + 2.25$$

$$a_A = -0.5\alpha\cos 30^\circ - 4.125$$

$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$= (0.5\alpha\cos 30^\circ + 4.125) \longleftarrow + \frac{L}{2}\alpha \searrow 30^\circ + \frac{L}{2}\omega^2 \measuredangle 60^\circ$$

$$= (0.5\alpha\cos 30^\circ + 4.125) \longleftarrow + \frac{0.5}{2}\alpha \searrow 30^\circ + \frac{0.5}{2}(3)^2 \measuredangle 60^\circ$$

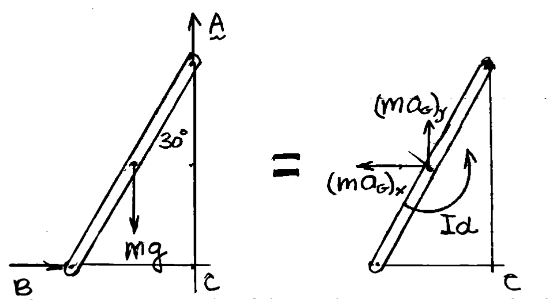
$$= (0.21651\alpha + 3.00) \longleftarrow + (1.94856 - 0.125\alpha) \uparrow$$

Kinetics:

$$m\mathbf{a}_G = (3\ \text{kg})\mathbf{a}_G$$

$$= [0.64952\alpha + 9.00] \longleftarrow + [5.8457 - 0.375\alpha] \uparrow$$

$$I\alpha = 0.0625\alpha$$



$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad mg\frac{L}{2}\sin 30^\circ = I\alpha + (ma_G)_x\frac{L}{2}\cos 30^\circ - (ma_G)_y\frac{L}{2}\sin 30^\circ$$

$$(3)(9.81)\frac{0.5}{2}\sin 30^\circ = 0.0625\alpha$$

$$+ (0.64952\alpha + 9.00)\frac{0.5}{2}\cos 30^\circ$$

$$- (5.8457 - 0.375\alpha)\frac{0.5}{2}\sin 30^\circ$$

$$3.67875 = 0.25\alpha + 1.21784$$

$$\alpha = 9.8436\ \text{rad/s}^2$$

$$m\mathbf{a}_G = [(0.64952)(9.8436) + 9.00] \longleftarrow + [5.8457 - (0.375)(9.8436)] \uparrow$$

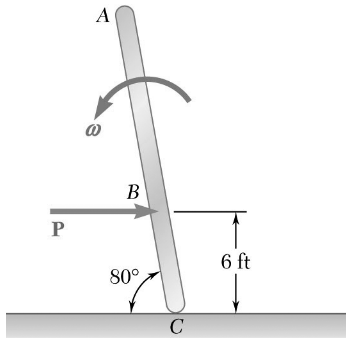
$$= [15.394\ \text{N}] \longleftarrow [2.1544\ \text{N}] \uparrow$$

PROBLEM 16.130 (Continued)

$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}} = m\mathbf{a}_G: \quad A\uparrow + B \longrightarrow + mg\downarrow = m\mathbf{a}_G$$

$+\uparrow$: $\quad A - (3 \text{ kg})(9.81 \text{ m/s}) = 2.1544 \text{ N}$ $\qquad \mathbf{A} = 31.6 \text{ N}\uparrow$ ◀

$+\longrightarrow$: $\quad B = -15.894 \text{ N}$ $\qquad \mathbf{B} = 15.89 \text{ N} \longleftarrow$ ◀



PROBLEM 16.131

At the instant shown, the 20 ft long, uniform 100-lb pole *ABC* has an angular velocity of 1 rad/s counterclockwise and Point *C* is sliding to the right. A 120-lb horizontal force **P** acts at *B*. Knowing the coefficient of kinetic friction between the pole and the ground is 0.3, determine at this instant (*a*) the acceleration of the center of gravity, (*b*) the normal force between the pole and the ground.

SOLUTION

Data: $l = 20 \text{ ft}, \quad W = 100 \text{ lb}, \quad P = 120 \text{ lb}, \quad \mu_k = 0.3$

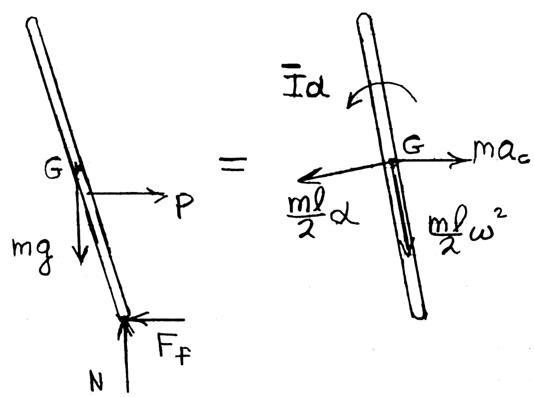
$$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{100}{32.2}\right)(20)^2 = 103.52 \text{ slug} \cdot \text{ft}^2$$

Kinematics: $\boldsymbol{\alpha} = \alpha \circlearrowright \qquad \mathbf{a}_C = a_C \longrightarrow$

$$\mathbf{a}_G = \mathbf{a}_C + (\mathbf{a}_{G/C})_t + (\mathbf{a}_{C/G})_n$$

$$= [a_C \longrightarrow] + \left[\frac{l}{2}\alpha \swarrow 10^\circ\right] + \left[\frac{l}{2}\omega^2 \searrow 80^\circ\right] \qquad (1)$$

Kinetics: Sliding to the right: $F_f = \mu_k N$



$$+\circlearrowright \Sigma M_G = +\circlearrowright \Sigma(M_G)_{\text{eff}}: \quad N\frac{l}{2}\sin 10^\circ - F_f\frac{l}{2}\cos 10^\circ + P\left(\frac{l}{2}\cos 10^\circ - h\right) = \bar{I}\alpha$$

$$\bar{I}\alpha - N\frac{l}{2}\sin 10^\circ + \mu_k N\frac{l}{2}\cos 10^\circ = P\left(\frac{l}{2}\cos 10^\circ - h\right)$$

$$103.519\alpha - (10)(\sin 10^\circ - 0.3\cos 10^\circ)N = 120(10\cos 10^\circ - 6) \qquad (2)$$

PROBLEM 16.131 (Continued)

$$+\uparrow \Sigma F_y = +\uparrow \Sigma(F_y)_{\text{eff}}: \quad N - mg = -\left(\frac{ml}{2}\alpha\right)\sin 10^\circ - \frac{ml}{2}\omega^2 \cos 10^\circ$$

$$\left(\frac{ml}{2}\sin 10^\circ\right)\alpha + N = mg - \frac{ml}{2}\omega^2 \cos 10^\circ$$

$$\left[\left(\frac{100}{32.2}\right)(10)\sin 10^\circ\right]\alpha + N = (100) - \left(\frac{100}{32.2}\right)(10)(1)^2 \cos 10^\circ \qquad (3)$$

Solving Eqs. (2) and (3) simultaneously,

$$\alpha = 3.8909 \text{ rad/s}^2 \qquad N = 48.433 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = \xrightarrow{+} \Sigma(F_x)_{\text{eff}}: \quad P - F_f = ma_C - \frac{ml}{2}\alpha\cos 10^\circ + \frac{ml}{2}\omega^2 \sin 10^\circ$$

$$ma_C = P - \mu_k N + \frac{ml}{2}\alpha\cos 10^\circ - \frac{ml}{2}\omega^2 \sin 10^\circ$$

$$\frac{100}{32.2}a_C = 120 - (0.3)(48.433) + \left(\frac{100}{32.2}\right)(10)(3.8909)\cos 10^\circ - \left(\frac{100}{32.2}\right)(10)(1)^2 \sin 10^\circ$$

$$a_C = 70.542 \text{ ft/s}^2$$

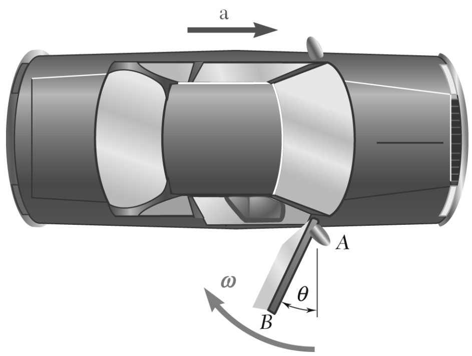
Using Eq. (1), $\quad \mathbf{a}_G = 70.542 + [(10)(3.8909) \swarrow 10^\circ] + [(10)(1)^2 \searrow 80^\circ]$

$$(a_G)_x = 70.542 - 38.909\cos 10^\circ + 10\cos 80^\circ$$
$$= 33.961 \text{ ft/s}^2$$
$$(a_G)_y = -38.909\sin 10^\circ - 10\sin 80^\circ$$
$$= -16.605 \text{ ft/s}^2$$
$$a_G = \sqrt{(33.961)^2 + (16.605)^2}$$
$$= 37.803 \text{ ft/s}^2$$

$$\tan\beta = \frac{16.605}{33.961} \qquad \beta = 26.055^\circ$$

(*a*) Acceleration at Point *G*. $\qquad \mathbf{a}_G = 37.8 \text{ ft/s}^2 \searrow 26.1^\circ$ ◀

(*b*) Normal force. $\qquad \mathbf{N} = 48.4 \text{ lb} \uparrow$ ◀

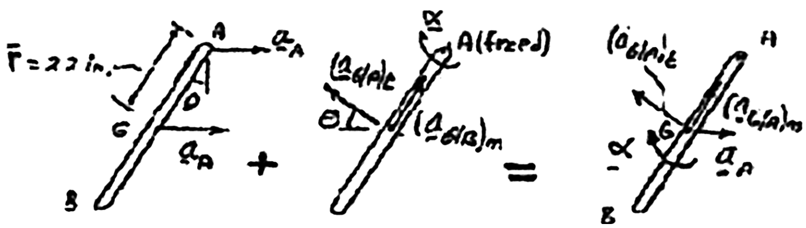


PROBLEM 16.132

A driver starts his car with the door on the passenger's side wide open ($\theta = 0$). The 80-lb door has a centroidal radius of gyration $\bar{k} = 12.5$ in., and its mass center is located at a distance $r = 22$ in. from its vertical axis of rotation. Knowing that the driver maintains a constant acceleration of 6 ft/s^2, determine the angular velocity of the door as it slams shut ($\theta = 90°$).

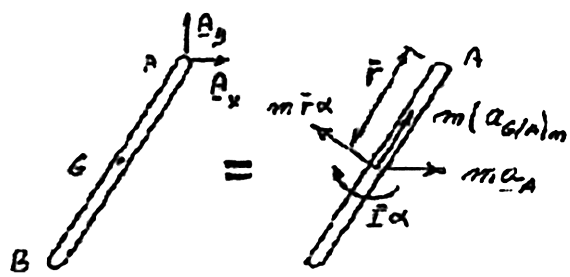
SOLUTION

Kinematics: $\mathbf{a} = a_A$



$$(\mathbf{a}_{G/A})_t = \bar{r}\alpha \searrow \theta$$

Kinetics:



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m\bar{r}\alpha)\bar{r} - ma_A(\bar{r}\cos\theta)$$

$$m\bar{k}^2\alpha + m\bar{r}^2\alpha = ma_A\bar{r}\cos\theta$$

$$\alpha = \frac{a_A\bar{r}}{\bar{k}^2 + \bar{r}^2}\cos\theta$$

Setting $\alpha = \omega\frac{d\omega}{d\theta}$, and using $\bar{r} = \frac{22}{12}$ ft, $\bar{k} = \frac{12.5}{12}$ ft

PROBLEM 16.132 (Continued)

$$\omega\frac{d\omega}{d\theta} = \frac{\left(\frac{22}{12}\text{ ft}\right)a_A}{\left[\left(\frac{12.5}{12}\text{ ft}\right)^2 + \left(\frac{22}{12}\text{ ft}\right)^2\right]}\cos\theta$$

$$= 0.41234a_A \cos\theta$$

$$\omega d\omega = 0.41234a_A \cos\theta\, d\theta$$

$$\int_0^{\omega_f} \omega d\omega = \int_0^{\pi/2} (0.4124a_A)\cos\theta\, d\theta$$

$$\left|\frac{1}{2}\omega^2\right|_0^{\omega_f} = 0.41234a_A |\sin\theta|_0^{\pi/2}$$

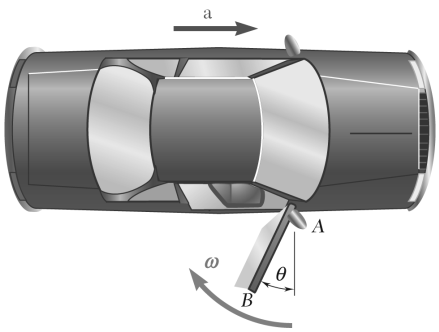
$$\omega_f^2 = 0.82468a_A \qquad (1)$$

Given data: $\mathbf{a}_A = 6\text{ ft/s}^2 \longrightarrow$

$$\omega_f^2 = 0.82468(6)$$

$$= 4.948\text{ rad}^2/\text{s}^2$$

$\omega_f = 2.22$ rad/s ◀

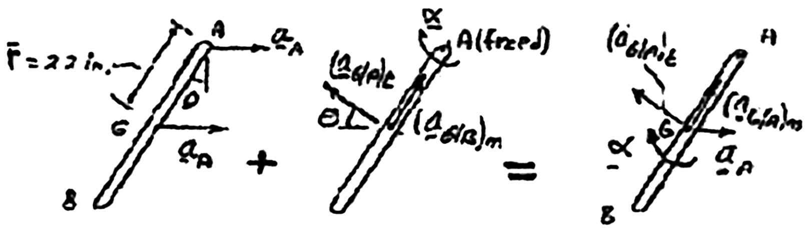


PROBLEM 16.133

For the car of Problem 16.132, determine the smallest constant acceleration that the driver can maintain if the door is to close and latch, knowing that as the door hits the frame its angular velocity must be at least 2 rad/s for the latching mechanism to operate.

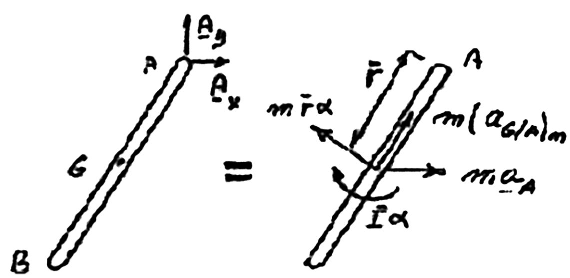
SOLUTION

Kinematics: $\mathbf{a} = a_A$



$$(\mathbf{a}_{G/A})_t = \bar{r}\alpha \measuredangle \theta$$

Kinetics:



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m\bar{r}\alpha)\bar{r} - ma_A(\bar{r}\cos\theta)$$

$$m\bar{k}^2\alpha + m\bar{r}^2\alpha = ma_A\bar{r}\cos\theta$$

$$\alpha = \frac{a_A\bar{r}}{\bar{k}^2 + \bar{r}^2}\cos\theta$$

Setting $\alpha = \omega\frac{d\omega}{d\theta}$, and using $\bar{r} = \frac{22}{12}\,\text{ft}$, $\bar{k} = \frac{12.5}{12}\,\text{ft}$

$$\omega\frac{d\omega}{d\theta} = \frac{\left(\frac{22}{12}\,\text{ft}\right)a_A}{\left[\left(\frac{12.5}{12}\,\text{ft}\right)^2 + \left(\frac{22}{12}\,\text{ft}\right)^2\right]}\cos\theta$$

$$= 0.41234a_A\cos\theta$$

$$\omega\, d\omega = 0.41234a_A\cos\theta\, d\theta$$

$$\int_0^{\omega_f} \omega d\omega = \int_0^{\pi/2} (0.4124 a_A) \cos\theta \, d\theta$$

$$\left| \frac{1}{2}\omega^2 \right|_0^{\omega_f} = 0.41234 a_A \mid \sin\theta \mid_0^{\pi/2}$$

$$\omega_f^2 = 0.82468 a_A \qquad (1)$$

Given data: $\omega_f = 2 \text{ rad/s}$

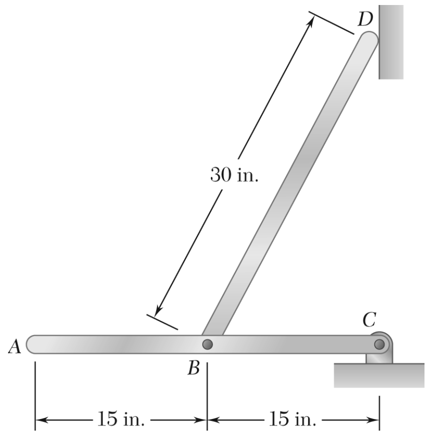
Eq. (1):

$$\omega_f^2 = 0.82468 a_A$$

$$(2)^2 = 0.82468 a_A$$

$$a_A = 4.85 \text{ ft/s}^2$$

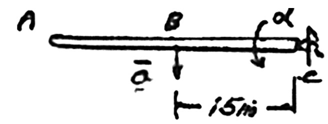
$\mathbf{a}_A = 4.85 \text{ ft/s}^2 \longrightarrow$ ◀



PROBLEM 16.134

Two 8-lb uniform bars are connected to form the linkage shown. Neglecting the effect of friction, determine the reaction at *D* immediately after the linkage is released from rest in the position shown.

SOLUTION



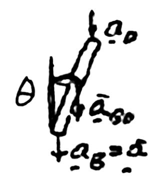
Kinematics:

Bar *AC*: Rotation about *C*

$$\bar{a} = (BC)\alpha = \left(\frac{15}{12}\text{ft}\right)\alpha$$

$$\bar{\mathbf{a}} = 1.25\alpha \downarrow$$

$$\sin\theta = \frac{15 \text{ in.}}{30 \text{ in.}} \qquad \theta = 30°$$

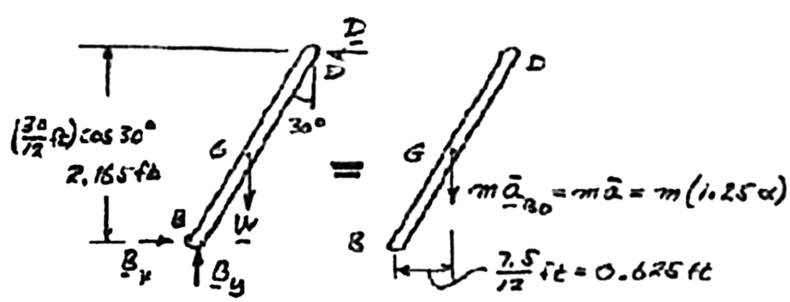


Bar *BC*: $\mathbf{a}_{D/B} = L\alpha \nwarrow$

Must be zero since $a_D \downarrow$ $\qquad \alpha_{BD} = 0 \quad$ and $\quad \bar{a}_{BD} = \bar{a}$

Kinetics:

Bar *BD*



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad B_y - W = -m\bar{a}$$

$$B_y - 8 \text{ lb} = -\frac{8 \text{ lb}}{32.2}(1.25\alpha)$$

$$B_y = 8 - 0.3105\alpha \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad D(2.165 \text{ ft}) - W(0.625 \text{ ft}) = -m\bar{a}(0.625 \text{ ft})$$

$$D(2.165 \text{ ft}) - (8 \text{ lb})(0.625 \text{ ft}) = -\frac{8 \text{ lb}}{32.2}(1.25\alpha)(0.625 \text{ ft})$$

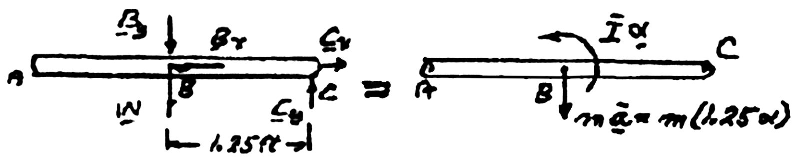
$$D = 2.309 - 0.08965\alpha \qquad (2)$$

Bar AC:

$$\bar{I} = \frac{1}{12}m(AC)^2$$

$$= \frac{1}{12}\frac{8 \text{ lb}}{32.2}(2.5 \text{ ft})^2$$

$$= 0.1294 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$



$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W(1.25 \text{ ft}) + B_y(1.25 \text{ ft}) = \bar{I}\alpha + m(1.25\alpha)(1.25)$$

Substitute from Eq. (1) for B_y

$$8(1.25) + (8 - 0.3105\alpha)(1.25) = (0.1294)\alpha + \frac{8}{32.2}(1.25)^2\alpha$$

$$10 + 10 - 0.3881\alpha = 0.1294\alpha + 0.3882\alpha$$

$$20 = 0.9057\alpha$$

$$\alpha = 22.08 \text{ rad/s}^2$$

Eq. (2),

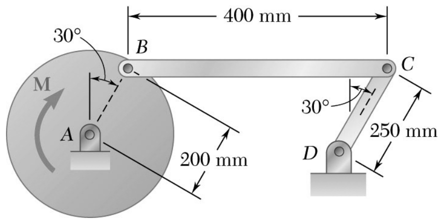
$$D = 2.309 - 0.08965\alpha$$

$$= 2.309 - 0.08965(22.08)$$

$$= 2.309 - 1.979$$

$$D = 0.330 \text{ lb}$$

$\mathbf{D} = 0.330 \text{ lb} \longleftarrow$ ◀



PROBLEM 16.135

The 6-kg rod *BC* connects a 10-kg disk centered at *A* to a 5-kg rod *CD*. The motion of the system is controlled by the couple **M** applied to disk *A*. Knowing that at the instant shown disk *A* has an angular velocity of 36 rad/s clockwise and no angular acceleration, determine (*a*) the couple **M**, (*b*) the components of the force exerted at *C* on rod *BC*.

SOLUTION

Kinematics: *Velocity analysis*. $\boldsymbol{\omega}_{AB} = 36 \text{ rad/s} \circlearrowright$

Disk *AB*: $\mathbf{v}_B = \overline{AB}\ \omega_{AB} \searrow 30° = (0.200)(36) \searrow 30° = 7.2 \text{ m/s} \searrow 30°$

Rod *BC*: $\mathbf{v}_C = v_C \searrow 30°$

Since $\mathbf{v}_C$ is parallel to $\mathbf{v}_B$, bar *BC* is in translation.

$$\mathbf{v}_C = 7.2 \text{ m/s} \searrow 30° \qquad \boldsymbol{\omega}_{BC} = 0$$

Rod *CD*:
$$\omega_{CD} = \frac{v_C}{l_{CD}} = \frac{7.2 \text{ m/s}}{0.25 \text{ m}} = 28.8 \text{ rad/s}$$

$$\boldsymbol{\omega}_{CD} = 28.8 \text{ rad/s} \circlearrowright$$

Acceleration analysis: $\boldsymbol{\alpha}_{AB} = 0$

Disk *AB*:
$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$
$$= 0 - (36)^2(0.2) \nearrow 60° = 259.2 \text{ m/s}^2 \nearrow 60°$$

Rod *BC*:
$$\boldsymbol{\alpha}_{BC} = \alpha_{BC} \circlearrowleft$$
$$\mathbf{a}_C = \mathbf{a}_B + (\mathbf{a}_{C/B})_t - \omega_{BC}^2 \mathbf{r}_{C/B}$$
$$\mathbf{a}_C = 259.2 \nearrow 60° + 0.4\alpha_{AB} \uparrow + 0 \tag{1}$$

Rod *CD*:
$$\boldsymbol{\alpha}_{CD} = \alpha_{CD} \circlearrowleft$$
$$\boldsymbol{\alpha}_C = (\mathbf{a}_{C/D})_t - \omega_{CD}^2 \mathbf{r}_{C/D} = [0.25\alpha_{CD} \nearrow 30°] - [(28.8)^2(0.25) \nearrow 60°]$$
$$\mathbf{a}_C = [0.25\alpha_{CD} \nearrow 30°] - [207.36 \nearrow 60°] \tag{2}$$

Equate components of two expressions (1) and (2) for $\mathbf{a}_C$.

$$+\!\longrightarrow: \quad -259.2\cos 60° = -0.25\alpha_{CD}\cos 30° - 207.36\cos 60°$$

$$\alpha_{CD} = 119.719 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{CD} = 119.719 \text{ rad/s}^2 \circlearrowleft$$

$$+\!\uparrow: \quad -259.2\sin 60° + 0.4\alpha_{BC} = 0.25\alpha_{CD}\sin 30° - 207.36\sin 60°$$

$$\alpha_{BC} = 149.649 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{BC} = 149.649 \text{ rad/s}^2 \circlearrowleft$$

Accelerations of the mass centers.

Disk *AB*: $\bar{\mathbf{a}}_{AB} = \mathbf{a}_A = 0$

Rod *BC*: Mass center at Point *P*. $\mathbf{r}_{P/B} = (0.2 \text{ m}) \longrightarrow$

$$\mathbf{a}_P = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{P/B} - \omega_{BC}^2 \mathbf{r}_{P/B}$$

$$= [259.2 \measuredangle 60°] + [0.2\alpha_{BC} \uparrow] + 0 \quad = [259.2 \measuredangle 60°] + [29.9298 \uparrow]$$

Rod *CD*: Mass center at Point *Q*. $\mathbf{r}_{Q/D} = 0.125 \text{ m} \measuredangle 60°$

$$\mathbf{a}_Q = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{Q/D} - \omega_{CD}^2 \mathbf{r}_{Q/D}$$

$$= [0.125\, \alpha_{CD} \measuredangle 30°] - [(28.8)^2 (0.125) \measuredangle 60°]$$

$$= [14.964875 \measuredangle 30°] - [103.60 \measuredangle 60°]$$

Masses: $m_{AB} = 10 \text{ kg}, \quad m_{BC} = 6 \text{ kg}, \quad m_{CD} = 5 \text{ kg}$

Effective forces at mass centers.

Disk *AB*: $m_{AB}\mathbf{a}_A = 0$

Rod *BC*: $m_{BC}\mathbf{a}_P = 6\mathbf{a}_P = [1555.2 \text{ N} \measuredangle 60°] + [179.58 \text{ N} \uparrow]$

Rod *CD*: $m_{CD}\mathbf{a}_a = 5\mathbf{a}_Q = [74.82 \text{ N} \measuredangle 30°] + [518 \text{ N} \measuredangle 60°]$

Moments of inertia:

Disk *AB*: $\bar{I}_{AB} = \frac{1}{2} m_{AB} r_{AB}^2 = \frac{1}{2}(10)(0.2)^2 = 0.2 \text{ kg} \cdot \text{m}^2$

Rod *BC*: $\bar{I}_{BC} = \frac{1}{12} m_{BC} l_{BC}^2 = \frac{1}{12}(6)(0.4)^2 = 0.08 \text{ kg} \cdot \text{m}^2$

Rod *CD*: $\bar{I}_{CD} = \frac{1}{12} m_{CD} l_{CD}^2 = \frac{1}{12}(5)(0.25)^2 = 0.0260417 \text{ kg} \cdot \text{m}^2$

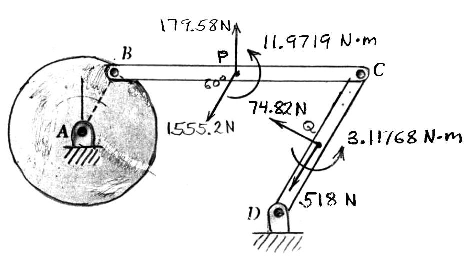
Effective couples at mass centers.

Disk *AB*: $\bar{I}_{AB}\boldsymbol{\alpha}_{AB} = 0$

Rod *BC*: $\bar{I}_{BC}\boldsymbol{\alpha}_{BC} = (0.08)(149.649) \circlearrowleft = 11.97192 \text{ N} \cdot \text{m} \circlearrowleft$

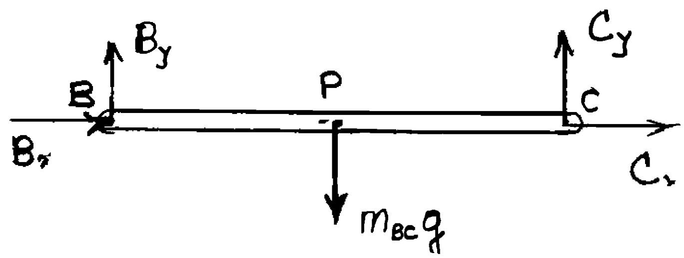
Rod *CD*: $\bar{I}_{CD}\boldsymbol{\alpha}_{CD} = (0.0260417)(119.719) \circlearrowleft = 3.11768 \text{ N} \cdot \text{m} \circlearrowleft$

Summary of effective forces and couples



Kinetics

Rod BC:



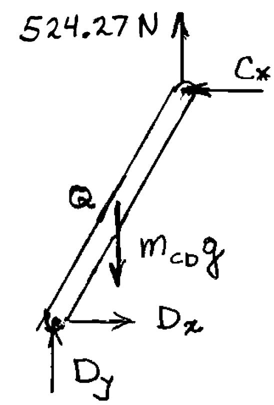
$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad l_{BC} C_y - \frac{1}{2} l_{BC} mg = \Sigma (M_B)_{\text{eff}}$$

$$(0.4 \text{ m}) C_y - (0.2 \text{ m})(6 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 11.9719 \text{ N} \cdot \text{m} + (0.2 \text{ m})(179.58 \text{ N})$$
$$-(0.2 \text{ m})(1555.2 \text{ N}) \sin 60°$$

$$C_y = -524.27 \text{ N}$$

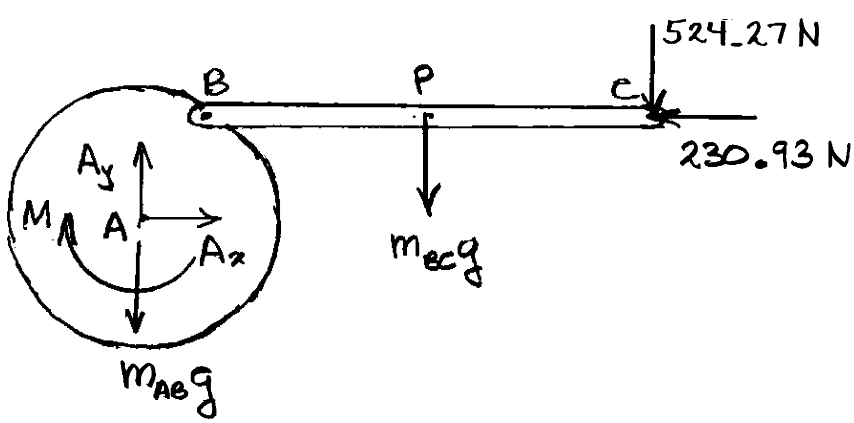
Rod CD:

$$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{\text{eff}}: \quad C_x l_{CD} \cos 30° + (524.27 \text{ N})(0.125 \text{ m})$$
$$-m_{CD} g (0.0625 \text{ m}) = \Sigma (M_D)_{\text{eff}}:$$
$$(0.25 \text{ m}) C_x \cos 30° + 65.534 \text{ N} \cdot \text{m}$$
$$-(5 \text{ kg})(9.81 \text{ m/s}^2)(0.0625 \text{ m})$$
$$= 3.11768 \text{ N} \cdot \text{m} + (0.125 \text{ m})(74.82 \text{ N})$$



$$C_x = -230.93 \text{ N}$$

Disk AB and rod BC:

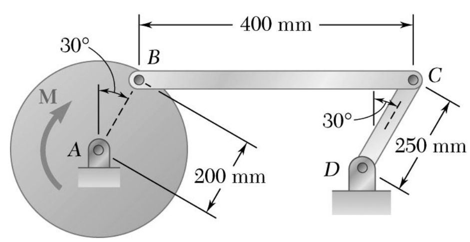


$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad -M - (524.27)(0.4 + 0.2\sin 30°) + (230.93)(0.2\cos 30°)$$
$$- (6\text{ kg})(9.81\text{ m/s}^2)(0.2 + 0.2\sin 30°)$$
$$= 11.9719 + (179.58)(0.2 + 0.2\sin 30°)$$
$$- (1555.2\cos 30°)(0.2)$$
$$-M - 262.135 + 40.0 - 17.658$$
$$= 11.9719 + 53.874 - 269.369$$
$$-M = 36.27\text{ N}\cdot\text{m}$$

(*a*) *Couple applied to disk A.* $\mathbf{M} = 36.3\text{ N}\cdot\text{m} \circlearrowleft$ ◀

(*b*) *Components of force exerted at C* on rod *BC*.

$\mathbf{C} = 231\text{ N} \longleftarrow + 524\text{ N}\downarrow$ ◀



PROBLEM 16.136

The 6-kg rod BC connects a 10-kg disk centered at A to a 5-kg rod CD. The motion of the system is controlled by the couple $\mathbf{M}$ applied to disk A. Knowing that at the instant shown disk A has an angular velocity of 36 rad/s clockwise and an angular acceleration of 150 rad/s^2 counterclockwise, determine (*a*) the couple $\mathbf{M}$, (*b*) the components of the force exerted at C on rod BC.

SOLUTION

Kinematics: *Velocity analysis.* $\boldsymbol{\omega}_{AB} = 36 \text{ rad/s} \circlearrowright$

Disk AB: $\mathbf{v}_B = \overline{AB}\omega_{AB} \nwarrow 30° = (0.200)(36) \nwarrow 30° = 7.2 \text{ m/s} \nwarrow 30°$

Rod BC: $\mathbf{v}_C = v_C \nwarrow 30°$

Since $\mathbf{v}_C$ is parallel to $\mathbf{v}_B$, bar BC is in translation

$$\mathbf{v}_C = 7.2 \text{ m/s} \nwarrow 30° \qquad \boldsymbol{\omega}_{BC} = 0$$

Rod CD:
$$\omega_{CD} = \frac{v_C}{l_{CD}} = \frac{7.2 \text{ m/s}}{0.25 \text{ m}} = 28.8 \text{ rad/s}$$

$$\boldsymbol{\omega}_{CD} = 28.8 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = 0$

Disk AB:
$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$
$$= [(150)(0.2) \searrow 30°] - [(36)^2(0.2) \nearrow 60°]$$
$$= [30 \text{ m/s}^2 \searrow 30°] + [259.2 \text{ m/s}^2 \swarrow 60°]$$

Rod BC:
$$\boldsymbol{\alpha}_{BC} = 150 \text{ rad/s}^2 \circlearrowleft$$
$$\mathbf{a}_C = \mathbf{a}_B + (\mathbf{a}_{C/B})_t - \omega_{BC}^2 \mathbf{r}_{C/B}$$
$$\mathbf{a}_C = [30 \searrow] + [259.2 \swarrow 60°] + [0.4\alpha_{AB}] + 0 \qquad (1)$$

Rod CD:
$$\boldsymbol{\alpha}_{CD} = \alpha_{CD} \circlearrowleft$$
$$\mathbf{a}_C = (\mathbf{a}_{C/D})_t - \omega_{CD}^2 \mathbf{r}_{C/D} = [0.25\alpha_{CD} \searrow 30°] - [(28.8)^2(0.25) \nearrow 60°]$$
$$\mathbf{a}_C = [0.25\alpha_{CD} \searrow 30°] - [207.36 \nearrow 60°] \qquad (2)$$

PROBLEM 16.136 (Continued)

Equate components of two expressions (1) and (2) for $\mathbf{a}_C$.

$$+\longrightarrow: \quad -30\cos 30° - 259.2\cos 60° = -0.25\alpha_{CD}\cos 30° - 207.36\cos 60°$$

$$\alpha_{CD} = 239.719 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{CD} = 239.719 \text{ rad/s}^2 \circlearrowleft$$

$$+\uparrow: \quad 30\cos 30° - 259.2\sin 60° + 0.4\alpha_{BC} = 0.25\alpha_{CD}\sin 30° - 207.36\sin 60°$$

$$\alpha_{BC} = 149.649 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{BC} = 149.649 \text{ rad/s}^2 \circlearrowleft$$

Accelerations of the mass centers.

Disk AB: $\quad \bar{\mathbf{a}}_{AB} = \mathbf{a}_A = 0$

Rod BC: Mass center at Point P. $\quad \mathbf{r}_{P/B} = (0.2 \text{ m}) \longrightarrow$

$$\mathbf{a}_P = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{P/B} - \omega_{BC}^2 \mathbf{r}_{P/B}$$

$$= [30 \searrow 30°] + [259.2 \swarrow 60°] + [0.2\alpha_{BC} \uparrow] + 0$$

$$= [30 \searrow 30°] + [259.2 \swarrow 60°] + [29.9298 \uparrow]$$

Rod CD: Mass center at Point Q. $\quad \mathbf{r}_{Q/D} = 0.125 \text{ m} \measuredangle 60°$

$$\mathbf{a}_Q = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{Q/D} - \omega_{CD}^2 \mathbf{r}_{Q/D}$$

$$= [0.125\ \alpha_{CD} \searrow 30°] - [(28.8)^2(0.125) \swarrow 60°]$$

$$= [29.964875 \searrow 30°] - [103.60 \swarrow 60°]$$

Masses: $\quad m_{AB} = 10 \text{ kg}, \quad m_{BC} = 6 \text{ kg}, \quad m_{CD} = 5 \text{ kg}$

Effective forces at mass centers.

Disk AB: $\quad m_{AB}\mathbf{a}_A = 0$

Rod BC: $\quad m_{BC}\mathbf{a}_P = 6\mathbf{a}_P = [180 \text{ N} \searrow 30°] + [1555.2 \text{ N} \swarrow 60°] + [179.58 \text{ N} \uparrow]$

Rod CD: $\quad m_{CD}\mathbf{a}_a = 5\mathbf{a}_Q = [149.82 \text{ N} \searrow 30°] + [518 \text{ N} \swarrow 60°]$

Moments of inertia:

Disk AB: $\quad \bar{I}_{AB} = \frac{1}{2} m_{AB} r_{AB}^2 = \frac{1}{2}(10)(0.2)^2 = 0.2 \text{ kg}\cdot\text{m}^2$

Rod BC: $\quad \bar{I}_{BC} = \frac{1}{12} m_{BC} l_{BC}^2 = \frac{1}{12}(6)(0.4)^2 = 0.08 \text{ kg}\cdot\text{m}^2$

Rod CD: $\quad \bar{I}_{CD} = \frac{1}{12} m_{CD} l_{CD}^2 = \frac{1}{12}(5)(0.25)^2 = 0.0260417 \text{ kg}\cdot\text{m}^2$

PROBLEM 16.136 (Continued)

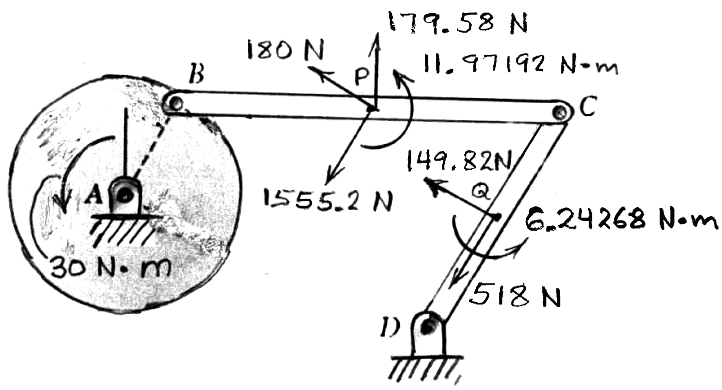
Effective couples at mass centers.

Disk AB: $\bar{I}_{AB}\boldsymbol{\alpha}_{AB} = (0.2)(150) = 30\ \text{N}\cdot\text{m}$

Rod BC: $\bar{I}_{BC}\boldsymbol{\alpha}_{BC} = (0.08)(149.649) = 11.97192\ \text{N}\cdot\text{m}$

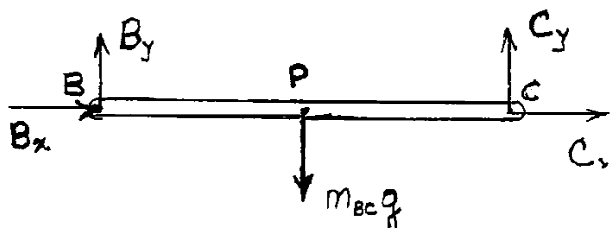
Rod CD: $\bar{I}_{CD}\boldsymbol{\alpha}_{CD} = (0.0260417)(239.719) = 6.24268\ \text{N}\cdot\text{m}$

Summary of effective forces and couples.



Kinetics

Rod BC:



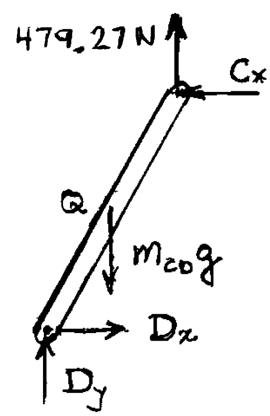
$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad l_{BC}C_y - \frac{1}{2}l_{BC}mg = \Sigma(M_B)_{\text{eff}}$$

$$(0.4\ \text{m})C_y - (0.2\ \text{m})(6\ \text{kg})(9.81\ \text{m/s}^2)$$

$$= (0.2\ \text{m})(180\ \text{N}\sin 30°) + 11.9719\ \text{N}\cdot\text{m} + (0.2\ \text{m})(179.58\ \text{N})$$

$$-(0.2\ \text{m})(1555.2\ \text{N})\sin 60°$$

$$C_y = -479.27\ \text{N}$$



Rod CD: $+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{\text{eff}}: \quad C_x l_{CD}\cos 30° + (479.27\ \text{N})(0.125\ \text{m})$

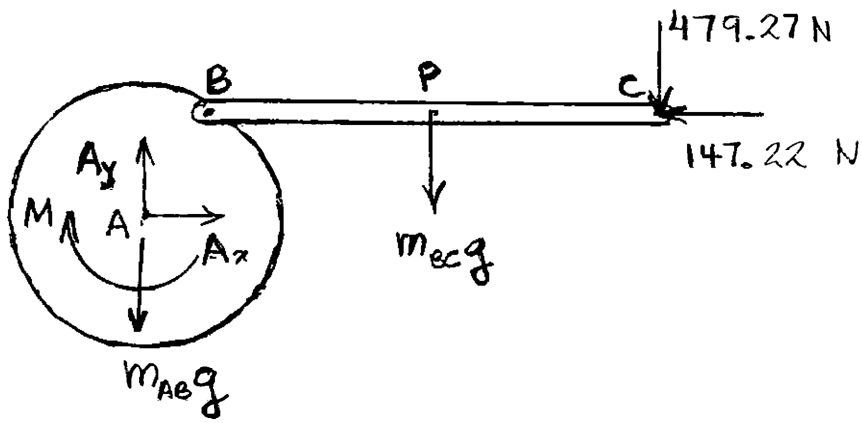
$$-m_{CD}g(0.0625\ \text{m}) = \Sigma(M_D)_{\text{eff}}:$$

$$(0.25\ \text{m})C_x\cos 30° + 59.909\ \text{N}\cdot\text{m} - (5\ \text{kg})(9.81\ \text{m/s}^2)(0.0625\ \text{m})$$

$$= 6.24268\ \text{N}\cdot\text{m} + (0.125\ \text{m})(149.82\ \text{N})$$

$$C_x = -147.22\ \text{N}$$

Disk *AB* and rod *BC*:

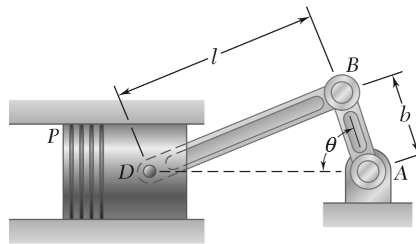


$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad -M - (479.27)(0.4 + 0.2\sin 30°) + (147.22)(0.2\cos 30°)$$
$$- (6\text{ kg})(9.81\text{ m/s}^2)(0.2 + 0.2\sin 30°)$$
$$= (180)(0.2 + 0.2\sin 30°) + 11.9719 + (179.58)(0.2 + 0.2\sin 30°)$$
$$- (1555.2\cos 30°)(0.2)$$
$$-M - 239.635 + 25.50 - 17.658$$
$$= 54 + 11.9719 + 53.874 - 269.369$$
$$-M = 82.27\text{ N}\cdot\text{m}$$

(*a*) *Couple applied to disk A.* $\mathbf{M} = 82.3\text{ N}\cdot\text{m} \circlearrowleft$ ◀

(*b*) *Components of force exerted at C* on rod *BC*.

$\mathbf{C} = 147.2\text{ N} \leftarrow + 479\text{ N} \downarrow$ ◀

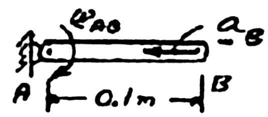


PROBLEM 16.137

In the engine system shown $l = 250$ mm and $b = 100$ mm. The connecting rod BD is assumed to be a 1.2-kg uniform slender rod and is attached to the 1.8-kg piston P. During a test of the system, crank AB is made to rotate with a constant angular velocity of 600 rpm clockwise with no force applied to the face of the piston. Determine the forces exerted on the connecting rod at B and D when $\theta = 180°$. (Neglect the effect of the weight of the rod.)

SOLUTION

Kinematics: Crank AB:



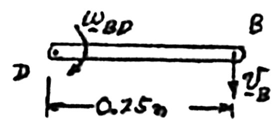
$$\omega_{AB} = 600 \text{ rpm}\left(\frac{2\pi}{60}\right) = 62.832 \text{ rad/s} \circlearrowright$$

$$a_B = (AB)\omega_{AB}^2 = (0.1 \text{ m})(62.832 \text{ rad/s}^2)$$

$$\mathbf{a}_B = 394.78 \text{ m/s}^2 \longleftarrow$$

Also: $$v_B = (AB)\omega_{AB} = (0.1 \text{ m})(62.832 \text{ rad/s}) = 6.2832 \text{ m/s} \downarrow$$

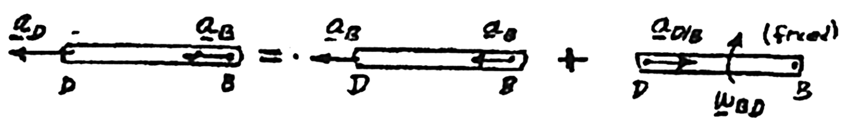
Connecting rod BD:



Velocity Instantaneous center at D.

$$\omega_{BD} = \frac{v_B}{BD} = \frac{6.2832 \text{ m/s}}{0.25 \text{ m}} = 25.133 \text{ rad/s}$$

Acceleration:



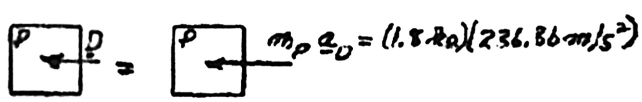
$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} = [a_B \longleftarrow] + [(BD)\omega_{BD}^2 \longrightarrow]$$

$$\mathbf{a}_D = [394.78 \text{ m/s}^2 \longleftarrow] + [(0.25 \text{ m})(25.133 \text{ rad/s})^2 \longrightarrow]$$

$$\mathbf{a}_D = [397.78 \text{ m/s}^2 \longleftarrow] + [157.92 \text{ m/s}^2 \longleftarrow] = 236.86 \text{ m/s}^2 \longleftarrow$$

$$\bar{\mathbf{a}}_{BD} = \frac{1}{2}(\mathbf{a}_B + \mathbf{a}_D) = \frac{1}{2}(394.78 \longleftarrow + 236.86 \longleftarrow) = 315.82 \text{ m/s}^2 \longleftarrow$$

Kinetics of piston

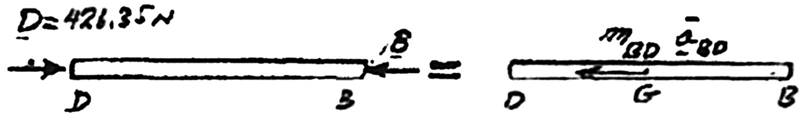


$$\mathbf{D} = 426.35 \text{ N} \longleftarrow$$

Force exerted on connecting rod at D is:

$\mathbf{D} = 426.35 \longrightarrow$

Kinetics of connecting rod: (neglect weight)



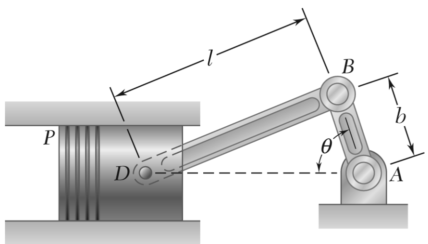
$$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad B - D = m_{BD}\bar{a}_{BD}$$

$$B - 426.34 \text{ N} = (1.2 \text{ kg})(315.82 \text{ m/s}^2)$$
$$B = 426.35 \text{ N} + 378.48 \text{ N}$$
$$= 805.33 \text{ N}$$

Forces exerted on connecting rod. $\mathbf{B} = 805 \text{ N} \longleftarrow$ ◀

$\mathbf{D} = 426 \text{ N} \longrightarrow$ ◀

PROBLEM 16.138



Solve Problem 16.137 when $\theta = 90°$.

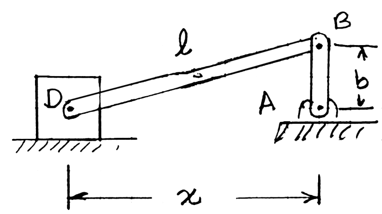
PROBLEM 16.137 In the engine system shown $l = 250$ mm and $b = 100$ mm. The connecting rod BD is assumed to be a 1.2-kg uniform slender rod and is attached to the 1.8-kg piston P. During a test of the system, crank AB is made to rotate with a constant angular velocity of 600 rpm clockwise with no force applied to the face of the piston. Determine the forces exerted on the connecting rod at B and D when $\theta = 180°$. (Neglect the effect of the weight of the rod.)

SOLUTION

Geometry: $l = 0.25$ m, $b = 0.1$ m, $x = \sqrt{l^2 - b^2} = 0.2291$ m

$$\mathbf{r}_{B/A} = (0.1\text{ m})\mathbf{j}, \quad \mathbf{r}_{D/B} = -(0.2291\text{ m})\mathbf{i} - (0.1\text{ m})\mathbf{j}$$

Kinematics: $\omega_{AB} = 600\text{ rev/min} = 62.832\text{ rad/s}$



Velocity:

$$\boldsymbol{\omega}_{AB} = -(62.832\text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= -(62.832\mathbf{k}) \times (0.1\mathbf{i})$$

$$= (6.2832\text{ m/s})\mathbf{i}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$$

$$v_D\mathbf{i} = 6.2832\mathbf{i} + \omega_{BD}\mathbf{k} \times (-0.2291\mathbf{i} - 0.1\mathbf{j})$$

$$= 6.2832\mathbf{i} + 0.1\omega_{BD}\mathbf{j} - 0.2291\omega_{BD}\mathbf{i}$$

Equating components gives

$$v_D = 6.2832\text{ m/s}, \quad \omega_{BD} = 0.$$

Acceleration:

$$\boldsymbol{\alpha}_{AB} = 0, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k} \quad \mathbf{a}_D = a_D\mathbf{i}$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = 0 - (62.832)^2(0.1\mathbf{j}) = -(394.78\text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2\mathbf{r}_{D/B}$$

$$a_D\mathbf{i} = -394.78\mathbf{j} + \alpha_{AB}\mathbf{k} \times (-0.2291\mathbf{i} - 0.1\mathbf{j}) - 0$$

$$= -394.78\mathbf{j} - 0.2291\alpha_{AB}\mathbf{j} + 0.1\alpha_{AB}\mathbf{i}$$

Equate like components.

$$\mathbf{j}: \quad 0 = -394.78 - 0.2291\alpha_{AB} \quad \alpha_{AB} = -1723\text{ rad/s}^2$$

$$\mathbf{i}: \quad a_D = (0.1)(-1723) = -172.3\text{ m/s}^2$$

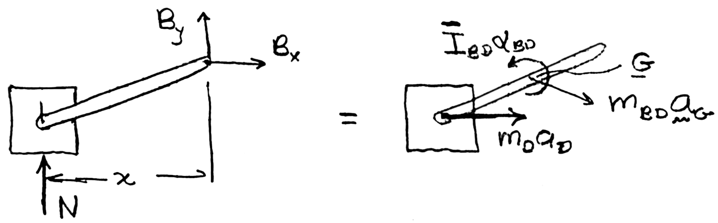
$$\boldsymbol{\alpha}_{AB} = -(1723\text{ rad/s}^2)\mathbf{k} \qquad \mathbf{a}_D = -(172.3\text{ m/s}^2)\mathbf{i}$$

Acceleration of mass center G of bar BD

$$\mathbf{r}_{G/D} = \frac{1}{2}(0.2291\mathbf{i} + 0.1\mathbf{j})$$
$$= (0.11455 \text{ m})\mathbf{i} + (0.05 \text{ m})\mathbf{j}$$
$$\mathbf{a}_G = \mathbf{a}_D + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{G/D} - \omega_{BD}^2 \mathbf{r}_{G/D}$$
$$= -172.3\mathbf{i} + (-1723\mathbf{k}) \times (0.11455\mathbf{i} + 0.05\mathbf{j}) - 0$$
$$= -172.3\mathbf{i} - 197.34\mathbf{j} + 86.15\mathbf{i}$$
$$= -(86.15 \text{ m/s}^2)\mathbf{i} - (197.34 \text{ m/s}^2)\mathbf{j}$$

Force on bar BD at P in B.

Use piston + bar BD as a free body



$$m_D \mathbf{a}_D = (1.8)(-172.3\mathbf{i})$$
$$= -(310.14 \text{ N})\mathbf{i}$$
$$m_{BD}\mathbf{a}_G = (1.2)(-86.15\mathbf{i} - 197.34\mathbf{j})$$
$$= -(103.38 \text{ N})\mathbf{i} - (236.88 \text{ N})\mathbf{j}$$
$$\bar{I}_{BD}\alpha_{BD} = \frac{1}{12}(1.2)(0.25)^2(-1723) = -10.769 \text{ N}\cdot\text{m}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad B_x = m_D a_D + (m_{BD} a_G)_x$$
$$B_x = -310.14 - 103.38 \qquad B_x = -413.52 \text{ N}$$

$$+\Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad -xN\mathbf{k} = \bar{I}_{BD}\alpha_{BD}\mathbf{k} + \mathbf{r}_{D/B} \times (m_D \mathbf{a}_D) + \mathbf{r}_{G/B} \times (m_{BD}\mathbf{a}_G)$$
$$-0.2291\,N\mathbf{k} = -10.769\mathbf{k} + (0.1)(-310.14)\mathbf{k} + (-0.11455\mathbf{i} - 0.05\mathbf{j}) \times (-103.38\mathbf{i} - 236.38\mathbf{j})$$
$$= -10.769\mathbf{k} - 31.014\mathbf{k} + 27.038\mathbf{k} - 5.169\mathbf{k}$$
$$N = 86.923 \text{ N}$$

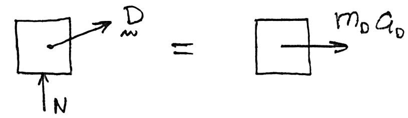
$$+\Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N + B_y = (m_{BD} a_G)_y$$
$$86.923 + B_y = -236.88 \qquad B_y = -323.80 \text{ N}$$
$$B = \sqrt{413.52^2 + 323.80^2} = 525.2 \text{ N}$$
$$\tan\beta = \frac{323.80}{413.52} \qquad \beta = 38.1°$$

$\mathbf{B} = 525 \text{ N} \measuredangle 38.1°$ ◀

Force exerted by bar *BD* as piston *D*.

Use piston *D* as a free body

$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}}$$



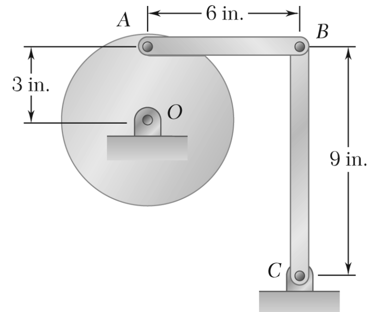
$$\mathbf{D} + N\mathbf{j} = m_D a_D \mathbf{i}$$

$$\mathbf{D} = m_D a_D \mathbf{i} - N\mathbf{j}$$

$$= -(310.14 \text{ N})\mathbf{i} + (86.923 \text{ N})\mathbf{j}$$

Force exerted by the piston on bar *BD*. By Newton's third law,

$$\mathbf{D}' = -\mathbf{D} = (310.14 \text{ N})\mathbf{i} - (86.923 \text{ N})\mathbf{j} \qquad \mathbf{D}' = 322 \text{ N} \searrow 15.7° \blacktriangleleft$$



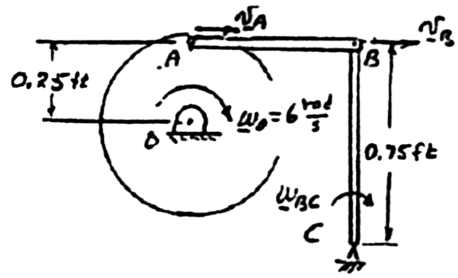
PROBLEM 16.139

The 4-lb rod AB and the 6-lb rod BC are connected as shown to a disk that is made to rotate in a vertical plane at a constant angular velocity of 6 rad/s clockwise. For the position shown, determine the forces exerted at A and B on rod AB.

SOLUTION

<u>Kinematics</u>:

<u>Velocity</u>



$$\omega_{AB} = 0$$

$$v_B = v_A$$

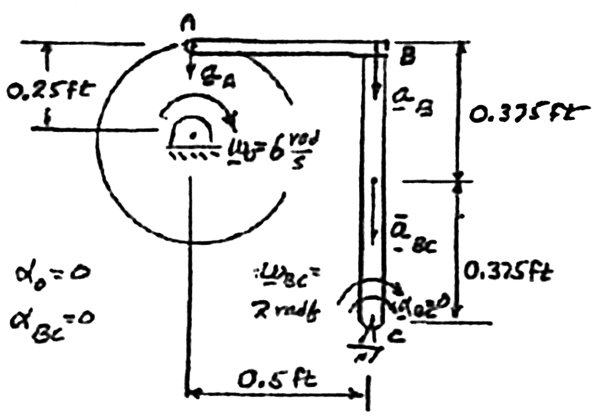
$$= (0.25 \text{ ft})(6 \text{ rad/s})$$

$$= 1.5 \text{ ft/s}^2$$

$$\omega_{BC} = \frac{v_B}{0.75 \text{ ft}} = \frac{1.5 \text{ m/s}^2}{0.75 \text{ ft}}$$

$$\boldsymbol{\omega}_{BC} = 2 \text{ rad/s} \circlearrowright$$

<u>Acceleration</u>



$$a_A = (0.25 \text{ ft})(6 \text{ rad/s})^2$$

$$\mathbf{a}_A = 9 \text{ ft/s}^2 \downarrow$$

$$\mathbf{a}_B = (0.75 \text{ ft})(2 \text{ rad/s})^2 = 3 \text{ ft/s}^2 \downarrow$$

$$\bar{a}_{BC} = (0.375 \text{ ft})(2 \text{ rad/s})^2$$

$$\bar{a}_{BC} = 1.5 \text{ ft/s}^2$$

PROBLEM 16.139 (Continued)

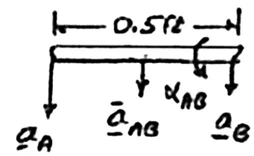
$$\bar{a}_{AB} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(9+3)$$

$$\bar{\mathbf{a}}_{AB} = 6 \text{ ft/s}^2 \downarrow$$

$$a_A = a_B + (0.5 \text{ ft})\alpha_{AB}$$

$$9 \text{ ft/s}^2 = 3 \text{ ft/s}^2 + (0.5 \text{ ft})\alpha_{AB}$$

$$\boldsymbol{\alpha}_{AB} = 12 \text{ rad/s}^2 \circlearrowleft$$



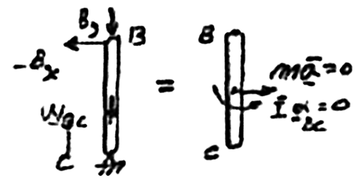
<u>Kinetics</u>:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB}(AB)^2 = \frac{1}{12}\frac{4 \text{ lb}}{32.2}(0.5 \text{ ft})^2$$

$$\bar{I}_{AB} = 2.588 \times 10^3 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

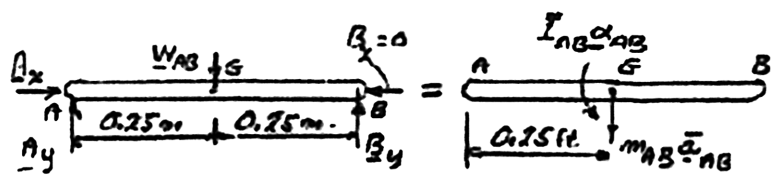
Rod BC:

Since $\alpha_{BC} = 0, \quad \bar{a} = 0$



$$\Sigma M_C = 0 \quad \text{yields} \quad B_x = 0$$

Rod AB:



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad B_y(0.5 \text{ ft}) - W_{AB}(0.25 \text{ ft}) = \bar{I}_{AB}\alpha_{AB} - m_{AB}\bar{a}_{AB}(0.25 \text{ ft})$$

$$0.5B_y - (4 \text{ lb})(0.25 \text{ ft}) = (2.588 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(12 \text{ rad/s})$$

$$-\frac{4 \text{ lb}}{32.2}(6 \text{ ft/s}^2)(0.25 \text{ ft})$$

$$0.5B_y - 1 = 0.03106 - 0.1863$$

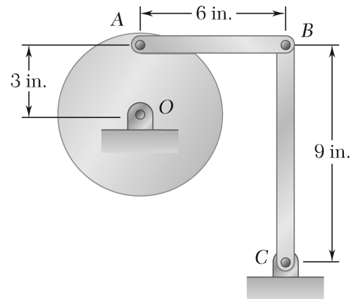
$$0.5B_y = 0.8447$$

$$B_y = 1.689 \text{ lb} \qquad \mathbf{B} = 1.689 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A_y - W_{AB} + B_y = -m_{AB}\bar{a}_{AB}$$

$$A_y - 4 \text{ lb} + 1.689 \text{ lb} = -\frac{4 \text{ lb}}{32.2}(6 \text{ ft/s}^2)$$

$$A_y = 1.565 \text{ lb} \qquad \mathbf{A} = 1.565 \text{ lb} \uparrow \blacktriangleleft$$



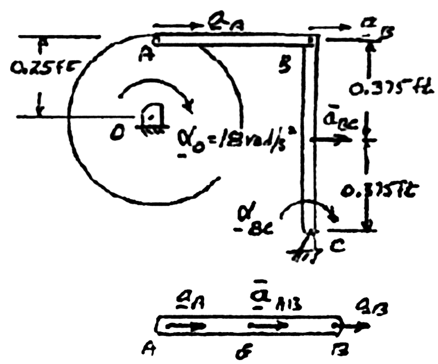
PROBLEM 16.140

The 4-lb rod AB and the 6-lb rod BC are connected as shown to a disk that is made to rotate in a vertical plane. Knowing that at the instant shown the disk has an angular acceleration of 18 rad/s^2 clockwise and no angular velocity, determine the components of the forces exerted at A and B on rod AB.

SOLUTION

<u>Kinematics</u>: Velocity of all elements $= C$

Acceleration:



$$\mathbf{a}_B = \mathbf{a}_A = (0.25\text{ ft})(18\text{ rad/s}^2) = 4.5\text{ ft/s}^2 \longrightarrow$$

$$\alpha_{BC} = \frac{a_B}{0.75\text{ ft}} = \frac{4.5\text{ ft/s}^2}{0.75\text{ ft}}$$

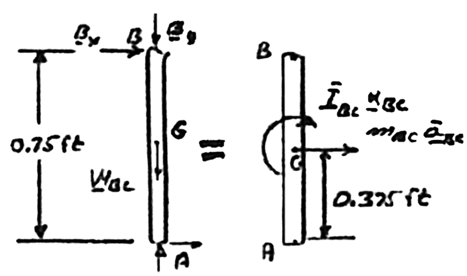
$$\boldsymbol{\alpha}_{BC} = 6\text{ rad/s}^2 \circlearrowright$$

$$\bar{\mathbf{a}}_{BC} = (0.375\text{ ft})(6\text{ rad/s}^2) = 2.25\text{ ft/s}^2 \longrightarrow$$

$$\bar{\mathbf{a}}_{AB} = \mathbf{a}_A = \mathbf{a}_B = 4.5\text{ ft/s}^2 \longrightarrow$$

<u>Kinetics</u>: $\quad \bar{I}_{BC} = \frac{1}{12} m_{BC}(BC)^2 = \frac{1}{12}\frac{6\text{ lb}}{32.2}(0.75\text{ ft})^2$

Rod BC: $\quad \bar{I}_{BC} = 8.734 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad B_x(0.75\text{ ft}) = \bar{I}_{BC}\alpha_{BC} + m_{BC}\bar{a}_{BC}(0.375\text{ ft})$$

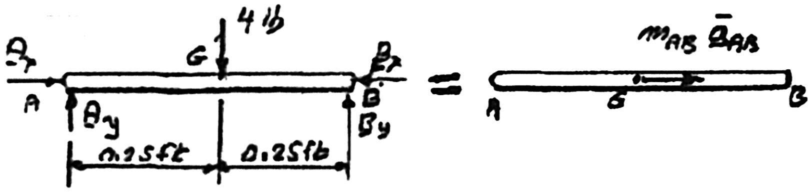
$$0.75B_y = (8.734 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2)(6\text{ rad/s}^2) + \left(\frac{6\text{ lb}}{32.2}\right)(2.25\text{ ft/s}^2)(0.375\text{ ft})$$

$$0.75B_x = 0.0524 + 0.1572$$

$$B_x = 0.2795\text{ lb}$$

$$(\text{on } AB) \quad \mathbf{B}_x = 0.280\text{ lb} \longleftarrow$$

Rod AB:



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad A_x - B_x = m_{AB}\bar{a}_{AB}$$

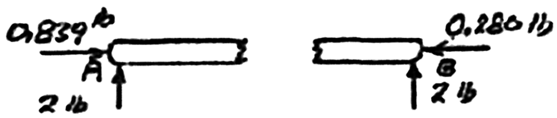
$$A_x - 0.2795 \text{ lb} = \left(\frac{4 \text{ lb}}{32.2}\right)(4.5 \text{ ft/s}^2)$$

$$A_x - 0.2795 \text{ lb} = 0.5590 \text{ lb}$$

$$A_x = 0.8385 \text{ lb} \qquad \mathbf{A}_x = 0.839 \text{ lb} \longrightarrow$$

$$\Sigma M_A: \quad \mathbf{B}_y = 2 \text{ lb} \uparrow$$

$$\Sigma M_B: \quad \mathbf{A}_y = 2 \text{ lb} \uparrow$$



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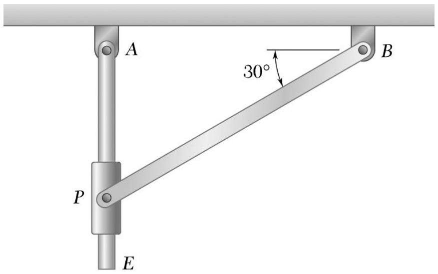


PROBLEM 16.141

Two rotating rods in the vertical plane are connected by a slider block *P* of negligible mass. The rod attached at *A* has a weight of 1.6 lb and a length of 8 in. Rod *BP* weighs 2 lb and is 10 in. long and the friction between block *P* and *AE* is negligible. The motion of the system is controlled by a couple **M** applied to rod *BP*. Knowing that rod *BP* has a constant angular velocity of 20 rad/s clockwise, determine (*a*) the couple **M**, (*b*) the components of the force exerted on *AE* by block *P*.

SOLUTION

Unit vectors: $\mathbf{i}=1\longrightarrow,\ \mathbf{j}=1\uparrow,\ \mathbf{k}=1\circlearrowleft.$

Geometry:

$$\mathbf{r}_{P/A}=-\left(\frac{10}{12}\sin 30°\text{ ft}\right)\mathbf{j}$$

$$\mathbf{r}_{P/B}=-\left(\frac{10}{12}\cos 30°\text{ ft}\right)\mathbf{i}-\left(\frac{10}{12}\sin 30°\text{ ft}\right)\mathbf{j}$$

$$\mathbf{r}_{P/A}=-\left(\frac{8}{12}\text{ ft}\right)\mathbf{j}$$

<u>Kinematics</u>: $\boldsymbol{\omega}_{BP}=-(20\text{ rad/s})\mathbf{k},\qquad \boldsymbol{\alpha}_{BP}=0$

Velocity analysis.

Rod *BP*:

$$\mathbf{v}_P=\boldsymbol{\omega}_{BP}\times\mathbf{r}_{P/B}$$

$$=-20\mathbf{k}\times\left(-\frac{10}{12}\cos 30°\mathbf{i}-\frac{10}{12}\sin 30°\mathbf{j}\right)$$

$$=-(8.3333\text{ ft/s})\mathbf{i}+(14.4338\text{ ft/s})\mathbf{j}$$

Rod *AE*: Use a frame of reference rotating with angular velocity $\boldsymbol{\omega}_{AE}=\omega_{AE}\mathbf{k}$. The collar *P* slides on the rod with relative velocity $\mathbf{v}_{P/A}=u\mathbf{j}$

$$\mathbf{v}_P=\mathbf{v}_{P'}+\mathbf{v}_{P/AE}=\boldsymbol{\omega}_{AE}\times\mathbf{r}_{P/A}+u\,\mathbf{j}$$

$$=\omega_{AE}\mathbf{k}\times\left(-\frac{10}{12}\sin 30°\mathbf{j}\right)+u\,\mathbf{j}=0.41667\,\omega_{AE}\mathbf{i}+u\,\mathbf{j}$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$\mathbf{i}$: $-8.3333=0.41667\,\omega_{AE}$ $\qquad\omega_{AE}=-20\text{ rad/s}$

$\mathbf{j}$: $14.4388=u$ $\qquad u=14.4338\text{ ft/s}$

Acceleration analysis.

Rod *BP*:

$$\mathbf{a}_P=\boldsymbol{\alpha}_{BP}\times\mathbf{r}_{P/B}-\omega_{BP}^2\mathbf{r}_{P/B}$$

$$=0-(20)^2\left(-\frac{10}{12}\cos 30°\mathbf{i}-\frac{10}{12}\sin 30°\mathbf{j}\right)$$

$$=(288.68\text{ ft/s}^2)\mathbf{i}+(166.67\text{ ft/s}^2)\mathbf{j}$$

Rod *AE*: $\boldsymbol{\alpha}_{AE} = \boldsymbol{\alpha}_{AE}\mathbf{k}, \qquad \mathbf{a}_{P/AE} = \dot{u}\,\mathbf{j}$

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AE} + 2\boldsymbol{\omega}\times\mathbf{v}_{P/AE}$$

where

$$\mathbf{a}_{P'} = \boldsymbol{\alpha}_{AB}\times r_{P/A} - \omega_{AE}^2\mathbf{r}_{P/A}$$
$$= \alpha_{AE}\mathbf{k}\times\left(-\frac{10}{12}\sin 30°\mathbf{j}\right) - (20)^2\left(-\frac{10}{12}\sin 30°\mathbf{j}\right)$$
$$= 0.41667\alpha_{AE}\mathbf{i} + (166.67\text{ ft/s}^2)\mathbf{j}$$

and

$$2\boldsymbol{\omega}_{AE}\times\mathbf{v}_{P/AE} = (2)(-20\mathbf{k})\times(14.4338\,\mathbf{j})$$
$$= 577.35\text{ ft/s}^2\mathbf{i}$$

Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

$$\mathbf{i}:\quad 288.68 = 0.41667\alpha_{AE} + 577.35$$

$$\alpha_{AE} = -692.8\text{ rad/s}^2$$

Summary:

$$\boldsymbol{\omega}_{BP} = -(20\text{ rad/s})\mathbf{k}, \quad \boldsymbol{\omega}_{AE} = -(20\text{ rad/s})\mathbf{k}$$
$$\boldsymbol{\alpha}_{BP} = 0, \qquad \boldsymbol{\alpha}_{AE} = -(692.8\text{ rad/s}^2)\mathbf{k}$$

Masses and moments of inertia.

$$m_{AE} = \frac{W_{AE}}{g} = \frac{1.6\text{ lb}}{32.2\text{ ft/s}^2} = 0.049689\text{ lb}\cdot\text{s}^2\text{/ft}$$
$$m_{BP} = \frac{W_{BP}}{g} = \frac{2\text{ lb}}{32.2\text{ ft/s}^2} = 0.062112\text{ lb}\cdot\text{s}^2\text{/ft}$$
$$\bar{I}_{AE} = \frac{1}{12}m_{AE}l_{AE}^2 = \frac{1}{12}(0.049689\text{ lb}\cdot\text{s}^2\text{/ft})\left(\frac{8}{12}\text{ ft}\right)^2 = 1.8403\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$
$$\bar{I}_{BP} = \frac{1}{12}m_{BP}l_{BP}^2 = \frac{1}{12}(0.062112\text{ lb}\cdot\text{s}^2\text{/ft})\left(\frac{10}{12}\text{ ft}\right)^2 = 3.5944\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Mass centers: Let Point *G* be the mass center of rod *AE* and Point *H* be that of rod *BP*.

$$\mathbf{r}_{G/A} = -\left(\frac{4}{12}\text{ ft}\right)\mathbf{j}$$
$$\mathbf{r}_{H/B} = -\left(\frac{10}{12}\cos 30°\text{ ft}\right)\mathbf{i} - \left(\frac{10}{12}\sin 30°\text{ ft}\right)\mathbf{j}$$

Acceleration of mass centers.

$$\mathbf{a}_G = \boldsymbol{\alpha}_{AE}\times\mathbf{r}_{G/A} - \omega_{AE}^2\mathbf{r}_{G/A}$$
$$= (-692.8\mathbf{k})\times(-0.33333\mathbf{j}) - (20)^2(-0.33333\mathbf{j}) = -(230.93\text{ ft/s}^2)\mathbf{i} + (133.33\text{ ft/s}^2)\mathbf{j}$$
$$\mathbf{a}_H = \boldsymbol{\alpha}_{BP}\times\mathbf{r}_{H/B} - \omega_{BP}^2\mathbf{r}_{H/B}$$
$$= 0 - (20)^2(-0.72169\mathbf{i} - 0.41667\mathbf{j}) = (288.68\text{ ft/s}^2)\mathbf{i} + (166.67\text{ ft/s}^2)\mathbf{j}$$

Effective forces at mass centers.

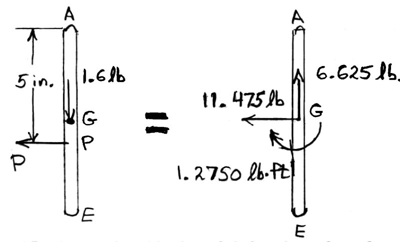
Rod *AE*: $m_{AE}\mathbf{a}_G = (0.049689)(-230.93\mathbf{i} + 133.33\mathbf{j}) = -(11.475\text{ lb})\mathbf{i} + (6.625\text{ lb})\,\mathbf{j}$

Rod *BP*: $m_{BP}\mathbf{a}_H = (0.062112)(288.68\mathbf{i} + 166.67\mathbf{j}) = (17.930\text{ lb})\mathbf{i} + (10.352\text{ lb})\,\mathbf{j}$

Effective couples at mass centers.

Rod *AE*: $\bar{I}_{AE}\boldsymbol{\alpha}_{AE} = (1.8403\times 10^{-3})(-692.8\mathbf{k}) = -(1.2750\text{ lb}\cdot\text{ft})\,\mathbf{k}$

Rod *BP*: $\bar{I}_{BP}\boldsymbol{\alpha}_{BP} = 0$



Kinetics.

Rod *AE*: $\Sigma\mathbf{M}_A = \Sigma(\mathbf{M}_A)_{\text{eff}}: \quad \mathbf{r}_{P/A}\times(-P\mathbf{i}) = \mathbf{r}_{G/A}\times(m_{AE}\mathbf{a}_G) + \bar{I}_{AE}\boldsymbol{\alpha}_{AE}$

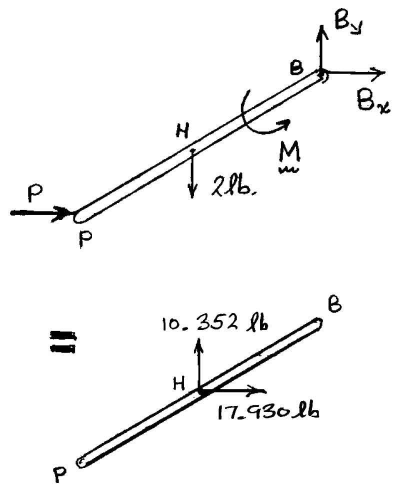
$$\left(-\frac{5}{12}\mathbf{j}\right)\times(-P\mathbf{i}) = \left(-\frac{4}{12}\mathbf{j}\right)\times(-11.475\mathbf{i} + 16.625\mathbf{j}) - 1.2750\mathbf{k}$$

$$-\frac{5}{12}P\mathbf{k} = -3.8249\mathbf{k} - 1.2750\mathbf{k} \qquad P = 12.240\text{ lb}$$

Rod *BP*: $\Sigma\mathbf{M}_B = \Sigma(\mathbf{M}_B)_{\text{eff:}} \quad \mathbf{r}_{P/B}\times P\mathbf{i} + \mathbf{r}_{H/B}\times(-W_{BP}\mathbf{j}) + M\,\mathbf{k} = \mathbf{r}_{H/B}\times(m_{BP}\mathbf{a}_H) + \bar{I}_{BP}\boldsymbol{\alpha}_{BP}$

$$\frac{5}{12}P\mathbf{k} + \left(\frac{5\cos 30^\circ}{12}\text{ ft}\right)(2\text{ lb})\mathbf{k} + M\,\mathbf{k}$$

$$= (-0.36084\mathbf{i} - 0.20833\mathbf{j})\times(17.930\mathbf{i} + 10.352\mathbf{j}) + 0$$



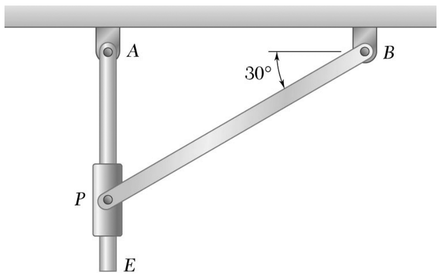
$$5.1\mathbf{k} + 0.72169\mathbf{k} + M\,\mathbf{k} = -3.7354\mathbf{k} + 3.7354\mathbf{k}$$

$$M = -5.82\text{ lb}\cdot\text{ft}$$

(*a*) *Couple* **M**. $\mathbf{M} = 5.82\text{ lb}\cdot\text{ft}$ ◀

(*b*) *Force exerted on AE by block P*. $\mathbf{P} = 12.24\text{ lb} \longleftarrow$ ◀

PROBLEM 16.142



Two rotating rods in the vertical plane are connected by a slider block P of negligible mass. The rod attached at A has a mass of 0.8 kg and a length of 160 mm. Rod BP has a mass of 1 kg and is 200 mm long and the friction between block P and AE is negligible. The motion of the system is controlled by a couple $\mathbf{M}$ applied to bar AE. Knowing that at the instant shown rod BP has an angular velocity of 20 rad/s clockwise and an angular acceleration of 80 rad/s^2 clockwise, determine (*a*) the couple $\mathbf{M}$, (*b*) the components of the force exerted on AE by block P.

SOLUTION

Geometry: $\mathbf{r}_{P/A} = (0.200 \text{ m})\sin 30° \downarrow = 0.100 \text{ m} \downarrow$

$\mathbf{r}_{P/B} = 0.200 \text{ m} \measuredangle 30°$

$\mathbf{r}_{E/A} = 0.160 \text{ m} \downarrow$

Kinematics: $\boldsymbol{\omega}_{BP} = 20 \text{ rad/s} \circlearrowright$

$\boldsymbol{\alpha}_{BP} = 80 \text{ rad/s}^2 \circlearrowright$

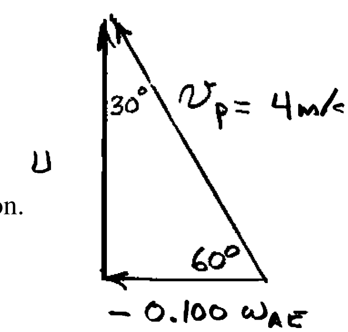
Velocity analysis.

Rod BP: $\mathbf{v}_P = \boldsymbol{\omega}_{BP} \times \mathbf{r}_{P/B} = (20 \text{ rad/s}) \cdot (0.200 \text{ m}) \searrow 60° = 4 \text{ m/s} \searrow 60°$

Rod AE: Use a frame of reference rotating with angular velocity $\boldsymbol{\omega}_{AE} = \omega_{AE} \circlearrowleft$. The collar slides on the rod with relative velocity $\mathbf{v}_{P/AE} = u \uparrow$.

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AE} = \omega_{AE} \times \mathbf{r}_{P/A} + u \uparrow$$

$$= 0.100\, \omega_{AE} \longrightarrow + u \uparrow$$



Equate the two expressions for $\mathbf{v}_P$ using a triangle construction for vector addition.

$$-0.100\omega_{AE} = 4\cos 60°$$

$$\omega_{AE} = -20 \text{ rad/s} \qquad \boldsymbol{\omega}_{AE} = 20 \text{ rad/s} \circlearrowright$$

$$u = 4\sin 60° = 3.461 \text{ m/s} \qquad \mathbf{v}_{P/AE} = 3.4641 \text{ m/s} \uparrow$$

Acceleration analysis. $\boldsymbol{\alpha}_{BP} = \alpha_{BP} \circlearrowleft$

Rod BP:

$$\mathbf{a}_P = \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{P/B} - \omega_{BP}^2 \mathbf{r}_{P/B}$$

$$= [(80 \text{ rad/s}^2)(0.200 \text{ m}) \searrow 60°] + [(20 \text{ rad/s})^2 (0.200 \text{ m}) \measuredangle 30°]$$

$$= [16 \text{ m/s}^2 \searrow 60°] + [80 \text{ m/s}^2 \measuredangle 30°]$$

Rod *AE*: $\boldsymbol{\alpha}_{AE} = \alpha_{AE}$ ↻, $\mathbf{a}_{P/AE} = \dot{u}\uparrow$

$$\boldsymbol{\alpha}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AE} + 2\boldsymbol{\omega}_{AE} \times \mathbf{v}_{P/AE}$$

where $\mathbf{a}_{P'} = \boldsymbol{\alpha}_{AE} \times \mathbf{r}_{P/A} - \omega_{AE}^2 \mathbf{r}_{P/A}$

$$= [(0.100 \text{ m})\alpha_{AE} \longrightarrow] + [(20 \text{ rad/s}^2)(0.100 \text{ m})\uparrow]$$

$$= [0.100\alpha_{AE} \longrightarrow] + [40 \text{ m/s}^2\uparrow]$$

and $2\boldsymbol{\omega}_{AE} \times \mathbf{v}_{P/AE} = [(2)(20 \text{ rad/s})(3.4641 \text{ m/s}) \longrightarrow] = 138.564 \text{ m/s}^2 \longrightarrow$

Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

$$\longrightarrow: -(16 \text{ m/s}^2)\cos 60° + (80 \text{ m/s}^2)\cos 30°$$

$$= 0.100\alpha_{AE} + 138.564 \text{ m/s}^2$$

$$\alpha_{AE} = -772.82 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{AE} = 772.82 \text{ rad/s}^2 \circlearrowright$$

Masses, weights, and moments of inertia.

$$m_{AE} = 0.8 \text{ kg} \quad W_{AE} = m_{AE}g = (0.8 \text{ kg})(9.81 \text{ m/s}^2) = 7.848 \text{ N}$$

$$m_{BP} = 1.0 \text{ kg} \quad W_{BP} = m_{BP}g = (1.0 \text{ kg})(9.81 \text{ m/s}^2) = 9.81 \text{ N}$$

$$\bar{I}_{AE} = \frac{1}{12}m_{AE}l_{AE}^2 = \frac{1}{12}(0.8 \text{ kg})(0.16 \text{ m})^2 = 1.70667 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{BP} = \frac{1}{12}m_{BP}l_{BP}^2 = \frac{1}{12}(1.0 \text{ kg})(0.20 \text{ m})^2 = 3.3333 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

Mass centers: Let Point *G* be the mass center of rod *AE* and Point *H* be that of rod *BP*.

$$\mathbf{r}_{G/A} = 0.08 \text{ m}\downarrow \qquad \mathbf{r}_{H/B} = 0.10 \text{ m} \measuredangle 30°$$

Accelerations of mass centers.

$$\mathbf{a}_G = \boldsymbol{\alpha}_{AE} \times \mathbf{r}_{G/A} - \omega_{AE}^2 \mathbf{r}_{G/A}$$

$$= (772.82 \text{ rad/s}^2)(0.08 \text{ m}) \longleftarrow + (20 \text{ rad/s}^2)(0.08 \text{ m})\uparrow$$

$$= [61.826 \text{ m/s}^2 \longleftarrow] + [32 \text{ m/s}^2\uparrow]$$

$$\mathbf{a}_H = \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{H/A} - \omega_{BP}^2 \mathbf{r}_{H/B}$$

$$= [(80 \text{ rad/s}^2)(0.10 \text{ m}) \searrow 60°] + [(20 \text{ rad/s})^2(0.10 \text{ m}) \measuredangle 30°]$$

$$= [8 \text{ m/s}^2 \searrow 60°] + [40 \text{ m/s}^2 \measuredangle 30°]$$

Effective forces at mass centers.

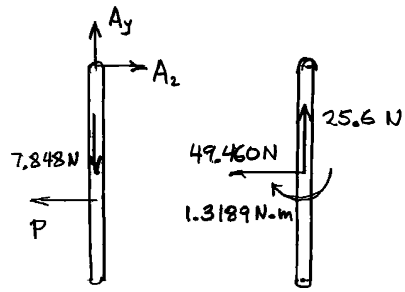
Rod *AE*: $m_{AE}\mathbf{a}_G = [49.460\text{ N} \longleftarrow] + [25.6\text{ N}\uparrow]$

Rod *BP*: $m_{BP}\mathbf{a}_H = [8\text{ N} \searrow 60°] + [40\text{ N} \measuredangle 30°]$

Effective couples at mass centers.

Rod *AE*: $\bar{I}_{AE}\boldsymbol{\alpha}_{AE} = 1.3189\text{ N}\cdot\text{m} \circlearrowright$

Rod *BP*: $\bar{I}_{BP}\boldsymbol{\alpha}_{BP} = 0.2667\text{ N}\cdot\text{m} \circlearrowright$



Kinetics:

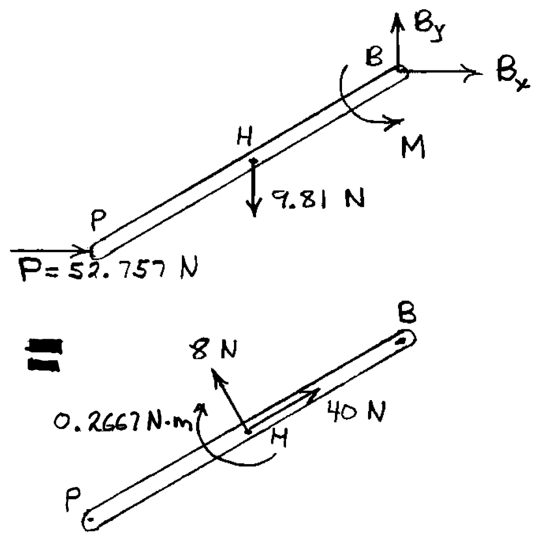
Rod *AE*: $+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}$: $-0.10P = -(0.08)(49.460) - 1.3189$

$$P = 52.757\text{ N}$$

Rod *BP*: $+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}$: $(52.757\text{ N})(0.1\text{ m}) + (9.81\text{ N})(0.086603\text{ m}) + M$

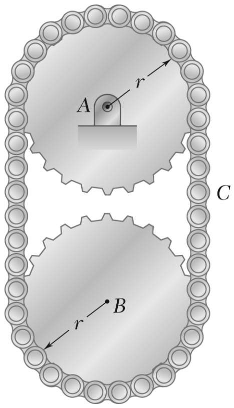
$$= -(8\text{ N})(0.1\text{ m}) - 0.2667\text{ N}\cdot\text{m}$$

$$M = 7.1919\text{ N}\cdot\text{m}$$



(*a*) *Couple* **M**. $M = 7.19\text{ N}\cdot\text{m} \circlearrowright$ ◀

(*b*) *Force exerted on AE by force by block P*. $P = 52.8\text{ N} \longleftarrow$ ◀

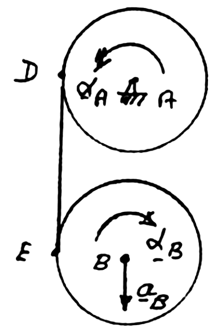


PROBLEM 16.143*

Two disks, each of mass m and radius r are connected as shown by a continuous chain belt of negligible mass. If a pin at Point C of the chain belt is suddenly removed, determine (a) the angular acceleration of each disk, (b) the tension in the left-hand portion of the belt, (c) the acceleration of the center of disk B.

SOLUTION

<u>Kinematics</u>: Assume $\boldsymbol{\alpha}_A \circlearrowleft$ and $\boldsymbol{\alpha}_B \circlearrowright$



$$\omega_A = \omega_B = 0$$

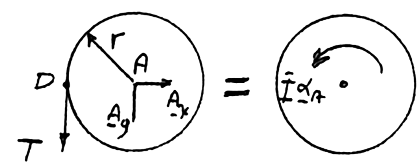
$$\mathbf{a}_D = r\alpha_A \downarrow$$

$$\mathbf{a}_E = a_D = r\alpha_A \downarrow$$

$$\bar{\mathbf{a}}_B = a_E + a_{B/E} = (r\alpha_A + r\alpha_B)$$

$$\bar{\mathbf{a}}_B = r(\alpha_A + \alpha_B)\downarrow$$

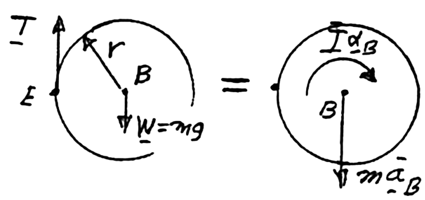
<u>Kinetics</u>: Disk A: $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}$: $Tr = \bar{I}\alpha_A$



$$Tr = \frac{1}{2}mr^2\alpha_A$$

$$\alpha_A = \frac{2T}{mr} \circlearrowleft \qquad (1)$$

Disk B: $+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}$: $Tr = \bar{I}\alpha_B$



$$Tr = \frac{1}{2}mr^2\alpha_B$$

$$\alpha_B = \frac{2T}{mr} \circlearrowright \qquad (2)$$

From Eqs. (1) and (2) we note that $\alpha_A = \alpha_B$

$$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{\text{eff}}: \quad Wr = \bar{I}\alpha_B + (m\bar{a}_B)r$$

$$Wr = \frac{1}{2}mr^2\alpha_B + mr(\alpha_A + \alpha_B)r$$

$$\alpha_A = \alpha_B: \quad Wr = \frac{5}{2}mr^2\alpha_A$$

$$\boldsymbol{\alpha}_A = \frac{2}{5}\frac{g}{r} \circlearrowleft \blacktriangleleft$$

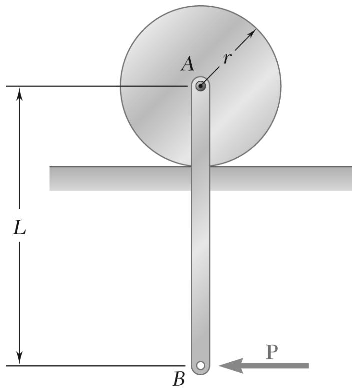
$$\boldsymbol{\alpha}_B = \frac{2}{5}\frac{g}{r} \circlearrowright \blacktriangleleft$$

Substitute for α_A into (1): $$\frac{2}{5}\frac{g}{r} = \frac{2T}{mr}$$

$$T = \frac{1}{5}mg \blacktriangleleft$$

$$a_B = r(\alpha_A + \alpha_B)$$
$$= r(2\alpha_A)$$
$$= 2r\left(\frac{2}{5}\frac{g}{r}\right)$$

$$\mathbf{a}_B = \frac{4}{5}g \downarrow \blacktriangleleft$$



PROBLEM 16.144*

A uniform slender bar *AB* of mass *m* is suspended as shown from a uniform disk of the same mass *m*. Neglecting the effect of friction, determine the accelerations of Points *A* and *B* immediately after a horizontal force **P** has been applied at *B*.

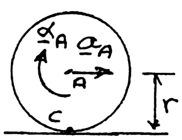
SOLUTION

<u>Kinematics</u>:

$$\omega = 0$$

Cylinder:

Rolling without sliding $\quad (a_C)_x = 0$

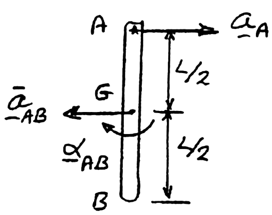


$$\xrightarrow{+} \mathbf{a}_A = (\mathbf{a}_C)_x + \mathbf{a}_{A/C} = 0 + r\alpha_A$$

$$\mathbf{a}_A = r\alpha_A \longrightarrow$$

$$\boldsymbol{\alpha}_A = \frac{a_A}{r}$$

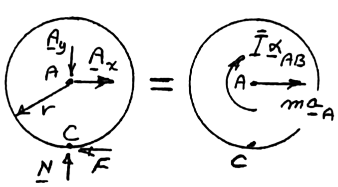
Rod *AB*:



$$\xrightarrow{+} a_A = \frac{L}{2}\alpha_{AB} - \bar{a}_{AB}$$

<u>Kinetics</u>:

Cylinder:



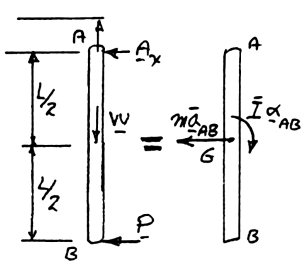
$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad A_x r = m a_A r + \bar{I}\alpha_A$$

$$A_x r = m a_A r + \frac{1}{2} m r^2 \left(\frac{a_A}{r}\right)$$

$$A_x = \frac{3}{2} m a_A$$

$$A_x = \frac{3}{2} m \left(\frac{L}{2}\alpha_{AB} - \bar{a}_{AB}\right) \qquad (1)$$

Rod *AB*:



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad PL = m\bar{a}_{AB}\frac{L}{2} + \bar{I}\alpha_{AB}$$

$$PL = m\bar{a}_{AB}\frac{L}{2} + \frac{m}{12}L^2\alpha_{AB} \qquad (2)$$

$$\xleftarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P + A_x = m\bar{a}_{AB}$$

Substitute from (1):

$$P + \frac{3}{2}m\left(\frac{L}{2}\alpha_{AB} - \bar{a}_{AB}\right) = m\bar{a}_{AB}$$

$$P = \frac{5}{2}m\bar{a}_{AB} - \frac{3}{4}mL\alpha_{AB} \qquad (3)$$

Multiply by $\frac{L}{9}$:

$$\frac{1}{9}PL = \frac{5L}{18}m\bar{a}_{AB} - \frac{1}{12}mL^2\alpha_{AB} \qquad (4)$$

(4) + (2):

$$\frac{10}{9}PL = \left(\frac{1}{2} + \frac{5}{18}\right)mL\bar{a}_{AB} = \frac{7}{9}mL\bar{a}_{AB}$$

$$\bar{\mathbf{a}}_{AB} = \frac{10}{7}\frac{P}{m} \longleftarrow \qquad (5)$$

(5) $\longrightarrow$ (3)

$$P = \frac{5}{2}m\left(\frac{10}{7}\frac{P}{m}\right) - \frac{3}{4}mL\alpha_{AB}$$

$$P = \frac{25}{7}P - \frac{3}{4}mL\alpha_{AB}$$

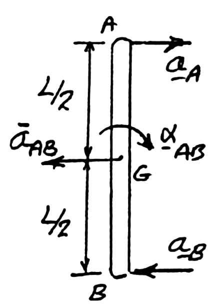
$$-\frac{18}{7}P = \frac{3}{4}mL\alpha_{AB}$$

$$\boldsymbol{\alpha}_{AB} = \frac{24}{7}\frac{P}{mL} \circlearrowright$$

$\xrightarrow{+}$:

$$a_A = \frac{L}{2}\alpha_{AB} - \bar{a}_{AB}$$

$$= \frac{L}{2}\left(\frac{24}{7}\frac{P}{mL}\right) - \frac{10}{7}\frac{P}{m}$$



$$a_A = \left(\frac{12}{7} - \frac{10}{7}\right)\frac{P}{m}$$

$$\mathbf{a}_A = \frac{2}{7}\frac{P}{m} \longrightarrow \blacktriangleleft$$

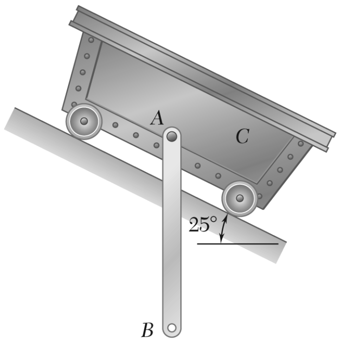
$\xleftarrow{+}$:

$$a_B = \frac{L}{2}\alpha_{AB} + \bar{a}_{AB}$$

$$= \frac{L}{2}\left(\frac{24}{7}\frac{P}{mL}\right) + \frac{10}{7}\frac{P}{m}$$

$$a_B = \left(\frac{12}{7} + \frac{10}{7}\right)\frac{P}{m}$$

$$\mathbf{a}_B = \frac{22}{7}\frac{P}{m} \longleftarrow \blacktriangleleft$$



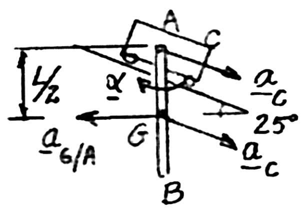
PROBLEM 16.145

A uniform rod AB, of mass 15 kg and length 1 m, is attached to the 20-kg cart C. Neglecting friction, determine immediately after the system has been released from rest, (a) the acceleration of the cart, (b) the angular acceleration of the rod.

SOLUTION

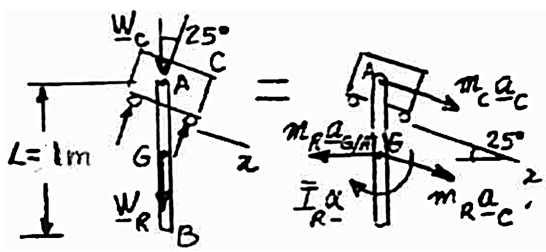
<u>Kinematics</u>: We resolve the acceleration of G into the acceleration of the cart and the acceleration of G relative to A:

$$\bar{\mathbf{a}}_R = \mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$$
$$\bar{\mathbf{a}}_R = \mathbf{a}_C + \mathbf{a}_{G/A}$$



where $$a_{G/A} = \frac{1}{2}L\alpha$$

<u>Kinetics</u>: Cart and rod



$$m_R = 15 \text{ kg}$$
$$m_C = 20 \text{ kg}$$
$$L = 1 \text{ m}$$
$$\bar{I}_R = \frac{1}{12}m_R L^2$$
$$a_{G/A} = \frac{1}{2}(1)\alpha = 0.5\alpha$$

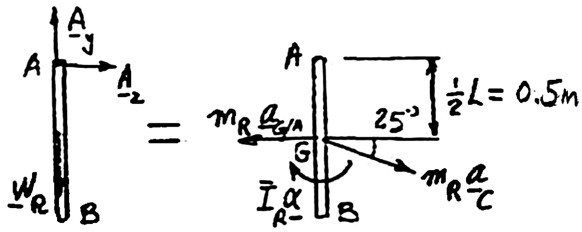
$$\searrow^{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad (m_C + m_R)g\sin 25° = (m_C + m_R)\mathbf{a}_C - m_R a_{G/A}\cos 25°$$

$$g\sin 25° = a_C - \left(\frac{m_g}{m_C + m_R}\right)\left(\frac{L}{2}\alpha\right)\cos 25°$$

$$a_C = (9.81)\sin 25° + \left(\frac{15}{20+15}\right)(0.5\cos 25°)\alpha$$

$$a_C = (9.81)\sin 25° + 0.19421\alpha \qquad (1)$$

Rod



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m_R a_{G/R})\frac{L}{2} - (m_R a_C \cos 25°)\frac{L}{2}$$

$$\frac{1}{12}15(1)^2\alpha + 15(0.5\alpha)(0.5) - (15a_C \cos 25°)(0.5) = 0$$

$$1.25\alpha + 3.75\alpha - (7.5\cos 25°)a_C = 0$$

$$\alpha = (1.50\cos 25°)a_C \tag{2}$$

(*a*) Acceleration of the cart.

Substitute for α from (2) into (1):

$$a_C = (9.81)\sin 25° + 0.19421(1.5\cos 25°)a_C$$

$$a_C = \frac{9.81\sin 25°}{1 - 0.19421(1.5\cos 25°)}$$

$$= 5.6331 \text{ m/s}^2$$

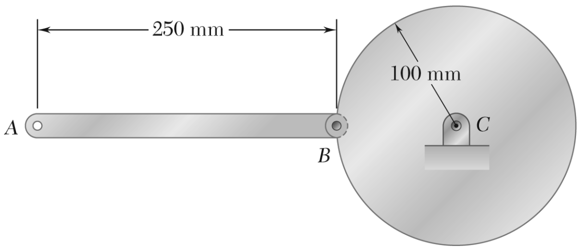
$\mathbf{a}_C = 5.63 \text{ m/s}^2 \searrow 25°$ ◀

(*b*) Angular acceleration.

From (2): $\alpha = (1.50\cos 25°)(5.6331)$

$$= 7.6580 \text{ rad/s}^2$$

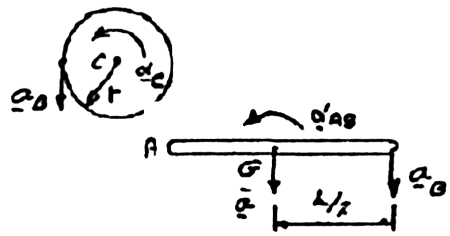
$\boldsymbol{\alpha} = 7.66 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 16.146*

The 5-kg slender rod AB is pin-connected to an 8-kg uniform disk as shown. Immediately after the system is released from rest, determine the acceleration of (a) Point A, (b) Point B.

SOLUTION



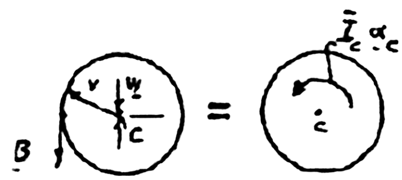
Kinematics:

$$\omega = 0$$

$$\mathbf{a}_B = r\alpha_C \downarrow$$

$$+\downarrow\ \bar{\mathbf{a}} = \mathbf{a}_B + \mathbf{a}_{G/B}$$

$$\bar{\mathbf{a}} = \left(r\alpha_C + \frac{L}{2}\alpha_{AB}\right)\downarrow$$



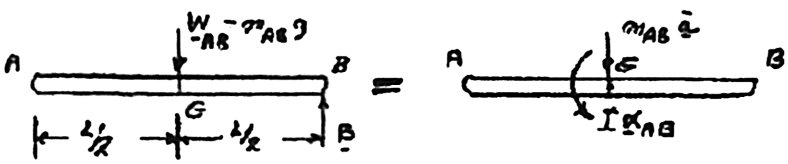
Kinetics: Disk

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad Br = \bar{I}\alpha_C$$

$$Br = \frac{1}{2}m_C r^2 \alpha_C$$

$$B = \frac{1}{2}m_C r\alpha_C$$

Rod AB:



$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad B\frac{L}{2} = \bar{I}\alpha_{AB}$$

$$\left(\frac{1}{2}m_C r\alpha_C\right)\frac{L}{2} = \frac{1}{12}m_{AB}L^2\alpha_{AB}$$

$$\alpha_C = \frac{1}{3}\frac{m_{AB}}{m_C}\cdot\frac{L}{r}\alpha_{AB} \qquad (1)$$

$$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad m_{AB}g - B = m_{AB}\bar{a}$$

$$m_{AB}g - \frac{1}{2}m_C r\alpha_C = m_{AB}\left(r\alpha_C + \frac{L}{2}\alpha_{AB}\right)$$

$$g = \frac{L}{2}\alpha_{AB} + \left(\frac{1}{2}\frac{m_C}{m_{AB}} + 1\right)r\alpha_C$$

$$\frac{g}{L} = \frac{1}{2}\alpha_{AB} + \left(\frac{1}{2}\frac{m_C}{m_{AB}} + 1\right)\frac{r}{L}\cdot\left(\frac{1}{3}\frac{m_{AB}}{m_C}\cdot\frac{L}{r}\right)\alpha_{AB}$$

$$\frac{g}{L} = \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{3}\frac{m_{AB}}{m_C}\right)\alpha_{AB} = \frac{1}{3}\left(2 + \frac{m_{AB}}{m_C}\right)\alpha_{AB}$$

$$\alpha_{AB} = \frac{3g}{L}\frac{1}{\left(2 + \frac{m_{AB}}{m_C}\right)} \qquad (2)$$

$$m_{AB} = 5\text{ kg}, \quad m_C = 8\text{ kg}, \quad r = 0.1\text{ m}, \quad L = 0.25\text{ m}$$

Eq. (1):

$$\boldsymbol{\alpha}_{AB} = \frac{3(9.81)\text{m/s}^2}{0.25\text{ m}}\cdot\frac{1}{2 + \frac{5\text{ kg}}{8\text{ kg}}} = 44.846\text{ rad/s}^2 \circlearrowright$$

Eq. (2):

$$\boldsymbol{\alpha}_C = \frac{1}{3}\frac{5\text{ kg}}{8\text{ kg}}\cdot\frac{0.25\text{ m}}{0.1\text{ m}}\cdot(44.846\text{ rad/s}^2) = 23.357\text{ rad/s}^2 \circlearrowright$$

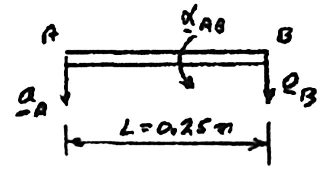
(*b*) Acceleration of *B*.

$$a_B = r\alpha_C$$
$$= (0.1\text{ m})(23.357\text{ rad/s}^2)$$
$$= 2.336\text{ m/s}^2$$

$$\mathbf{a}_B = 2.34\text{ m/s}^2 \downarrow \blacktriangleleft$$

(*a*) Acceleration of *A*.

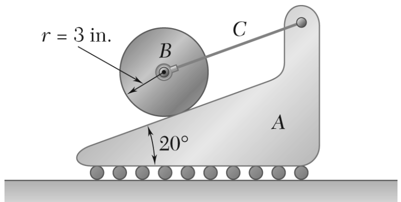
$$+\!\downarrow \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A}$$



$$\mathbf{a}_A = \mathbf{a}_B + [L\alpha_{AB}\downarrow]$$
$$a_A = 2.336\text{ m/s}^2$$
$$+ (0.25\text{ m})(44.846\text{ rad/s}^2)$$
$$a_A = 2.336 + 11.212$$

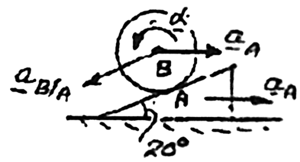
$$\mathbf{a}_A = 13.55\text{ m/s}^2 \downarrow \blacktriangleleft$$

PROBLEM 16.147*



The 6-lb cylinder B and the 4-lb wedge A are held at rest in the position shown by cord C. Assuming that the cylinder rolls without sliding on the wedge and neglecting friction between the wedge and the ground, determine, immediately after cord C has been cut, (*a*) the acceleration of the wedge, (*b*) the angular acceleration of the cylinder.

SOLUTION



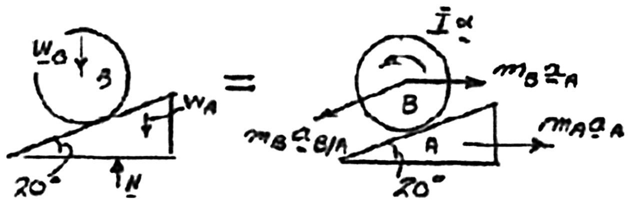
<u>Kinematics</u>: We resolve $\mathbf{a}_B$ into $\mathbf{a}_A$ and a component parallel to the incline

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Where $a_{B/A} = r\alpha,$ since the cylinder rolls on wedge A.

$$a_{B/A} = (0.25 \text{ ft})\alpha$$

<u>Kinetics</u>: Cylinder and wedge

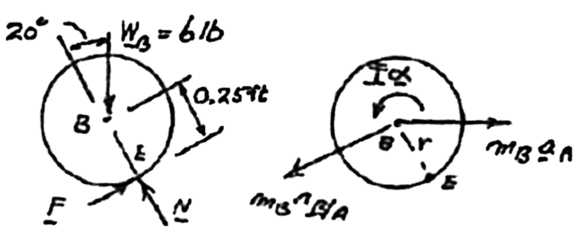


$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad 0 = m_A a_A + m_B a_A - m_B a_{B/A} \cos 20°$$

$$0 = \frac{(4+6)\text{lb}}{g} a_A - \frac{6 \text{ lb}}{g}\left(\frac{3}{12}\text{ft}\right)\alpha \cos 20°$$

$$a_n = (0.15 \cos 20°)\alpha \qquad (1)$$

Cylinder



$$\bar{I} = \frac{1}{2}\frac{W}{g}r^2 = \frac{1}{2}\frac{6 \text{ lb}}{g}(0.25 \text{ ft})^2$$

$$\bar{I} = \frac{3}{16g}$$

(*b*) Angular acceleration of the cylinder.

$$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{\text{eff}}: \quad (6\text{ lb})\sin 20°(0.25\text{ ft}) = \bar{I}\alpha + m_B a_{B/A}(0.25\text{ ft}) - m_B a_A \cos 20°(0.25\text{ ft})$$

$$1.5\sin 20° = \frac{3}{16(32.2)}\alpha + \frac{6\text{ lb}}{32.2}(0.25\alpha)(0.25)$$

$$-\frac{6\text{ lb}}{32.2}a_A \cos 20°(0.25)$$

$$0.51303 = 0.00582\alpha + 0.01165\alpha - 0.04378 a_A$$

Substitute from (1):

$$0.51303 = 0.01747\alpha - 0.04378(0.15\cos 20°)\alpha$$

$$0.51303 = (0.01747 - 0.00617)\alpha$$

$$\alpha = 45.41\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 45.4\text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

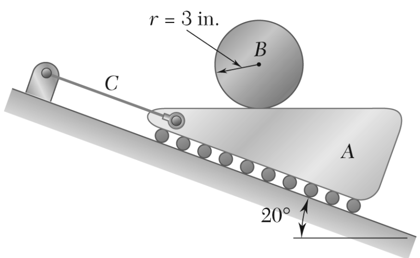
(*a*) Acceleration of the wedge.

Eq. (1):

$$a_A = (0.15\cos 20°)\alpha$$

$$= (0.15\cos 20°)(45.41)$$

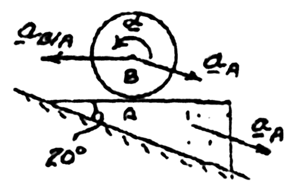
$$a_A = 6.401\text{ ft/s}^2 \qquad \mathbf{a}_A = 6.40\text{ ft/s}^2 \longrightarrow \blacktriangleleft$$



PROBLEM 16.148*

The 6-lb cylinder *B* and the 4-lb wedge *A* are held at rest in the position shown by cord *C*. Assuming that the cylinder rolls without sliding on the wedge and neglecting friction between the wedge and the ground, determine, immediately after cord *C* has been cut, (*a*) the acceleration of the wedge, (*b*) the angular acceleration of the cylinder.

SOLUTION



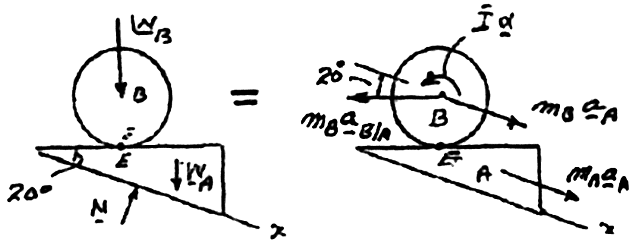
Kinematics: We resolve $\mathbf{a}_B$ into $\mathbf{a}_A$ and a horizontal component $\mathbf{a}_{B/A}$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Where $a_{B/A} = r\alpha$, since the cylinder *B* rolls on wedge *A*.

$$a_{B/A} = (0.25 \text{ ft})\alpha$$

Kinetics: Cylinder and wedge:



$$+\searrow \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad (W_A + W_B)\sin 20° = (m_A + m_B)a_A - m_B a_{B/A}\cos 20°$$

$$(10 \text{ lb})\sin 20° = \left(\frac{10}{g}\right)a_A - \left(\frac{6}{g}\right)(0.25\alpha)\cos 20°$$

$$a_A = g\sin 20° + \frac{6}{10}(0.25)\cos 20°\alpha$$

$$a_A = g\sin 20° + 0.15\cos 20°\alpha \qquad (1)$$

Cylinder: $\quad +\circlearrowright \Sigma M_E = \Sigma(M_E)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m_B a_{B/A})(0.25 \text{ ft}) - (m_B a_A \cos 20°)(0.25 \text{ ft})$

$$0 = \frac{1}{2}\frac{6 \text{ lb}}{g}(0.25 \text{ ft})^2\alpha + \frac{6 \text{ lb}}{g}(0.25\alpha)(0.25)$$

$$-\frac{6 \text{ lb}}{g}a_A \cos 20°(0.25)$$

$$0 = \frac{1}{g}[0.1875\alpha + 0.325\alpha - 1.4095a_A]$$

$$0 = 0.5625\alpha - 1.4095a_A$$

$$\alpha = 2.506a_A \qquad (2)$$

(*a*) Acceleration of the wedge.

Substitute for α from (2) into (1):

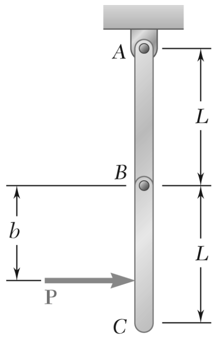
$$a_A = g\sin 20° + 0.15\cos 20°(2.506a_A)$$
$$a_A = 11.013 + 0.3532a_A$$
$$(1-0.3532)a_A = 11.013$$
$$a_A = 17.027\ \text{ft/s}^2$$

$\mathbf{a}_A = 17.03\ \text{ft/s}^2 \searrow 20°$ ◀

(*b*) Angular acceleration of the cylinder.

Eq. (2):
$$\alpha = 2.506a_A$$
$$= 2.506(17.027)$$
$$\alpha = 42.7\ \text{rad/s}^2$$

$\boldsymbol{\alpha} = 42.7\ \text{rad/s}^2 \circlearrowright$ ◀

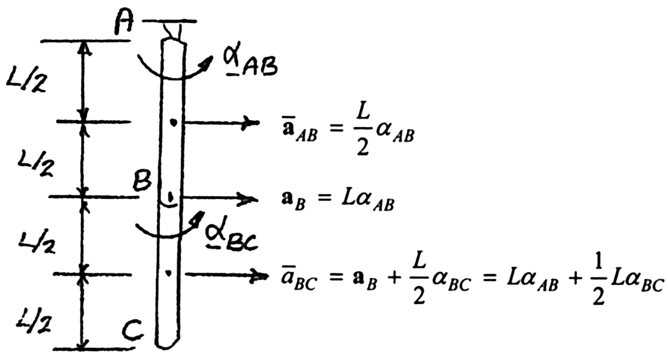


PROBLEM 16.149*

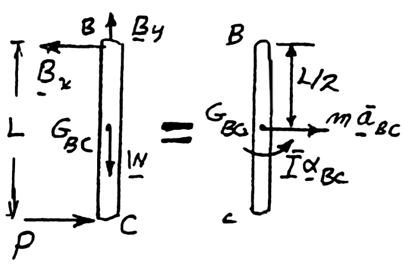
Each of the 3-kg bars AB and BC is of length $L = 500$ mm. A horizontal force $\mathbf{P}$ of magnitude 20 N is applied to bar BC as shown. Knowing that $b = L$ ($\mathbf{P}$ is applied at C), determine the angular acceleration each bar.

SOLUTION

Kinematics: Assume $\alpha_{AB} \circlearrowright$ $\alpha_{BC} \circlearrowright$ and $\omega_{AB} = \omega_{BC} = 0$



Kinetics: Bar BC



$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad PL = \bar{I}\alpha_{BC} + (m\bar{a}_{BC})\frac{L}{2}$$

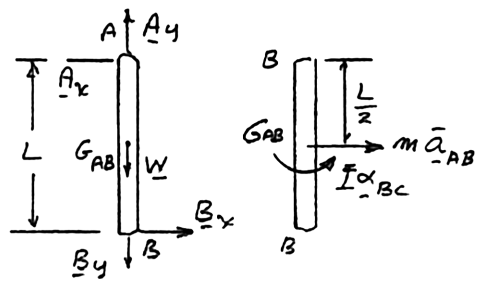
$$= \frac{m}{12}L^2\alpha_{BC} + m\left(L\alpha_{AB} + \frac{L}{2}\alpha_{BC}\right)\frac{L}{2}$$

$$P = \frac{1}{2}mL\alpha_{AB} + \frac{1}{3}mL\alpha_{BC} \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P - B_x = m\bar{a}_{BC}$$

$$P - B_x = m\left(L\alpha_{AB} + \frac{1}{2}L\alpha_{BC}\right) \qquad (2)$$

Bar *AB*:



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad B_x L = \bar{I}\alpha_{AB} + (m\bar{a}_{AB})\frac{L}{2}$$

$$= \frac{m}{12}L^2\alpha_{AB} + m\left(\frac{L}{2}\alpha_{AB}\right)\frac{L}{2}$$

$$B_x = \frac{1}{3}mL\alpha_{AB} \qquad (3)$$

Add (2) and (3):

$$P = \frac{4}{3}mL\alpha_{AB} + \frac{1}{2}mL\alpha_{BC} \qquad (4)$$

Subtract (1) from (4)

$$0 = \frac{5}{6}mL\alpha_{AB} + \frac{1}{6}mL\alpha_{BC}$$

$$\alpha_{BC} = -5\alpha_{AB} \qquad (5)$$

Substitute for α_{BC} in (1):

$$P = \frac{1}{2}mL\alpha_{AB} + \frac{1}{3}mL(-5\alpha_{AB}) = -\frac{7}{6}mL\alpha_{AB}$$

$$\alpha_{AB} = -\frac{6}{7}\frac{P}{mL} \qquad (6)$$

Eq. (5)

$$\alpha_{BC} = -5\left(-\frac{6}{7}\frac{P}{mL}\right) \qquad \alpha_{BC} = \frac{30}{7}\frac{P}{mL} \qquad (7)$$

Data:

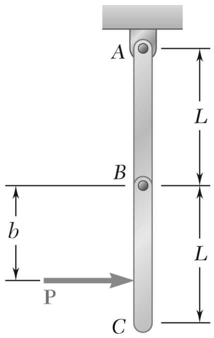
$$L = 500 \text{ mm} = 0.5 \text{ m}, \quad m = 3 \text{ kg}, \quad P = 20 \text{ N}$$

$$\alpha_{AB} = -\frac{6}{7}\frac{P}{mL} = -\frac{6}{7}\frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})}$$

$$= -11.249 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{AB} = 11.43 \text{ rad/s}^2 \blacktriangleleft$$

$$\alpha_{BC} = \frac{30}{7}\frac{P}{mL} = \frac{30}{7}\frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})}$$

$$= 57.143 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{BC} = 57.1 \text{ rad/s}^2 \blacktriangleleft$$

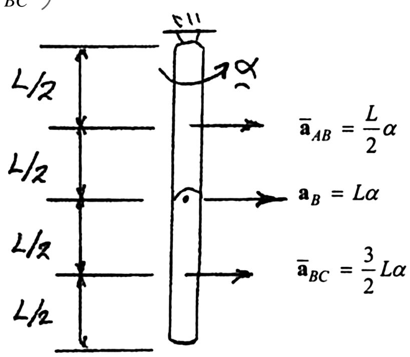


PROBLEM 16.150*

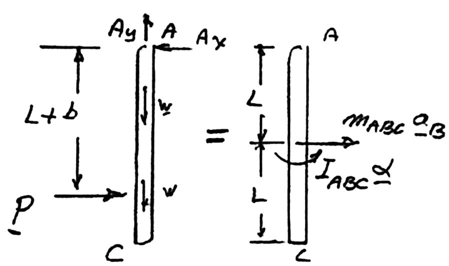
Each of the 3-kg bars AB and BC is of length $L = 500$ mm. A horizontal force **P** of magnitude 20 N is applied to bar BC. For the position shown, determine (*a*) the distance b for which the bars move as if they formed a single rigid body, (*b*) the corresponding angular acceleration of the bars.

SOLUTION

Kinematics: We choose $\alpha = \alpha_{AB} = \alpha_{BC}$ ↻



Kinetics: Bars AB and BC (acting as rigid body)



$$m_{ABC} = 2m$$

$$\bar{I} = \frac{1}{12}(2m)(2L)^2$$

$$\bar{I} = \frac{2}{3}mL^2$$

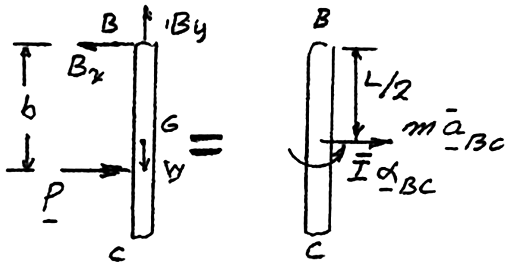
$$+↻\ \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad P(L+b) = \bar{I}_{ABC}\alpha + m_{ABC}a_B L$$

$$P(L+b) = \frac{2}{3}mL^2\alpha + (2m)(L\alpha)L$$

$$P(L+b) = \frac{8}{3}mL^2\alpha \qquad (1)$$

PROBLEM 16.150* (Continued)

Bar *BC*:



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad Pb = \bar{I}_{BC}\alpha + (m\bar{a}_{BC})\frac{L}{2}$$

$$= \frac{m}{12}L^2\alpha + m\left(\frac{3}{2}L\alpha\right)\frac{L}{2}$$

$$Pb = \frac{5}{6}mL^2\alpha$$

$$\alpha = \frac{6}{5}\frac{Pb}{mL^2} \qquad (2)$$

Substitute for α into (1):

$$P(L+b) = \frac{8}{3}mL^2\left(\frac{6}{5}\frac{Pb}{mL^2}\right)$$

$$PL + Pb = \frac{16}{5}Pb$$

$$L = \left(\frac{16}{5} - 1\right)b = \frac{11}{5}b$$

$$b = \frac{5}{11}L$$

Eq. (2)

$$\alpha = \frac{6}{5}\frac{P}{mL^2}\left(\frac{5}{11}L\right) \qquad \alpha = \frac{6}{11}\frac{P}{mL}$$

Data:

$$L = 500 \text{ mm} = 0.5 \text{ m}$$

$$m = 3 \text{ kg}$$

$$P = 20 \text{ N}$$

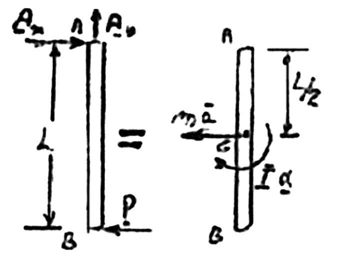
(*a*) $b = \dfrac{5}{11}L = \dfrac{5}{11}(0.5) = 0.22727 \text{ m}$ $\qquad b = 227 \text{ mm}$ ◀

(*b*) $\alpha = \dfrac{6}{11}\dfrac{P}{mL} = \dfrac{6}{11}\dfrac{(20 \text{ N})}{(3 \text{ kg})(0.5 \text{ m})} = 7.2727 \text{ rad/s}^2$ $\qquad \boldsymbol{\alpha} = 7.27 \text{ rad/s}^2 \circlearrowleft$ ◀

PROBLEM 16.151*

(*a*) Determine the magnitude and the location of the maximum bending moment in the rod of Problem 16.78.
(*b*) Show that the answer to Part *a* is independent of the weight of the rod.

SOLUTION



Rod *AB*:

$$\bar{a} = \frac{L}{2}\alpha$$

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$$

$$= \left(m\frac{1}{2}\alpha\right)\frac{L}{2} + \frac{1}{12}mL^2\alpha$$

$$\boldsymbol{\alpha} = \frac{3P}{mL} \circlearrowright \qquad (1)$$

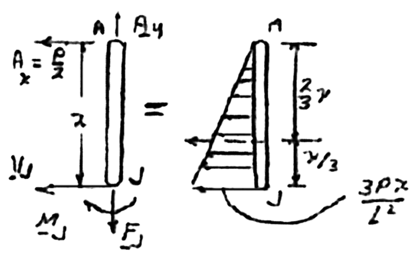
$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A_x - P = -m\bar{a}$$

$$A_x = P - m\frac{L}{2}\alpha = P - m\frac{L}{2}\left(\frac{3P}{mL}\right) = -\frac{P}{2} \qquad \mathbf{A}_x = \frac{1}{2}P \longleftarrow$$

<u>Portion *AJ* of Rod</u>:

<u>External forces</u>: A_x, W_{AJ}, axial force F_J, shear V_J, and bending moment M_J.

<u>Effective forces</u>: Since acceleration at any point is proportional to distance from *A*, effective forces are linearly distributed. Since mass per unit length is *m*/*L*, at Point *J* we find



$$\left(\frac{m}{L}\right)a_J = \frac{m}{L}(x\alpha)$$

Using (1):

$$\frac{m}{L}a_J = \frac{m}{L}\left(\frac{3P}{mL}\right)$$

$$\frac{m}{L}a_J = \frac{3Px}{L^2}$$

$$+\circlearrowright \Sigma M_J = \Sigma(M_J)_{\text{eff}}: \quad M_J - A_x x = -\frac{1}{2}\left(\frac{3Px}{L^2}\right)\left(\frac{2x}{3}\right)$$

$$M_J = \frac{1}{2}Px - \frac{1}{2}\frac{P}{L^2}x^3 \qquad (2)$$

For $M_{\max}$:

$$\frac{dM_J}{dx} = \frac{P}{2} - \frac{3}{2}\frac{P}{L^2}x^2 = 0$$

$$x = \frac{L}{\sqrt{3}} \qquad (3)$$

Substituting into (2)

$$(M_J)_{\max} = \frac{1}{2}\frac{PL}{\sqrt{3}} - \frac{1}{2}\frac{P}{L^2}\left(\frac{L}{\sqrt{3}}\right)^3 = \frac{1}{2}\frac{PL}{\sqrt{3}}\left(\frac{2}{3}\right)$$

$$(M_J)_{\max} = \frac{PL}{3\sqrt{3}} \qquad (4)$$

Note: Eqs. (3) and (4) are independent of W.

Data: $L = 36 \text{ in.}, \quad P = 1.5 \text{ lb}$

Eq. (3): $$x = \frac{L}{\sqrt{3}} = \frac{36 \text{ in.}}{\sqrt{3}} = 20.78 \text{ in.}$$

Eq. (4): $$(M_J)_{\max} = \frac{(1.5 \text{ lb})(36 \text{ in.})}{3\sqrt{3}}$$

$$= 10.392 \text{ lb} \cdot \text{in.}$$

$M_{\max} = 10.39 \text{ lb} \cdot \text{in.}$ located 20.8 in. below A ◀

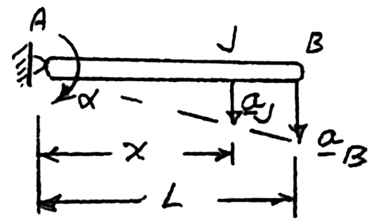
PROBLEM 16.152*

Draw the shear and bending-moment diagrams for the beam of Problem 16.84 immediately after the cable at B breaks.

SOLUTION

From answers to Problem 16.84:

$$\mathbf{a}_B = \frac{3}{2}g\downarrow \qquad \mathbf{A} = \frac{1}{4}mg\uparrow$$



We now find

$$\boldsymbol{\alpha} = \frac{a_B}{L} \circlearrowright = \frac{3g}{2L} \circlearrowright$$

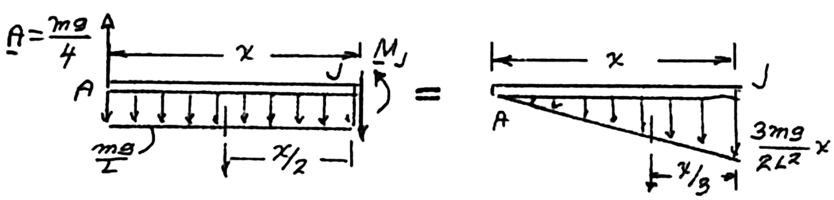
$$\mathbf{a}_J = x\alpha = \frac{3g}{2L}x\downarrow$$

Portion AJ of rod:

External forces: Reaction A, distributed load per unit length mg/L, shear V_J, bending moment M_J.

Effective forces: Since a is proportional to x, the effective forces are linearly distributed. The effective force per unit length at J is:

$$\frac{m}{L}a_J = \frac{m}{L}\cdot\frac{3g}{2L}x = \frac{3mg}{2L^2}x$$



$$+\downarrow\Sigma F_y = \Sigma(F_y)_{\text{eff}}:\quad \frac{mg}{L}x - \frac{mg}{4} + V_J = \frac{1}{2}\left(\frac{3mg}{2L^2}x\right)x$$

$$V_J = \frac{mg}{4} - \frac{mg}{L}x + \frac{3}{4}\frac{mg}{L^2}x^2$$

$$+\circlearrowright\Sigma M_J = \Sigma(M_J)_{\text{eff}}:\quad \left(\frac{mg}{L}x\right)\frac{x}{2} - \frac{mg}{4}x + M_J = \frac{1}{2}\left(\frac{3mg}{2L^2}x\right)x\left(\frac{x}{3}\right)$$

$$M_J = \frac{mg}{4}x - \frac{1}{2}\frac{mg}{L}x^2 + \frac{1}{4}\frac{mg}{L^2}x^3$$

Find $V_{\min}$:

$$\frac{dV_J}{dx} = -\frac{mg}{L} + \frac{3}{2}\frac{mg}{L^2}x = 0; \qquad x = \frac{2}{3}L$$

$$V_{\min} = \frac{mg}{4} - \frac{mg}{L}\left(\frac{2}{3}L\right) + \frac{3}{4}\frac{mg}{L^2}\left(\frac{2}{3}L\right)^2; \qquad V_{\min} = -\frac{mg}{12}$$

PROBLEM 16.152* (Continued)

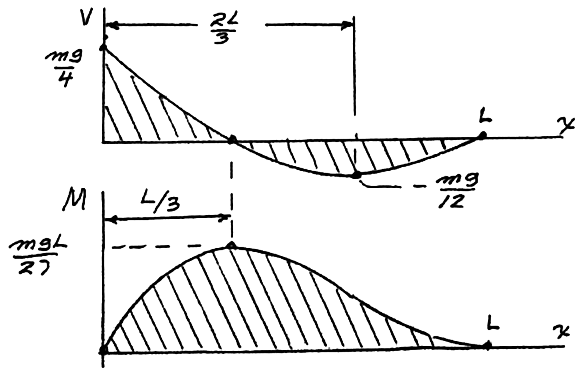
Find $M_{\max}$ where $\qquad V_J = 0\text{:} \quad V_J = \frac{mg}{4} - \frac{mg}{L}x + \frac{3}{4}\frac{mg}{L}x^2 = 0$

$$3x^2 - 4Lx + L^2 = 0$$

$$(3x - L)(x - L) = 0 \qquad x = \frac{L}{3} \quad \text{and} \quad x = L$$

$$M_{\max} = \frac{mg}{4}\left(\frac{L}{3}\right) - \frac{1}{2}\frac{mg}{L}\left(\frac{L}{3}\right)^2 + \frac{1}{4}\left(\frac{mg}{L}\right)\left(\frac{L}{3}\right)^3 = \frac{mgL}{27}$$

$$M_{\min} = \frac{mg}{4}L - \frac{1}{2}\frac{mg}{L}L^2 + \frac{1}{4}\frac{mg}{L}L^3 = 0$$

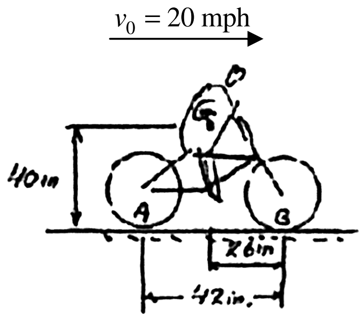


$M_{\max} = \frac{mgL}{27}$ at $\frac{L}{3}$ from A ◀

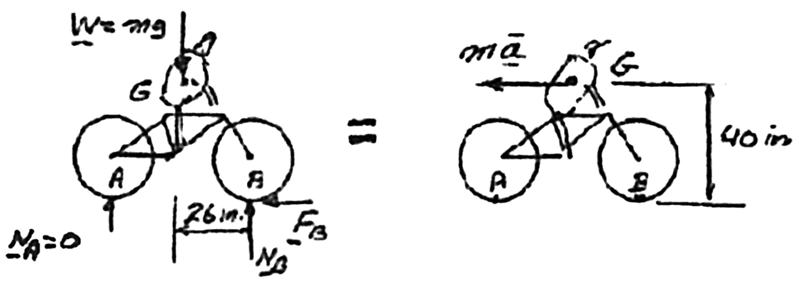
PROBLEM 16.153

A cyclist is riding a bicycle at a speed of 20 mph on a horizontal road. The distance between the axles is 42 in., and the mass center of the cyclist and the bicycle is located 26 in. behind the front axle and 40 in. above the ground. If the cyclist applies the brakes only on the front wheel, determine the shortest distance in which he can stop without being thrown over the front wheel.

SOLUTION



When cyclist is about to be thrown over the front wheel, $N_A = 0$



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad mg(26\text{ in.}) = m\bar{a}(40\text{in.})$$

$$a = \frac{26}{40}g = \frac{26}{40}(32.2\text{ ft/s}^2) = 20.93\text{ ft/s}^2$$

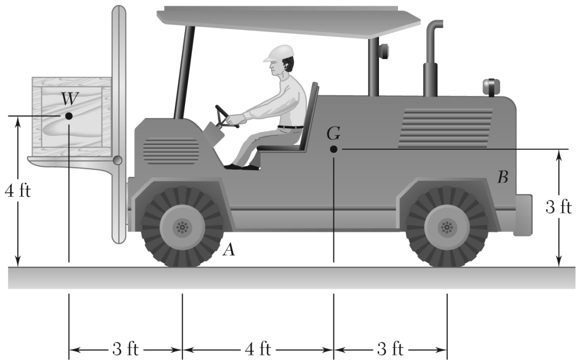
Uniformly accelerated motion:

$$v_0 = 20\text{ mph} = 29.333\text{ ft/s}$$

$$v^2 - v_0^2 = 2as: \quad 0 - (29.333\text{ ft/s})^2 = 2(-20.93\text{ ft/s}^2)s$$

$$s = 20.555\text{ ft}$$

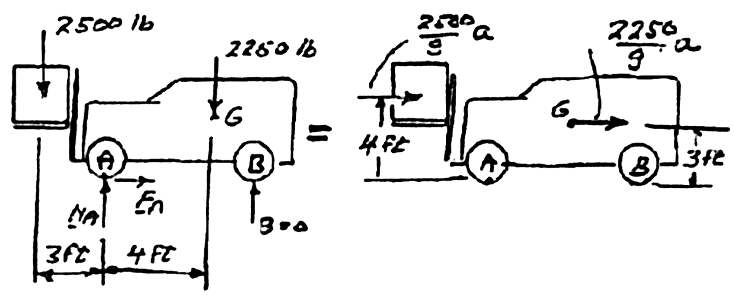
$s = 20.6$ ft ◀



PROBLEM 16.154

The forklift truck shown weighs 2250 lb and is used to lift a crate of weight $W = 2500$ lb. The truck is moving to the left at a speed of 10 ft/s when the brakes are applied on all four wheels. Knowing that the coefficient of static friction between the crate and the fork lift is 0.30, determine the smallest distance in which the truck can be brought to a stop if the crate is not to slide and if the truck is not to tip forward.

SOLUTION



Assume crate does not slide and that tipping impends about A. $(B = 0)$

$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$(2500 \text{ lb})(3 \text{ ft}) - (2250 \text{ lb})(4 \text{ ft}) = -\left(2500\frac{a}{g}\right)(4 \text{ ft}) - \left(2250\frac{a}{g}\right)(3 \text{ ft})$$

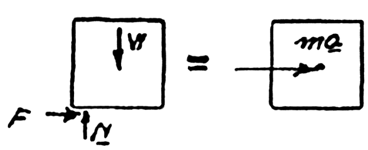
$$7500 - 9000 = -(10{,}000 + 6750)\frac{a}{g}$$

$$\frac{a}{g} = 0.09; \qquad a = 0.09(32.2 \text{ ft/s}^2) \qquad\qquad \mathbf{a} = 2.884 \text{ ft/s}^2 \longrightarrow$$

Uniformly accelerated motion

$$v^2 = v_0^2 + 2ax; \quad 0 = (10 \text{ ft/s})^2 - 2(2.884 \text{ ft/s}^2)x \qquad\qquad x = 17.34 \text{ ft} \blacktriangleleft$$

Check whether crate slides

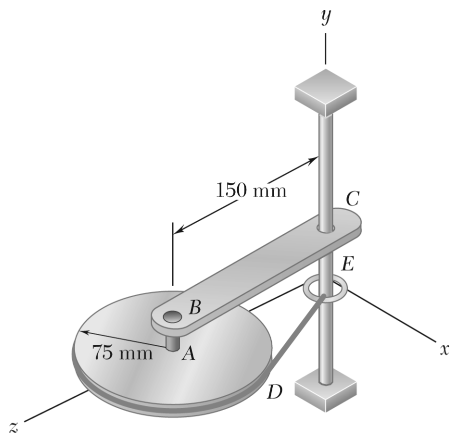


$$N = W$$

$$F = ma = \frac{W}{g}a$$

$$\mu_{\text{req}} = \frac{F}{N} = \frac{a}{g} = \frac{2.884 \text{ ft/s}^2}{32.2 \text{ ft/s}^2}$$

$\mu_{\text{req}} = 0.09 < 0.30.$ The crate does not slide. ◀

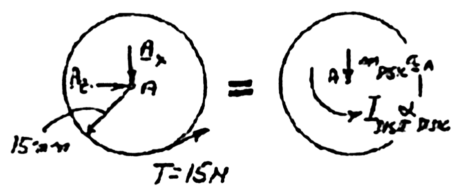


PROBLEM 16.155

A 5-kg uniform disk is attached to the 3-kg uniform rod BC by means of a frictionless pin AB. An elastic cord is wound around the edge of the disk and is attached to a ring at E. Both ring E and rod BC can rotate freely about the vertical shaft. Knowing that the system is released from rest when the tension in the elastic cord is 15 N, determine (a) the angular acceleration of the disk, (b) the acceleration of the center of the disk.

SOLUTION

(a) Angular acceleration of the disk.



Disk: $$\bar{I}_{\text{disk}} = \frac{1}{2} m_{\text{disk}} r^2 = \frac{1}{2}(5\text{ kg})(0.075\text{ m})^2$$

$$\bar{I}_{\text{disk}} = 14.06 \times 10^{-3} \text{kg} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad (15\text{ N})(0.075\text{ N}) = \bar{I}_{\text{disk}}\, \alpha_{\text{disk}}$$

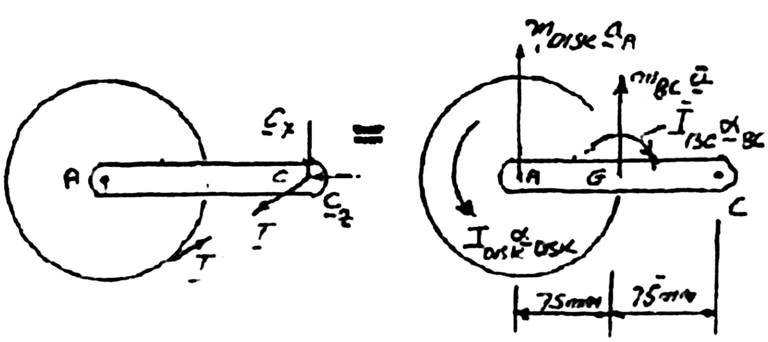
$$1.125\text{ N} \cdot \text{m} = (14.06 \times 10^3\text{ kg} \cdot \text{m}^2)\alpha_{\text{disk}}$$

$$\alpha_{\text{disk}} = 80.0\text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{\text{disk}} = 80.0\text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(b) Acceleration of center of disk.

Entire assembly

$$\bar{I}_{BC} = \frac{1}{12} m_{BC}(BC)^2 = \frac{1}{12}(3\text{ kg})(0.15\text{ m})^2 = 5.625 \times 10^{-3}\text{ kg} \cdot \text{m}^2$$



PROBLEM 16.155 (Continued)

Assume $\boldsymbol{\alpha}_{BC}$ is $\circlearrowright$ $\quad \mathbf{a}_A = (0.15\text{ m})\alpha_{BC} \uparrow$

$$\bar{a} = (0.075\text{ m})\alpha_{BC}$$

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad 0 = I_{\text{disk}}\alpha_{\text{disk}} - m_{\text{disk}}a_A(0.15\text{ m}) - m_{BC}\bar{a}(0.075\text{ m}) - \bar{I}_{BC}\alpha_{BC}$$

$$0 = (14.06\times 10^{-3}\text{ kg}\cdot\text{m}^2)(80\text{ rad/s}^2) - (5\text{ kg})(0.15\text{ m})^2\alpha_{BC}$$

$$-(3\text{ kg})(0.075\text{ m})^2\alpha_{BC} - (5.625\times 10^3\text{ kg}\cdot\text{m}^2)\alpha_{BC}$$

$$0 = 1.125 - 0.1125\alpha_{BC} - 16.875\times 10^{-3}\alpha_{BC} - 5.625\times 10^{-3}\alpha_{BC}$$

$$0 = 1.125 - 0.135\alpha_{BC}$$

$$\alpha_{BC} = +8.333\text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{BC} = 8.33\text{ rad/s}^2 \circlearrowright$$

$$a_A = (AC)\alpha_{BC} = (0.15\text{ m})(8.333\text{ rad/s}^2)$$

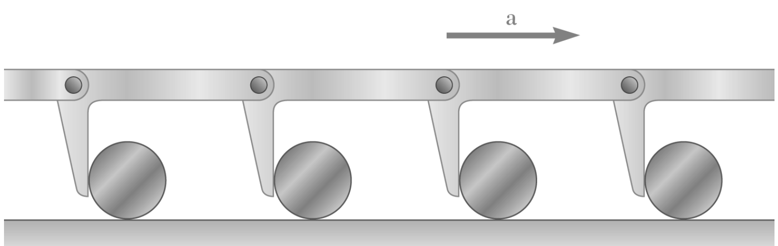
$$a_A = +1.25\text{ m/s}^2 \qquad \mathbf{a}_A = 1.250\text{ m/s}^2 \uparrow \blacktriangleleft$$

Note: Answers can also be written:

$$\boldsymbol{\alpha}_{\text{disk}} = (80\text{ rad/s}^2)\mathbf{j} \qquad \mathbf{a}_A = -(1.25\text{ m/s}^2)\mathbf{i}$$

PROBLEM 16.156

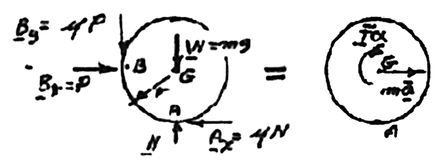
Identical cylinders of mass m and radius r are pushed by a series of moving arms. Assuming the coefficient of friction between all surfaces to be $\mu < 1$ and denoting by a the magnitude of the acceleration of the arms, derive an expression for (a) the maximum allowable value of a if each cylinder is to roll without sliding, (b) the minimum allowable value of a if each cylinder is to move to the right without rotating.



SOLUTION

(a) <u>Cylinder rolls</u> without sliding $a = r\alpha$ or $\alpha = \dfrac{a}{r}$

P is horizontal component of force that the arm exerts on cylinder.



$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad Pr - (\mu_k P) r = I\alpha + (m\bar{a})r$$

$$P(1-\mu)r = \frac{1}{2}mr^2\left(\frac{\bar{a}}{r}\right) + (m\bar{a})r$$

$$P = \frac{3}{2}\frac{m\bar{a}}{(1-\mu)} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \qquad N - \mu P - mg = 0 \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \qquad P - \mu N = m\bar{a} \qquad (3)$$

Solve (2) for N and substitute for N into (3).

$$P - \mu^2 P - \mu mg = m\bar{a}$$

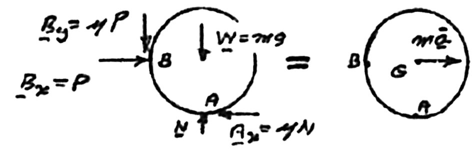
Substitute P from (1): $$(1-\mu^2)\frac{3}{2}\frac{m\bar{a}}{(1-\mu)} - \mu mg = m\bar{a}$$

$$3(1+\mu)\bar{a} - 2\mu g = 2\bar{a}$$

$$\bar{a}(1+3\mu) - 2\mu g = 0$$

$$\bar{a} = \frac{2\mu}{1+3\mu}g \quad ◀$$

(*b*) Cylinder translates: $\alpha = 0$



Sliding occurs at A: $A_x = \mu N$

Assume sliding impends at B: $B_y = \mu P$

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad Pr - \mu Pr = (m\bar{a})r$$

$$P(1-\mu)r = m\bar{a}\,r$$

$$P = \frac{m\bar{a}}{1-\mu} \tag{4}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P - \mu N = m\bar{a} \tag{5}$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - \mu P - mg = 0 \tag{6}$$

Solve (5) for N and substitute for N into (6).

$$P - P\mu^2 - \mu mg = m\bar{a}$$

Substitute for P from (4):

$$\frac{m\bar{a}}{1-\mu}(1-\mu^2) - \mu mg = m\bar{a}$$

$$\bar{a}(1+\mu) - \mu g = \bar{a}$$

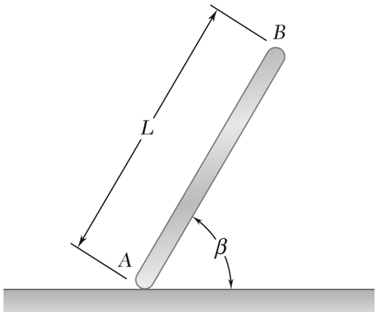
$$\bar{a}\mu - \mu g = 0$$

$\bar{a} = g$ ◀

Summary: $a < \dfrac{2\mu}{1+3\mu}g$: Rolling

$\dfrac{3\mu}{1+3\mu}g < a < g$: Rotating and sliding

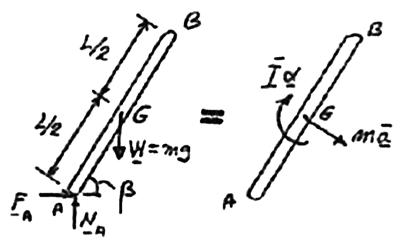
$a > g$: Translation



PROBLEM 16.157

The uniform rod AB of weight W is released from rest when $\beta = 70°$. Assuming that the friction force between end A and the surface is large enough to prevent sliding, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the normal reaction at A, (*c*) the friction force at A.

SOLUTION



We note that rod rotates about A. $\omega = 0$

$$\bar{I} = \frac{1}{12}mL^2$$

$$\bar{a} = \frac{L}{2}\alpha$$

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad mg\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha + (m\bar{a})\frac{L}{2}$$

$$\frac{1}{2}mgL\cos\beta = \frac{1}{12}mL^2\alpha + \left(m\frac{L}{2}\alpha\right)\frac{L}{2}$$

$$= \frac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3}{2}\frac{g\cos\beta}{L} \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F_A = m\bar{a}\sin\beta$$

$$F_A = m\frac{L}{2}\alpha\sin\beta = m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\sin\beta$$

$$F_A = \frac{3}{4}mg\sin\beta\cos\beta \qquad (2)$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N_A - mg = -m\bar{a}\cos\beta = -m\left(\frac{L}{2}\alpha\right)\cos\beta$$

$$N_A - mg = -m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\cos\beta$$

$$N_A = mg\left(1 - \frac{3}{4}\cos^2\beta\right) \qquad (3)$$

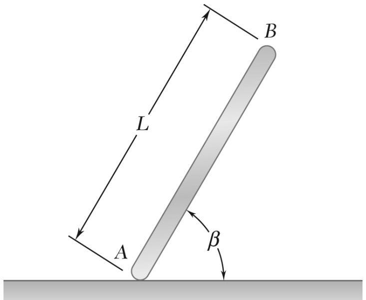
PROBLEM 16.157 (Continued)

For $\beta = 70°$:

(*a*) Eq. (1): $\alpha = \frac{3}{2}\frac{g\cos 70°}{L}$ $\boldsymbol{\alpha} = 0.513\frac{g}{L}$ ◀

(*b*) Eq. (3): $N_A = mg\left(1 - \frac{3}{4}\cos^2 70°\right)$ $\mathbf{N}_A = 0.912mg \uparrow$ ◀

(*c*) Eq. (2) $F_A = \frac{3}{4}mg\sin 70°\cos 70°$ $\mathbf{F}_A = 0.241mg \longrightarrow$ ◀

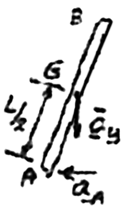


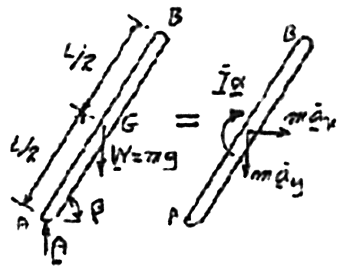
PROBLEM 16.158

The uniform rod AB of weight W is released from rest when $\beta = 70°$. Assuming that the friction force is zero between end A and the surface, determine immediately after release (a) the angular acceleration of the rod, (b) the acceleration of the mass center of the rod, (c) the reaction at A.

SOLUTION

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \qquad a = m\bar{\mathbf{a}}_x \quad \bar{\mathbf{a}}_x = 0$$





$$\mathbf{a}_A = \bar{\mathbf{a}}_y + \frac{L}{2}\alpha \nwarrow \beta$$

$$+\!\downarrow 0 = a_y - \frac{L}{2}\alpha\cos\beta$$

$$\mathbf{a}_y = \frac{L}{2}\alpha\cos\beta \downarrow$$

$$+\!\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \qquad mg - A = m\bar{a}_y = m\left(\frac{L}{2}\alpha\cos\beta\right) \tag{1}$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad A\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha = \frac{1}{12}mL^2\alpha$$

$$A = \frac{mL}{6}\frac{\alpha}{\cos\beta} \tag{2}$$

Substitute (2) into (1):

$$mg - \frac{mL}{6}\frac{\alpha}{\cos\beta} = m\frac{L}{2}\alpha\cos\beta$$

$$g = \left(\frac{L}{2}\cos\beta + \frac{L}{6\cos\beta}\right)\alpha$$

$$g = \frac{L}{6}\left(3\cos\beta + \frac{1}{\cos\beta}\right)\alpha$$

$$g = \frac{L}{6}\left(\frac{3\cos^2\beta + 1}{\cos\beta}\right)\alpha$$

$$\alpha = \frac{6g}{L}\left(\frac{\cos\beta}{1 + 3\cos^2\beta}\right) \circlearrowright$$

PROBLEM 16.158 (Continued)

$$\bar{\mathbf{a}} = \frac{L}{2}\alpha\cos\beta = \frac{L}{2}\left(\frac{6g}{L}\cdot\frac{\cos\beta}{1+3\cos^2\beta}\right)\cos\beta = 3g\left(\frac{\cos^2\beta}{1+3\cos^2\beta}\right) \longleftarrow$$

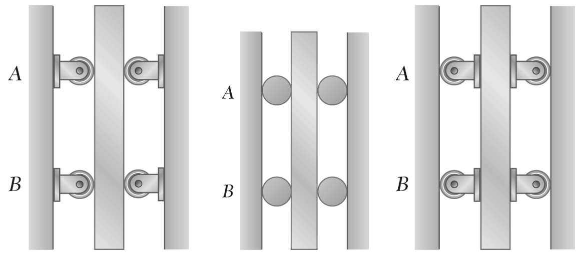
$$\mathbf{A} = \frac{mL}{6}\cdot\frac{\alpha}{\cos\beta} = \frac{mL}{6}\cdot\left(\frac{6g}{L}\cdot\frac{\cos\beta}{1+3\cos^2\beta}\right)\frac{1}{\cos\beta} = mg\frac{1}{1+3\cos^2\beta} \uparrow$$

For $\beta = 70°$:

(*a*) $$\alpha = \frac{6g}{L}\frac{\cos 70°}{1+3\cos^2 70°}$$ $\boldsymbol{\alpha} = 1.519\frac{g}{L}$ ◀

(*b*) $$\bar{a} = 3g\frac{\cos^2 70}{1+3\cos^2 70°}$$ $\bar{\mathbf{a}} = 0.260g \downarrow$ ◀

(*c*) $$A = mg\frac{1}{1+3\cos^2 70°}$$ $\mathbf{A} = 0.740mg \uparrow$ ◀



(*a*) (*b*) (*c*)

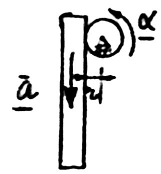
PROBLEM 16.159

A bar of mass $m = 5$ kg is held as shown between four disks, each of mass $m' = 2$ kg and radius $r = 75$ mm. Knowing that the normal forces on the disks are sufficient to prevent any slipping, for each of the cases shown determine the acceleration of the bar immediately after it has been released from rest.

SOLUTION

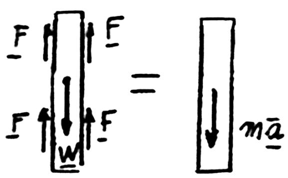
(*a*) Configuration (*a*)

Kinematics:



$$\bar{a} = r\alpha \qquad \alpha = \frac{\bar{a}}{r}$$

Kinetics of bar

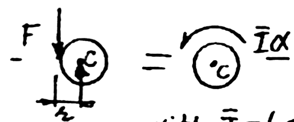


with $W = mg$

$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$:

$$W - 4F = m\bar{a} \qquad (1)$$

Kinetics of one disk



with $\bar{I} = \frac{1}{2} m' r^2$

$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}$: $\quad Fr = \bar{I}\alpha$

$$Fr = \frac{1}{2} m' r^2 \left(\frac{\bar{a}}{r}\right)$$

$$F = \frac{1}{2} m' \bar{a} \qquad (2)$$

Substitute for F from (2) into (1).

$$mg - 4\left(\frac{1}{2} m' \bar{a}\right) = m\bar{a}$$

$$mg = (m + 2m')\bar{a} \qquad \bar{a} = \frac{mg}{m + 2m'}$$

$$m = 5 \text{ kg}, \quad m' = 2 \text{ kg}: \quad \bar{a} = \frac{5}{5 + 2(2)} g$$

$$\mathbf{\bar{a}} = \frac{5}{9} g \downarrow \blacktriangleleft$$

(*b*) Configuration (*b*)

Kinematics:

Disk is rolling on vertical wall

$$\bar{a}' = a_C = r\alpha$$

$$\bar{a} = a_r = 2r\alpha$$

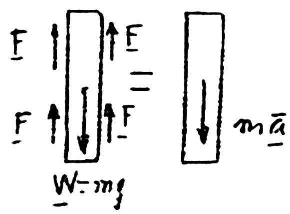
Therefore:

$$\alpha = \frac{\bar{a}}{2r}$$

$$\bar{a}' = r\alpha = \frac{1}{2}\bar{a}$$

PROBLEM 16.159 (Continued)

Kinetics of bar



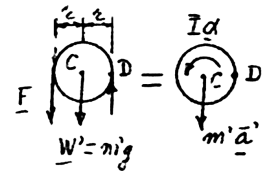
$$+\!\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}:$$

$$W - 4F = m\bar{a}$$

$$mg - 2(2F) = m\bar{a} \qquad (1)$$

Kinetics of one disk

$$\bar{I} = \frac{1}{2}m'r^2$$



$$+\!\circlearrowleft \Sigma M_D = \Sigma (M_D)_{\text{eff}}:$$

$$F(2r) + m'gr = \bar{I}\alpha + m'\bar{a}'r$$

$$2Fr = m'gr = \frac{1}{2}m'r^2\left(\frac{\bar{a}}{2r}\right) + m'\frac{\bar{a}}{2}r$$

$$2F = m'\left(\frac{1}{4}\bar{a} + \frac{1}{2}\bar{a}\right) - m'g \qquad (2)$$

Substitute for $2F$ from (2) into (1):

$$mg - 2m'\left(\frac{3}{4}\bar{a}\right) + 2m'g = m\bar{a}$$

$$\left(m + \frac{3}{2}m'\right)\bar{a} = (m + 2m')g \qquad \bar{a} = \frac{m + 2m'}{m + \frac{3}{2}m'}g$$

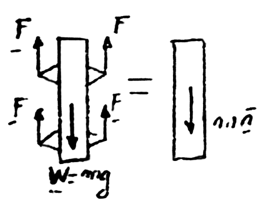
$$m = 5\text{ kg}, \quad m' = 2\text{ kg}: \quad \bar{a} = \frac{5+4}{5+3}g \qquad \mathbf{\bar{a}} = \frac{9}{8}g\downarrow \blacktriangleleft$$

(*c*) Configuration (*c*)

Kinematics:

Disk is rolling on vertical wall $\qquad \bar{a} = \bar{a}' = r\alpha$

Kinetics of bar



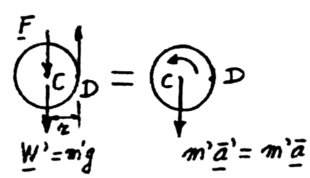
$$+\!\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}:$$

$$W - 4F = m\bar{a}$$

$$mg - 4F = m\bar{a} \qquad (1)$$

Kinetics of one disk

$$\bar{I}\alpha = \frac{1}{2}m'r^2\left(\frac{\bar{a}}{r}\right)$$



$$+\!\circlearrowleft \Sigma M_D = \Sigma (M_D)_{\text{eff}}:$$

$$(F + w')r = \bar{I}\alpha + m'\bar{a}'r$$

$$(F + m'g)r = \frac{1}{2}m'r'\left(\frac{\bar{a}}{r}\right) + m'\bar{a}t$$

$$F = \frac{3}{2}m'\bar{a} - m'g \qquad (2)$$

Substitute for F from (2) into (1):

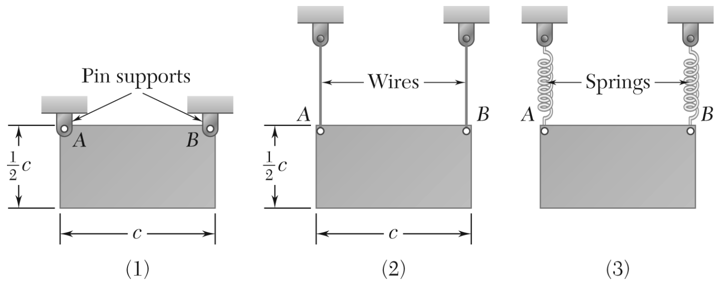
$$mg - 6m'\overline{a} + 4m'g = m\overline{a}$$

$$(m + 6m')\overline{a} = (m + 4m')g \quad \overline{a} = \frac{m + 4m'}{m + 6m'}g$$

$$m = 5 \text{ kg}, \quad m' = 2 \text{ kg}: \quad \overline{a} = \frac{5 + 8}{5 + 12} \qquad \overline{\mathbf{a}} = \frac{13}{17}g \downarrow \blacktriangleleft$$

PROBLEM 16.160

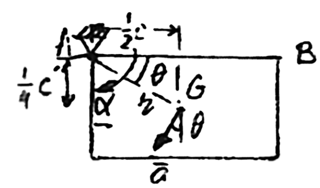
A uniform plate of mass m is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released (*a*) the angular acceleration of the plate, (*b*) the acceleration of its mass center.



SOLUTION

(1) Plate attached to pins

Kinematics: Assume α ↻ $(\omega = 0)$



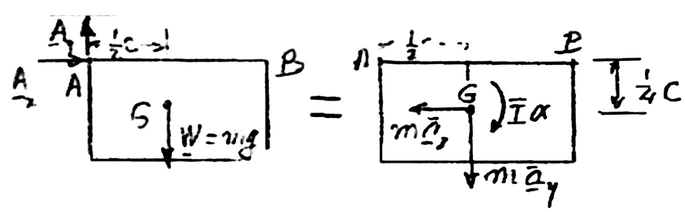
$$\bar{\mathbf{a}} = r\alpha \measuredangle \theta$$

$\xleftarrow{+}$ x comp: $\bar{a}_x = r\alpha \sin\theta = (r\sin\theta)\alpha$

$+\downarrow$ y comp: $\bar{a}_y = r\alpha\cos\theta = (r\cos\theta)\alpha$

Thus: $\bar{\mathbf{a}}_x = \dfrac{1}{4}c\alpha \longleftarrow; \quad \bar{\mathbf{a}}_y = \dfrac{1}{2}c\alpha \downarrow$ (1)

Kinetics: $\bar{I} = \dfrac{1}{12}m\left[c^2 + \left(\dfrac{c}{2}\right)^2\right] = \dfrac{5}{48}mc^2$



(*a*) $+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}:\quad W\left(\dfrac{c}{2}\right) = \bar{I}\alpha + (m\bar{a}_x)\left(\dfrac{c}{4}\right) + (m\bar{a}_y)\left(\dfrac{c}{2}\right)$

$$\frac{1}{2}mgc = \frac{5}{48}mc^2\alpha + m\left(\frac{1}{4}c\alpha\right)\left(\frac{c}{4}\right) + m\left(\frac{1}{2}c\alpha\right)\left(\frac{c}{2}\right)$$

$$\frac{1}{2}mgc = \frac{20}{48}mc^2\alpha \qquad \alpha = 1.2\frac{g}{c}$$

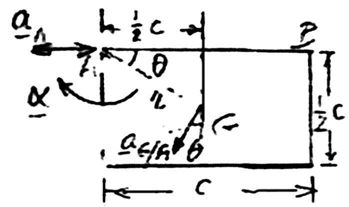
$\boldsymbol{\alpha} = 1.2\dfrac{g}{c}$ ↻ ◀

(*b*) From (1): $\bar{a}_x = \frac{1}{4}c\alpha = \frac{1}{4}(1.2g)$ $\quad \bar{\mathbf{a}}_x = 0.3g \longleftarrow$

$\bar{a}_y = \frac{1}{2}c\alpha = \frac{1}{2}(1.2g)$ $\quad \bar{\mathbf{a}}_y = 0.6g \downarrow$ $\qquad \bar{\mathbf{a}} = 0.671g \nearrow 63.4° \blacktriangleleft$

(2) Plate suspended from wires.

<u>Kinematics</u>: Assume $\alpha \circlearrowright$ $(\omega = 0)$



$$\bar{\mathbf{a}} = \mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{G/A}$$

$$= a_A \leftrightarrow + r\alpha \measuredangle \theta$$

$+\uparrow$ y comp.

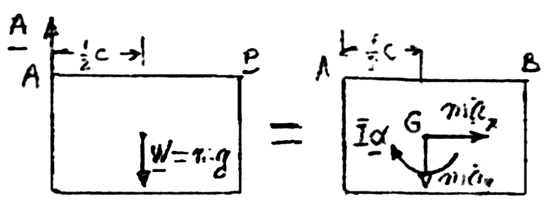
$$\bar{a}_y = 0 - r\alpha\cos\theta$$

$$= -(r\cos\theta)\alpha$$

$$r\cos\theta = \frac{1}{2}c$$

Thus: $\bar{a}_y = -\frac{1}{2}c\alpha \qquad \bar{\mathbf{a}}_y = \frac{1}{2}c\alpha \downarrow$ (2)

<u>Kinetics</u>: $\bar{I} = \frac{1}{12}m\left[c^2 + \left(\frac{c}{2}\right)^2\right] = \frac{I}{48}mc^2$



$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\quad 0 = m\bar{a}_2$

$$\bar{a}_x = 0$$

$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}$: $\quad W\left(\frac{1}{2}c\right) = \bar{I}\alpha + (m\bar{a}_y)\left(\frac{1}{2}c\right)$

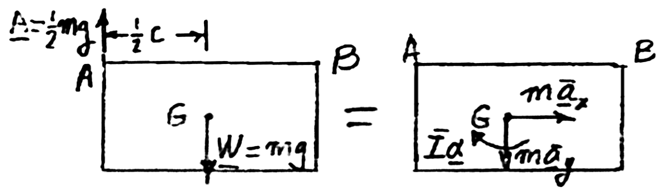
Recalling (1): $\frac{1}{2}mgc = \frac{5}{48}mc^2\alpha + \left(\frac{1}{2}mc\alpha\right)\left(\frac{1}{2}c\right)$

$\frac{1}{2}mgc = \frac{17}{48}mc^2\alpha \quad \alpha = \frac{24}{17}\frac{g}{c}$ $\qquad \alpha = \frac{24}{17}\frac{g}{c} \circlearrowright \blacktriangleleft$

$\bar{a}_y = \frac{1}{2}c\alpha = \frac{1}{2}c\left(\frac{24}{17}\frac{g}{c}\right) = \frac{12}{17}g$ $\qquad \bar{\mathbf{a}} = \frac{12}{17}g \downarrow \blacktriangleleft$

PROBLEM 16.160 (Continued)

(3) Plate suspended from springs. Immediately after spring B is released, the tension in spring A is still $\frac{1}{2}mg$ since its elongation is unchanged.



(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad \left(\frac{1}{2}mg\right)\left(\frac{1}{2}c\right) = \bar{I}\alpha$$

$$\frac{1}{4}mgc = \frac{5}{48}mc^2\alpha \qquad \boldsymbol{\alpha} = 2.4\frac{g}{c} \circlearrowright \blacktriangleleft$$

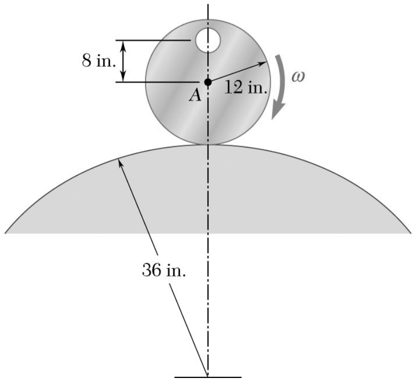
(*b*) Acceleration at mass center.

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad 0 = m\bar{a}_x \quad \bar{a}_x = 0$$

$$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad mg - \frac{1}{2}mg = m\bar{a}_y$$

$$\bar{a}_y = \frac{1}{2}g \qquad \bar{\mathbf{a}} = 0.5g \downarrow \blacktriangleleft$$

PROBLEM 16.161



A cylinder with a circular hole is rolling without slipping on a fixed curved surface as shown. The cylinder would have a weight of 16 lb without the hole, but with the hole it has a weight of 15 lb. Knowing that at the instant shown the disk has an angular velocity of 5 rad/s clockwise, determine (*a*) the angular acceleration of the disk, (*b*) the components of the reaction force between the cylinder and the ground at this instant.

SOLUTION

Geometry: Let the mass center G of the cylinder lie a distance b below the geometric center for the position shown. Let C, the contact point between the cylinder and the fixed curved surface, be the origin of a coordinate system, as shown. The position vector of a point is

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

Let r be the radius of the cylinder and R that of the fixed curved surface

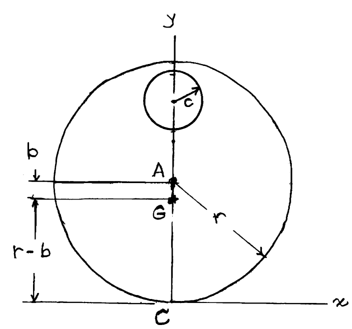
Kinematics:

The acceleration $\mathbf{a}_P$ at a point is given by

$$\mathbf{a}_P = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{P/C} - \omega^2 \mathbf{r}_{P/C}$$

Let

$$\boldsymbol{\alpha} = \alpha \circlearrowright$$



Then, using the coordinate system

$$\mathbf{a}_P = [(a_C)_x \longrightarrow] + [(a_C)_y \uparrow]$$
$$+ [y\alpha \longrightarrow] + [x\alpha \downarrow]$$
$$+ [y\omega^2 \downarrow] + [x\omega^2 \longleftarrow]$$

Since the cylinder rolls without slipping on a fixed surface,

$$(a_C)_x = 0$$

For Points G and A,

$$\mathbf{a}_G = [(a_C)_y \uparrow] + [(r-b)\alpha \longrightarrow] + [(r-b)\omega^2 \downarrow] \tag{1}$$

$$\mathbf{a}_A = [(a_C)_y \uparrow] + [r\alpha \longrightarrow] + [r\omega^2 \downarrow] \tag{2}$$

Subtract Eq. (2) from Eq. (1) to eliminate $(a_C)_y$

$$\mathbf{a}_G = \mathbf{a}_A = [b\alpha \longleftarrow] + [b\omega^2 \uparrow]$$

$$\mathbf{a}_G = \mathbf{a}_A + [b\alpha \longleftarrow] + [b\omega^2 \uparrow]$$
$$= [(a_A)_x \longrightarrow] + [(\mathbf{a}_A)_y \uparrow] + [b\alpha \longleftarrow] + [b\omega^2 \uparrow]$$
$$= [(r-b)\alpha \longrightarrow] + [(\mathbf{a}_A)_y \uparrow] + [b\omega^2 \uparrow] \tag{3}$$

PROBLEM 16.161 (Continued)

Point C is the instantaneous center, so that

$$\mathbf{v}_A = r\omega \longrightarrow$$

Point A is constrained to move on a circle of radius

$$\rho = R + r$$

so its vertical component of acceleration is

$$(\mathbf{a}_A)_y = \frac{v_A^2}{\rho}\downarrow = \frac{r^2\omega^2}{\rho}\downarrow$$

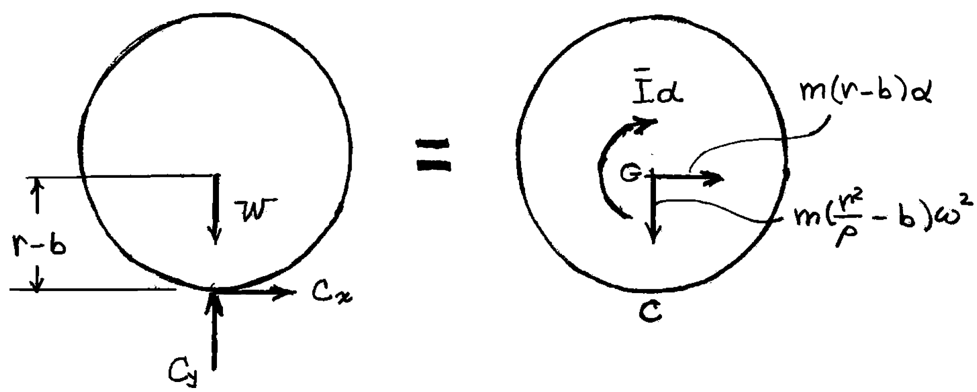
Using Eq. 3, $$\mathbf{a}_G = [(r-b)\alpha \longrightarrow] + \left(\frac{r^2}{\rho} - b\right)\downarrow$$

The effective force at the mass center is

$$m\mathbf{a}_G = [m(r-b)\alpha \longrightarrow] + \left(\frac{r^2}{\rho} - b\right)\omega^2\downarrow$$

Kinetics:

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad 0 + \bar{I}\alpha + (r-b)m(r-b)\alpha$$
$$= [\bar{I} + m(r-b)^2]\alpha$$



(*a*) *Angular acceleration.* $\boldsymbol{\alpha} = 0$ ◀

(*b*) *Force components at C.*

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad C_x = m(r-b)\alpha = 0$$

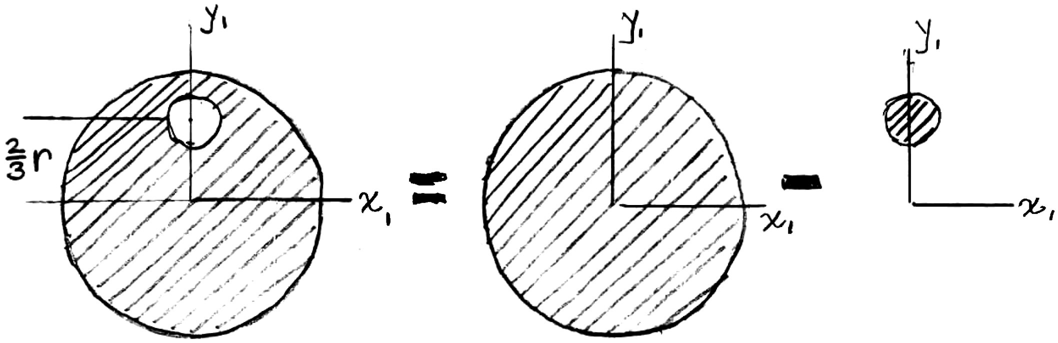
$\mathbf{C}_x = 0$ ◀

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad C_y - W = -m\left(\frac{r^2}{\rho} - b\right)\omega^2$$

$$C_y = W - \frac{W}{g}\left(\frac{r^2}{\rho} - b\right)\omega^2$$

It remains to determine the distance b from the mass distribution of the cylinder.

The mass center G coincides with the centroid of a circular cylinder of area $A_1 = \pi r^2$ with a circular cut out of area $A_2 = \frac{1}{16}A$, with its center located $\frac{2}{3}r$ above the center of A_1.



	A_1	$\bar{y}_1$	$A_1\bar{y}_1$
(1)	πr^2	0	0
(2)	$-\frac{1}{16}\pi r^2$	$\frac{2}{3}r$	$-\frac{1}{24}\pi r^3$
Σ	$\frac{15}{16}\pi r^2$		$-\frac{1}{24}\pi r^3$

$$\bar{Y}\Sigma A = \Sigma A\bar{y}_1$$

$$\frac{15}{16}\pi r^2 \bar{Y} = -\frac{1}{24}\pi r^3$$

$$\bar{Y} = -\frac{2}{45}r$$

$$b = \frac{2}{45}r$$

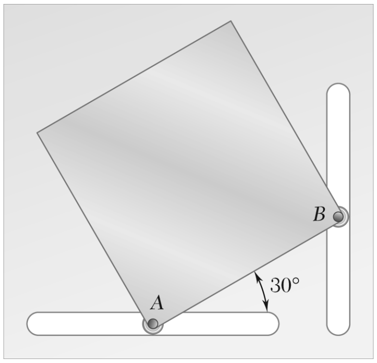
Data: $r = 12 \text{ in.} = 1 \text{ ft}, \quad R = 36 \text{ in.}, \quad \rho = 48 \text{ in.}$

$$b = \frac{2}{45}(12) = 0.53333 \text{ in.}$$

$$\left(\frac{r^2}{\rho} - b\right) = \frac{144}{48} - 0.53333 = 2.4667 \text{ in.} = 0.20556 \text{ ft}$$

$$C_y = 15 \text{ lb} - \frac{15 \text{ lb}}{32.2 \text{ ft/s}^2}(0.20556 \text{ ft})(5 \text{ rad/s})^2$$

$\mathbf{C} = 12.61 \text{ lb} \uparrow$ ◀

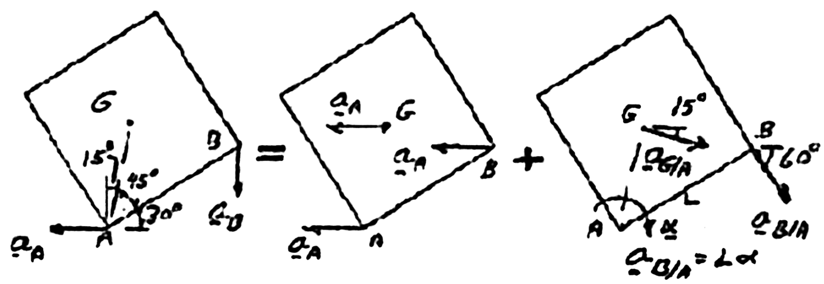


PROBLEM 16.162

The motion of a square plate of side 150 mm and mass 2.5 kg is guided by pins at corners A and B that slide in slots cut in a vertical wall. Immediately after the plate is released from rest in the position shown, determine (a) the angular acceleration of the plate, (b) the reaction at corner A.

SOLUTION

<u>Kinematics</u>:
$$AG = \frac{L}{2}\sqrt{2} = \frac{L}{\sqrt{2}} \qquad a_{G/A} = (AB)\alpha = \frac{L\alpha}{\sqrt{2}}$$

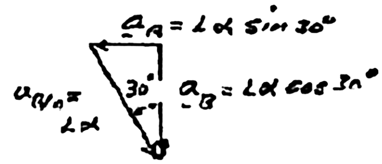


<u>Plane motion</u> = <u>Translation</u> + <u>Rotation</u>

$$a_B\downarrow = a_A \longleftarrow + a_{B/A} \nwarrow 60°$$

$$a_B\downarrow = a_A \longleftarrow + L\alpha \nwarrow 60°$$

$$\bar{a} = a_A \longleftarrow + a_{G/A} \nwarrow 15°$$



$$\bar{\mathbf{a}} = [L\alpha \sin 30° \longleftarrow] + \left[\frac{L\alpha}{\sqrt{2}} \nwarrow 15°\right] = [0.5L\alpha \longleftarrow] + [0.707L\alpha \nwarrow 15°]$$

<u>Law of cosines</u>

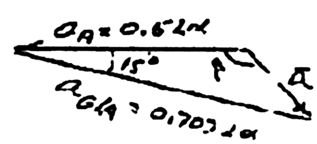
$$\bar{a}^2 = a_A^2 \cdot a_{G/A}^2 - 2a_A a_{G/A}\cos 15°$$

$$\bar{a}^2 = (0.5L\alpha)^2 + (0.707L\alpha)^2$$
$$-2(0.5L\alpha)(0.707L\alpha)\cos 15°$$

$$\bar{a}^2 = L^2\alpha^2(0.25 + 0.5 - 0.68301)$$

$$a^2 = L^2\alpha^2(0.06699)$$

$$\bar{a} = 0.25882L\alpha$$



<u>Law of sines</u>.

$$\frac{\bar{a}}{\sin 15°} = \frac{a_{G/A}}{\sin\beta}; \quad \sin\beta = \frac{a_{G/A}}{\bar{a}}\sin 15° = \frac{0.707L\alpha}{0.25950L\alpha}\sin 15°$$

$$\sin\beta = 0.707; \quad \beta = 135°$$

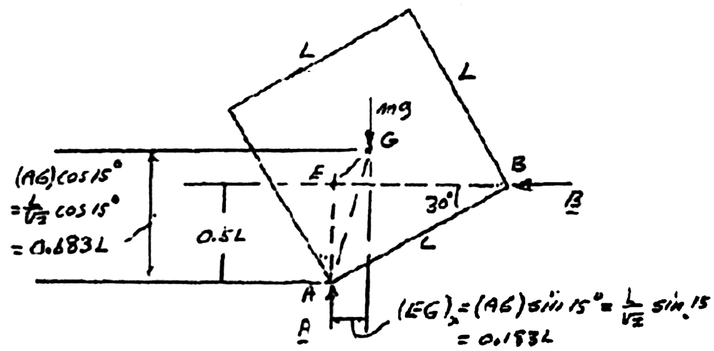
$$\bar{\mathbf{a}} = 0.2583L\alpha \searrow 45°$$

<u>Kinetics</u>: $(\omega = 0)$

We find the location of Point E where lines of action of A and B intersect.

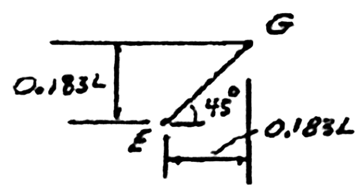
$$MG = \frac{L}{\sqrt{2}}$$

$$\measuredangle EAG = 15°$$

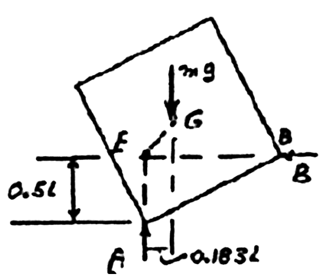


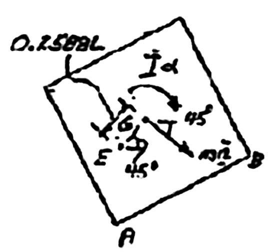
$$(EG)_y = 0.6829L - 0.5L = 0.183L$$

$$(EG) = (0.183L)\sqrt{2} = 0.2588L$$



$$\bar{I} = \frac{1}{6}mL^2$$





(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad mg(0.183L) = \bar{I}\alpha + (m\bar{a})(0.2588L)$$

$$0.183mgL = \frac{1}{6}mL^2\alpha + m(0.2588L\alpha)(0.2588L)$$

$$0.183gL = L^2\alpha\left(\frac{1}{6} + 0.06698\right)$$

$$0.183\frac{g}{L} = 0.2336\alpha; \quad \alpha = 0.7834\frac{g}{L}$$

$$\alpha = 0.7834\frac{9.81 \text{ m/s}^2}{0.15 \text{ m}} \qquad \boldsymbol{\alpha} = 51.2 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(*b*) Reaction at corner *A*.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A - mg = -m\bar{a}\sin 45°$$

$$= -m(0.2588L\alpha)\sin 45°$$

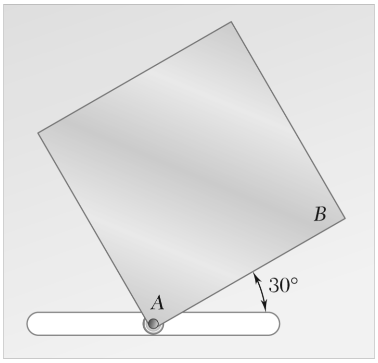
$$= -m(0.2588L)\left(0.7834\frac{g}{L}\right)\sin 45°$$

$$A - mg = 0.1434mg$$

$$A = 0.8566mg$$

$$= 0.8566(2.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 21.01 \text{ N} \qquad \mathbf{A} = 21.0 \text{ N}\uparrow \blacktriangleleft$$

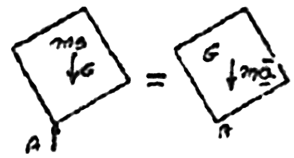


PROBLEM 16.163

Solve Problem 16.162, assuming that the plate is fitted with a single pin at corner *A*.

Problem 16.162 The motion of a square plate of side 150 mm and mass 2.5 kg is guided by pins at corners *A* and *B* that slide in slots cut in a vertical wall. Immediately after the plate is released from rest in the position shown, determine (*a*) the angular acceleration of the plate, (*b*) the reaction of corner *A*.

SOLUTION

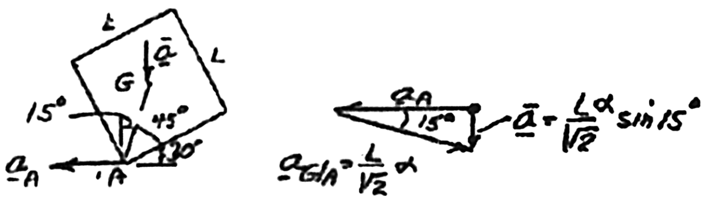


Since both *A* and *mg* are vertical, $\bar{a}_x = 0$ and $\bar{\mathbf{a}}$ is $\downarrow$

Kinematics:

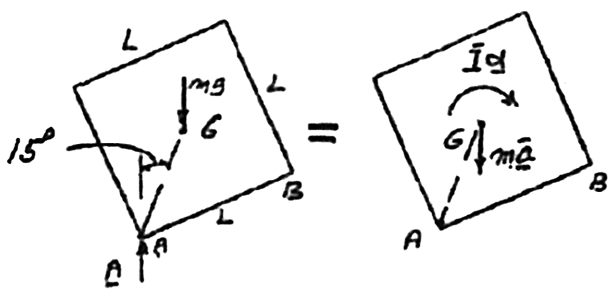
$$AG = \frac{L}{\sqrt{2}} \measuredangle 15^\circ \quad \mathbf{a}_{G/A} = (AG)\alpha \searrow 15^\circ$$

$$\bar{\mathbf{a}}\downarrow = \mathbf{a}_A \longleftarrow + \mathbf{a}_{G/A} \searrow 15^\circ$$



$$\bar{\mathbf{a}} = 0.183L\alpha \searrow 15^\circ$$

Kinetics: $\bar{I} = \frac{1}{6}mL^2$



PROBLEM 16.163 (Continued)

(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad mg(AG)\sin 15° = \bar{I}\alpha + m\bar{a}(AG)\sin 15°$$

$$mg\left(\frac{L}{\sqrt{2}}\right)\sin 15° = \frac{1}{6}mL^2\alpha + m(0.183L\alpha)\left(\frac{L}{\sqrt{2}}\right)\sin 15°$$

$$0.183\frac{g}{L} = \left(\frac{1}{6} + 0.033494\right)\alpha$$

$$0.183\frac{g}{L} = 0.2002\alpha$$

$$\alpha = 0.9143\frac{g}{L}$$

$$= 0.9143\frac{9.81 \text{ m/s}^2}{0.15 \text{ m}}$$

$\boldsymbol{\alpha} = 59.8 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) Reaction at corner *A*.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A - mg = -m\bar{a}$$

$$A - mg = -m(0.183L\alpha)$$

$$= -m(0.183L)\left(0.9143\frac{g}{L}\right)$$

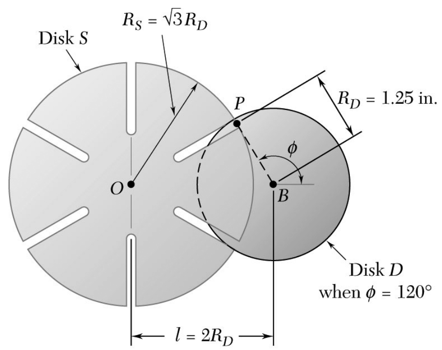
$$A - mg = -0.1673mg$$

$$A = 0.8326mg$$

$$A = 0.8326(2.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$\mathbf{A} = 20.4 \text{ N}\uparrow$ ◀

PROBLEM 16.164

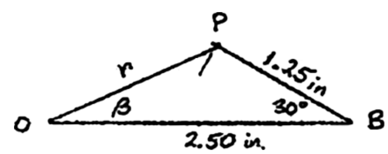


The Geneva mechanism shown is used to provide an intermittent rotary motion of disk *S*. Disk *D* weighs 2 lb and has a radius of gyration of 0.9 in. and disk *S* weighs 6 lb and has a radius of gyration of 1.5 in. The motion of the system is controlled by a couple **M** applied to disk *D*. A pin *P* is attached to disk *D* and can slide in one of the six equally spaced slots cut in disk *S*. It is desirable that the angular velocity of disk *S* be zero as the pin enters and leaves each of the six slots; this will occur if the distance between the centers of the disks and the radii of the disks are related as shown. Knowing disk *D* rotates with a constant counterclockwise angular velocity of 8 rad/s and the friction between the slot and pin *P* is negligible, determine when $\phi = 150°$ (*a*) the couple **M**, (*b*) the magnitude of the force pin *P* applies to disk *S*.

SOLUTION

Geometry:

Law of cosines.
$$r^2 = 1.25^2 + 2.50^2 - (2)(1.25)(2.50)\cos 30°$$
$$r = 1.54914 \text{ in.}$$



Law of sines.
$$\frac{\sin\beta}{1.25} = \frac{\sin 30°}{r}$$
$$\beta = 23.794°$$

Let disk *S* be a rotating frame of reference.

$$\mathbf{\Omega} = \omega_S \circlearrowright, \quad \dot{\mathbf{\Omega}} = \alpha_S \circlearrowright$$

Motion of coinciding Point P′ on the disk.

$$\mathbf{v}_{P'} = r\omega_S = 1.54914\omega_S \searrow \beta$$

$$\mathbf{a}_{P'} = -\alpha_S \mathrm{k} \times \mathrm{r}_{P/O} - \omega_S^2 \mathrm{r}_{P/O} = [1.54914\alpha_S \searrow \beta] + [1.54914\omega_S^2 \swarrow \beta]$$

Motion relative to the frame.

$$\mathbf{v}_{P/S} = u \swarrow \beta \qquad \mathrm{a}_{P/S} = \dot{u} \swarrow \beta$$

Coriolis acceleration. $\qquad 2\omega_S u \searrow \beta$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/S} = [1.54914\omega_S \searrow \beta] + [u \swarrow \beta]$$

$$\mathbf{a}_P = \mathbf{a}_P + \mathbf{a}_{P/S} + 2\omega_S u \searrow$$

$$= [1.54914\alpha_S \searrow \beta] + [1.54914\omega_S^2 \swarrow \beta] + [\dot{u} \swarrow \beta] + [2\omega_S u \searrow \beta]$$

PROBLEM 16.164 (Continued)

Motion of disk D. (rotation about B)

$$\mathbf{v}_P = (BP)\omega_D = (1.25)(8) = 10 \text{ in./s} \swarrow 30^\circ$$

$$\mathbf{a}_P = [(BP)\alpha_D \swarrow 60^\circ] + [(BP)\omega_S^2 \searrow 30^\circ] = 0 + [(1.25)(8)^2 \searrow 30^\circ]$$

$$= 80 \text{ in./s}^2 \searrow 30^\circ$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\searrow \beta: \quad 1.54914\omega_S = 10\cos(30^\circ + \beta)$$

$$\omega_S = \frac{10\cos 53.794^\circ}{1.54914}$$

$$= 3.8130 \text{ rad/s}$$

$$\swarrow \beta: \quad u = 10\sin(30^\circ + \beta) = 10\sin 53.794^\circ = 8.0690 \text{ in./s}$$

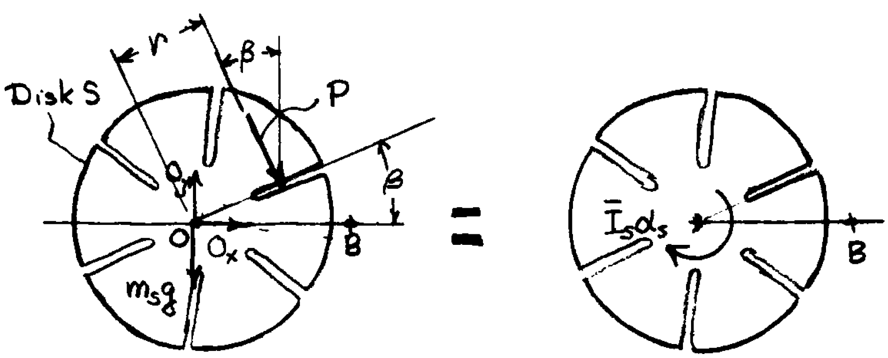
Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

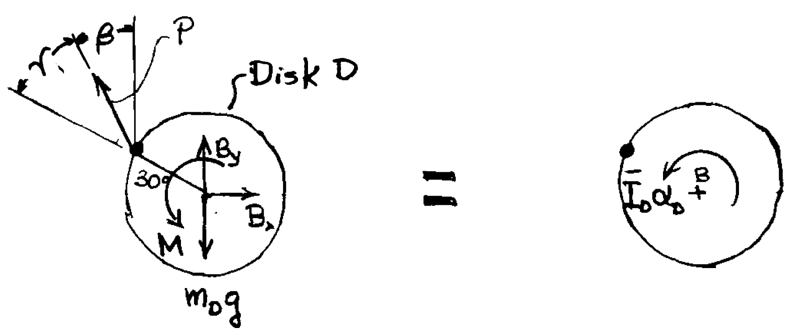
$$\searrow \beta: \quad 1.54914\alpha_S - 2\omega_S u = 80\sin(30^\circ + \beta)$$

$$\alpha_S = \frac{80\sin 53.794^\circ + (2)(3.8130)(8.0690)}{1.54914}$$

$$= 81.391 \text{ rad/s}^2$$

Kinetics:





$$\bar{I}_s\alpha_s = \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)\left(\frac{1.5}{12}\text{ ft}\right)^2(81.391\text{ rad/s}^2) = 0.23697\text{ lb}\cdot\text{ft}$$

$$\bar{I}\alpha_D = 0 \qquad \text{since} \quad \boldsymbol{\alpha}_D = 0$$

Disk S:

$$\Sigma M_0 = \Sigma M_{\text{eff}}: \quad Pr = \bar{I}_S\alpha_S$$

$$P\left(\frac{1.54914}{12}\text{ ft}\right) = 0.23697\text{ lb}\cdot\text{ft} \qquad P = 1.8356\text{ lb}$$

Disk D:

$$r = 90° - 30° - \beta = 36.206°$$

$$+\Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad M - (P\sin r)\left(\frac{1.25}{12}\right)\text{ft}$$

$$M = \frac{(1.25)(1.8356\sin 36.206°)}{12} = 0.11294\text{ lb}\cdot\text{ft}$$

(*a*) *Couple* **M**. $\mathbf{M} = 1.355\text{ lb}\cdot\text{in}$ ◀

(*b*) *Magnitude of contact force*. $P = 1.836\text{ lb}$ ◀