Ch.11 : Displacement Method

{ force Method / flexibility Method }
flexibility: deformation due to unit load

{ Displacemend method / Stiffness Method }
Stiffness: force due to unit deformation

- Displacement method :

simplifications
- slope - deflection equations
- Moment distribution method

} flextural systems

- General procedure:-

- Direct Stiffness Method $\{F\} = [K]\{D\}$
- Finit Element Method

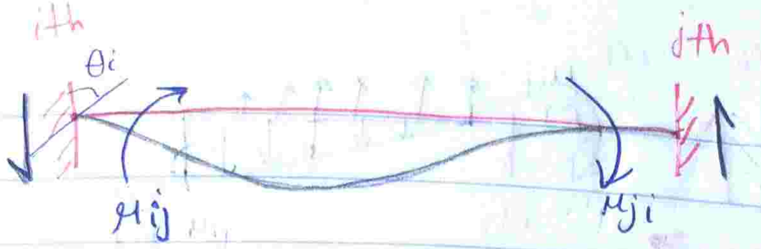
- Slope - deflection Equations:-

→ to derive stiffness cofficients (Beam element)

Kinematically det.

<force method>
. Static indet.:- unknown forces

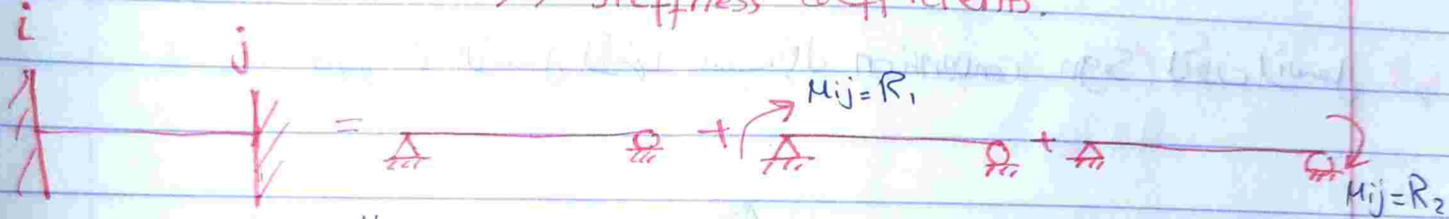
. Kinematic indet. :-
unknown deformations
<deformation method>



- M_{ij} , M_{ji} :- end-moment.

  - $M_{ij} = \boxed{}\ \theta_i$
  - $M_{ji} = \boxed{}\ \theta_i$

    ↳ steffness coefficients.



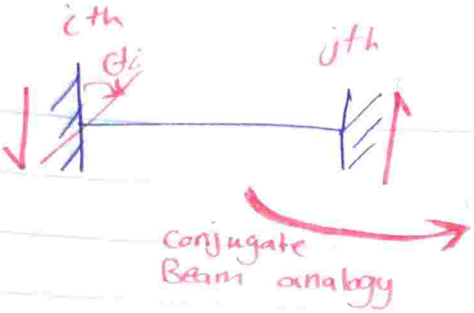
- rotational Sett at i^{th} θ_i cw
- compatability Equ. at i :

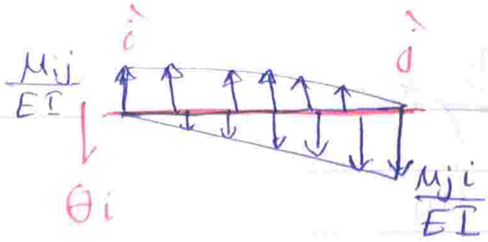
$$= D_{10} + R_1 d_{11} + R_2 d_{12}$$

- compatabilit Equ. at j :

$$0 = D_{20} + R_1 d_{21} + R_2 d_{22}$$

or



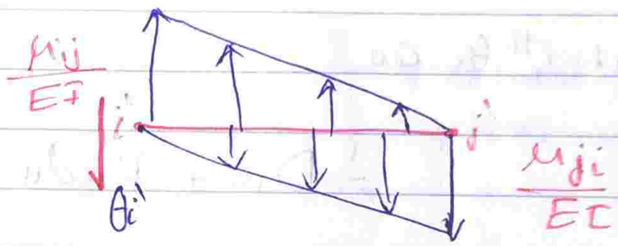


Real Beam — Conjugate Beam

→ θ C.C.W — Shear (+)

→ Δ ↑ — Moment (+)

C.W نضع θ_i للأسفل لأنها حسب ال(sign convention) تكون للأسفل لأنها

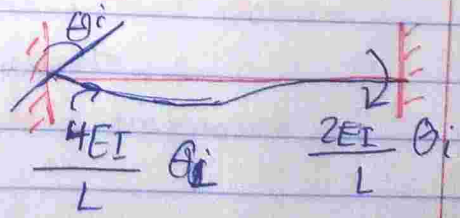


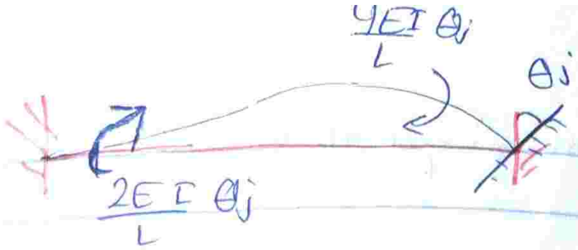
- $\Sigma M_i = 0 = \frac{1}{2}\left(\frac{M_{ij}}{EI}\right)(L)\left(\frac{L}{3}\right) - \frac{1}{2}\left(\frac{M_{ji}}{EI}\right)L\left(\frac{2}{3}L\right)$

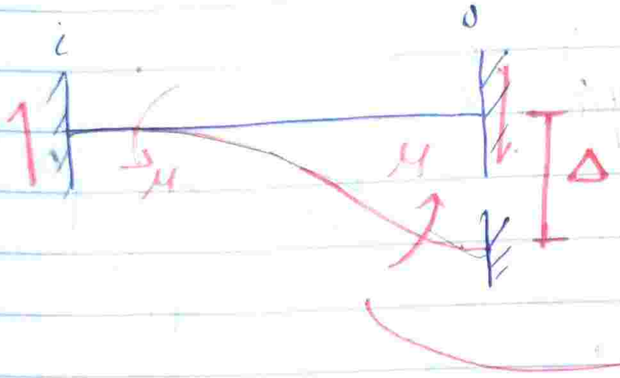
- $M_{ji} = \frac{1}{2} M_{ij}$

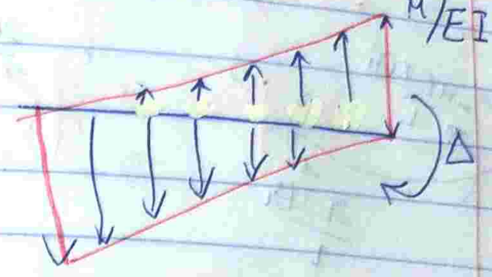
- $M_j' = 0 = \frac{1}{2}\left(\frac{M_{ij}}{EI}\right)(L)\left(\frac{2}{3}L\right) - \frac{1}{2}\left(\frac{M_{ji}}{EI}\right)(L)\left(\frac{L}{3}\right) - \theta_i(L)$

- $M_{ij} = \frac{4EI}{L}\theta_i$
- $M_{ji} = \frac{2EI}{L}\theta_i$



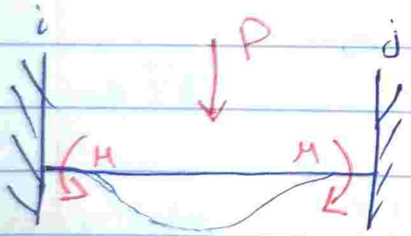






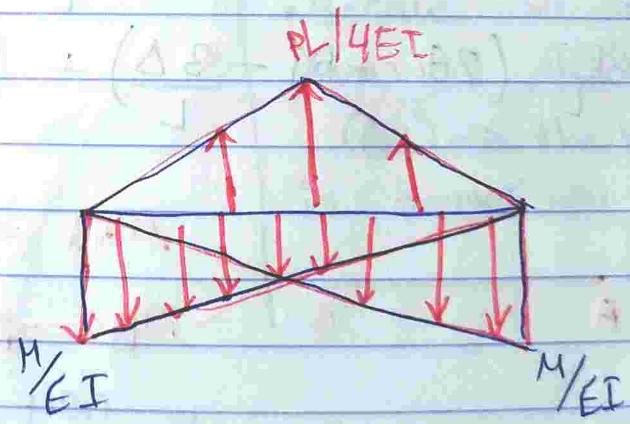
Conjugat Beam anology

- $\Sigma M_j' = 0$
- $M = \dfrac{6EI}{L^2}\Delta$



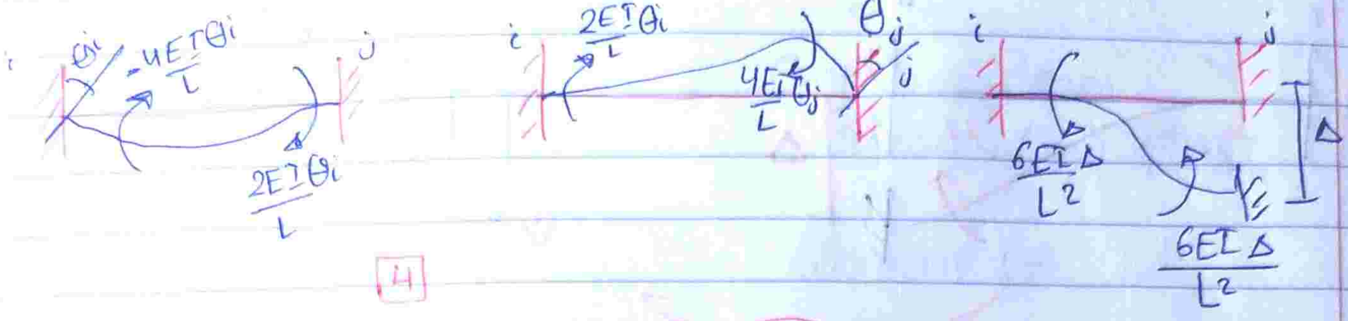
$\Sigma F_y = 0$

$M = PL/8$



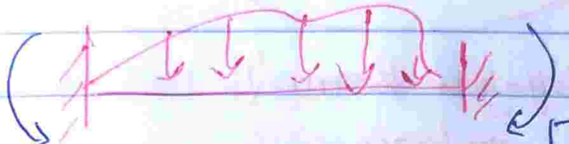
$\rightarrow M_{ij} = (\quad)\theta_i + (\quad)\theta_i + (\quad)\Delta +$ Moment applied load

• slope-deflection Equations



[4]

• fixed-end moment :-



⇒ Equation for the End moment M_{ij}

CW +

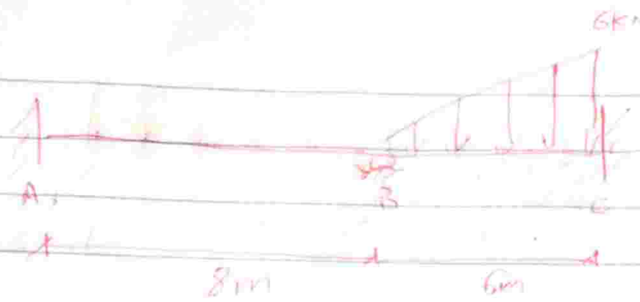
CCW −

$$M_{ij} = \frac{4EI}{L}\theta_i + \frac{2EI}{L}\theta_j - \frac{6EI}{L^2}\Delta + FEM_{ij}$$

$$M_{ij} = \frac{2EI}{L}\left(2\theta_i + \theta_j - \frac{3\Delta}{L}\right) + FEM_{ij}$$

$M_{AB} \Rightarrow i = A, j = B$

$M_{BA} \Rightarrow i = B, j = A$



- Draw BM diagram Indicating Key Values
- Sketch Deformed Shape

① divide the system into discrete element

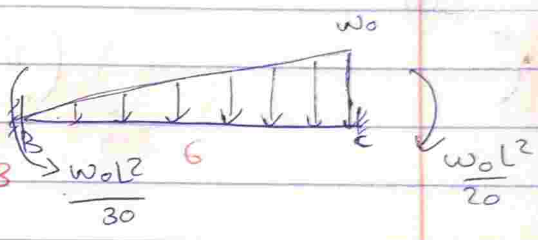
- we have two elements AB + BC

② write end-moment equations

zero → fixed support; zero → No diffrential stettlement

- $M_{AB} = \frac{2EI}{8}\left(2\cancel{\theta_A} + \theta_B - \cancel{\frac{3\Delta_{AB}}{8}}\right) + \cancel{FEM_{AB}}$
- $M_{BA} = \frac{2EI}{8}\left(2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\Delta_{AB}}{8}}\right) + \cancel{FEM_{BA}}$

- $M_{BC} = \frac{2EI}{6}\left(2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\Delta_{BC}}{6}}\right) + FEM_{BC}$
- $M_{CB} = \frac{2EI}{6}\left(2\cancel{\theta_C} + \theta_B - \cancel{\frac{3\Delta_{BC}}{6}}\right) + FEM_{CB}$

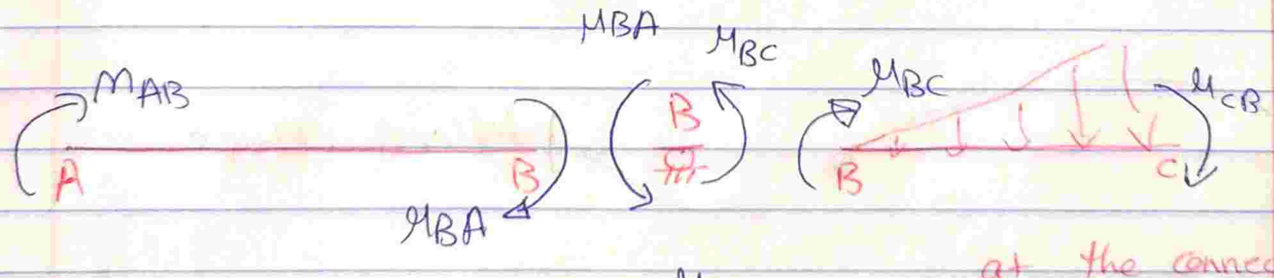


→ $FEM_{BC} = - \frac{6(6)^2}{30} =$

→ $FEM_{CB} = + \frac{6(6)^2}{20} =$

⇒ Solve for $\boxed{\theta_B}$

- Becaus we don't knew 1 deformation (θ_B) → the system is kinmetically Ind.

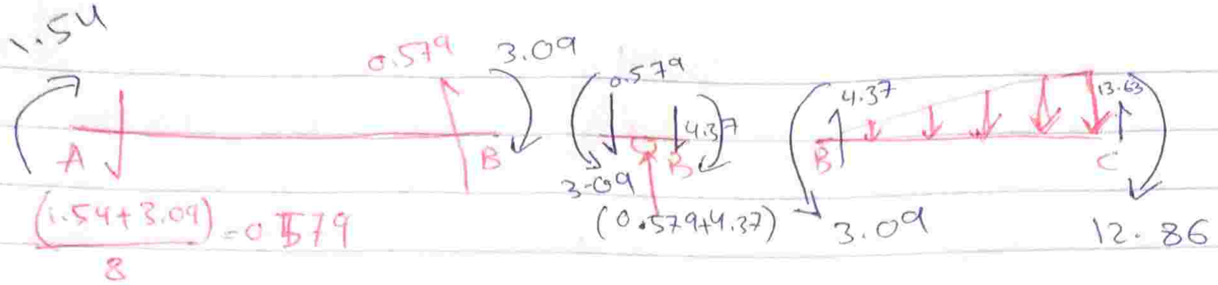


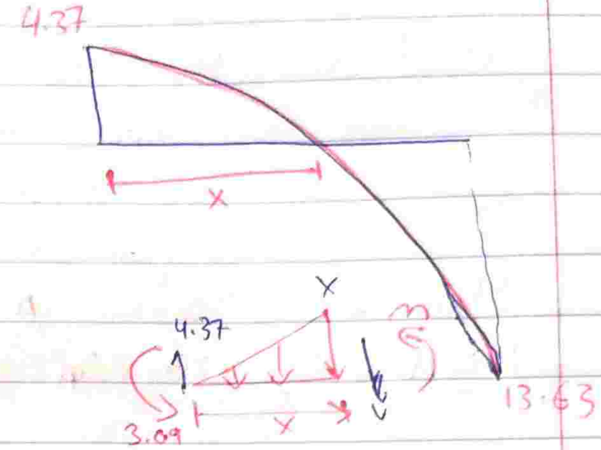
- $\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0$ ⇒ at the connected joint

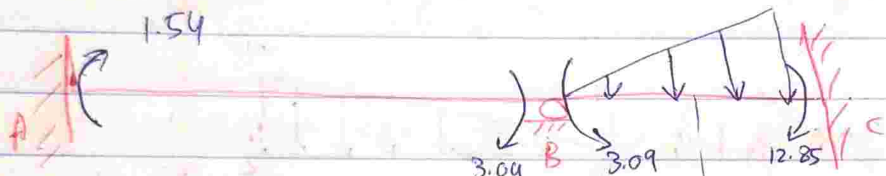
$$\frac{4EI}{8}\theta_B + \frac{4EI}{6}\theta_B - 7.2 = 0$$

$$\theta_B = 6.17/EI$$

- $M_{AB} = 1.54$ kN.m
- $M_{BA} = 3.09$ kN.m
- $M_{BC} = -3.08$
- $M_{CB} = 12.85$







- $\Sigma F_y = 0$

$$4.37 - \tfrac{1}{2}(x)(x) = 0$$

$$4.37 = x^2/2$$

$$X = 2.96\ m$$

- $\Sigma M_{cut} = 0$

$$m = 5.5$$