

## 8.1 Population Mean when $\sigma$ is known

(100)

- Recall that the sample mean  $\bar{x}$  is the point estimator for the population mean  $\mu$ .
- $\bar{x}$  can not be expected to provide the exact estimate of  $\mu$ .

$\Rightarrow$  An Interval Estimate of a population Mean when  $\sigma$  is known is used as

Point estimate  $\pm$  Margin of error

$$\bar{x} \pm Z_{\alpha/2} \sigma_{\bar{x}} = \left[ \bar{x} - Z_{\alpha/2} \sigma_{\bar{x}}, \bar{x} + Z_{\alpha/2} \sigma_{\bar{x}} \right]$$

confidence interval

where  $(1 - \alpha)$  is the confidence coefficient

- $Z_{\alpha/2}$  is the z value providing an area of  $\frac{\alpha}{2}$  in the upper tail of the standard normal prob. distribution

- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$  is the standard error.



Values of  $Z_{\alpha/2}$  for common used confidence levels:

| Confidence Level | $\alpha$ | $\frac{\alpha}{2}$ | $Z_{\alpha/2}$ |
|------------------|----------|--------------------|----------------|
| 90 %             | 0.10     | 0.05               | 1.645          |
| 95 %             | 0.05     | 0.025              | 1.96           |
| 99 %             | 0.01     | 0.005              | 2.576          |

2.576  $\rightarrow$  2.575

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Example: A simple random sample of 40 items resulted in a sample mean of 25. The population standard deviation is  $\sigma = 5$

[a] what is the standard error of the mean?

$$n = 40, \bar{x} = 25$$

$$\sigma = 5$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{40}} = 0.79$$

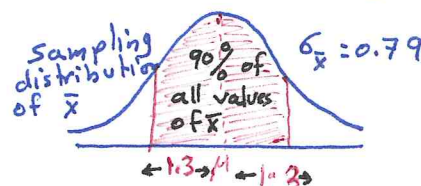
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[b] At 95 % confidence, what is the margin of error? Uploaded By: Jibreel Bornat

$$\text{The margin of error} = Z_{\alpha/2} \sigma_{\bar{x}} = (1.96)(0.79) = 1.55$$

[c] Provide a 90 % confidence interval for the population mean?

$$\bar{x} \pm Z_{\alpha/2} \sigma_{\bar{x}} = 25 \pm (1.645)(0.79) = 25 \pm 1.3 = [23.7, 26.3]$$



lol

Sampling distribution of  $\bar{x}$

95% of all values of  $\bar{x}$

$\bar{x} = 0.79$

$z = 1.55 \rightarrow M = 1.55$

[e] Provide a 99% confidence interval for the population Mean?

99% of all values of  $\bar{x}$

$\mu$

2 2

Q. what is the relation between the confidence level and

- 
- 90%
- 23.7 26.3
- 95%
- 23.45 26.55
- 99%
- 23 27

9] What is the relationship between the confidence level and the margin of error?

✓ Positive relationship

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h) what is the relationship between the sample size and the confidence interval?

As  $n \uparrow \Rightarrow \frac{\sigma}{\sqrt{n}} \downarrow \Rightarrow$  the margin of error  $= z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \downarrow$   
 $\Rightarrow$  the confidence interval  $\downarrow$

As  $n \downarrow \Rightarrow$  the confidence interval  $\uparrow$

- Interval estimate (or Confidence interval): is an estimate of a population parameter that provides an interval believed to contain the value of the parameter. It has the form: point estimate  $\pm$  margin of error.
- Margin of error: The  $\pm$  value added and subtracted from a point estimate in order to develop an interval estimate of a population parameter.
- $\sigma$  known: The case when historical data or other information provides a good value for the population standard deviation  $\sigma$  prior taking a sample. The interval estimation uses this known value of  $\sigma$  in computing the margin of error.
- Confidence level: The confidence associated with an interval estimate. 90%, 95%, 99%.
- Confidence coefficient: The confidence level expressed as decimal value 0.90, 0.95, 0.99