# Equilibrium of Rigid Bodies

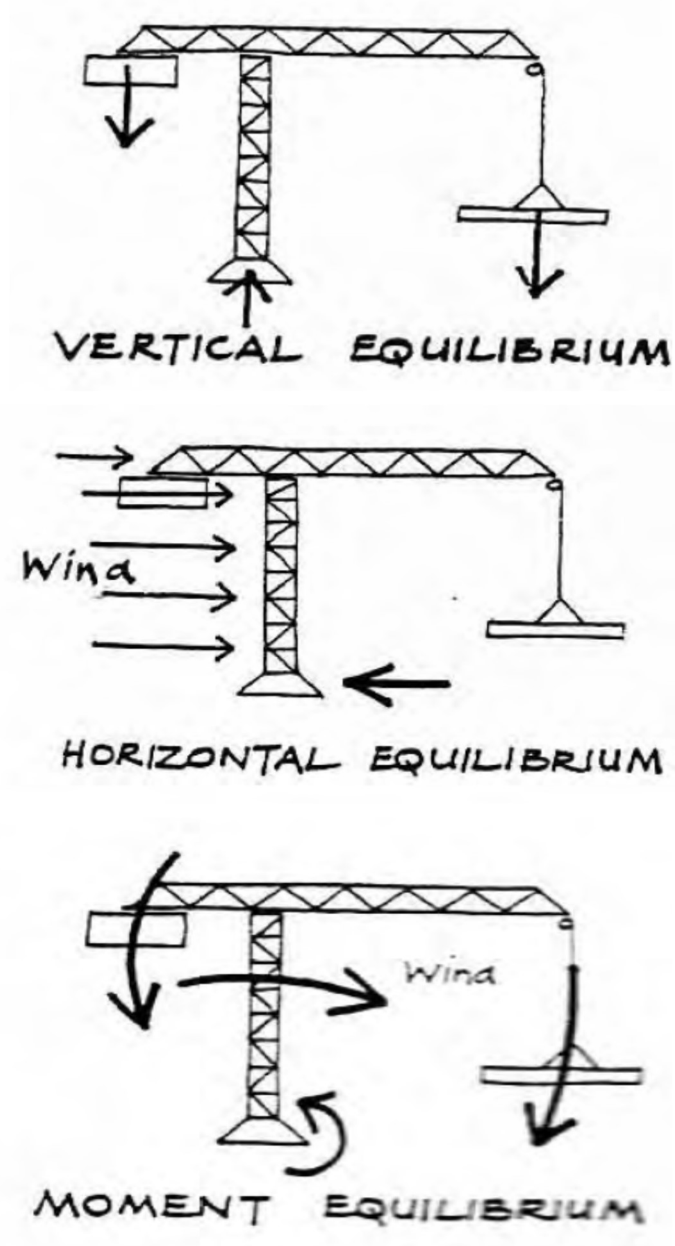
## Chapter 4

# Objectives

- Analyze the static equilibrium of rigid bodies in two and three dimensions.
- Consider the attributes of a properly drawn free-body diagram, an essential tool for the equilibrium analysis of rigid bodies.
- Examine rigid bodies supported by statically indeterminate reactions and partial constraints.
- Study two cases of particular interest: the equilibrium of two force and three-force bodies.

## Introduction

- For a rigid body, the condition of static equilibrium means that the body under study does not translate or rotate under the given loads that act on the body.
- On real-life structures, this can be achieved if the body is supported by a proper supporting system that can provide for counteracting forces (reactions) to the applied loads so that the body remains at rest.
- So what shall the equilibrium equations look like?



# Equations of Equilibrium

$$\sum \boldsymbol{Forces} = \mathbf{0}$$

**2D**

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_z = 0$$

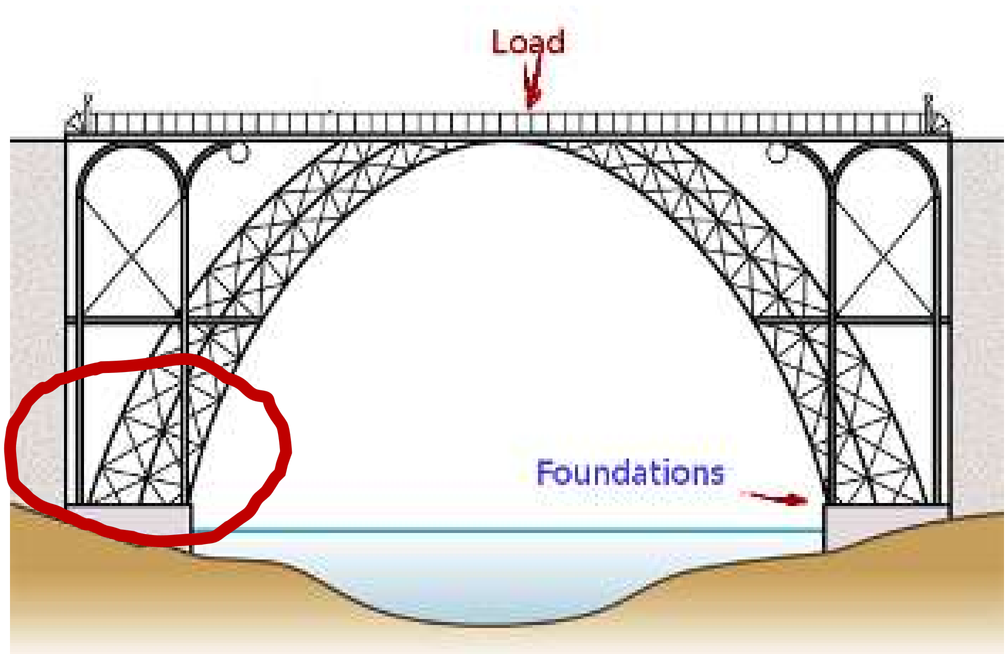
**3D**

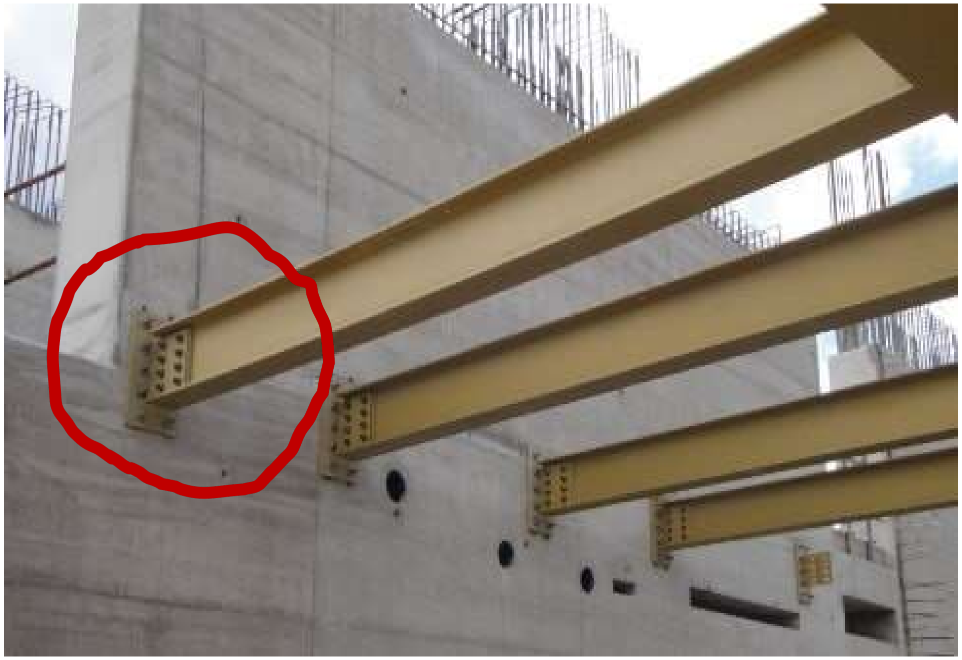
$$\sum F_x = 0 \qquad \sum M_x = 0$$

$$\sum F_y = 0 \qquad \sum M_y = 0$$

$$\sum F_z = 0 \qquad \sum M_z = 0$$

# Supporting Systems

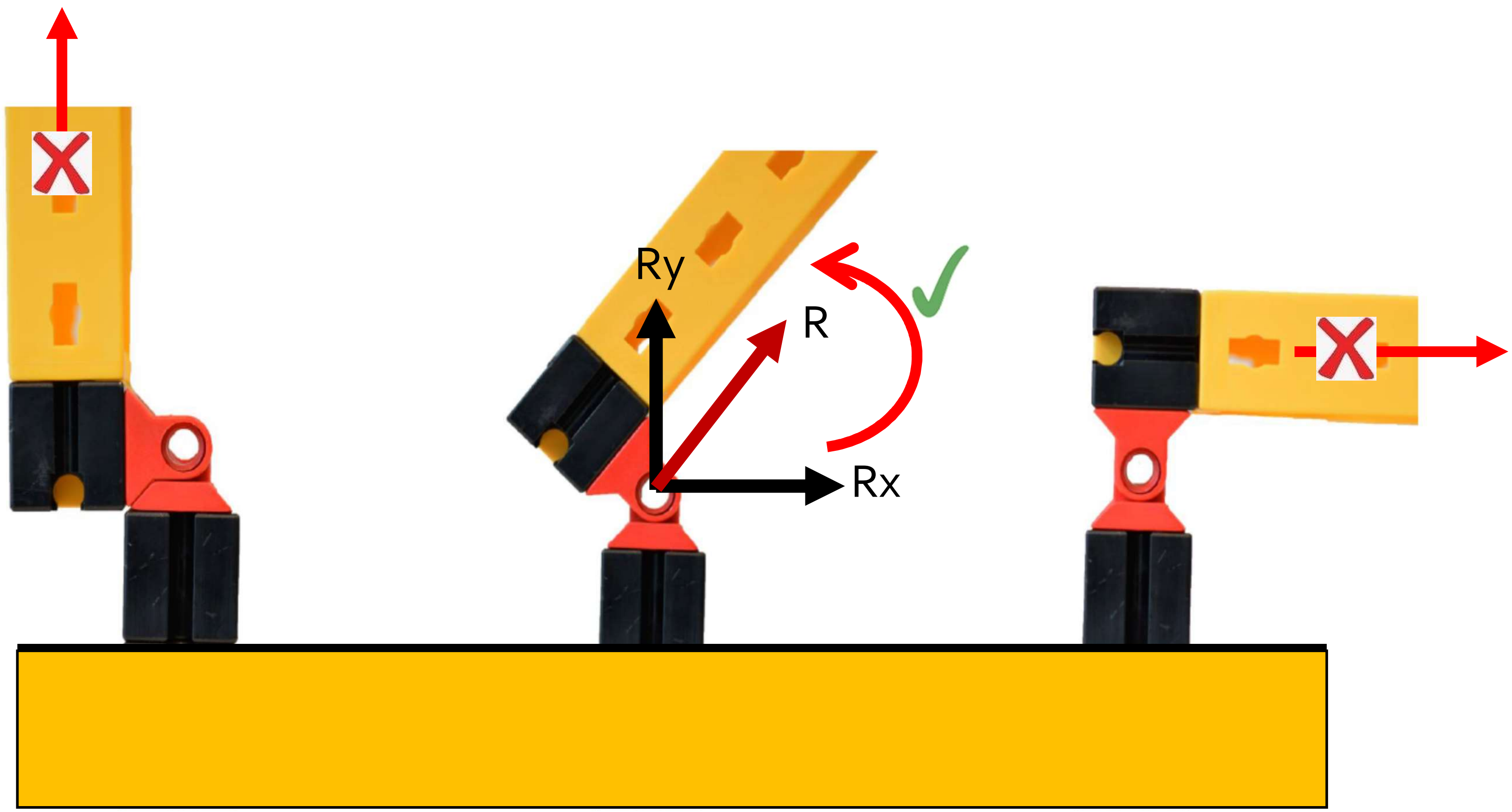


- There are several forms of the supporting system that can be used to provide the reactions necessary to achieve equilibrium.
- Different forms of supporting system produce different type of reactions

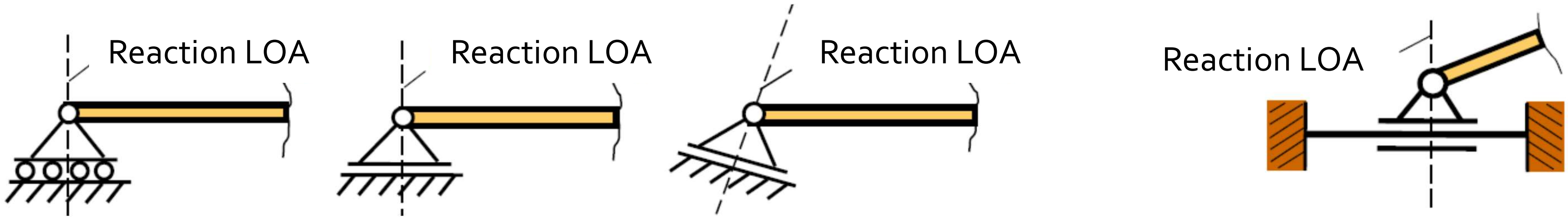
# Supporting Systems

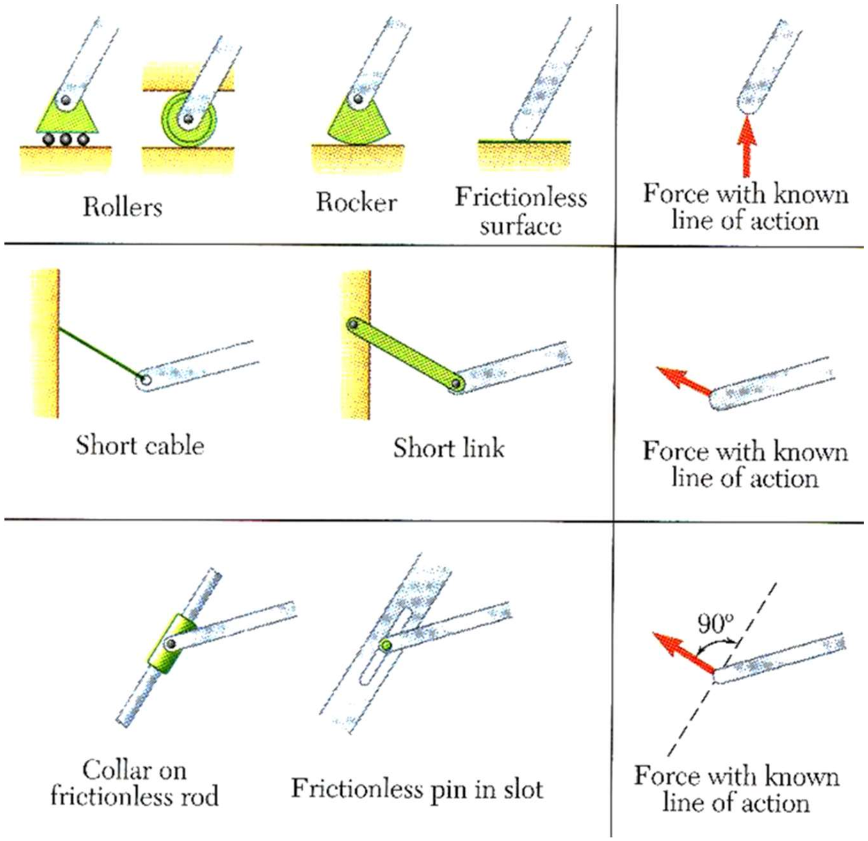
- The reactions for a particular support may be determined by considering the motion the support prevents.



# 4.1A Reactions for a Two-Dimensional Structure

1. Reactions equivalent to a force with known line of action



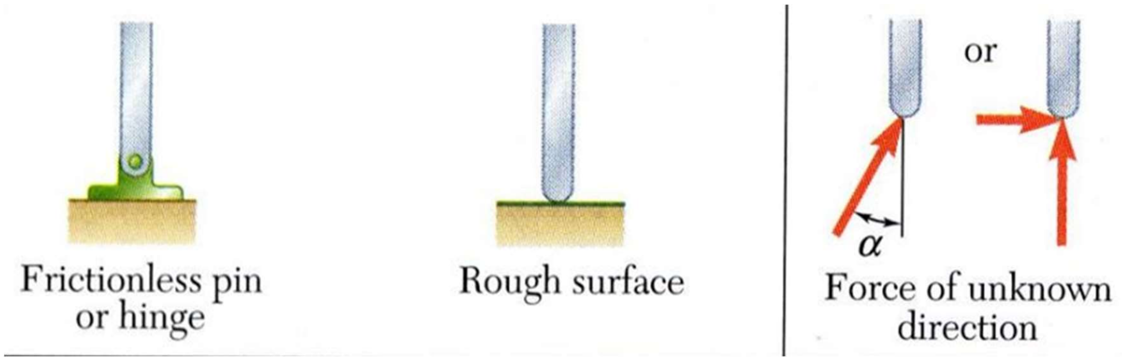




This rocker bearing supports the weight of a bridge. The convex surface of the rocker allows the bridge to move slightly horizontally.

Force applied to the slider exerts a normal force on the rod, causing the window to open.

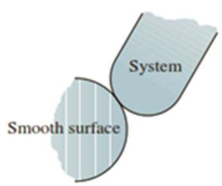
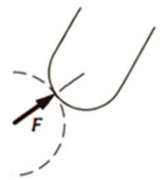
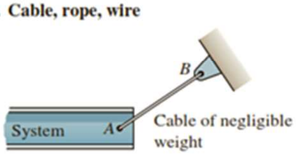
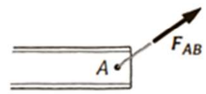
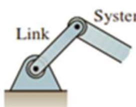
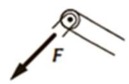
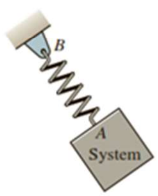
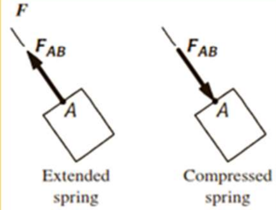
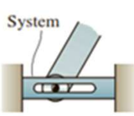
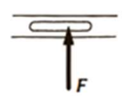
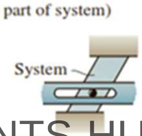
# 4.1A Reactions for a Two-Dimensional Structure

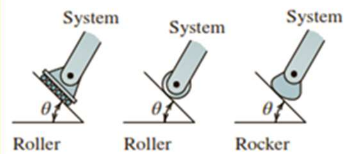
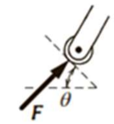
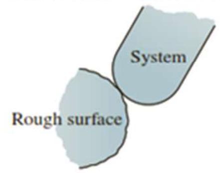
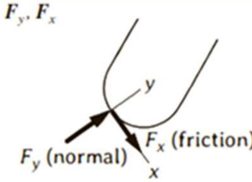
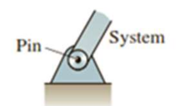
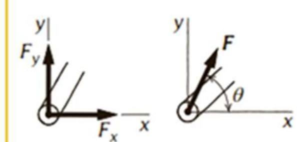
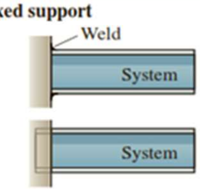
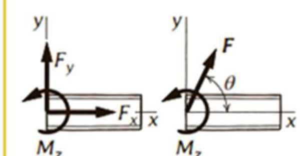
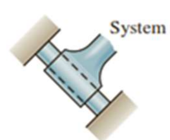
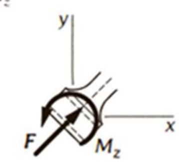
2. Reactions equivalent to a force of unknown direction and magnitude.



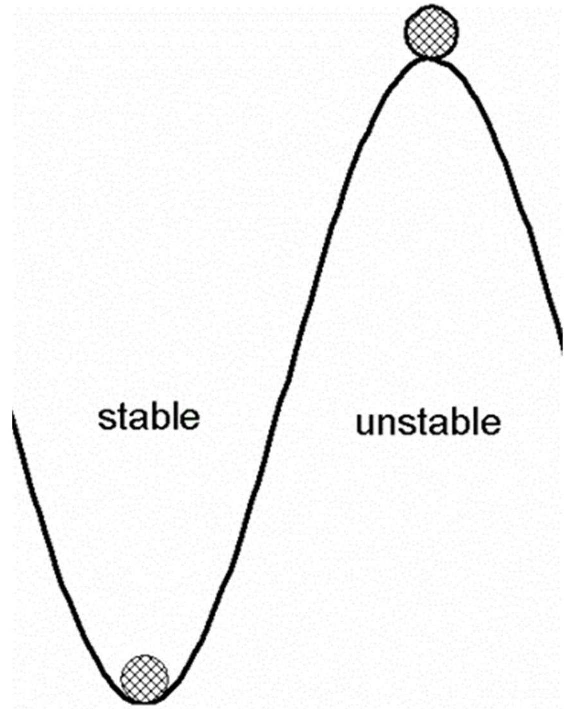

3. Reactions equivalent to a force of unknown direction and magnitude and a couple of unknown magnitude


# 4.1A Reactions for a Two-Dimensional Structure

| (A) Supports | Description of Loads | (B) Loads to Be Shown on Free-Body Diagram |
|---|---|---|
| **1. Normal contact without friction**   | **Force** (*F*) oriented normal to surface on which system rests. Direction is such that force pushes on system. | *F*   |
| **2. Cable, rope, wire**   | **Force** (*F*) oriented along cable. Direction is such that cable pulls on the system. | *F*   |
| **3. Link**   | **Force** (*F*) oriented along link length; force can push or pull on the system. | *F*   |
| **4. Spring**   | **Force** (*F*) oriented along long axis of spring. Direction is such that spring pulls on system if spring is in tension, and pushes if spring is in compression. | *F*   |
| **5. Slot-on-pin (frictionless)** (slotted member is part of system)   | **Force** (*F*) oriented normal to long axis of slot. Direction is such that force can pull or push on system. The slot is frictionless. Therefore no forces act parallel to the slot. | *F*   |
| **6. Pin-in-slot (frictionless)** (pin is part of system)   | **Force** (*F*) oriented normal to long axis of slot. Direction is such that force can pull or push on system. The slot is frictionless. Therefore no forces act parallel to the slot. | *F*   |

| (A) Supports | Description of Loads | (B) Loads to Be Shown on Free-Body Diagram |
|---|---|---|
| **7. Roller or rocker**   | **Force** (*F*) oriented normal to surface on which system rests. Direction is such that force pushes on system. | *F*   |
| **8. Normal contact with friction**   | **Two force components**, one ($F_y$) oriented normal to surface on which the system rests so as to push on system, other force ($F_x$) is tangent to surface. | $F_y$, $F_x$   |
| **9. Pin connection** (pin or hole is part of system)   | **Force** perpendicular to pin axis represented in terms of components $F_x$ and $F_y$. Point of application is at center of pin.<br><br>Alternative representation: **Force** (*F*) oriented at **angle** θ with respect to coordinate system. Point of application is at center of pin. | $F_x$, $F_y$   *F*, θ   |
| **10. Fixed support**   | **Force** in *x*-*y* plane represented in terms of components $F_x$ and $F_y$. **Moment** about *z* axis ($M_z$).<br><br>Alternative representation: **Force** (*F*) oriented at **angle** θ with respect to coordinate system. **Moment** about *z* axis ($M_z$). | $F_x$, $F_y$, $M_z$   *F*, θ, $M_z$   |
| **11. Smooth collar on smooth shaft**   | **Force** (*F*) oriented perpendicular to long axis of shaft. Direction is such that force can pull or push on system. **Moment** ($M_z$) about *z* axis. | *F*, $M_z$   |

# 4.1C Stability and Determinacy



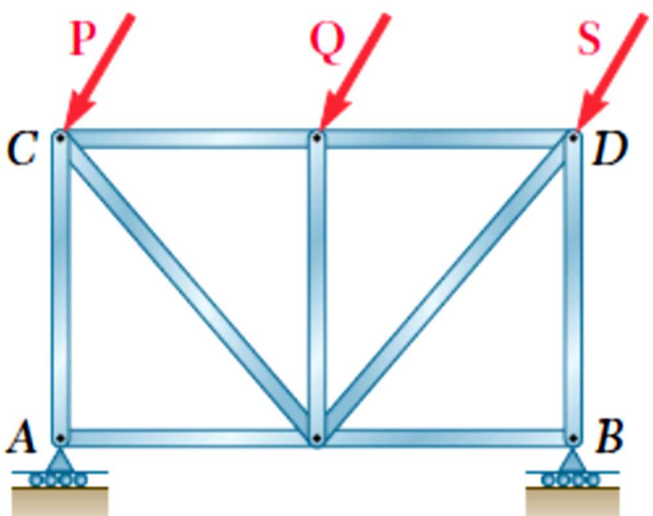

- This is equilibrium but is it sufficient?

# 4.1C Stability and Determinacy

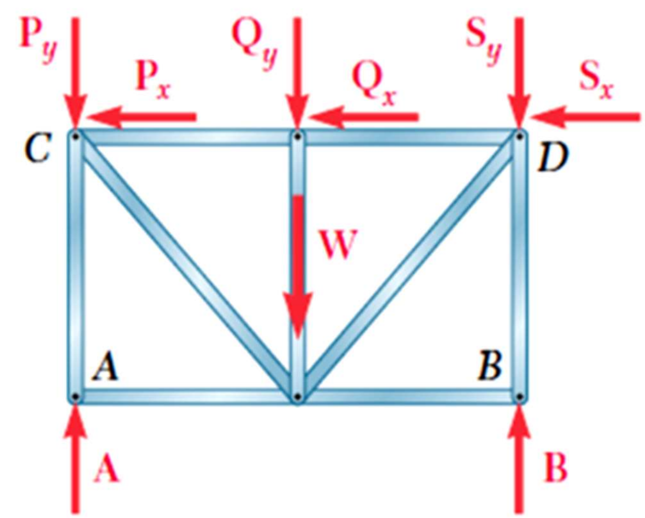
- Structures shall be in a stable equilibrium status.
- In two dimension structures we have 3 equations of equilibrium that can used to solve 3 unknowns, if R (number of reactions), then:

1. If R < 3, then we have two unknowns and three equations

→ Structure is <u>partial constraints or Unstable</u>



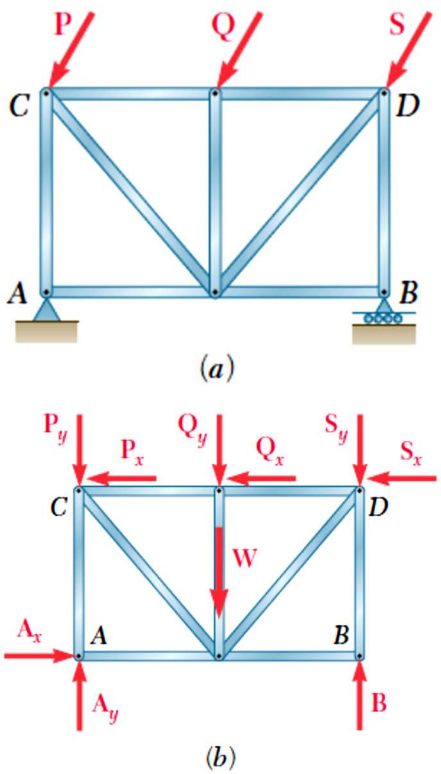

(a)

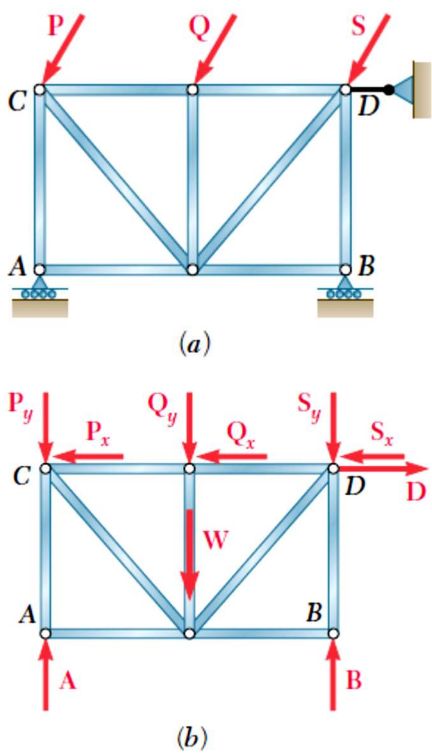



(b)

# 4.1C Stability and Determinacy

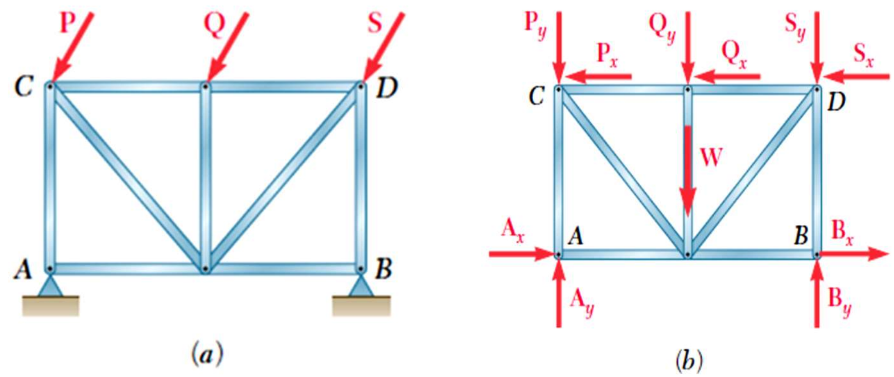
2. If R = 3 and the structure is proper restrained, the structure is stable and determinate




Three unknowns; Three equations
→ statically determinate

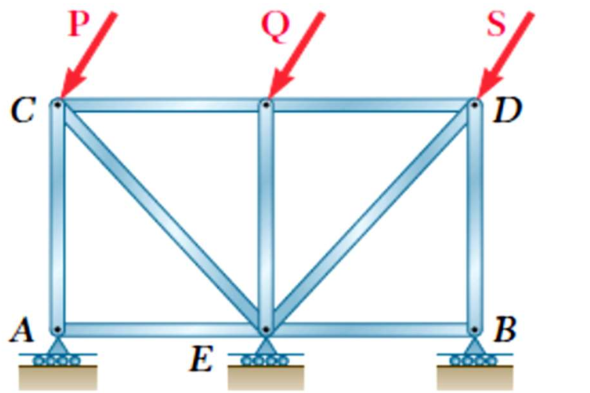
3. If R > 3 and the structure is proper restrained, the structure is stable and indeterminate



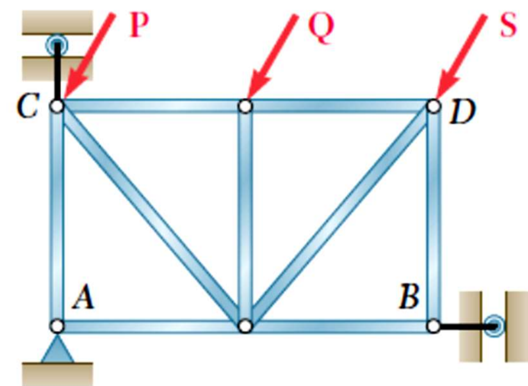

Four unknowns, Three equations
→ statically indeterminate.

# 4.1C Improper Constraints

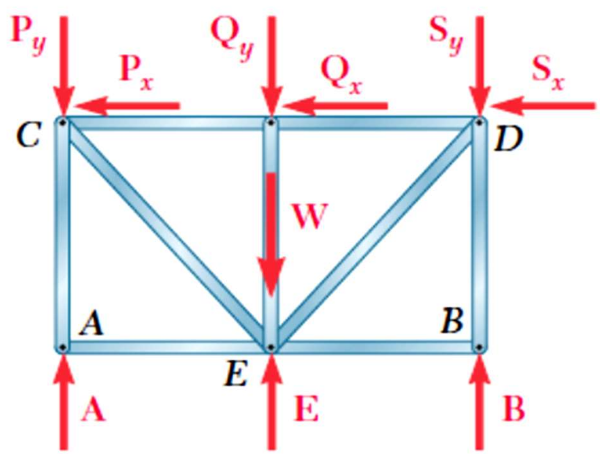
A rigid body is improperly constrained whenever the supports (even though they may provide a sufficient number of reactions) are arranged in such a way that the reactions must be either concurrent or parallel.



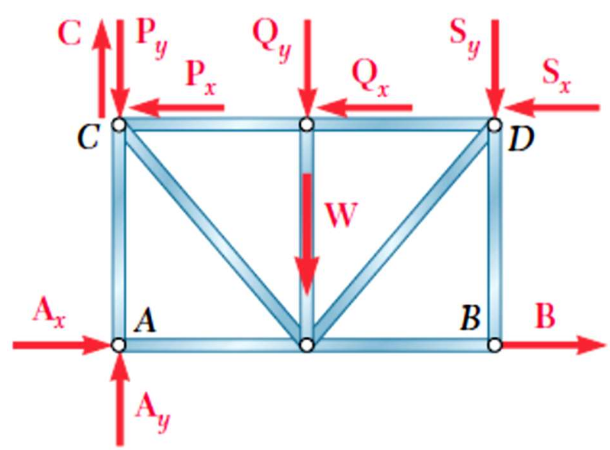

(a)




(a)

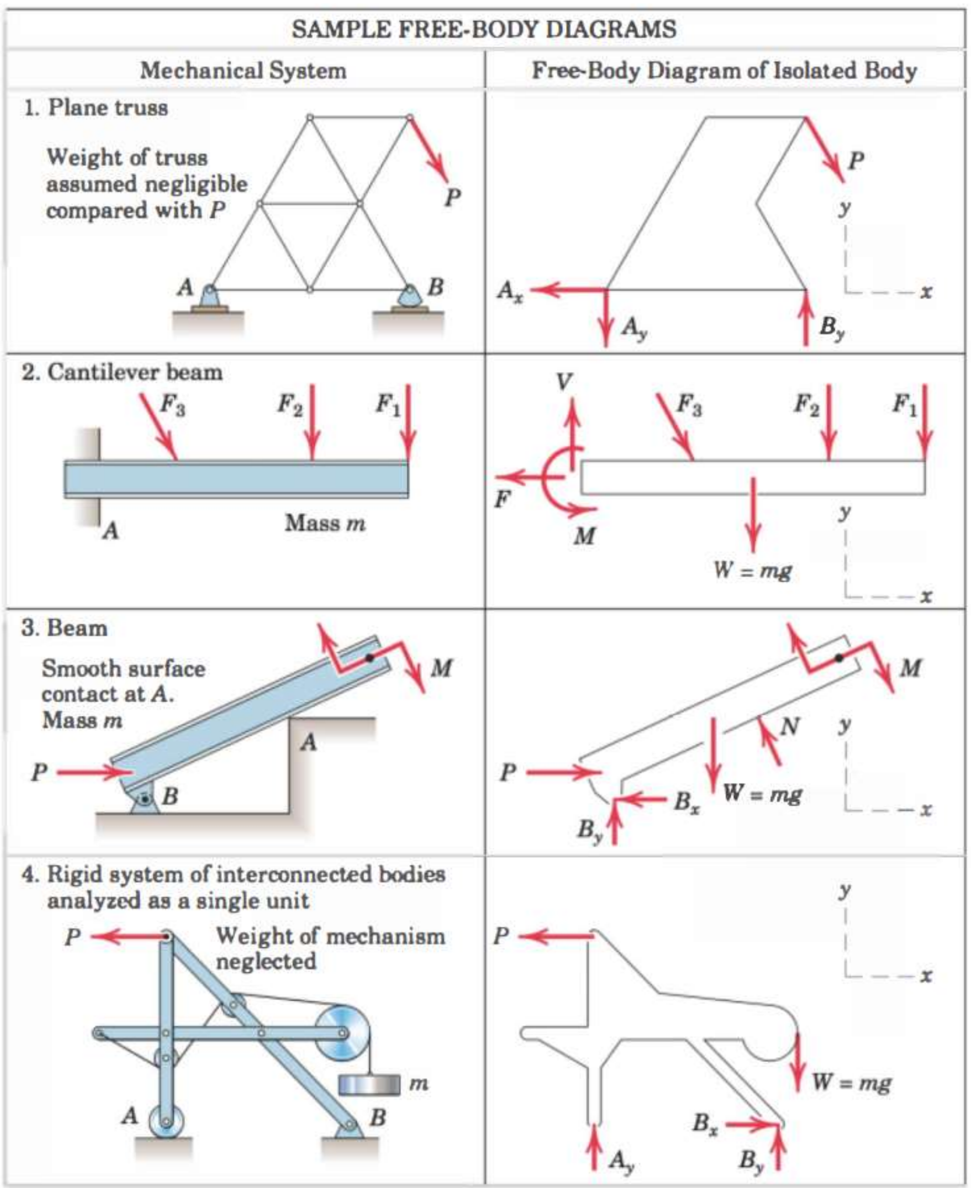



(b)




(b)

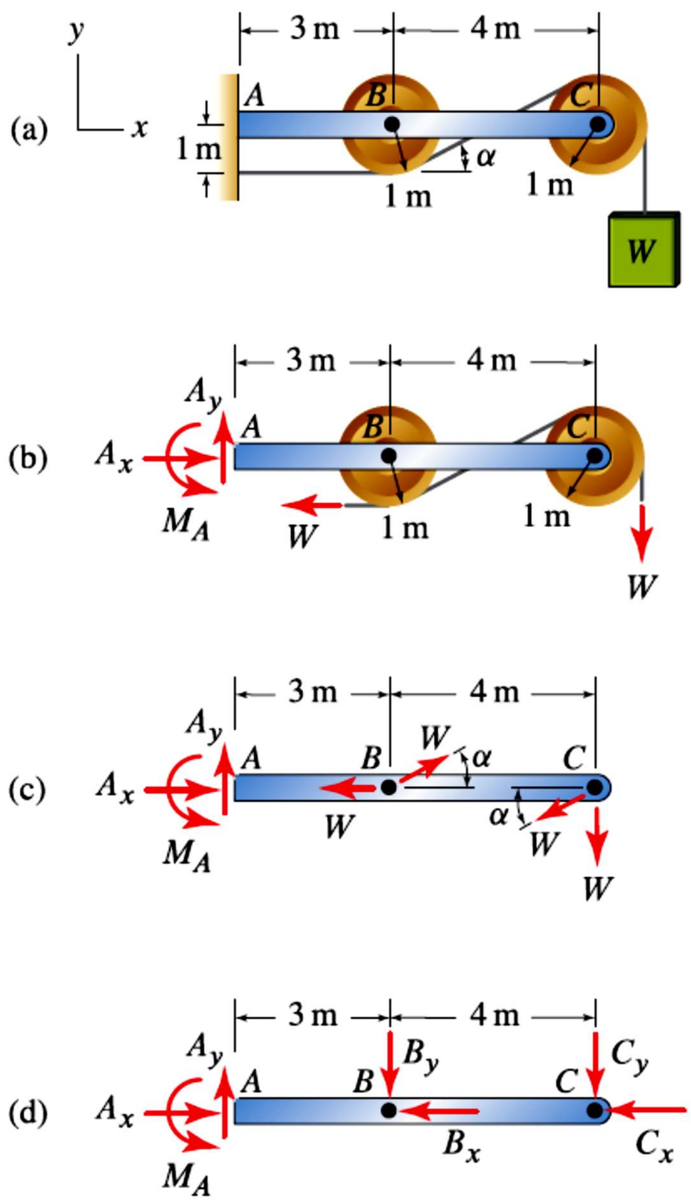
# Free-body diagram (FBD)



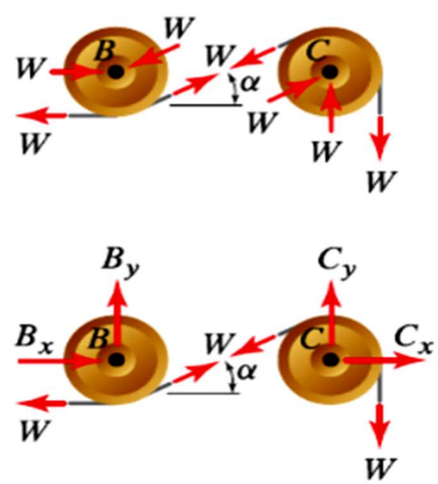

**The free-body** diagram is **the most important single step in the solution of problems** in mechanics. It aims to identify all forces acting on the body.

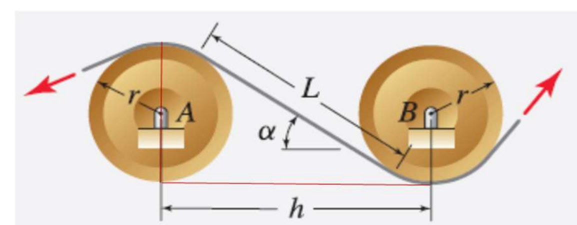
- Select the extent of the free-body and detach it from the ground and all other bodies.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown applied forces. These usually consist of reactions through which the ground and other bodies oppose the possible motion of the rigid body.
- Include the dimensions necessary to compute the moments of the forces.

# FBD of Cables and Pulleys




- Since the pulleys are idealized as frictionless and the cable is continuous, and weightless, all portions of the cable support the same tensile force which is equal to W.
- Cable forces can be transferred to pulley central pin and then to the bar as shown.
- All FBDs shown can be used to determine reactions at A. ((b) is the most useful).



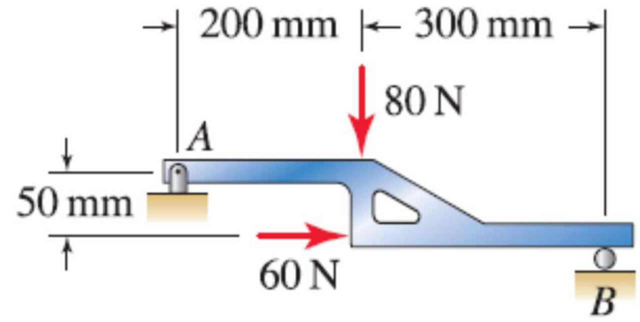




$$\alpha = \sin^{-1}\frac{2r}{h},$$

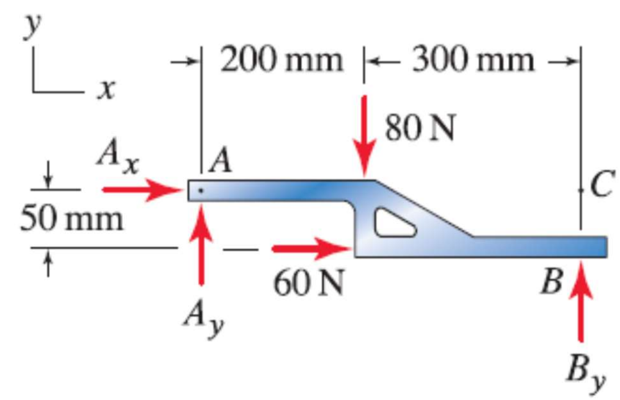
$$L = h\cos\alpha.$$

# Equilibrium problems – Important Notes

- **Coordinate system**. Indicate the problem coordinate system. While we will most often use a coordinate system whose directions are horizontal and vertical, occasionally other choices may be more convenient and will be used.

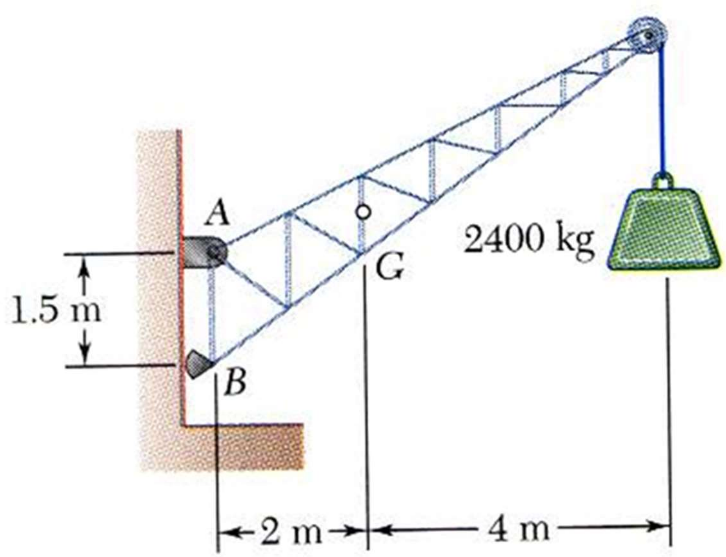



- **Direction of reactions in FBD**. When putting reaction forces and moments in the FBD, we often do not know the actual directions these forces will have until after the equilibrium equations are solved.




- **Number of unknowns**. After you draw the FBD, it is a good idea to count the number of unknowns.
- **Selection of moment summation point**. While any point can be used, it is more practical to select a point that eliminate the larger numbers of unknowns.

# Sample Problem 4.1



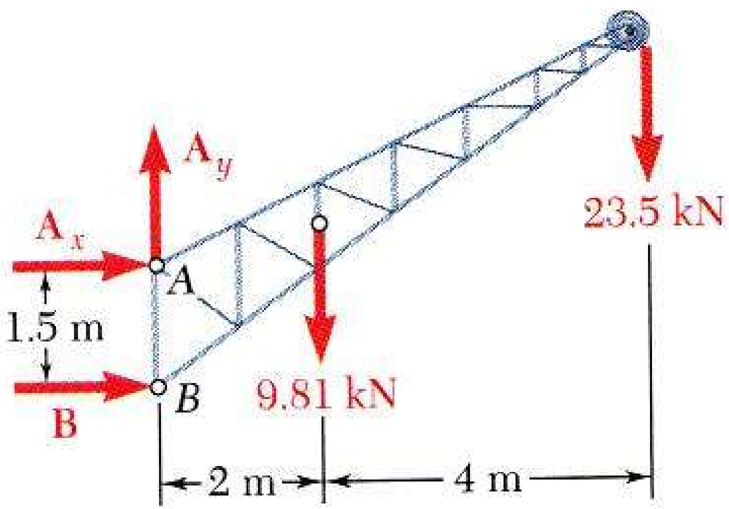

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G.

Determine the components of the reactions at A and B.

SOLUTION:

- Create a free-body diagram for the crane.
- Determine B by solving the equation for the sum of the moments of all forces about A. Note there will be no contribution from the unknown reactions at A.
- Determine the reactions at A by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about B of all forces is zero.

# Sample Problem 4.1




- Create the free-body diagram.

- Determine *B* by solving the equation for the sum of the moments of all forces about *A*.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$\boxed{B = +107.1\,\text{kN}}$$

- Determine the reactions at *A* by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\sum F_x = 0: \quad A_x + B = 0$$

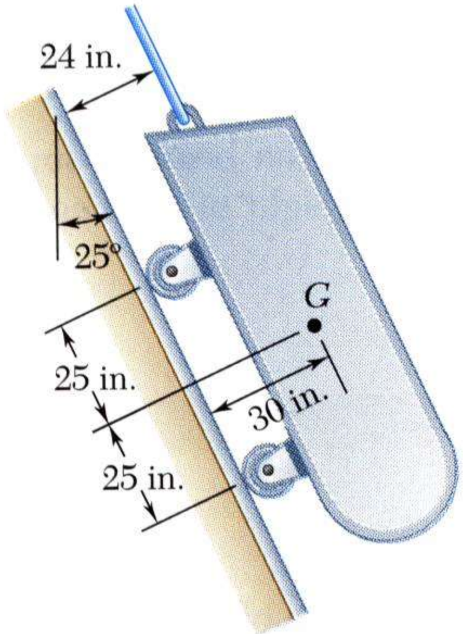
$$\boxed{A_x = -107.1\,\text{kN}}$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$\boxed{A_y = +33.3\,\text{kN}}$$

- Check the values obtained.

# Sample Problem 4.3



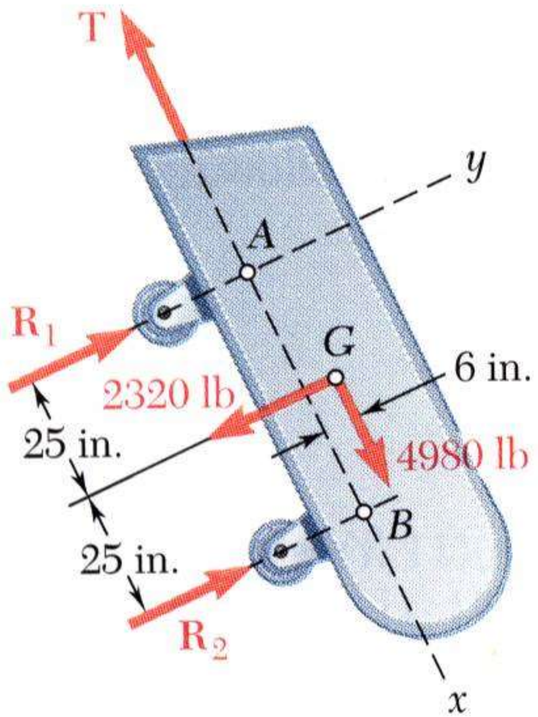

A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb, and it is applied at *G*. The cart is held in position by the cable.

Determine the tension in the cable and the reaction at each pair of wheels.

SOLUTION:

- Create a free-body diagram for the car with the coordinate system aligned with the track.
- Determine the reactions at the wheels by solving equations for the sum of moments about points above each axle.
- Determine the cable tension by solving the equation for the sum of force components parallel to the track.
- Check the values obtained by verifying that the sum of force components perpendicular to the track are zero.

# Sample Problem 4.3




- Create a free-body diagram

$$W_x = +(5500\,\text{lb})\cos 25^\circ$$
$$= +4980\,\text{lb}$$

$$W_y = -(5500\,\text{lb})\sin 25^\circ$$
$$= -2320\,\text{lb}$$

- Determine the reactions at the wheels.

$$\sum M_A = 0: \quad -(2320\,\text{lb})25\text{in.} - (4980\,\text{lb})6\text{in.} + R_2(50\text{in.}) = 0$$

$$\boxed{R_2 = 1758\,\text{lb}}$$

$$\sum M_B = 0: \quad +(2320\,\text{lb})25\text{in.} - (4980\,\text{lb})6\text{in.} - R_1(50\text{in.}) = 0$$

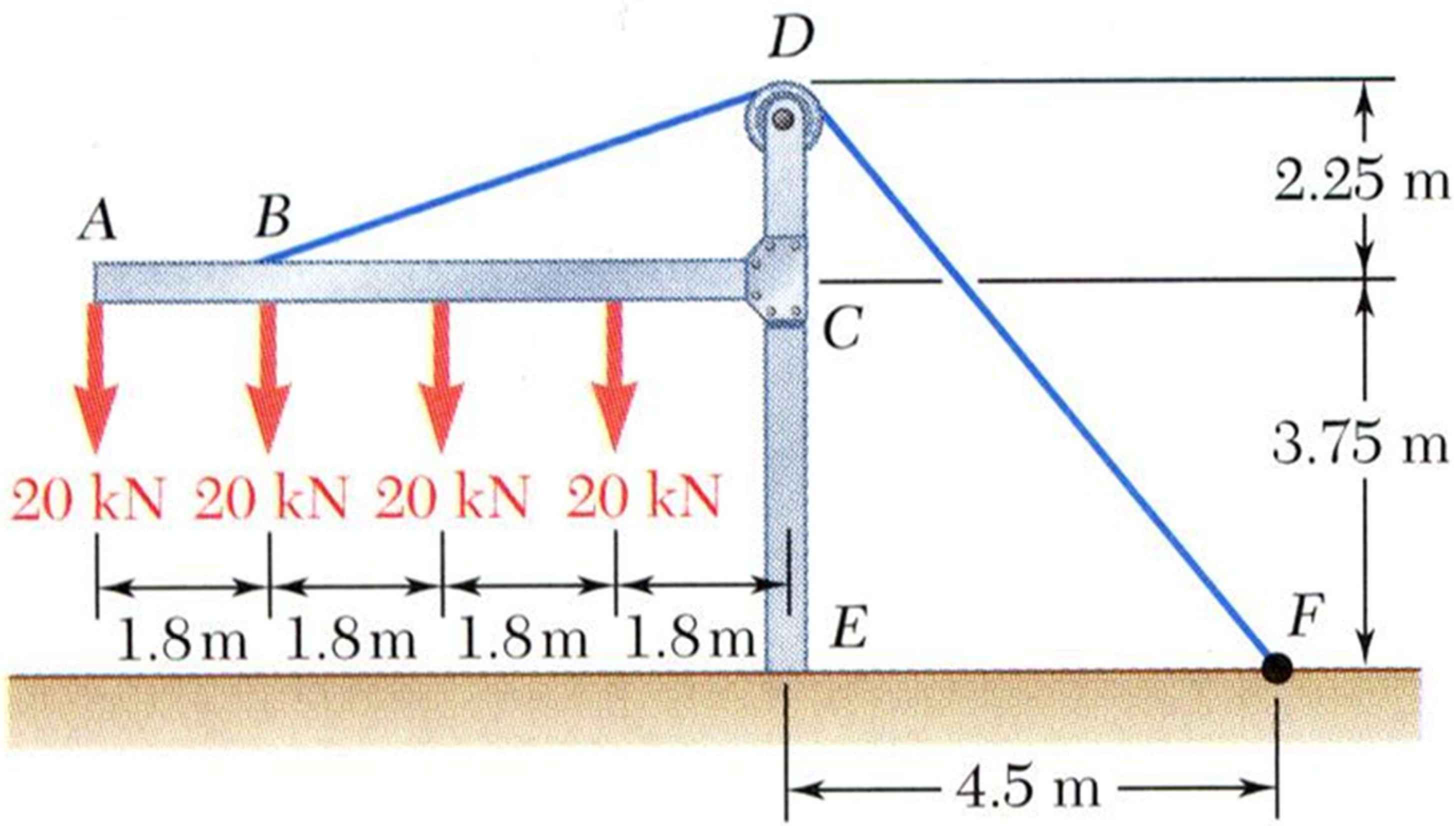
$$\boxed{R_1 = 562\,\text{lb}}$$

- Determine the cable tension.

$$\sum F_x = 0: \quad +4980\,\text{lb} - \text{T} = 0$$

$$\boxed{T = +4980\,\text{lb}}$$

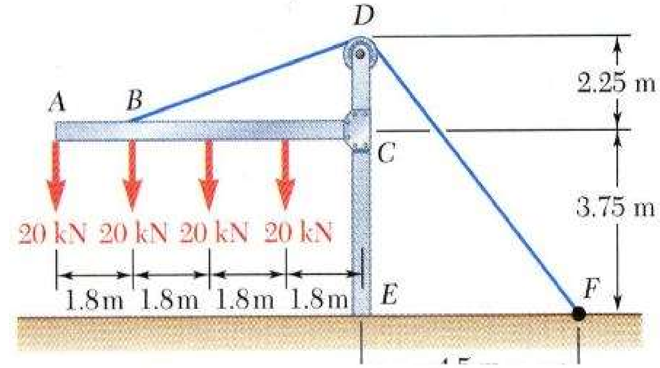
# Sample Problem 4.4



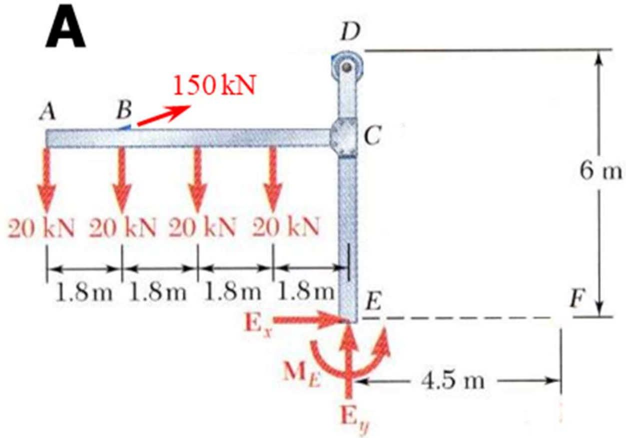

The frame shown supports part of the roof of a small building. If the tension in the cable is 150 kN. Determine the reaction at the fixed end E.

# Sample Problem 4.4

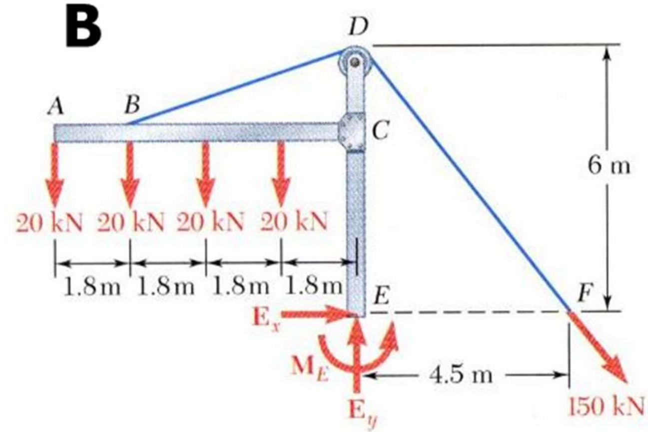
Choose the most correct FBD for the original problem.



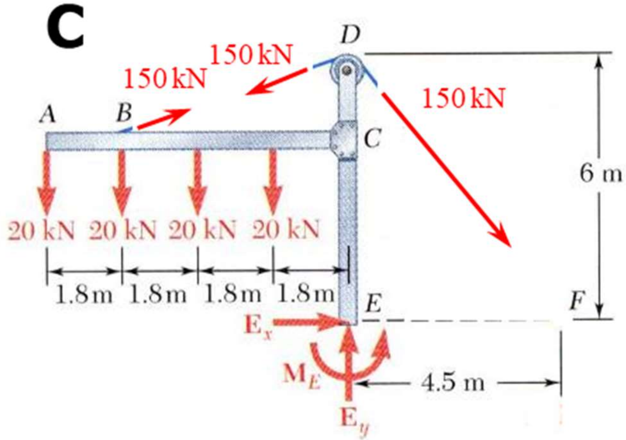

A



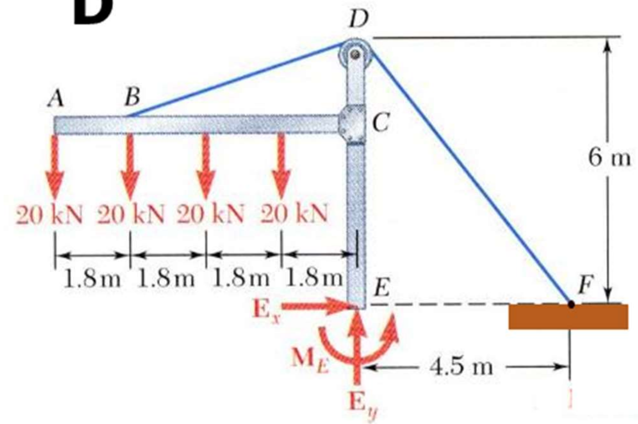

B




C




D

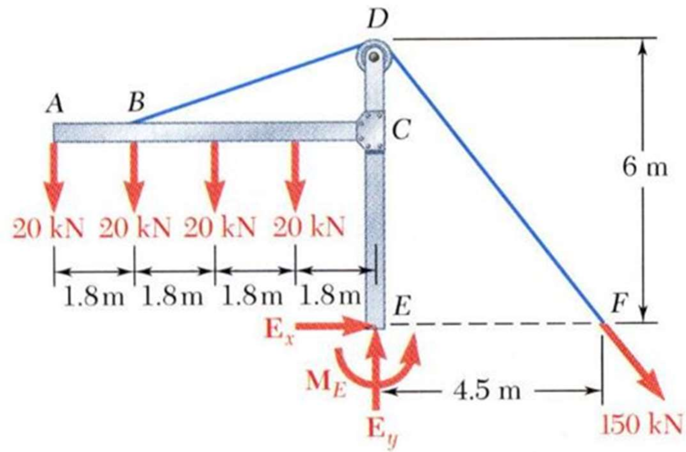



**B is the most correct, though C is also correct. A & D are incorrect; why?**

# Sample Problem 4.4

$$\sum F_x = 0: \quad E_x + \sin 36.9^o (150\text{ kN}) = 0$$

$$\boxed{E_x = -90.0\text{ kN}}$$




$$\sum F_y = 0: E_y - 4(20\text{kN}) - \cos 36.9^o (150\text{kN}) = 0$$

$$\boxed{E_y = +200\text{kN}}$$

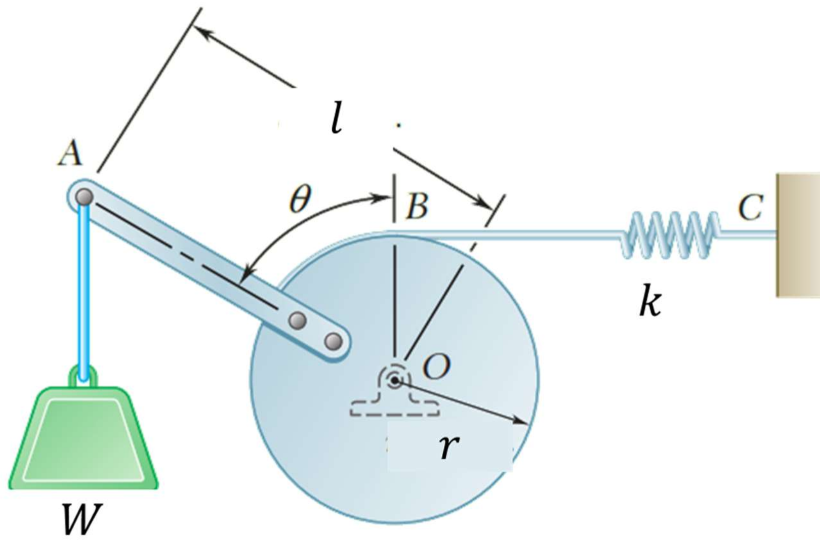
$$\sum M_E = 0: \quad +20\text{ kN}(7.2\text{ m}) + 20\text{ kN}(5.4\text{ m})$$

$$+20\text{ kN}(3.6\text{ m}) + 20\text{ kN}(1.8\text{ m})$$

$$-\frac{6}{7.5}(150\text{ kN})4.5\text{ m} + M_E = 0$$

$$\boxed{M_E = 180.0\text{ kN}\cdot\text{m}}$$

# Sample Problem 4.5

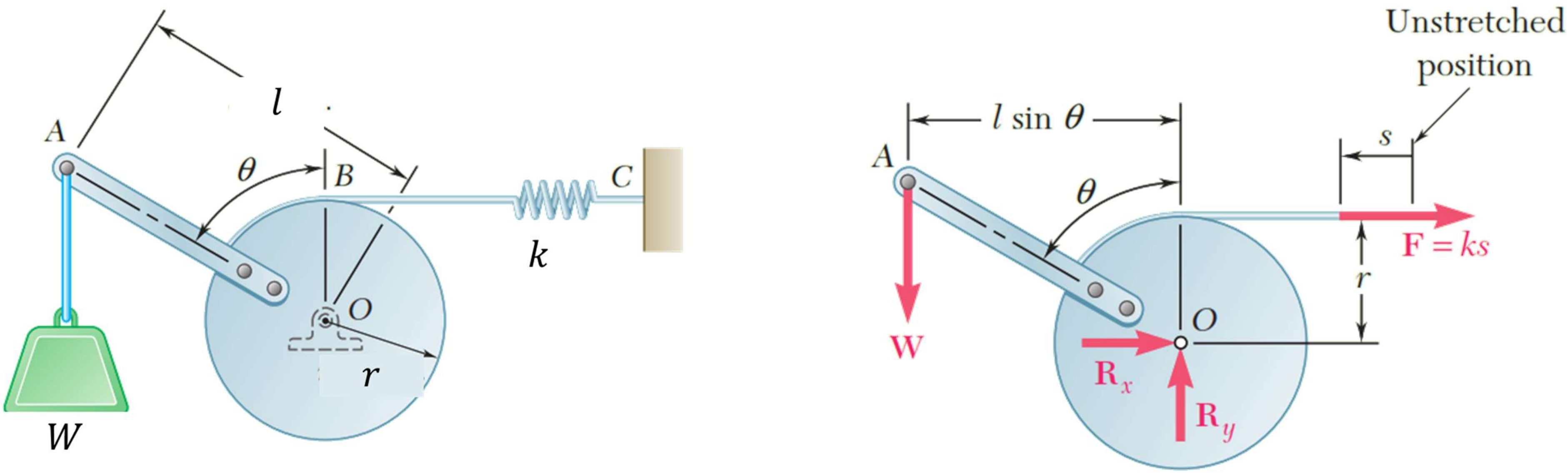



A weight (W) is attached at A to the lever shown. The constant of the spring BC is (k) and the spring is unstretched when θ = 0.

Determine the position of equilibrium as a function of θ.

- Draw a free-body diagram of the lever and cylinder to
- show all forces acting on the body.
- Sum moments about O. Your final answer should be the angle θ.

# Sample Problem 4.5



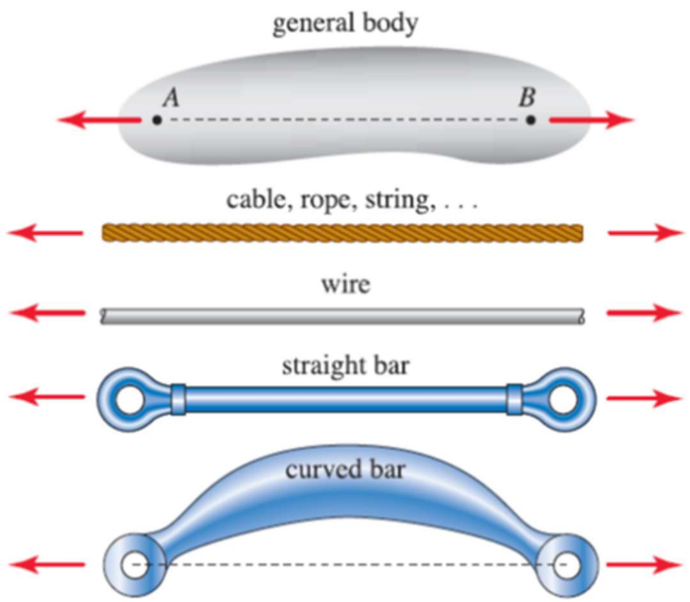

If (s) is the deflection of the spring from its unstretched position then:

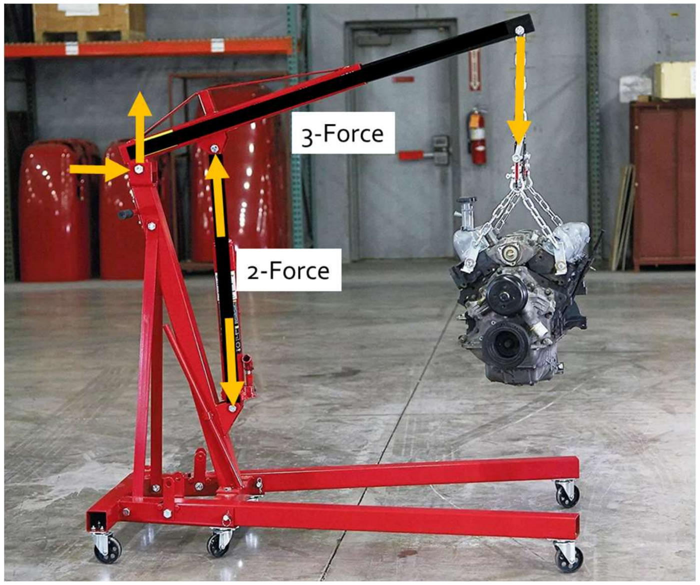
$$s = r\theta$$

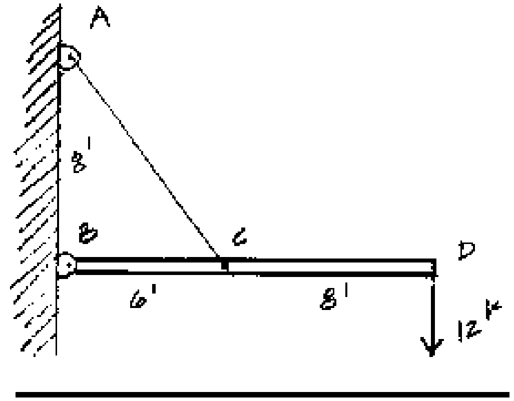
$$F = ks = kr\theta.$$

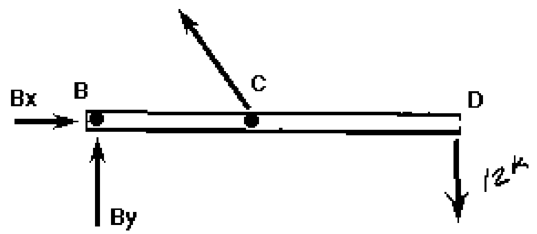
$$+\circlearrowleft \Sigma M_O = 0: \qquad Wl \sin\theta - r(kr\theta) = 0 \qquad \sin\theta = \frac{kr^2}{Wl}\theta$$

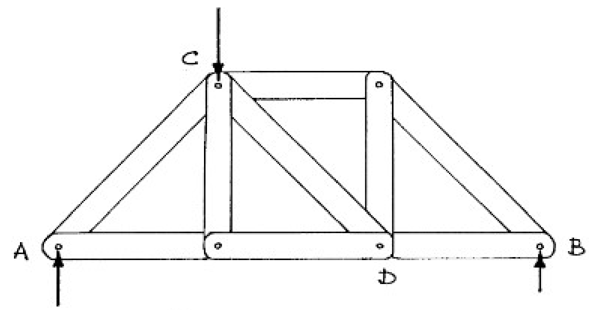
# 4.2 Special cases of equilibrium



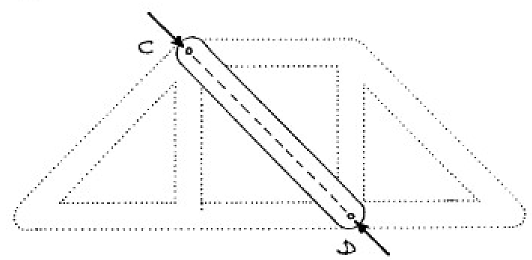




(a)

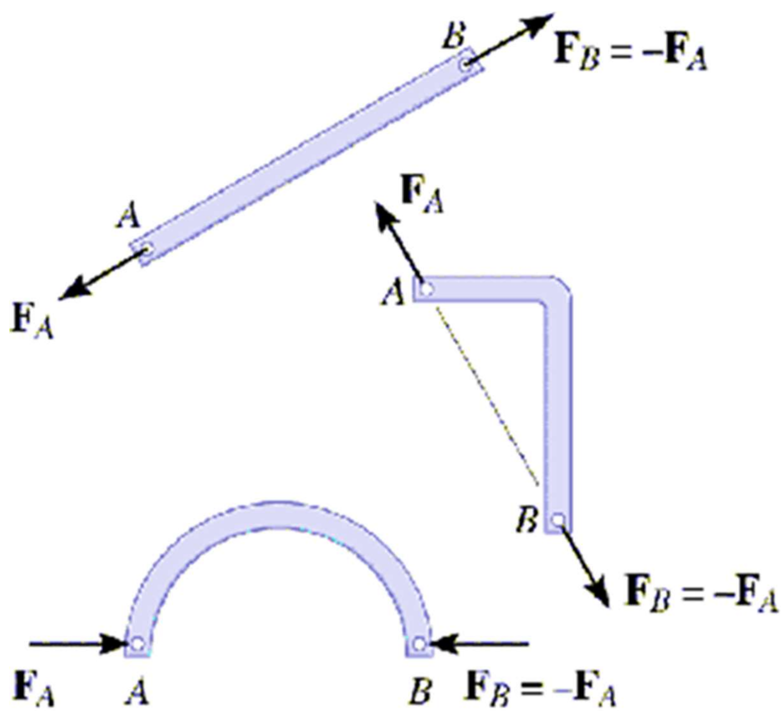



(b)

- Two-Force Body
- Three-Force Body

# 4.2A Equilibrium of a Two-Force Body

- If a body has pins or hinge supports at both ends and carries no load in-between (through its length), it is called a two-force member.
- If only two forces act on a body that is in equilibrium, then they must be equal in magnitude, co-linear and opposite in sense.



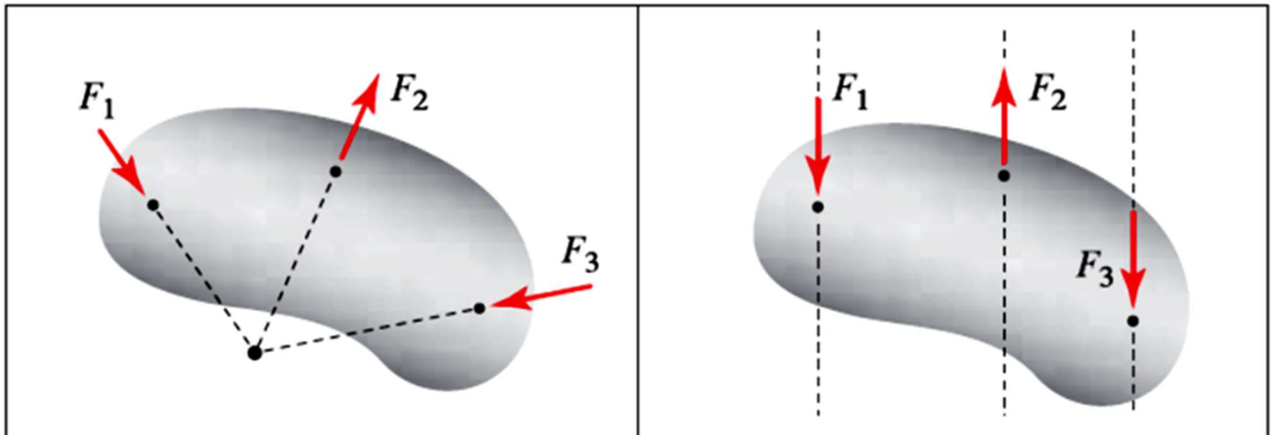

Two-force members

# 4.2B Equilibrium of a Three-Force Body

Three-force member. A body subjected to forces at three points (no moment loading and no distributed forces such as weight) is called a three-force member. The special feature of a three-force member is that, when in equilibrium:

I. The lines of action of all three forces intersect at a common point.

II. If the three forces are parallel (this is called a parallel force system), then their point of intersection can be thought of as being at infinity. Examples are shown in the figure.




three-force member subjected to a concurrent force system

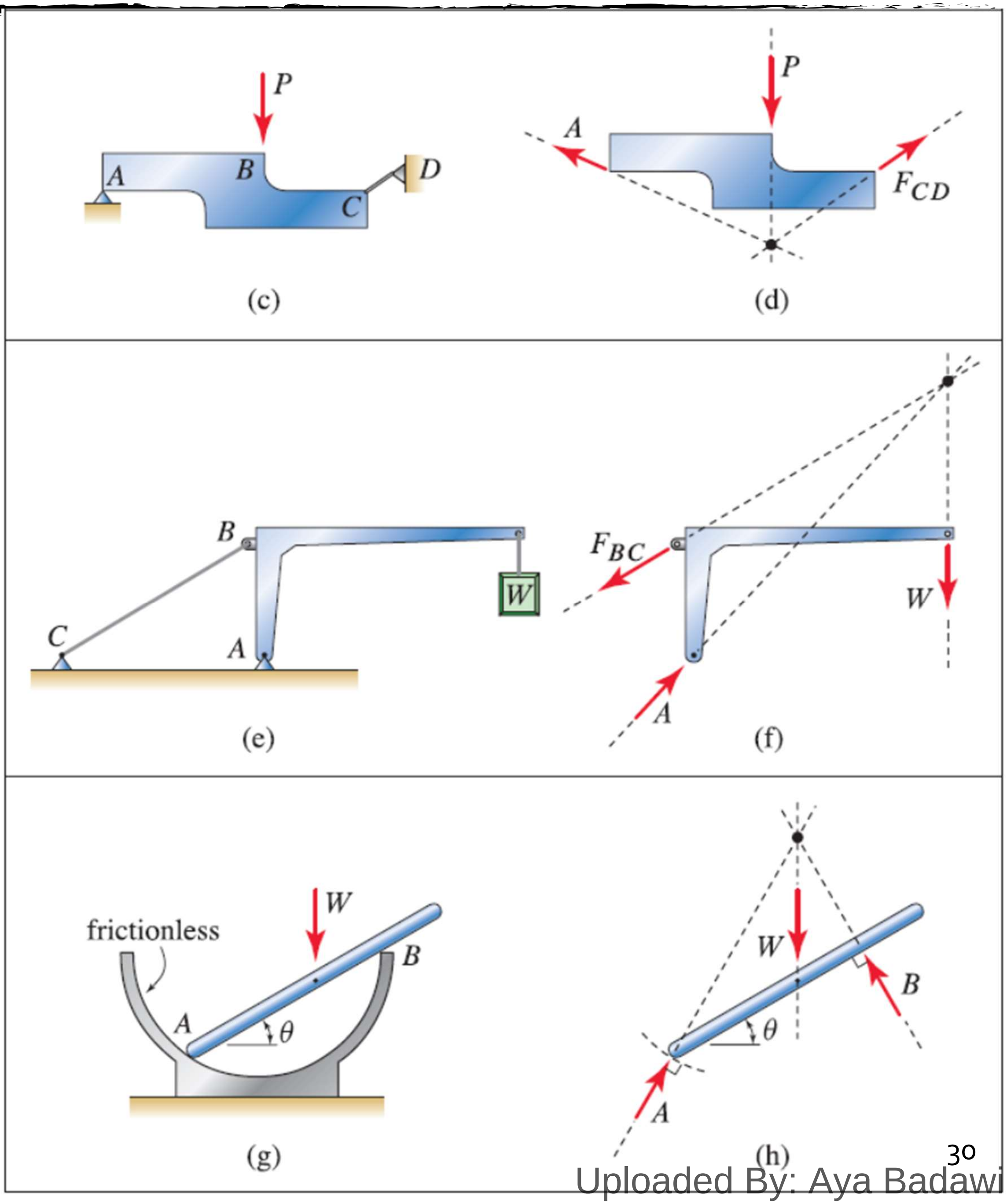
(a)

three-force member subjected to a parallel force system

(b)

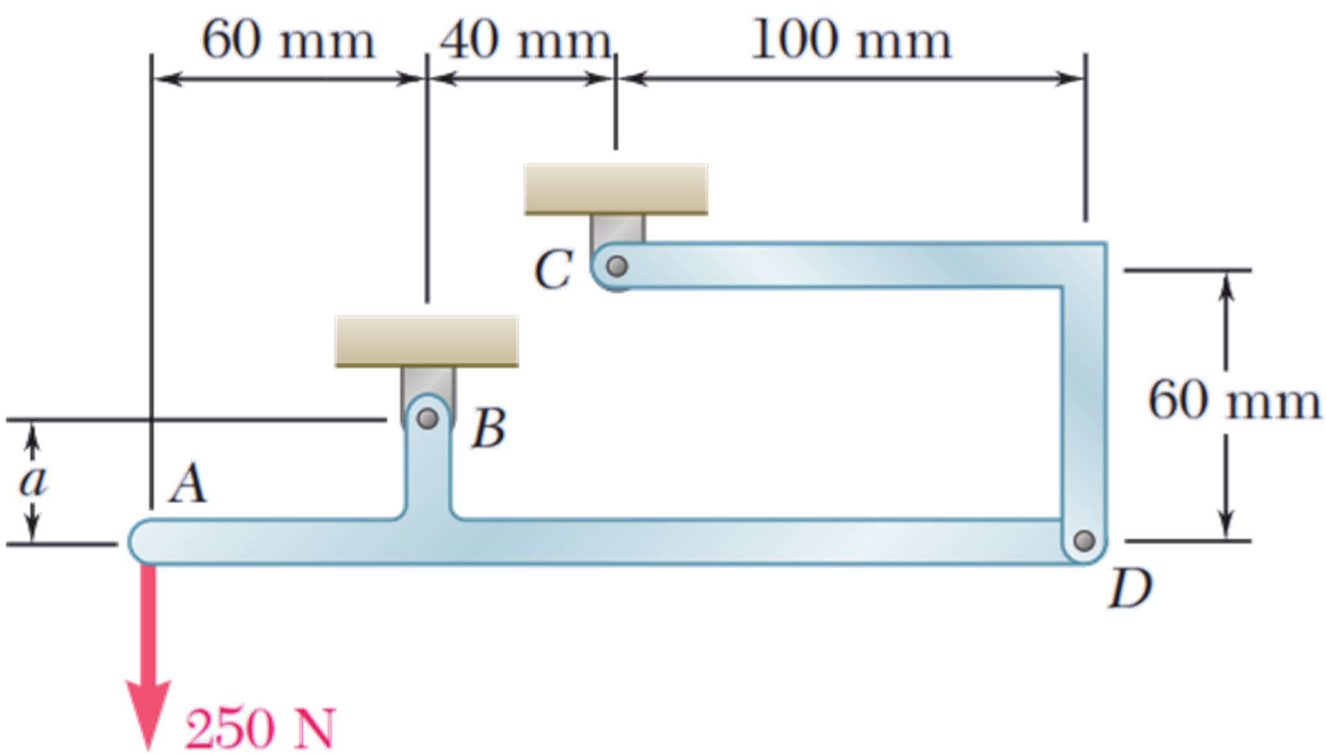
# 4.2B Equilibrium of a Three-Force Body

Examples of three-force members with concurrent force systems.

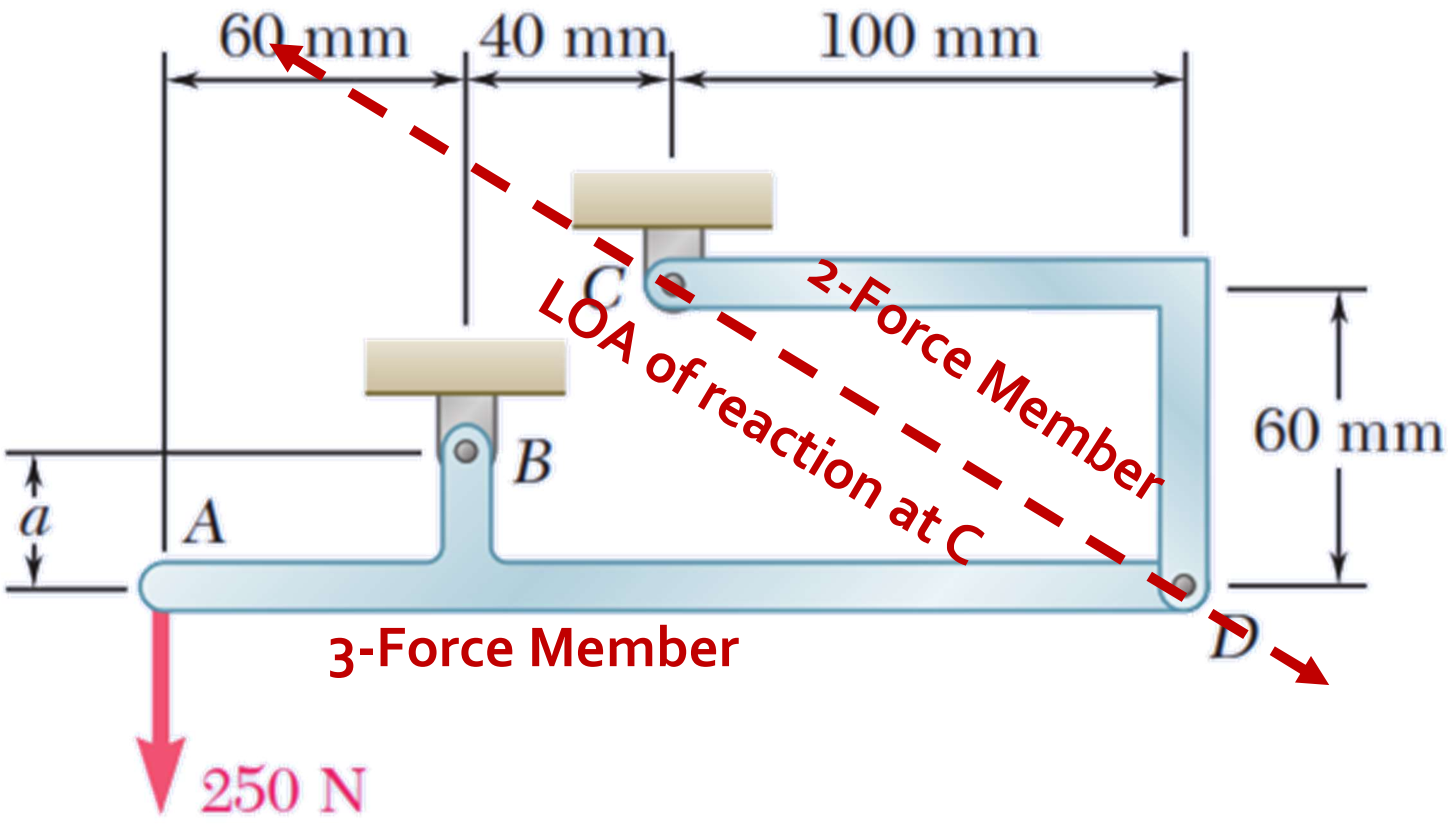


# Example

Determine the reactions at B and C when a = 30 mm.



# Example



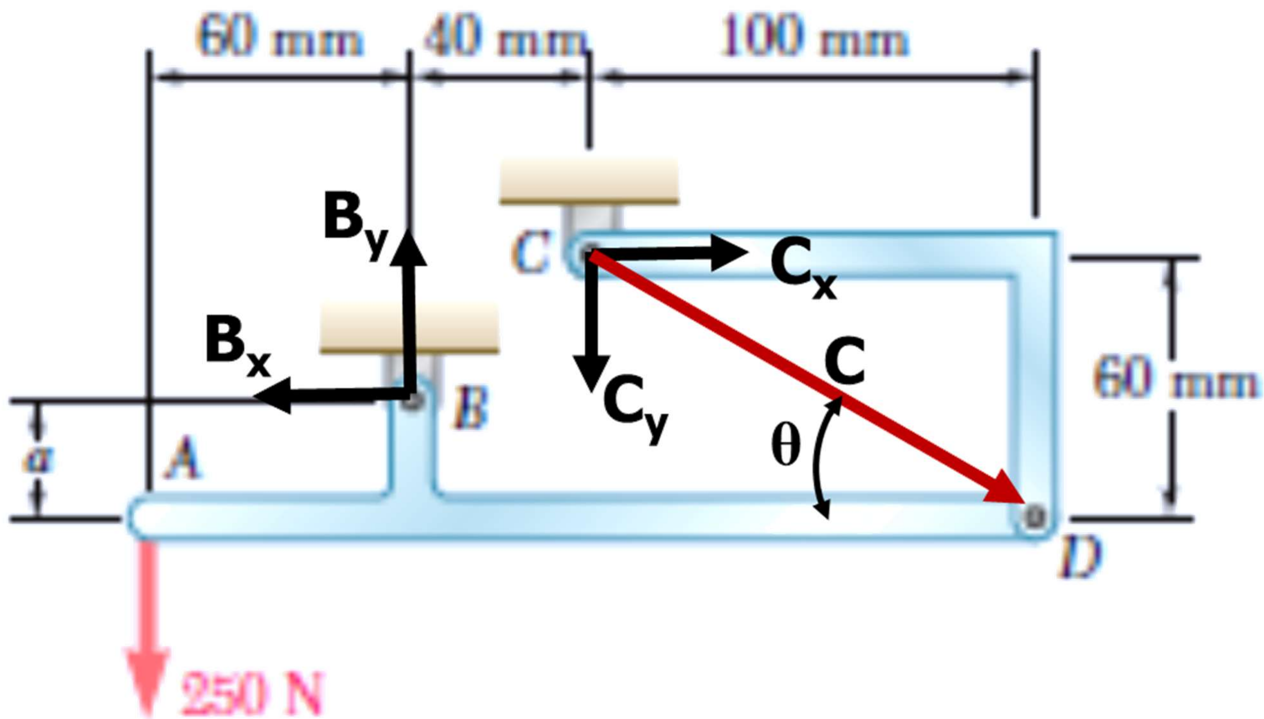
60 mm
40 mm
100 mm
60 mm
a
A
B
C
D
250 N
2-Force Member
LOA of reaction at C
3-Force Member

# Example

$C_x = C\cos\theta$

$C_y = C\sin\theta$

$\theta = \tan^{-1}\dfrac{60}{100} = 31°$




$$\sum M_B = 0$$

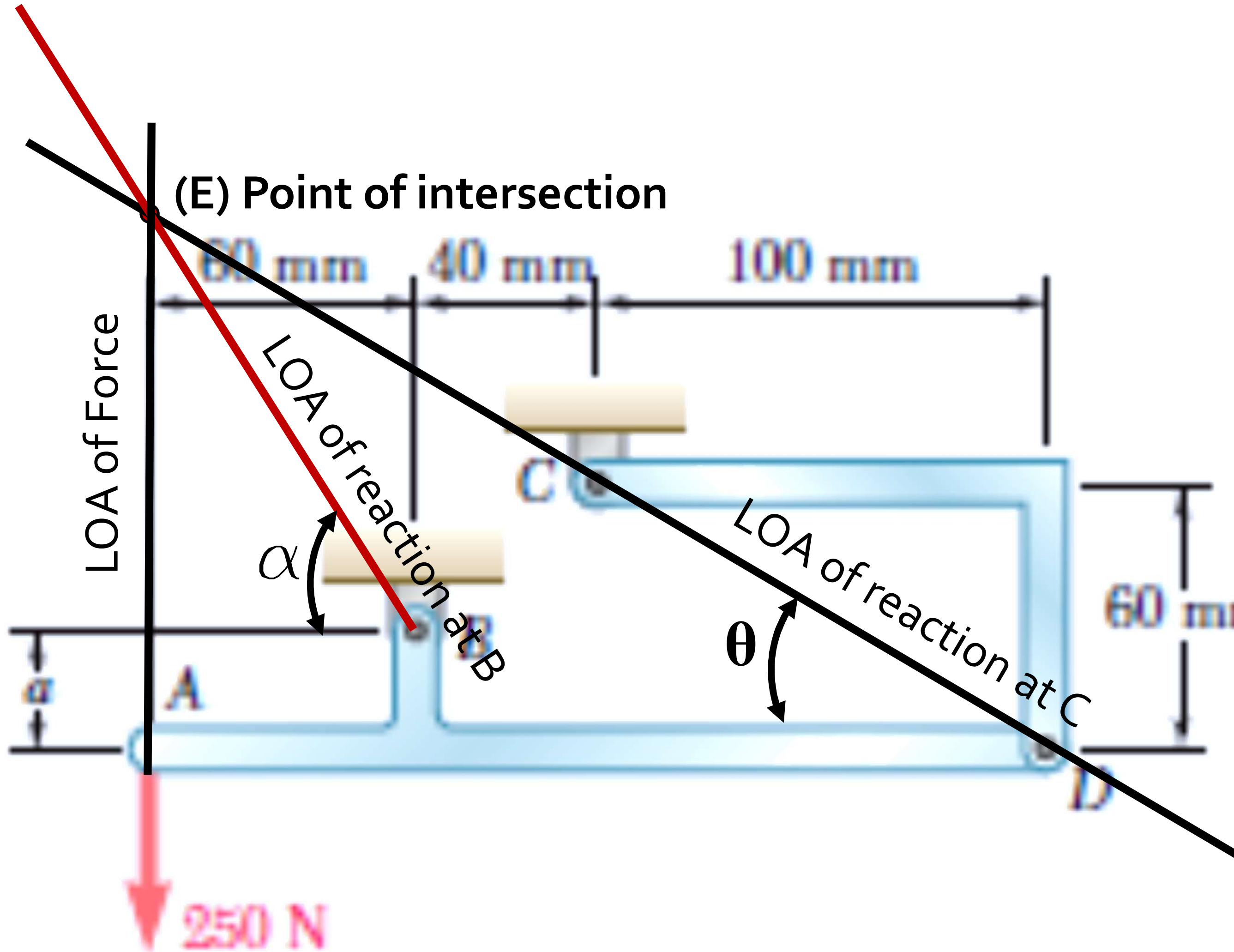
$$250 \times 60 - C\sin\theta \times 40 - C\cos\theta \times 30 = 0$$

$$\rightarrow C = 323.86\ N, \rightarrow C_x = 277.6\ N, C_y = 166.8\ N$$

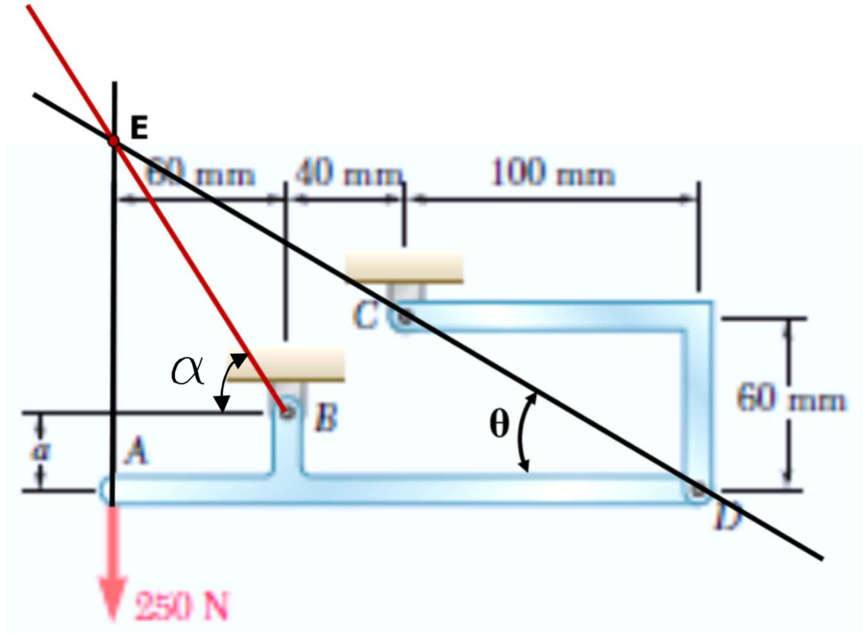
$$\sum F_x = 0, \rightarrow B_x = C_x = 277.6\ N,$$

$$\sum F_y = 0, \rightarrow B_y = C_y + 250 = 416.8\ N$$

# Example – Graphical Solution



# Example – Graphical Solution

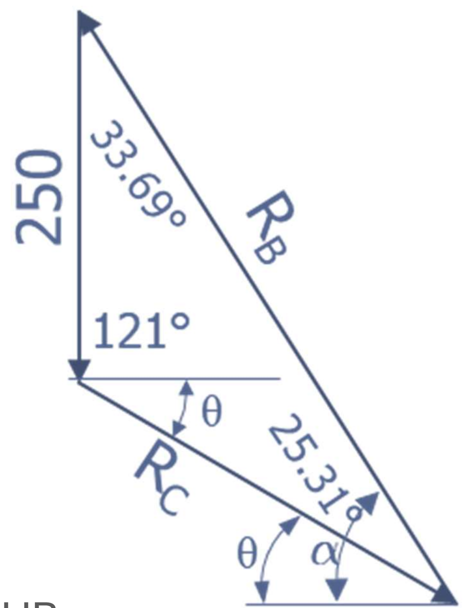



$$\theta = \tan^{-1}\frac{60}{100} = 31°$$

$$\tan\theta = \frac{60}{100} = \frac{AE}{200}$$

$$\rightarrow AE = 120\ mm$$

$$\alpha = \tan^{-1}\frac{(120-30)}{60} = 56.31$$




$$\frac{250}{\sin 25.31} = \frac{R_B}{\sin 121} = \frac{R_C}{\sin 33.69}$$

$$R_B = 501\ N, R_C = 324\ N$$

# 4.3 Equilibrium in Three Dimensions

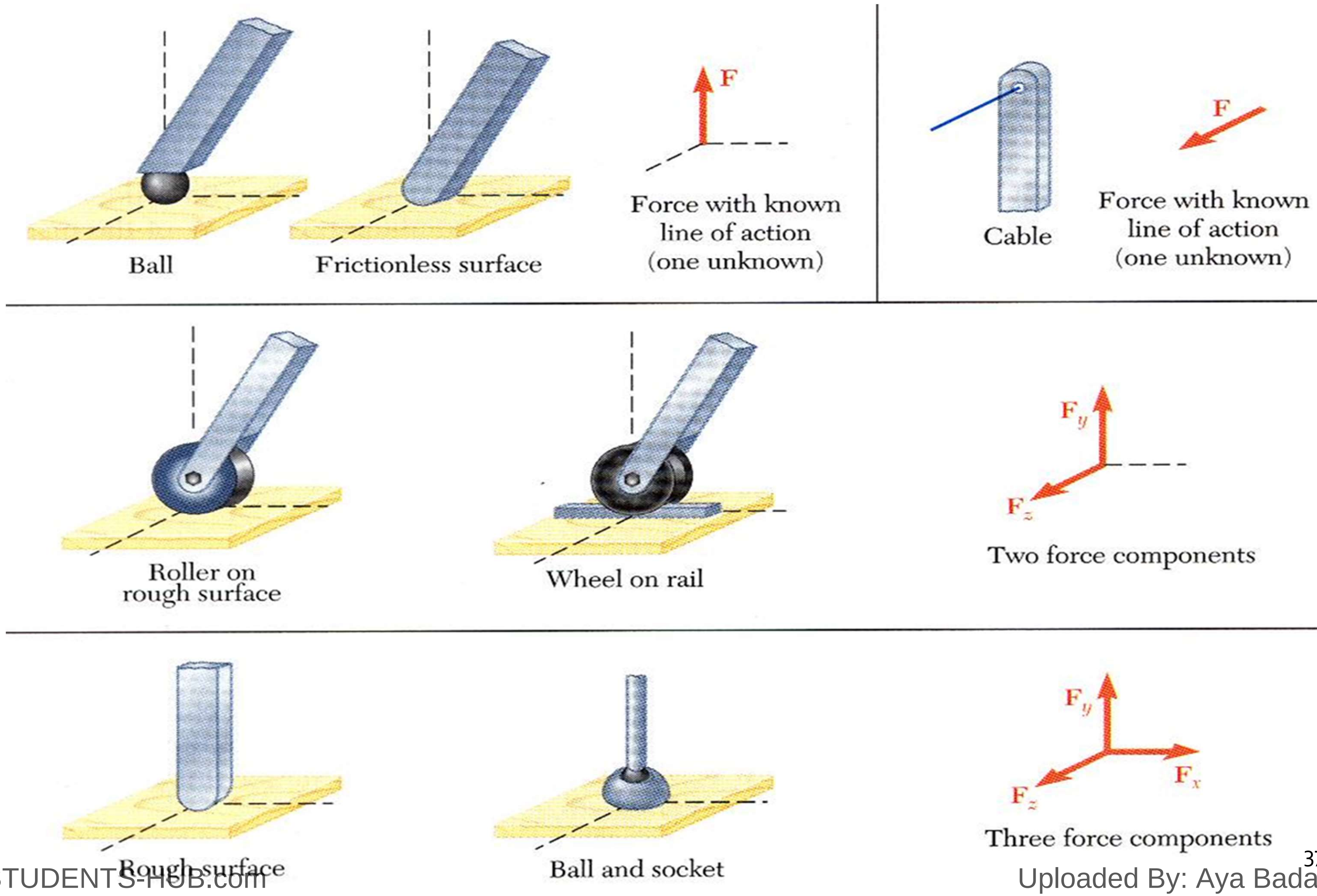
- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

$$\sum F = 0 \quad \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

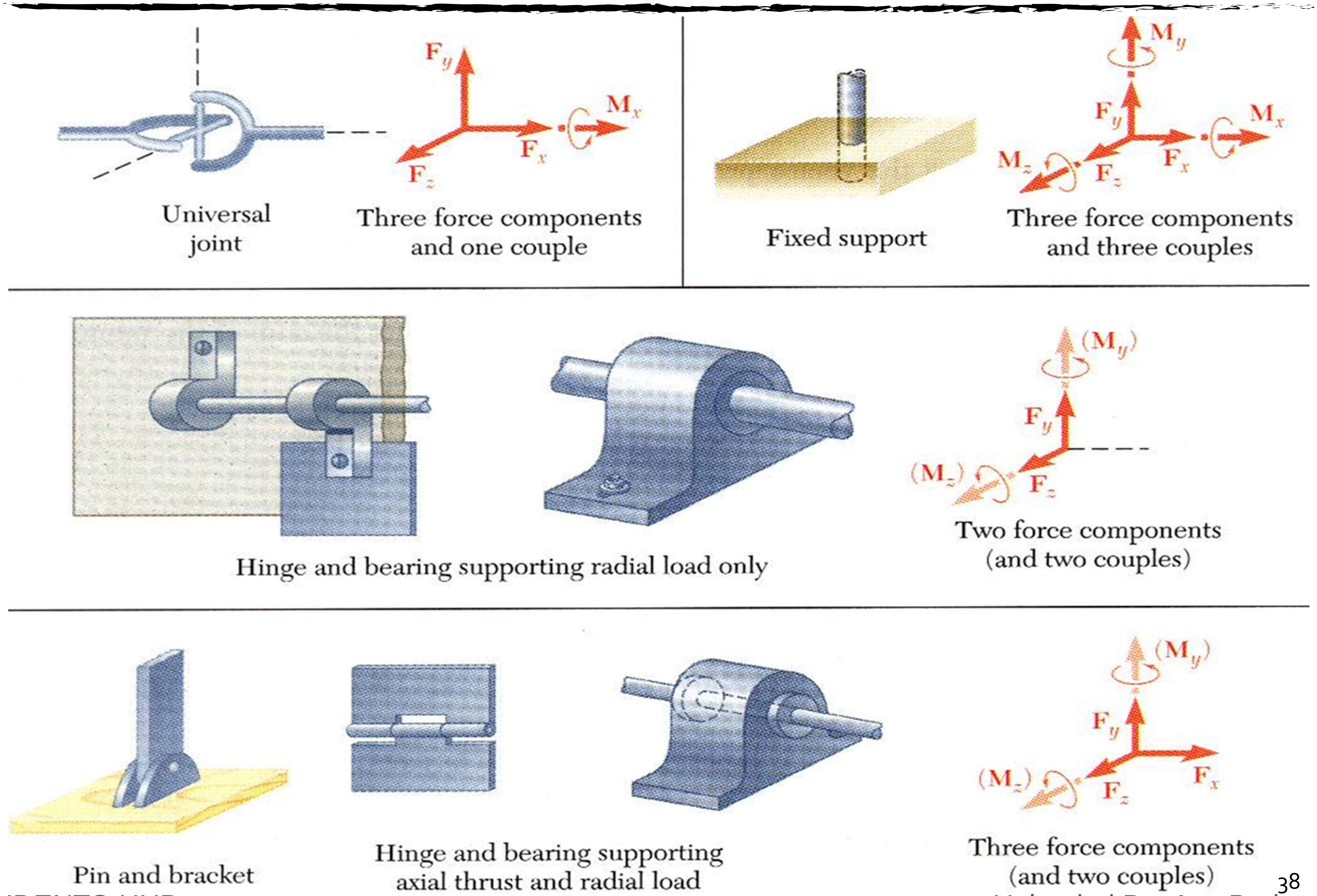
$$\sum M = 0 \quad \begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases}$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections or unknown applied forces.

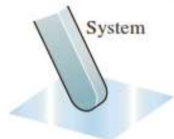
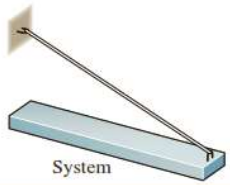
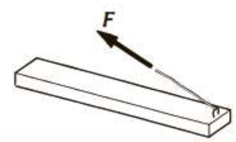
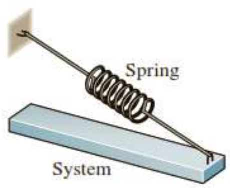
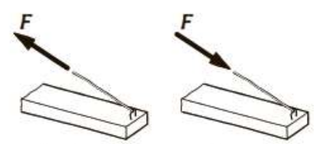
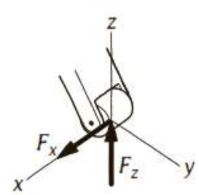
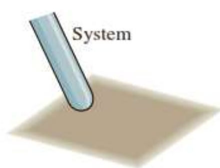
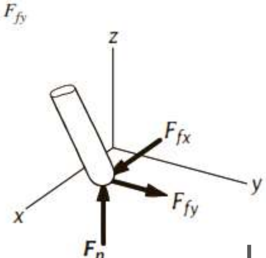
# 4.3B Reactions at Supports and Connections for a Three-Dimensional Structure



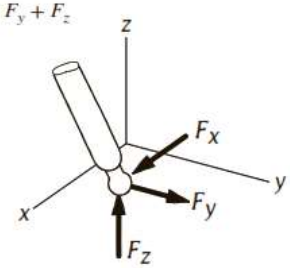
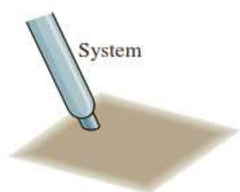
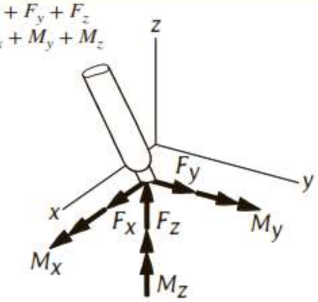
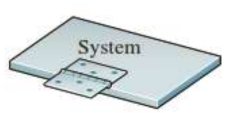
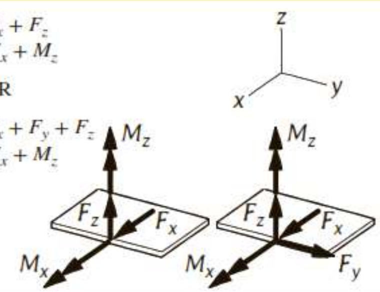
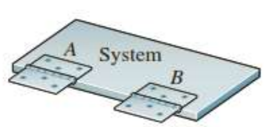
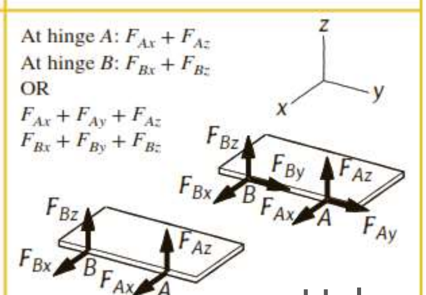
# 4.3B Reactions at Supports and Connections for a Three-Dimensional Structure



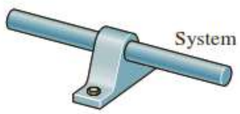
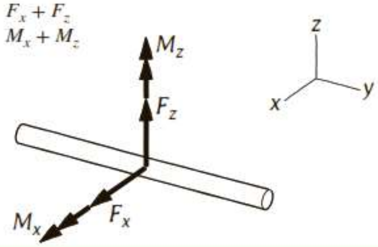
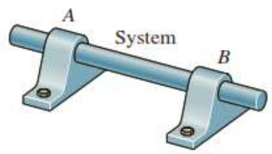
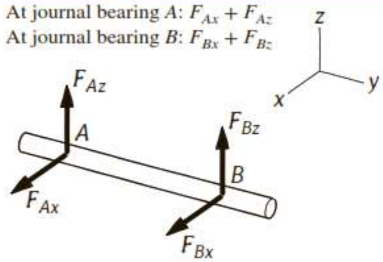
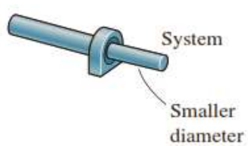
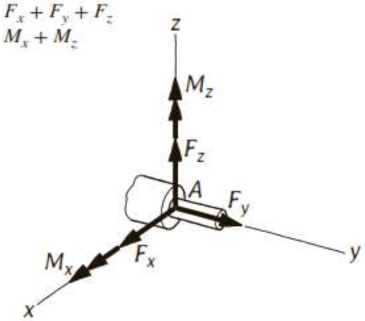
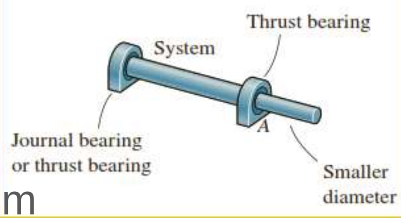
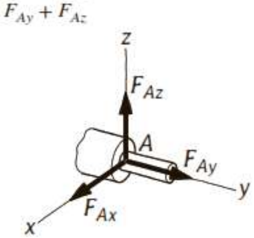
# 4.3B Reactions at Supports and Connections for a Three-Dimensional Structure

| (A) Supports | Description of Boundary Loads | (B) Loads to Be Shown in Free-Body Diagram |
|---|---|---|
| **1. Normal contact without friction**  | **Force** ($F$) oriented normal to surface on which system rests. Direction is such that force pushes on system. | $F$  |
| **2. Cable, rope, wire**  | **Force** ($F$) oriented along cable. Direction is such that force pulls on system. | $F$  |
| **3. Spring**  | **Force** ($F$) oriented along long axis of spring. Direction is such that force pulls on system if spring is in tension and pushes if spring is in compression. | $F$  Spring in tension $+\Delta$; Spring in compression $-\Delta$ |
| **4. Smooth roller in guide**  | **Force** represented as two components. One component ($F_z$) normal to surface on which system rests; the other is perpendicular to rolling direction ($F_x$). | $F_x + F_z$  |
| **5. Normal contact with friction**  | **Two forces**, one ($F_n$) oriented normal to surface so as to push on system, other force is tangent to surface on which the system rests and is represented in terms of its components ($F_{fx} + F_{fy}$). | $F_n$<br>$F_{fx} + F_{fy}$  |

# 4.3B Reactions at Supports and Connections for a Three-Dimensional Structure

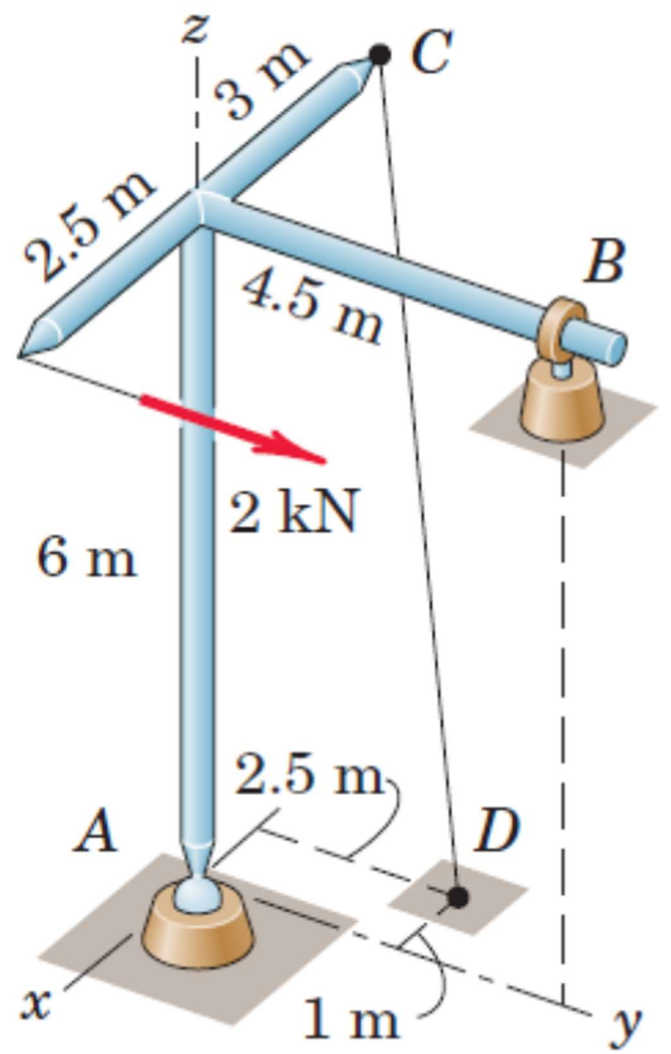
| (A) Supports | Description of Boundary Loads | (B) Loads to Be Shown in Free-Body Diagram |
|---|---|---|
| **6. Ball and socket support** (ball or socket as part of system)   | **Force** represented as three components. | $F_x + F_y + F_z$   |
| **7. Fixed support**   | **Force** represented in terms of components $(F_x + F_y + F_z)$. **Moment** represented in terms of components $(M_x + M_y + M_z)$. | $F_x + F_y + F_z$<br>$M_x + M_y + M_z$   |
| **8A. Single hinge** (shaft and articulated collar)   | **Force** in plane perpendicular to shaft axis; represented as $x$ and $z$ components $(F_x + F_z)$. **Moment** with components about axes perpendicular to shaft axis $(M_x + M_z)$. Depending on the hinge design, may also have a **force** component along axis of shaft, $(F_y)$. | $F_x + F_z$<br>$M_x + M_z$<br>OR<br>$F_x + F_y + F_z$<br>$M_x + M_z$   |
| **8B. Multiple hinges** (one of two or more properly aligned hinges)   | **Force** in plane normal to shaft axis represented in terms of components $(F_x + F_z)$. Point of application at center of shaft. Depending on design, may also apply **force** component along axis of shaft $(F_y)$. | At hinge $A$: $F_{Ax} + F_{Az}$<br>At hinge $B$: $F_{Bx} + F_{Bz}$<br>OR<br>$F_{Ax} + F_{Ay} + F_{Az}$<br>$F_{Bx} + F_{By} + F_{Bz}$   |

# 4.3B Reactions at Supports and Connections for a Three-Dimensional Structure

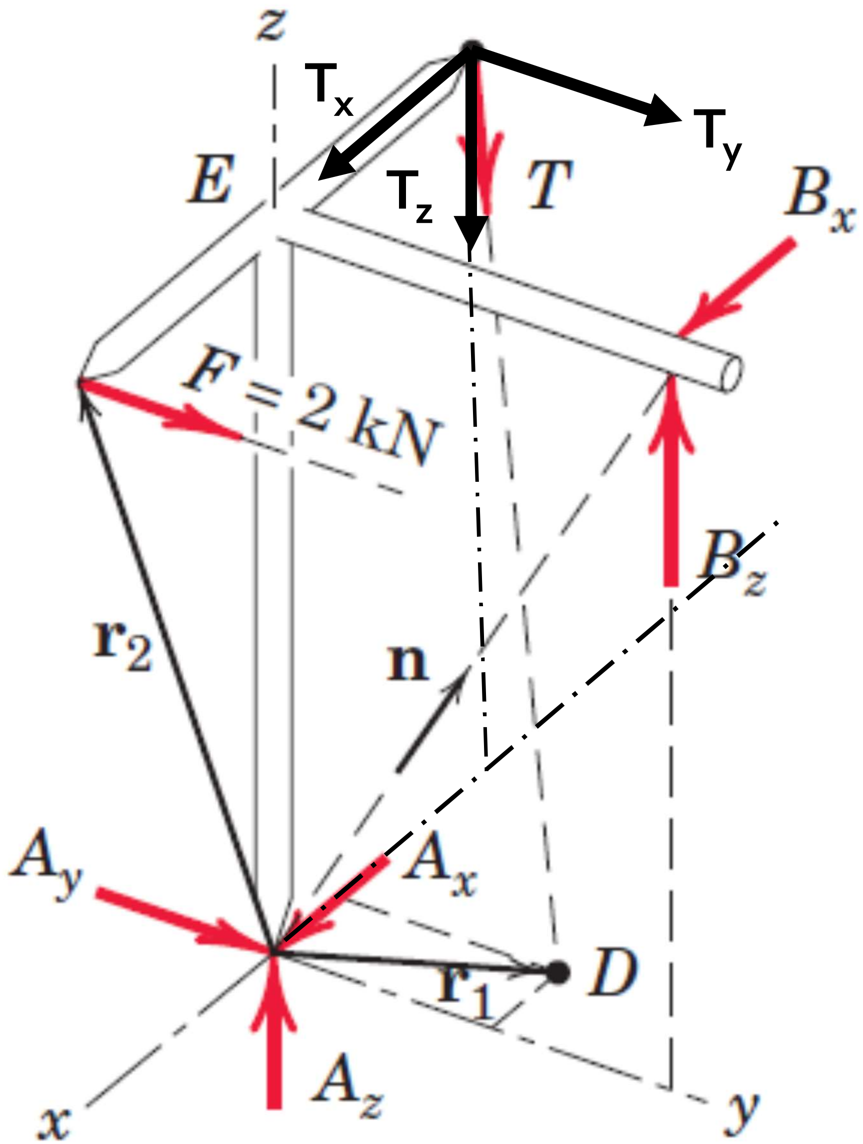
| (A)<br>Supports | Description of Boundary Loads | (B)<br>Loads to Be Shown in<br>Free-Body Diagram |
|---|---|---|
| **9A. Single journal bearing**<br>(frictionless collar that holds a shaft)<br> | **Force** in plane perpendicular to shaft axis; represented as $x$ and $z$ components ($F_x + F_z$).<br>**Moment** with components about axes perpendicular to shaft axis ($M_x + M_z$). | $F_x + F_z$<br>$M_x + M_z$<br> |
| **9B. Multiple journal bearings**<br>(two or more properly aligned journal bearings holding a shaft)<br> | **Force** in plane perpendicular to shaft axis represented in terms of components ($F_{Ax} + F_{Az}$). Point of application at center of shaft. | At journal bearing $A$: $F_{Ax} + F_{Az}$<br>At journal bearing $B$: $F_{Bx} + F_{Bz}$<br> |
| **10A. Single thrust bearing**<br>(journal bearing that also restricts motion along axis of shaft)<br> | **Force** represented in terms of three components ($F_x + F_y + F_z$). Component in direction of shaft axis ($F_y$) is sometimes referred to as the "thrust force." Point of application is at center of shaft.<br>**Moment** with components perpendicular to shaft axis ($M_x + M_z$). | $F_x + F_y + F_z$<br>$M_x + M_z$<br> |
| **10B. Multiple thrust bearings**<br>(one of two or more properly aligned thrust bearings)<br> | **Force** represented in terms of three components ($F_x + F_y + F_z$). Component in direction of shaft axis ($F_y$) is sometimes referred to as the "thrust force." Point of application is at center of shaft. | At thrust bearing $A$:<br>$F_{Ax} + F_{Ay} + F_{Az}$<br> |

# Example

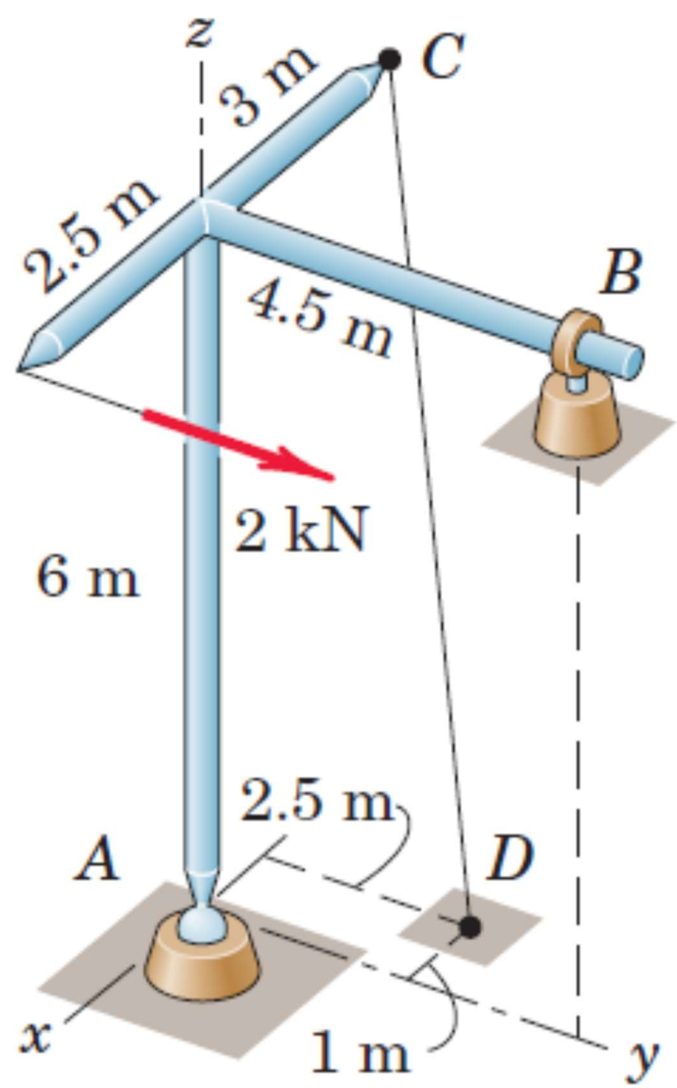
The frame shown is secured to the horizontal x-y plane by a ball and-socket joint at A and receives support from the loose-fitting ring at B. Under the action of the 2-kN load, rotation about a line from A to B is prevented by the cable CD (no couple moments are required at support B), and the frame is stable in the position shown. Neglect the weight of the frame compared with the applied load and determine the tension T in the cable, the reaction at the ring, and the reaction components at A.



# Example

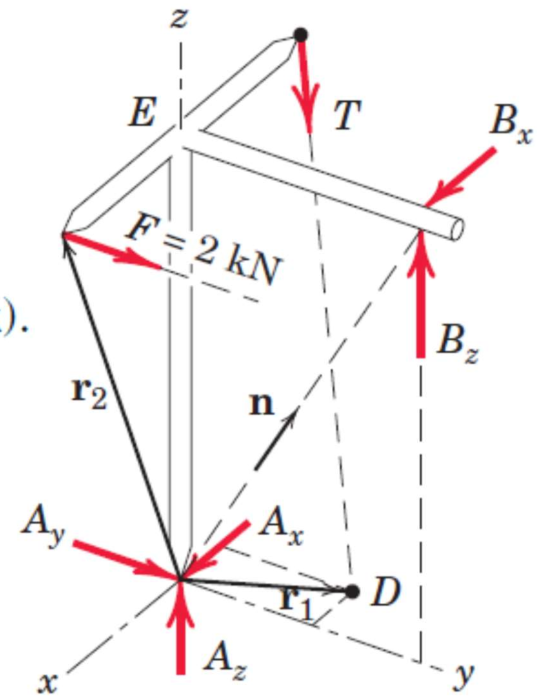

z
$T_x$
$T_y$
$T_z$
T
E
$B_x$
F = 2 kN
$B_z$
$r_2$
n
$A_y$
$A_x$
D
$r_1$
$A_z$
x
y

F.B.D


z
C
3 m
2.5 m
B
4.5 m
2 kN
6 m
2.5 m
A
D
x
1 m
y

# Example

**1. Forces:**




$$\mathbf{T} = \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \qquad \mathbf{F} = 2\mathbf{j}\ \text{kN}$$

**2.** $\sum M_{AB} = 0$

$$\vec{\lambda}_{AB} = \frac{1}{\sqrt{6^2 + 4.5^2}}(4.5\mathbf{j} + 6\mathbf{k}) = \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}).$$

$$\mathbf{r}_1 = -\mathbf{i} + 2.5\mathbf{j}\ \text{m} \qquad \mathbf{r}_2 = 2.5\mathbf{i} + 6\mathbf{k}\ \text{m}$$

$$(-\mathbf{i} + 2.5\mathbf{j}) \times \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k})$$

$$+ (2.5\mathbf{i} + 6\mathbf{k}) \times (2\mathbf{j}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}) = 0$$

$$-\frac{48T}{\sqrt{46.2}} + 20 = 0 \qquad T = 2.83\ \text{kN}$$

and the components of *T* become

$$T_x = 0.833\ \text{kN} \qquad T_y = 1.042\ \text{kN} \qquad T_z = -2.50\ \text{kN}$$

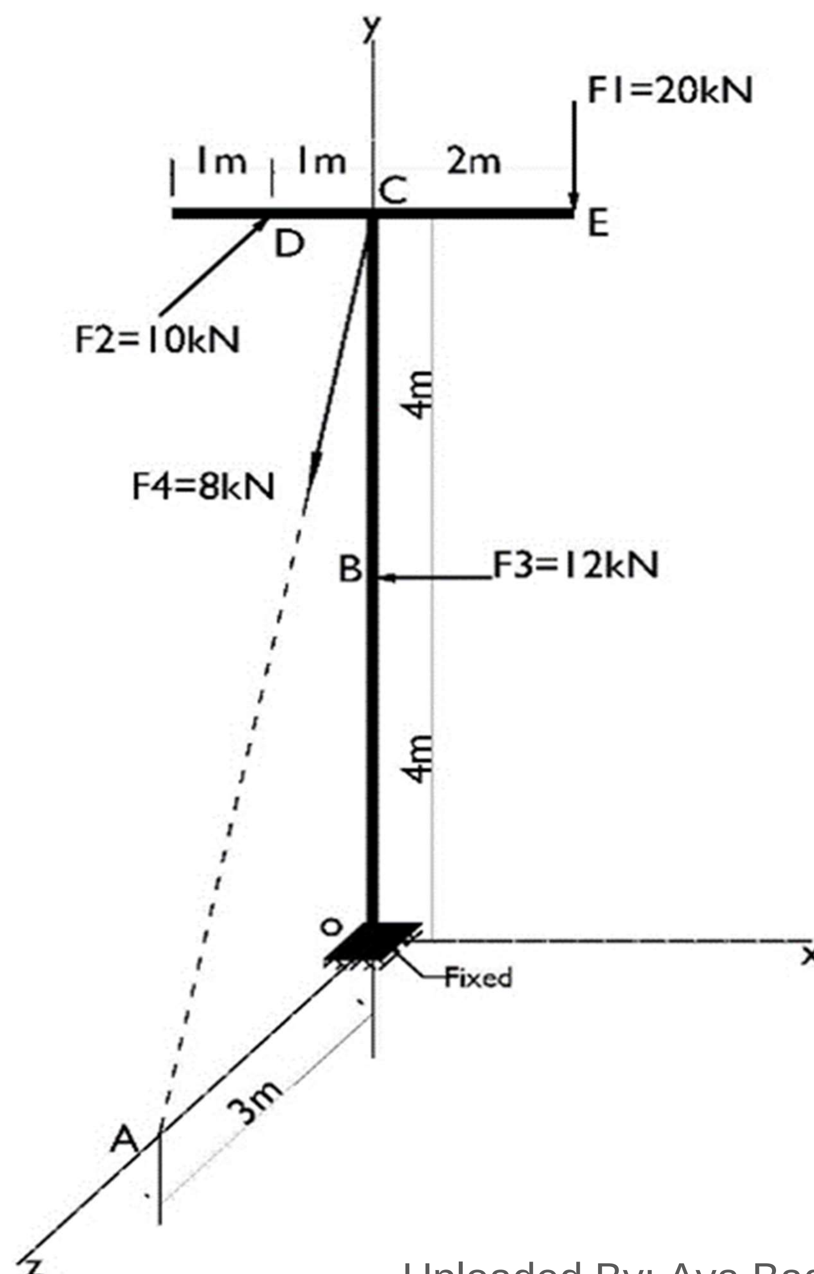
# Example

3. We may find the remaining unknowns by moment and force summations as follows:

| | | |
|---|---|---|
| $[\Sigma M_z = 0]$ | $2(2.5) - 4.5B_x - 1.042(3) = 0$ | $B_x = 0.417 \text{ kN}$ |
| $[\Sigma M_x = 0]$ | $4.5B_z - 2(6) - 1.042(6) = 0$ | $B_z = 4.06 \text{ kN}$ |
| $[\Sigma F_x = 0]$ | $A_x + 0.417 + 0.833 = 0$ | $A_x = -1.250 \text{ kN}$ |
| $[\Sigma F_y = 0]$ | $A_y + 2 + 1.042 = 0$ | $A_y = -3.04 \text{ kN}$ |
| $[\Sigma F_z = 0]$ | $A_z + 4.06 - 2.50 = 0$ | $A_z = -1.556 \text{ kN}$ |

# Example

For the fixed post at O and the loading shown, determine the reactions at point O.



# Example – Graphical Solution

$F_1 = -20j \quad \vec{r} = 2i + 8j$

$M_{F_{1}O} = (2i + 8j) \times -20j = \boxed{-40k}$

$F_2 = -10k \quad \vec{r}_2 = -i + 8j$

$M_{F_2 O} = (-i + 8j) \times (-10k) = \boxed{-10j - 80i}$

$F_3 = -12i \quad \vec{r} = 4j$

$M_{F_3 O} = 4j \times -12i = \boxed{48k}$

$F_4 = 8\left(-\frac{8}{\sqrt{73}}j + \frac{3}{\sqrt{73}}k\right) \quad \vec{r} = 3k$

$M_{F_4 O} = 3k \times \left(\frac{-64}{\sqrt{73}}j + \frac{24}{\sqrt{73}}k\right)$

$= +\boxed{\frac{3 \times 64}{\sqrt{73}} i}$

$\Sigma M_x = 0 \quad +80 - \frac{3 \times 64}{\sqrt{73}} = M_x = 57.52$

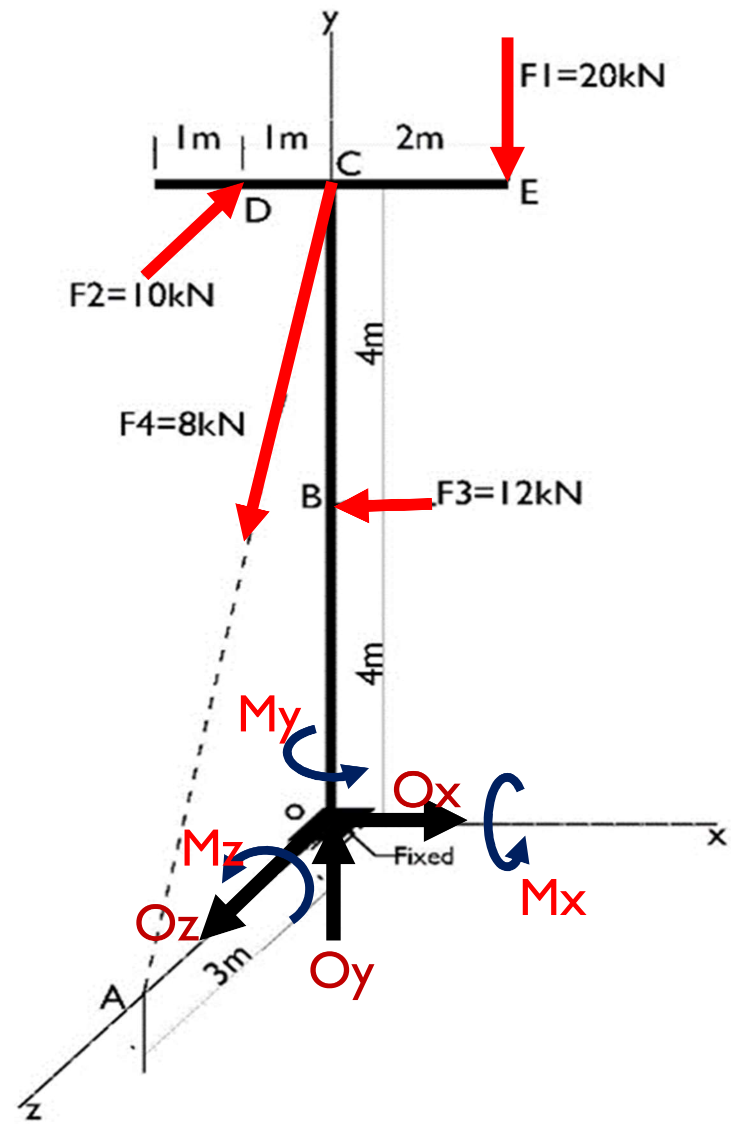
$\Sigma M_y = 0 \quad -10 + M_y = 0, \; M_y = 10$

$\Sigma M_z = 0 \quad -40 + 48 + M_z = 0$

$M_z = 8$ kN

$\Sigma F = 0$

$O_x = 12, \; O_y = 27.5, \; O_z = 7.19$ kN



# Example

- The light right-angle boom which supports the 400-kg cylinder is supported by three cables and a ball-and-socket joint at o attached to the vertical x-y surface. Determine the reactions at o and the cables tension.