

## 2.1 Quadratic Equations

A quadratic equation of one variable has a general form  $ax^2 + bx + c = 0$ , where  $a \neq 0$

Example: 1)  $x^2 - 3x + 2 = 0$

2)  $3x^2 + 6x + 12 = 0$

3)  $4x^2 - 9 = 0$

4)  $2x^2 + 1 = x^2 - x$   $\xrightarrow{\text{إعادة ترتيب}}$   $x^2 + x + 1 = 0$

Solving quadratic equations:

1) Using quadratic formula:

Once the equation is in the general form ( $ax^2 + bx + c = 0$ ) then the solution(s) can be found using the

quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac$  is called the discriminant (ميزان)

| Discriminant       | Solutions                            |
|--------------------|--------------------------------------|
| Discriminant $> 0$ | $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| Discriminant $= 0$ | $-\frac{b}{2a}$                      |
| Discriminant $< 0$ | no real solutions                    |

Example: Solve the following system of linear equations

$$\begin{cases} 3x + 4y = 1 & \text{--- ①} \\ 2x - 3y = 12 & \text{--- ②} \end{cases}$$

By elimination: multiply eq ① by 3 and multiply eq. ② by 4

$$\begin{aligned} \text{You have } 9x + 12y &= 3 \\ 8x - 12y &= 48 \end{aligned}$$

$$\text{add the two equations: } \frac{17x}{17} = \frac{51}{17} \rightarrow \boxed{x = 3}$$

Substitute  $x = 3$  in eq ②:  $8x - 12y = 48$

$$8(3) - 12y = 48 \rightarrow 24 - 12y = 48$$

$\quad \quad -24 \quad \quad -24$

$$\rightarrow \frac{-12y}{-12} = \frac{24}{-12} \rightarrow \boxed{y = -2}$$

The solution is  $(3, -2)$

check the solution: substitute  $(3, -2)$  in eq ①

$$9x + 12y = 3 \rightarrow 9(3) + 12(-2) \stackrel{??}{=} 3$$

$$27 - 24 \stackrel{??}{=} 3$$

$$3 = 3 \quad \checkmark$$

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Example: Solve the following equation using the quadratic formula  $x^2 - 3x + 2 = 0 \rightarrow a = 1$   
 $b = -3$   
 $c = 2$

$$\text{discriminant: } b^2 - 4ac \\ = (-3)^2 - 4(1)(2) = 9 - 8 = 1 > 0$$

$$\text{Solutions: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{1}}{2(1)}$$

$$x = \frac{3 \pm 1}{2}$$

$$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2, \quad x_2 = \frac{3-1}{2} = \frac{2}{2} = 1$$

solutions : 1, 2

Example: Solve the equation using the quadratic formula  $3x^2 + 6x + 12 = 0$   $a = 3, b = 6, c = 12$

$$\text{discriminant} = b^2 - 4ac = (6)^2 - 4(3)(12) \\ = 36 - 144 = -108 < 0$$

The equation has no real solutions.

Example:- Solve the following using the quadratic formula  $x^2 + 2x + 1 = 0 \rightarrow a=1, b=2, c=1$

$$\begin{aligned}\text{discriminant} &= b^2 - 4ac \\ &= (2)^2 - 4(1)(1) \\ &= 4 - 4 = 0 \quad \text{one real solution}\end{aligned}$$

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm 0}{2a} = \frac{-b}{2a} \\ &= \frac{-2}{2(1)} = \frac{-2}{2} = \boxed{-1}\end{aligned}$$

2) Using Factoring

If the quadratic equation can be factored, then we can use the zero product property.

if  $ab = 0$ , then  $a = 0$  or  $b = 0$  or both.

Example:- Solve using factoring  $x^2 - 3x + 2 = 0$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

$$\begin{array}{cc} x - 2 = 0 & \text{or} & x - 1 = 0 \\ +2 & & +1 \end{array}$$

$$\boxed{x = 2} \quad \text{or} \quad \boxed{x = 1}$$

Example: Solve the equations using factoring

1)  $x^2 + 2x + 1 = 0$

$$(x + 1)(x + 1) = 0$$

$$x + 1 = 0 \quad \text{or} \quad x + 1 = 0$$

$-1 \quad -1 \qquad -1 \quad -1$

$$x = -1 \quad \text{or} \quad x = -1 \quad \text{one solution } (-1)$$

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