Morphological Image Processing

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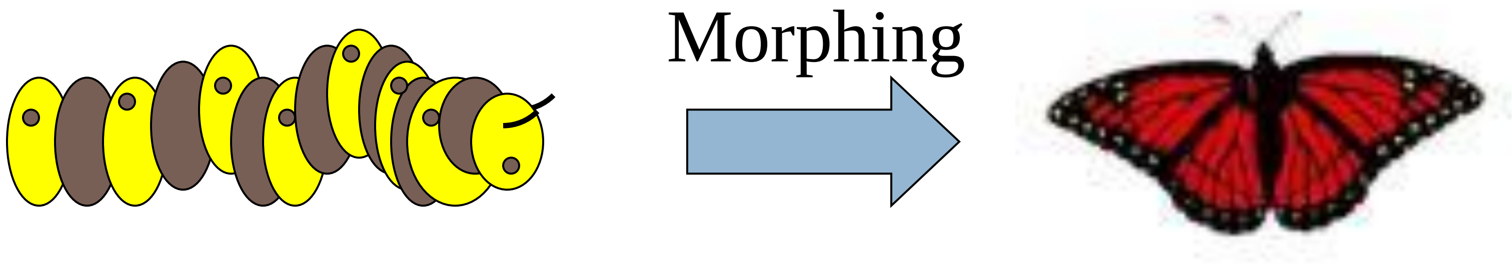
First Semester 2015/2016

Outline

- Introduction
- Erosion and Dilation
- Opening and Closing
- Boundary Extraction
- Hole Filling
- Hit-or-Miss Transformation

Introduction

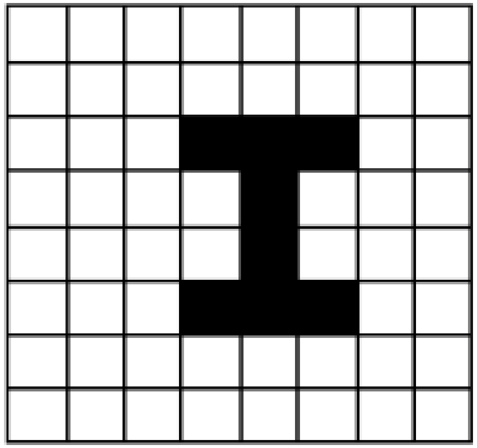
- Morphological operations come from the word "morphing" in Biology which means "changing a shape".



- In the context of image processing , it refers to a set of mathematical tools that can be used to
  - Extract useful description and representation of regions in images (boundaries, skeletons, convex hull)
  - Remove imperfections introduced during segmentation (thinning, regions filling)

Introduction

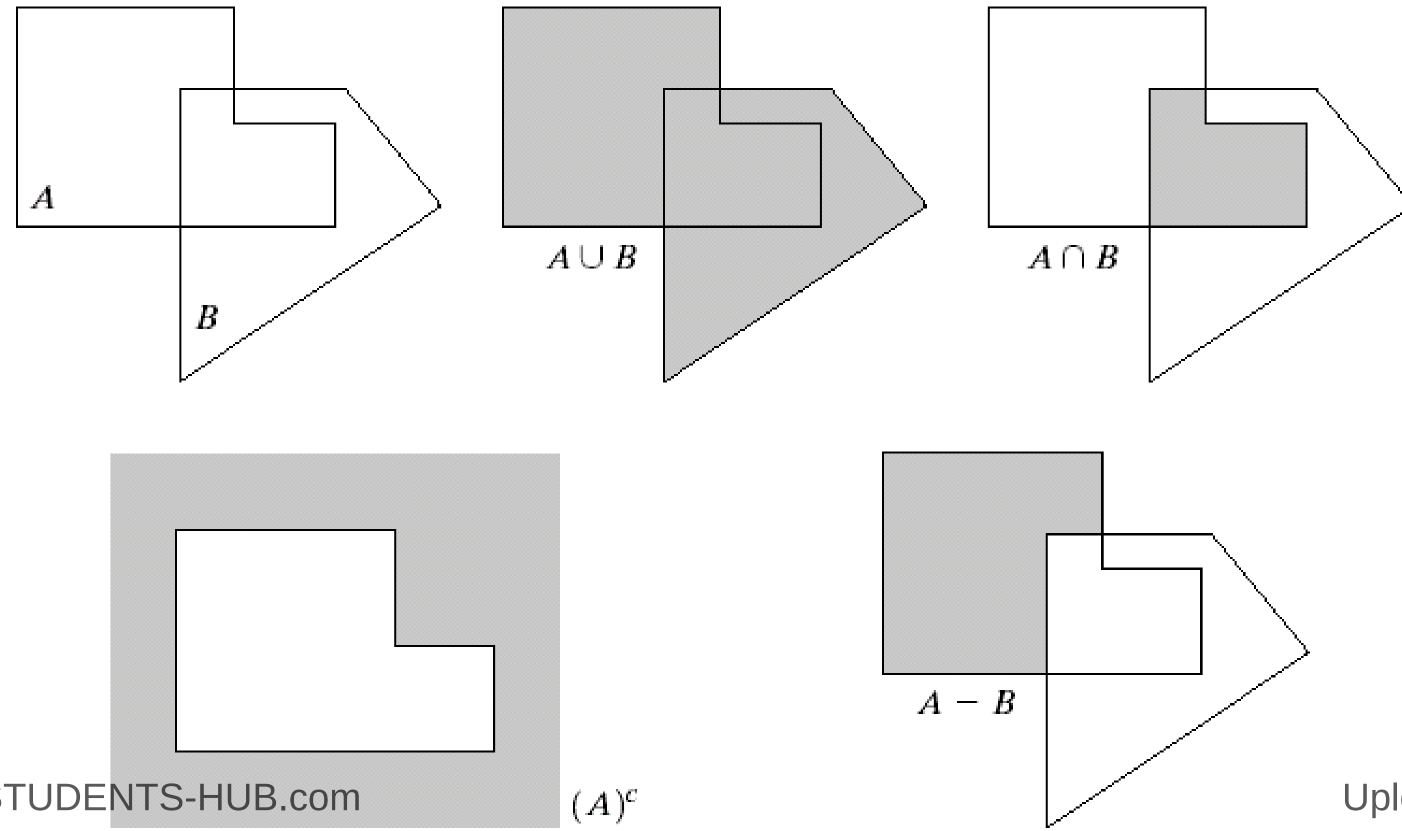
- A binary image can be considered as a set by considering "black" pixels (with image value "1") as elements in the set and "white" pixels (with value "0") as outside the set, or vice versa.



- Morphological filters are essentially set operations

Basic Set Operations

- Let x, y, z, … represent locations of 2D pixels, e.g. x = (x1, x2), S denote the complete set of all pixels in an image, let A, B, … represent subsets of S
  - Each set may represent one object. Each pixel (x,y) has its status: belong to a set or not belong to a set.

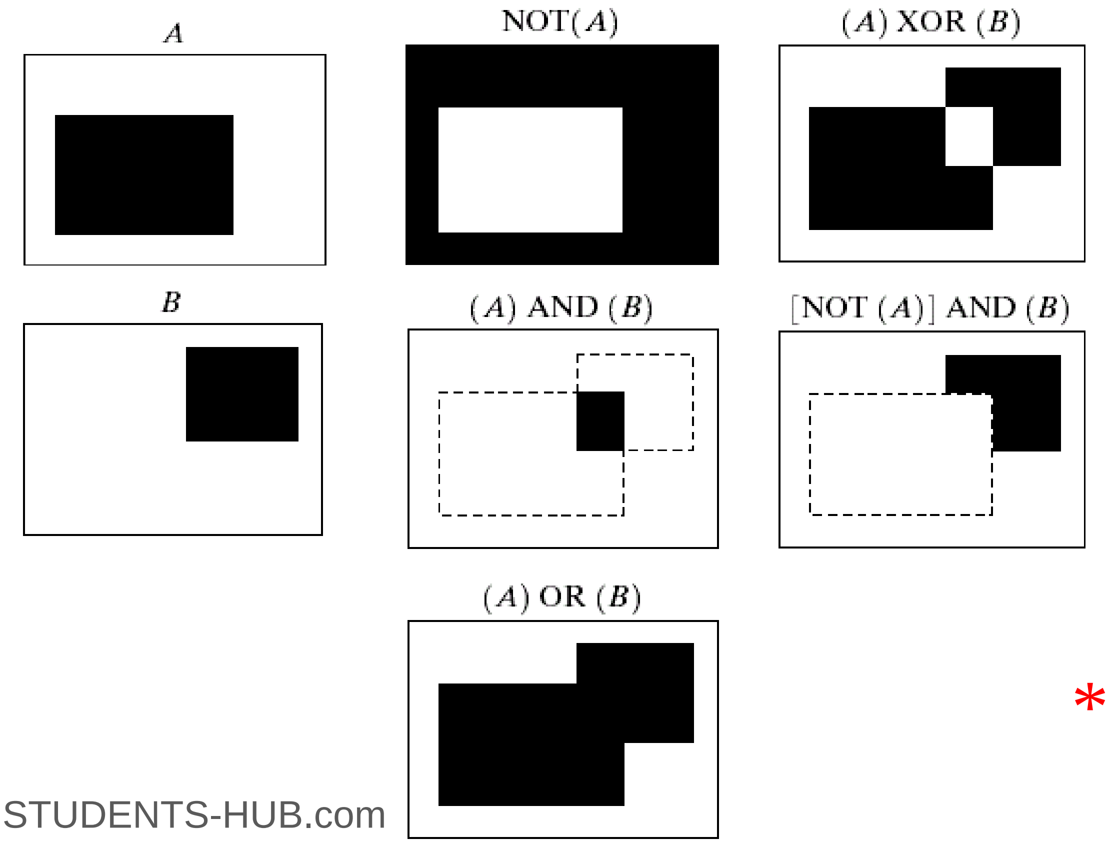


a b c
d e

FIGURE 9.1 (a) Two sets *A* and *B*. (b) The union of *A* and *B*. (c) The intersection of *A* and *B*. (d) The complement of *A*. (e) The difference between *A* and *B*.

Basic Set Operations

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

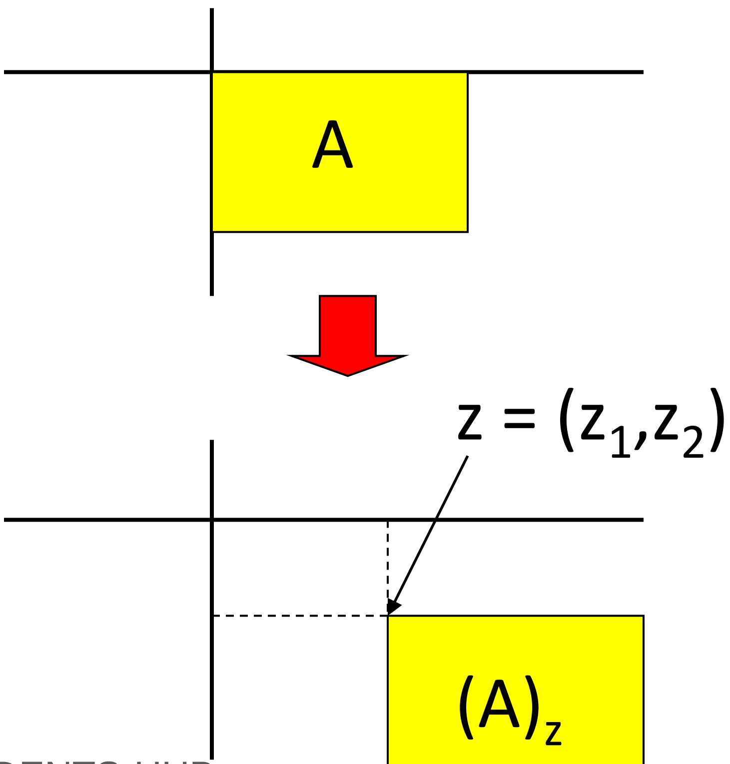


*For binary images only

Basic Set Operations

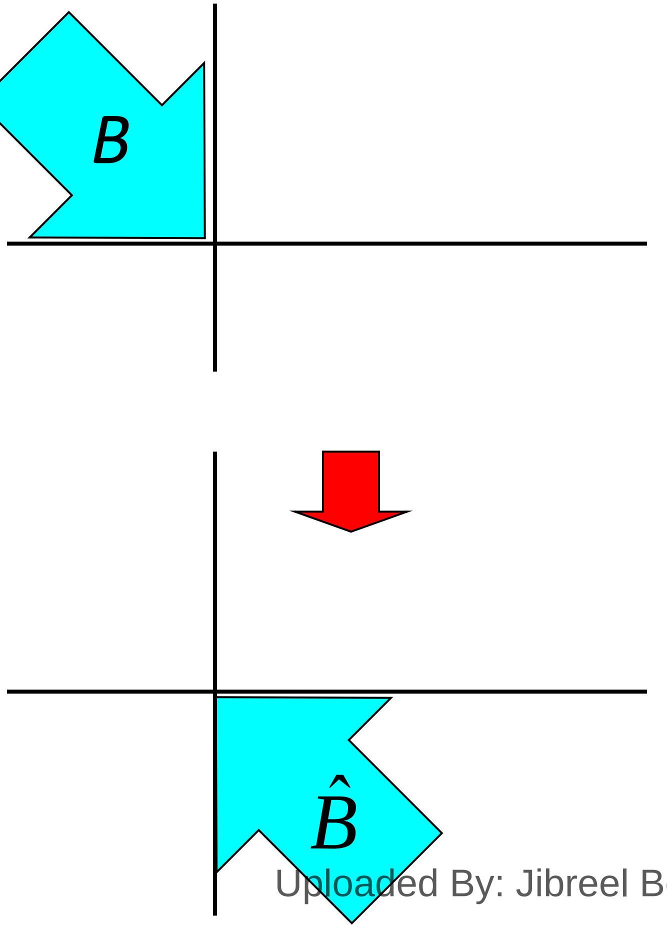
Translation

$$(A)_z = \{c \mid c = a + z, \quad \text{for } a \in A\}$$



Reflection

$$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$$



Structuring Elements

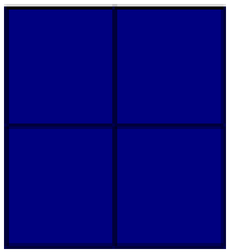
- All morphological operations are based on using structuring elements (similar to filter masks)

1	1	1
1	**1**	1
1	1	1

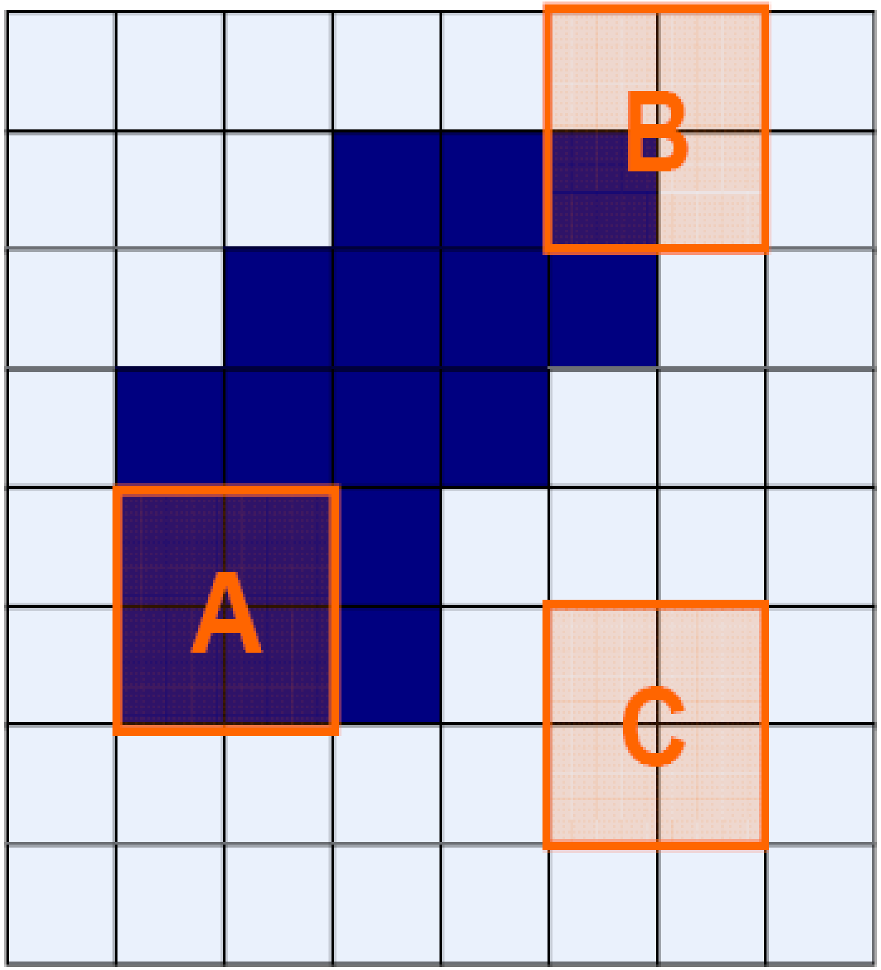
0	1	0
1	**1**	1
0	1	0

- Morphological operations are based moving the structuring element over the image pixels to check for a HIT or FIT
  - FIT - all ON *pixels* in the structuring element cover ON *pixels* in the image
  - HIT - any ON pixel in the structuring element covers an ON pixel in the image

Structuring Elements



- A – Fit
- B – Hit
- C – Miss



Structuring Elements

- Structuring elements can be of any size and make any shape
- Rectangular shapes are usually used with the origin being at the center pixel

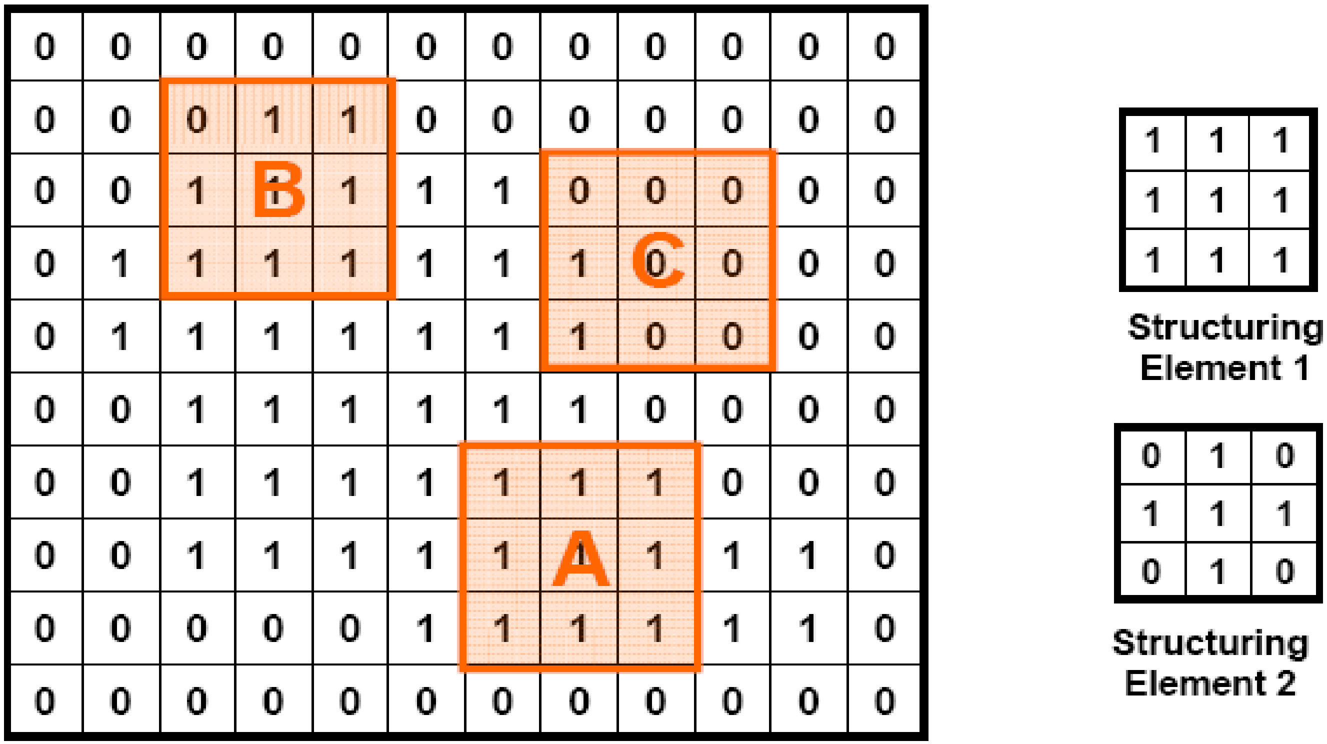
1	1	1
1	1	1
1	1	1

0	1	0
1	1	1
0	1	0

0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0

- Fundamentally morphological image processing is very like spatial filtering
- The structuring element is moved across every pixel in the original image to give a pixel in a new processed image
- The value of this new pixel depends on the operation performed

Structuring Elements



Location	Structuring Element 1	Structuring Element 2
A	FIT	FIT
B	HIT	FIT
C	HIT	HIT

Dilation

- Dilation of image A by structuring element B is given by $A \oplus B$ *such that*

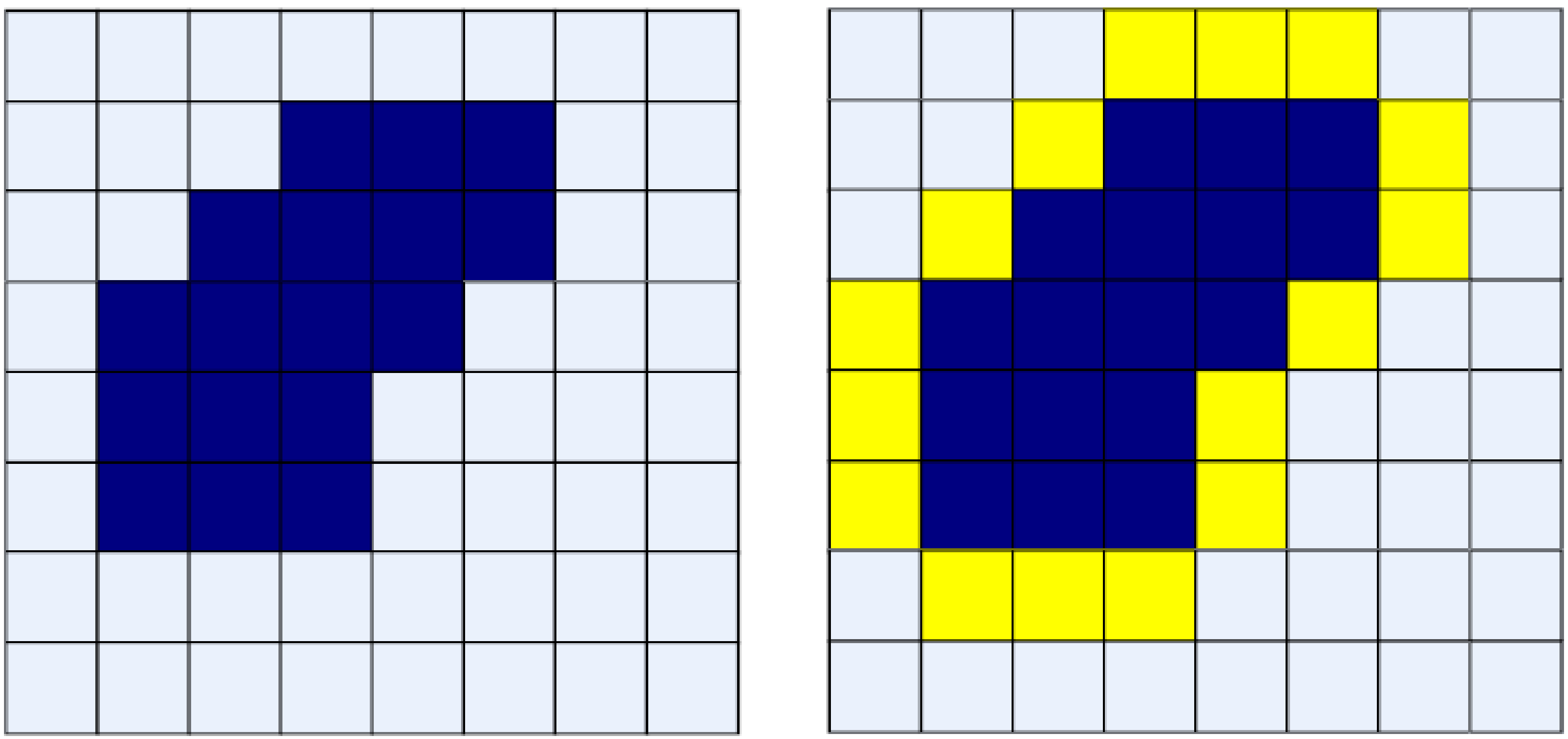
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

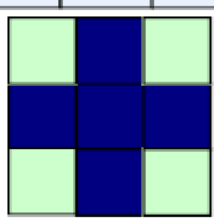
- The structuring element B is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

$$g(x,y) = \begin{cases} 1 & \text{, if } B \text{ hits } A \\ 0 & \text{, otherwise} \end{cases}$$

Dilation

Original Image　　　　　　　　Processed Image

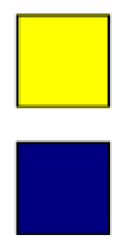




Structuring Element

* Keep pixels at which the structuring element hits at least one ON pixel

* Dilation enlarges objects



Pixels added to object

Kept Pixels

Dilation

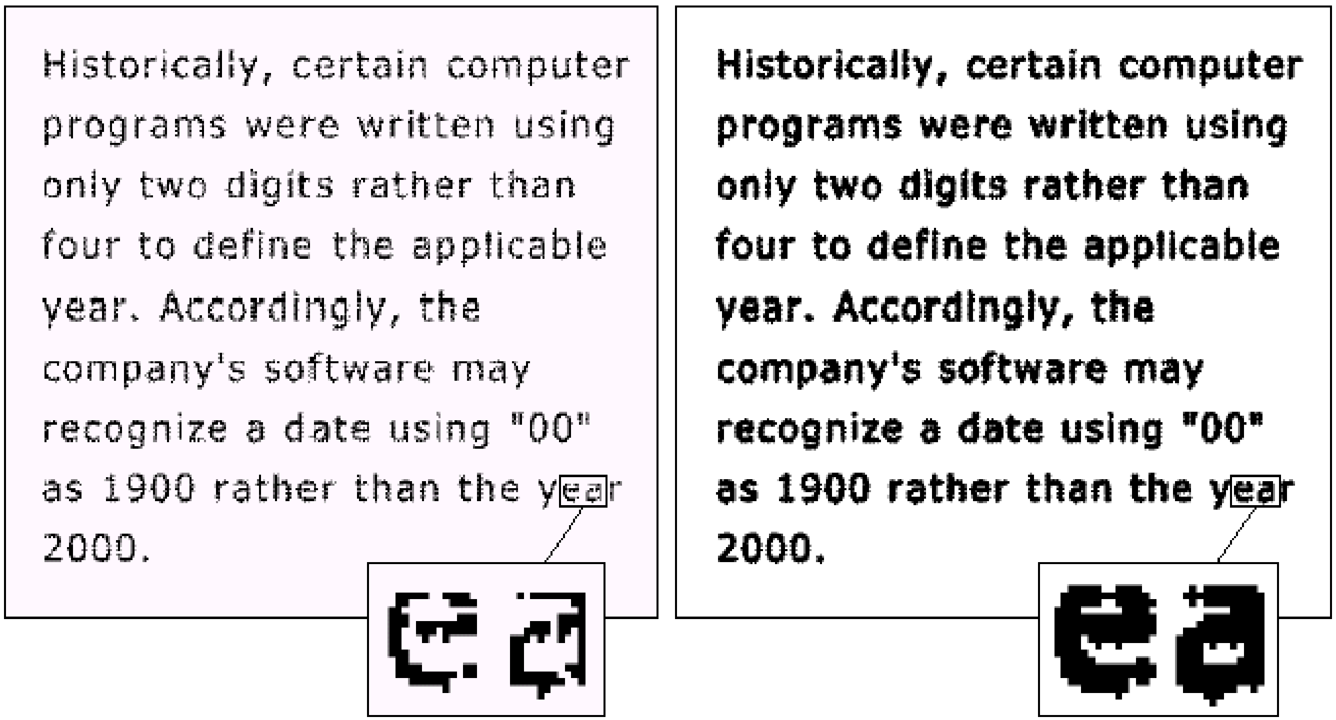
Original image

Dilation by 3*3 square structuring element

Dilation by 5*5 square structuring element

Watch out: In these examples a 1 refers to a black pixel!

Dilation



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Dilation

What Is Dilation For?

- Dilation can repair breaks

- Dilation can repair intrusions

Erosion

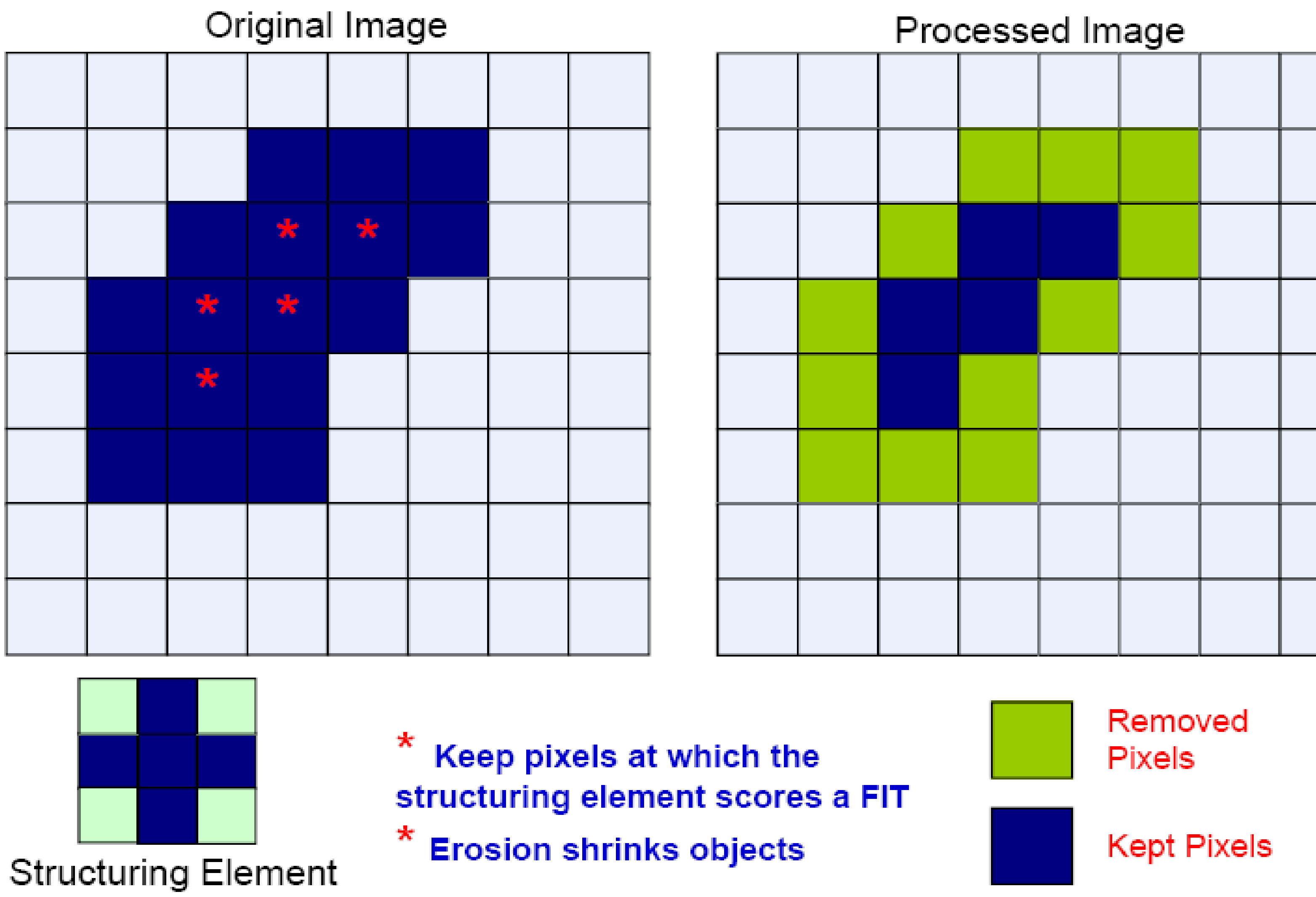
- Erosion of image A by structuring element B is given by $A \ominus B$ *such that*

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

- The structuring element B is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

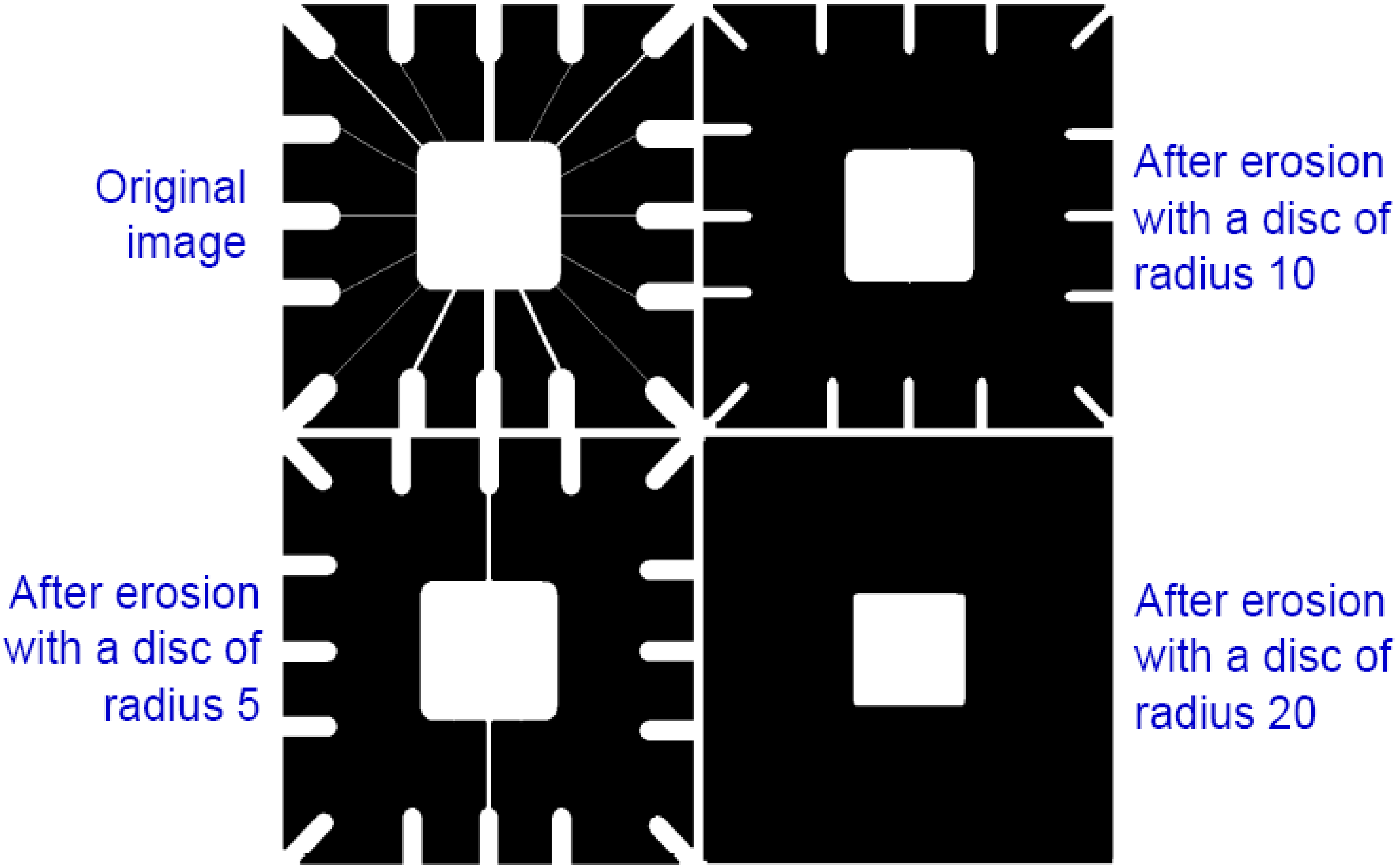
$$g(x,y) = \begin{cases} 1 & \text{, if } B \text{ fits } A \\ 0 & \text{, otherwise} \end{cases}$$

Erosion



Original Image
Processed Image
Structuring Element
* Keep pixels at which the structuring element scores a FIT
* Erosion shrinks objects
Removed Pixels
Kept Pixels

Using Erosion to remove image component



Erosion

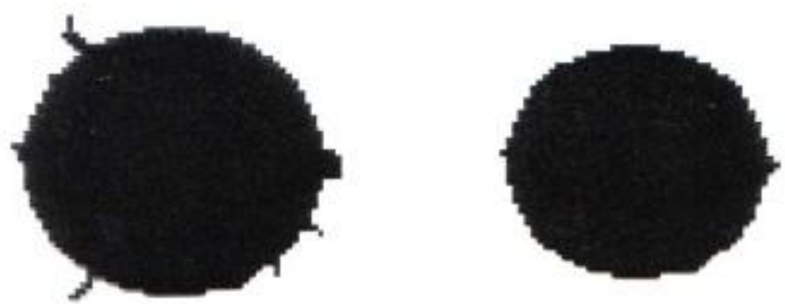
Original image — Erosion by 3*3 square structuring element — Erosion by 5*5 square structuring element

- **Watch out:** In these examples a 1 refers to a black pixel!

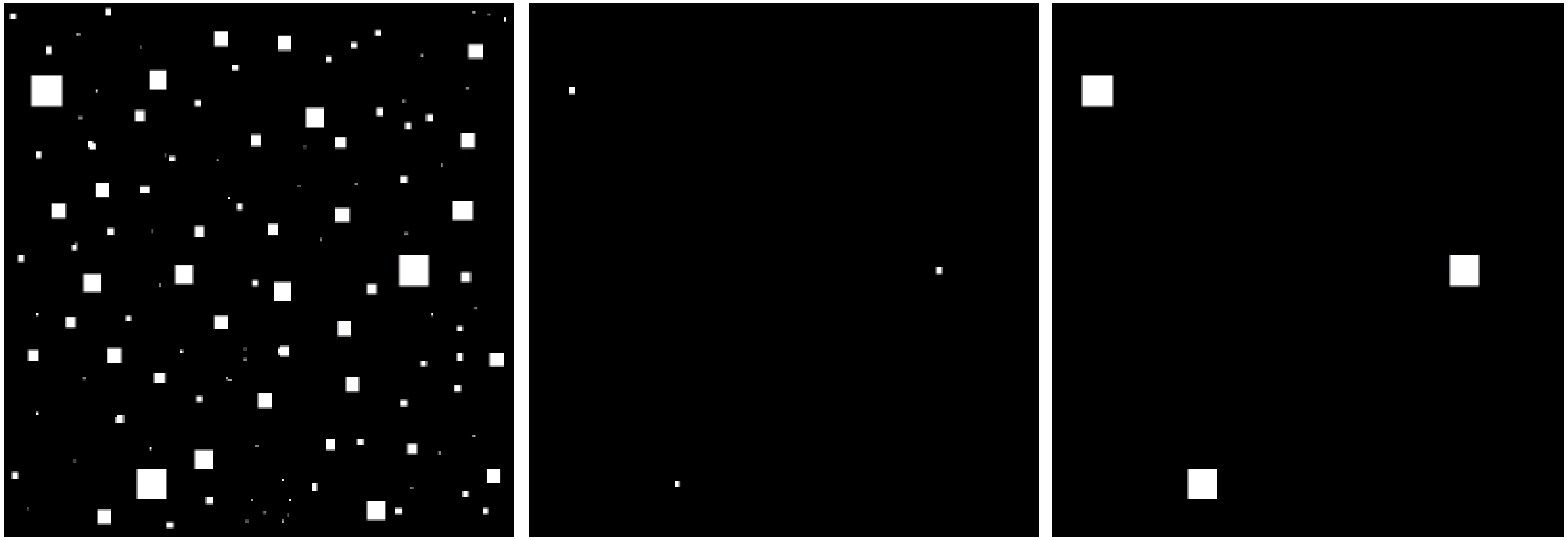
What Is Erosion For?

- Erosion can split apart joined objects

- Erosion can strip away extrusions



Application of Dilation and Erosion



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Erosion thins objects in a binary image

- Image details smaller than SE are removed

Dilation grows object in a binary image

Opening

- The opening operation on image A by structuring element B is defined as

$$A \circ B = (A \ominus B) \oplus B$$

in other words, it is erosion followed by dilation

- Generally, opening is used to
  - Smooth the contour of an object
  - Break narrow paths between large objects
  - Eliminate thin protrusions

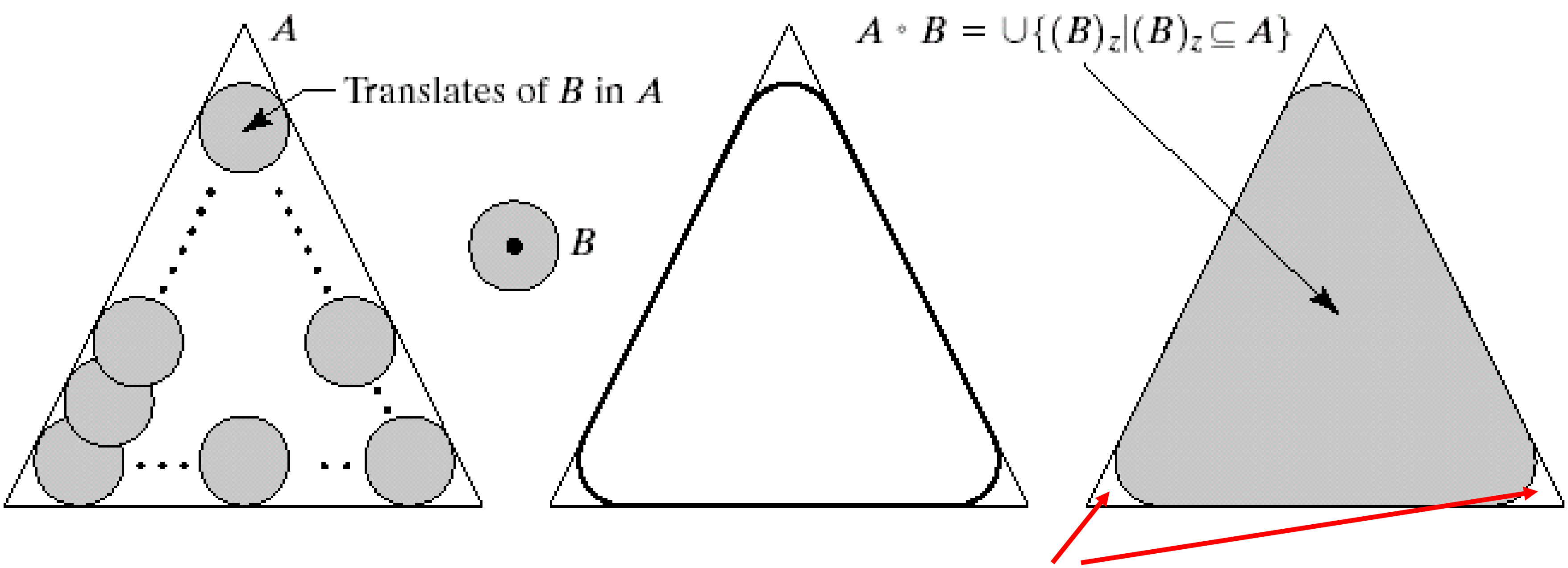
Opening

$$A \circ B = (A \ominus B) \oplus B$$

or

$$A \circ B = \cup \{(B)_z | (B)_z \subseteq A\}$$

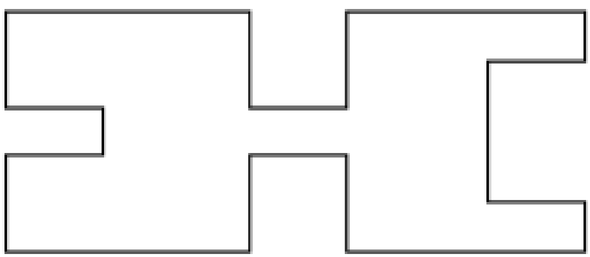
= Combination of all parts of A that can completely contain B



Opening eliminates narrow and small details and corners.

Opening

- **Example**

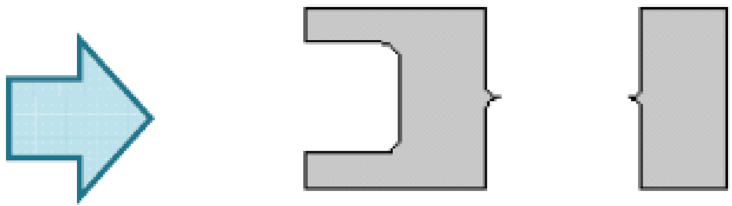


A

Original shape

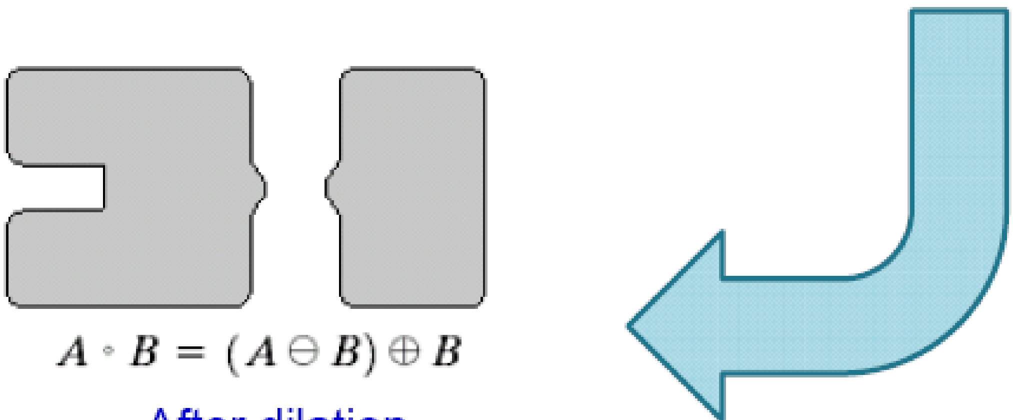
0	0	1	0	0
0	1	1	1	0
1	1	**1**	1	1
0	1	1	1	0
0	0	1	0	0

Structuring Element



$A \ominus B$

After erosion



$$A \circ B = (A \ominus B) \oplus B$$

After dilation
(opening)

Closing

- The closing operation on image A by structuring element B is defined as

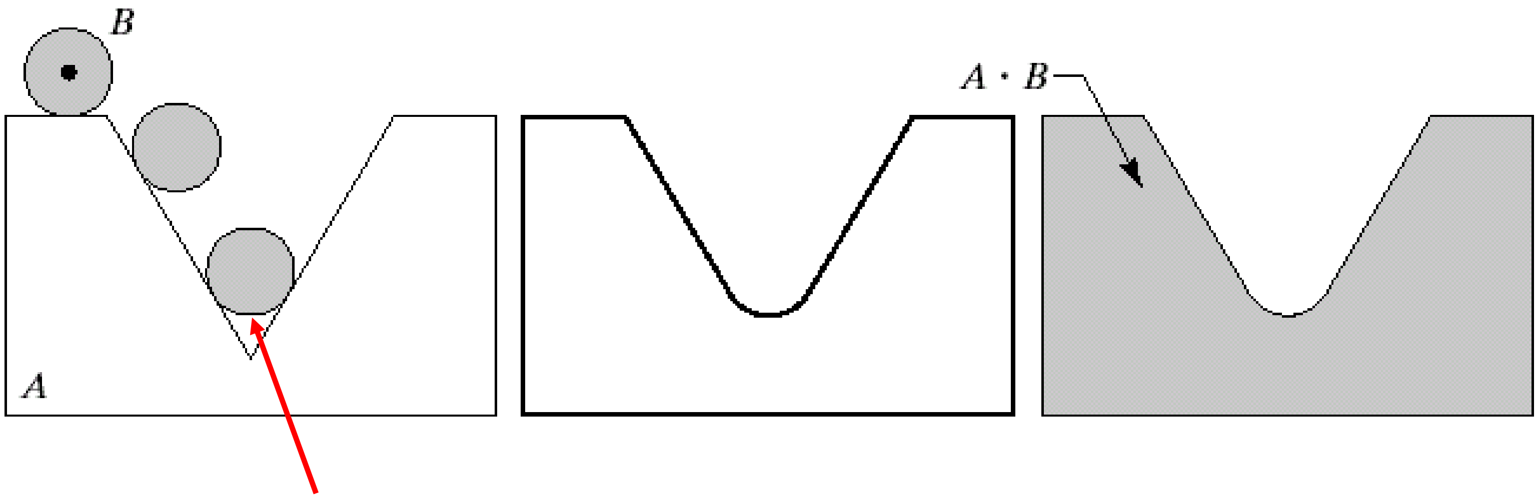
$$A \bullet B = (A \oplus B) \ominus B$$

in other words, it is dilation followed by erosion

- Generally, closing is used to
  - Smooth the contour of an object
  - Fuse narrow paths between large objects
  - Eliminate thin small holes and fill gaps in the contour

Closing

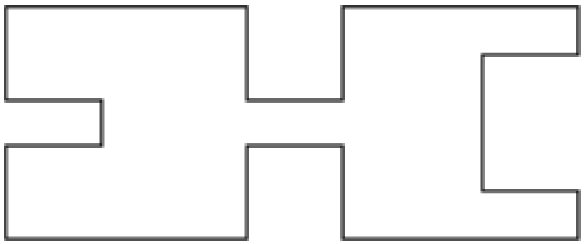
$$A \bullet B = (A \oplus B) \ominus B$$



Closing fills narrow gaps and notches

Closing

- **Example**

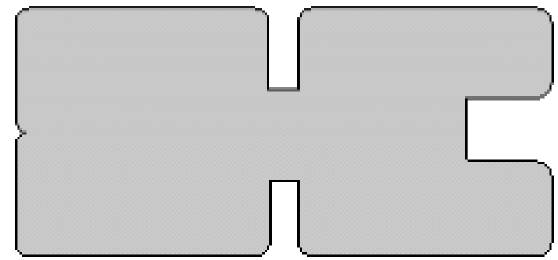


A

Original shape

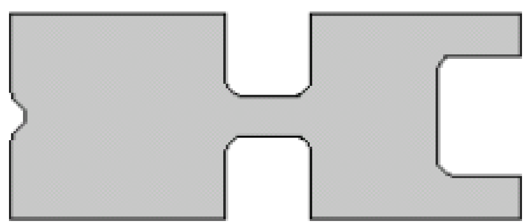
0	0	1	0	0
0	1	1	1	0
1	1	**1**	1	1
0	1	1	1	0
0	0	1	0	0

Structuring Element



$A \oplus B$

After dilation

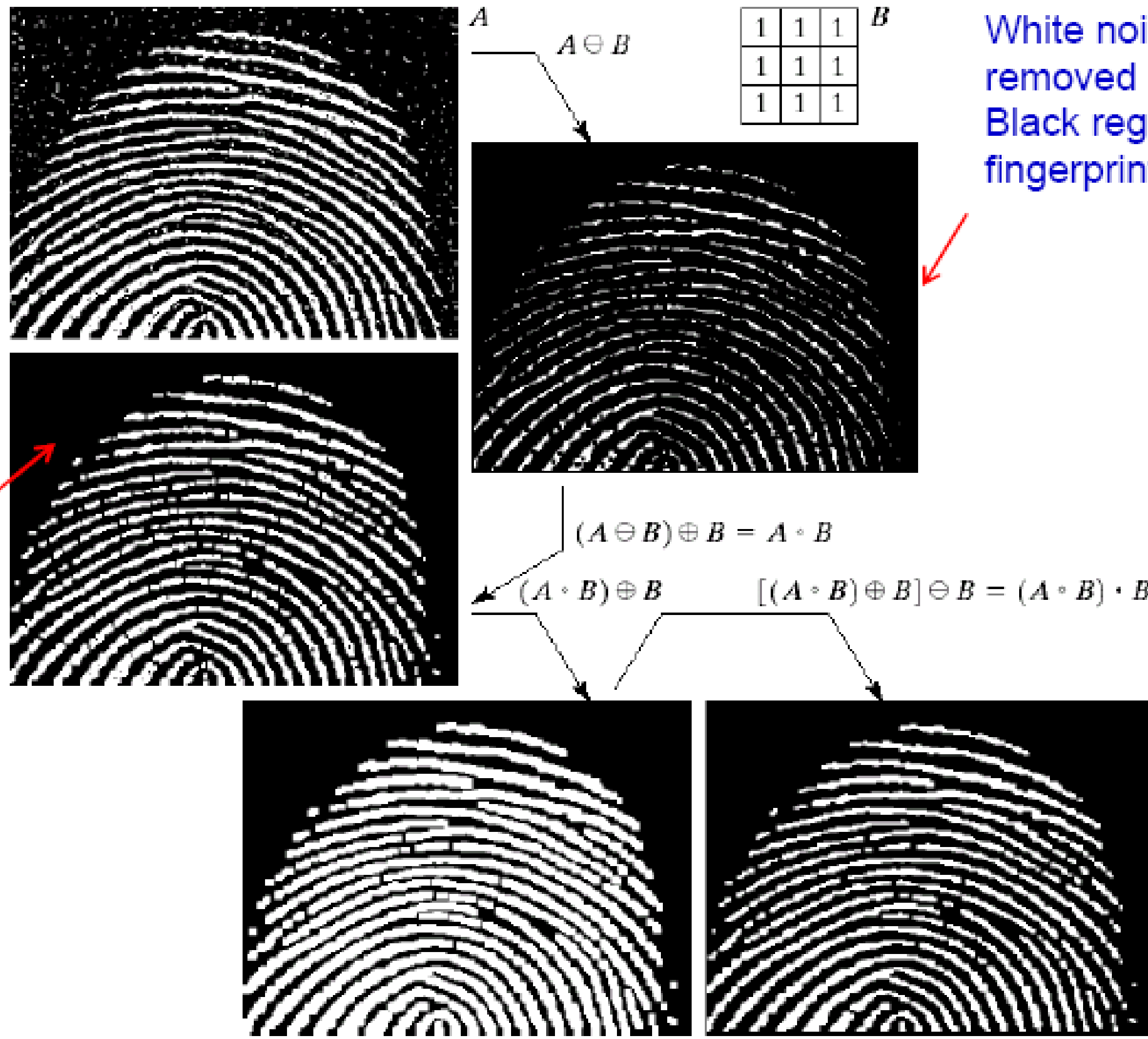


$A \cdot B = (A \oplus B) \ominus B$

After erosion
(Closing)

Example

White noise regions removed
Black regions within the fingerprint have enlarged



Black regions within the fingerprint shrunken
But gaps are introduced between ridges

Hit-or-Miss Transformation

- A basic tool for shape detection
- Suppose we have an image A that consists of three objects C, D, and E and we want to detect the presence of shape D; assuming we know its shape

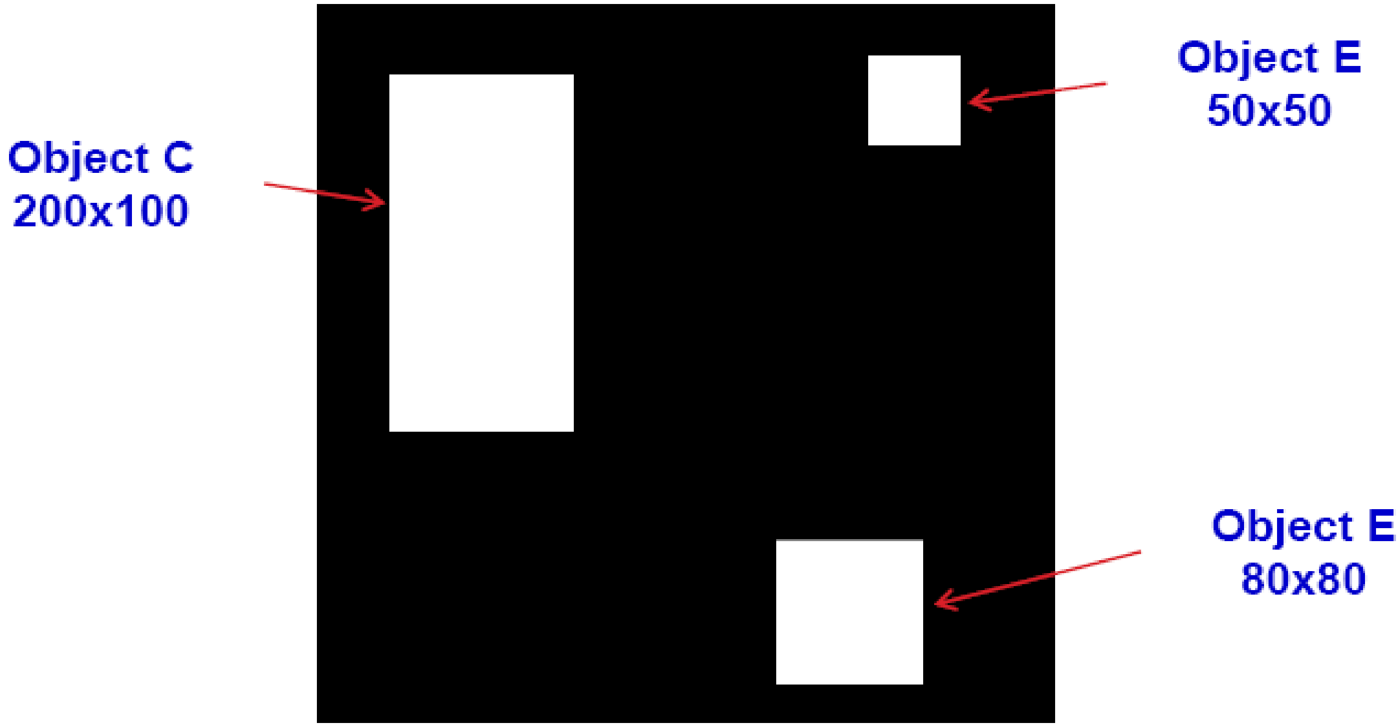
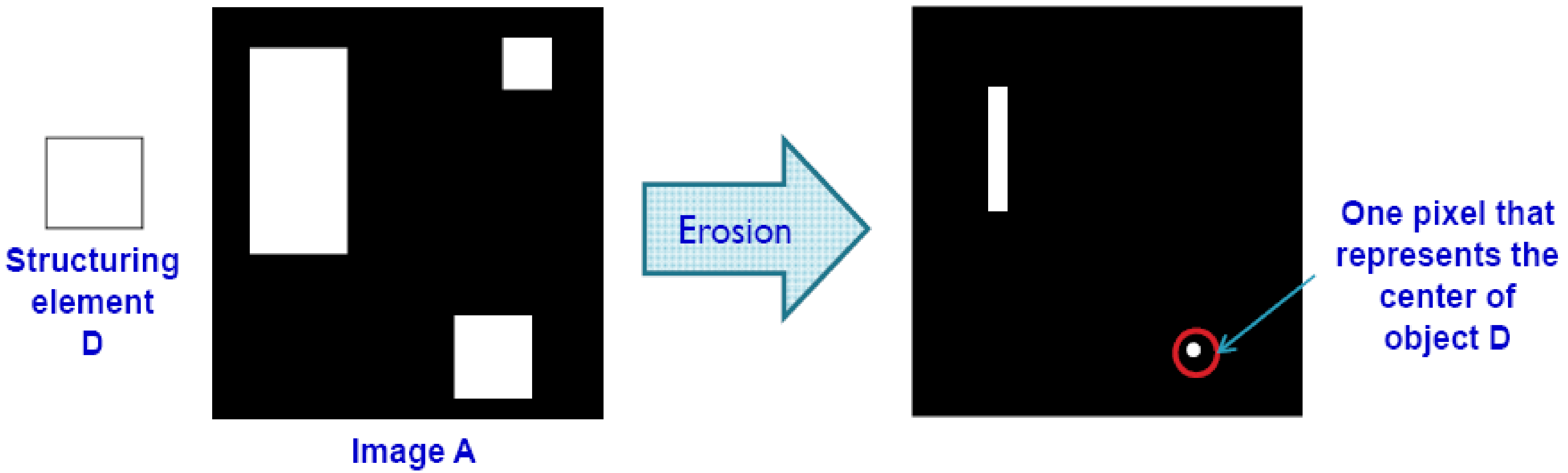


Image A

Hit-or-Miss Transformation

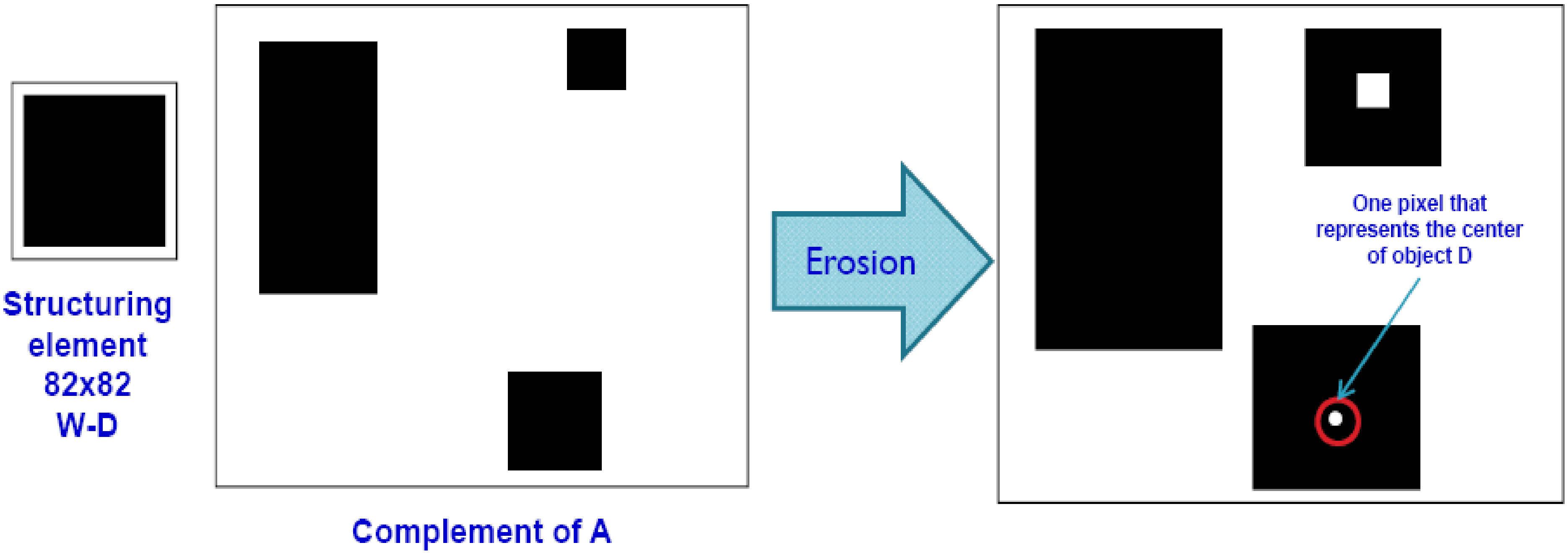
- We can start by eroding A with a structuring element that with the same shape as D



- However, the result may contain parts of larger objects

Hit-or-Miss Transformation

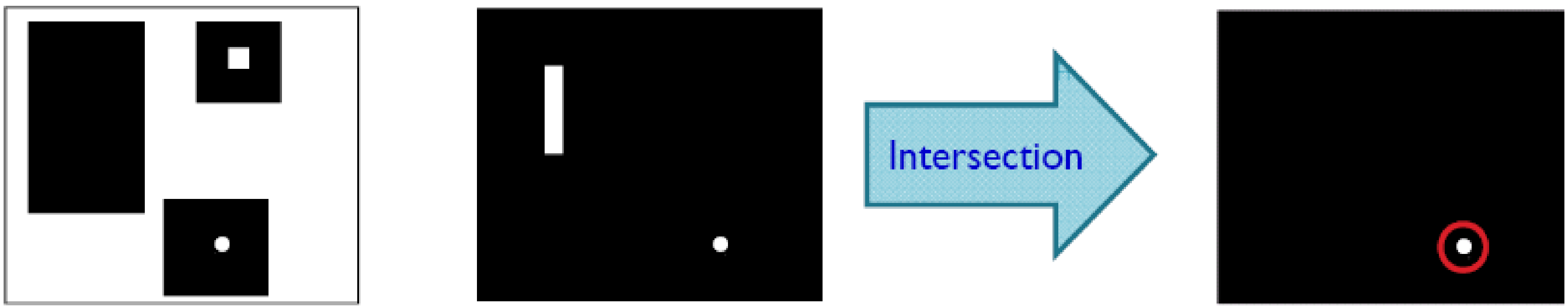
- Consider the complement of A and a new structuring element that is one pixel thicker than the object



- Note how the center of object D was detected again !

Hit-or-Miss Transformation

- Consider the intersection of the erosion results from the previous two slides



- Object is successfully located !
- Formally, the hit-or-miss transformation on image A to detect some object X, is performed by

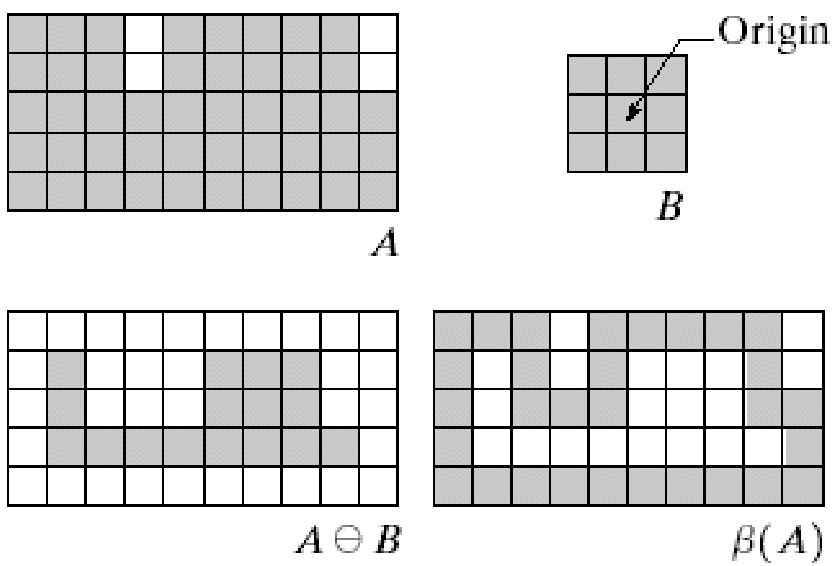
$$A \circledast X = (A \ominus X) \cap [Ac \ominus (W - X)]$$

where W is a the object X thickened by one pixel

Boundary Extraction

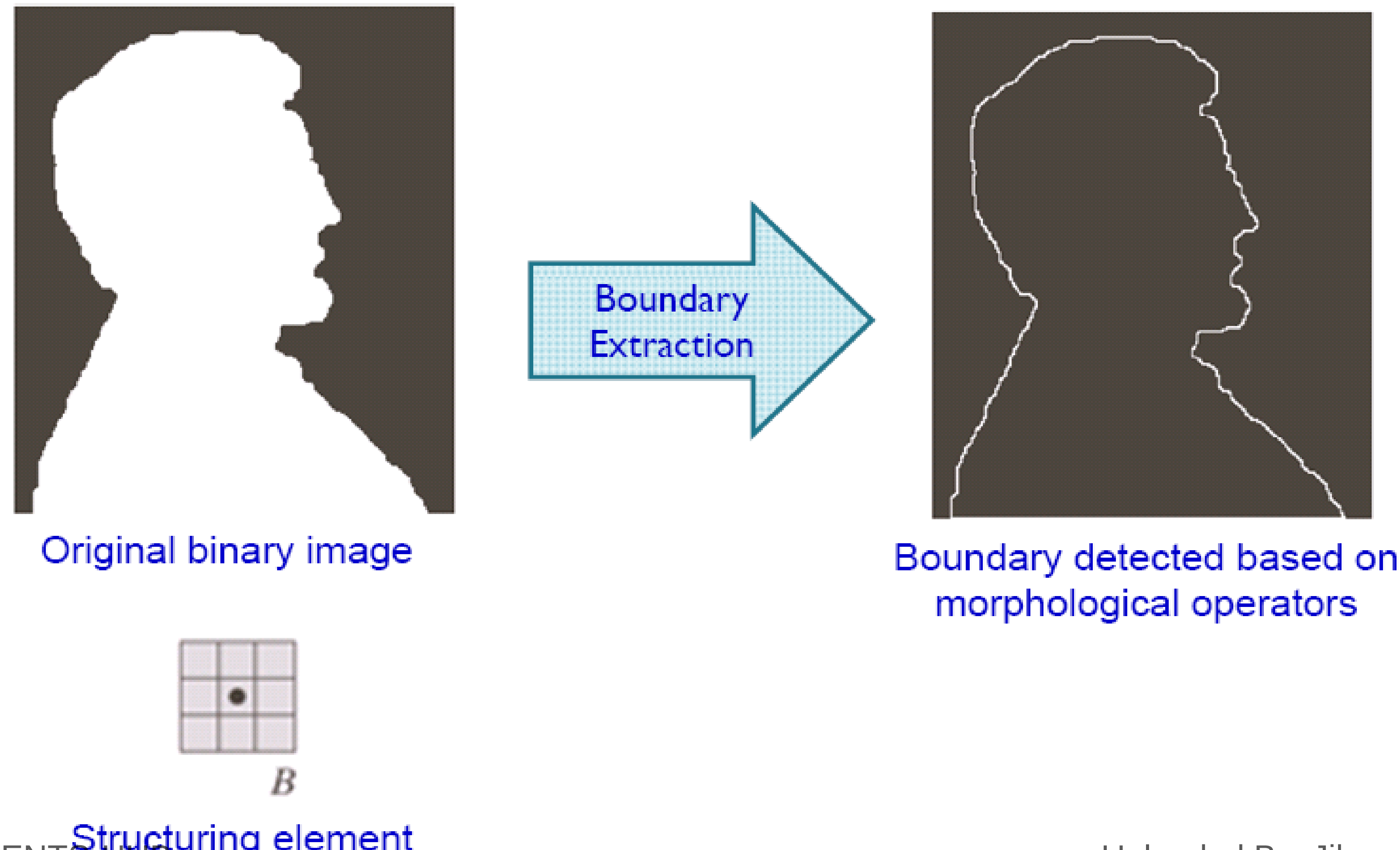
- The boundary of set A denoted by $\beta(A)$ can be obtained by first eroding A by a suitable structuring element B then perform the set difference between A and its erosion

$$\beta(A) = A - (A \ominus B)$$



Boundary Extraction

- **Example**



Hole Filling

- A hole may be defined as a background region surrounded by a connected border of foreground pixels
- The algorithm presented here assumes that we know one pixel for each hole in the image
- **Algorithm**
  - Form an array X_0 of the same size as A and initialize it with zeros except locations that correspond to pixels inside the regions to be filled
  - Apply the following equation iteratively on array X until $X_k = X_{k-1}$ to form the filled holes

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

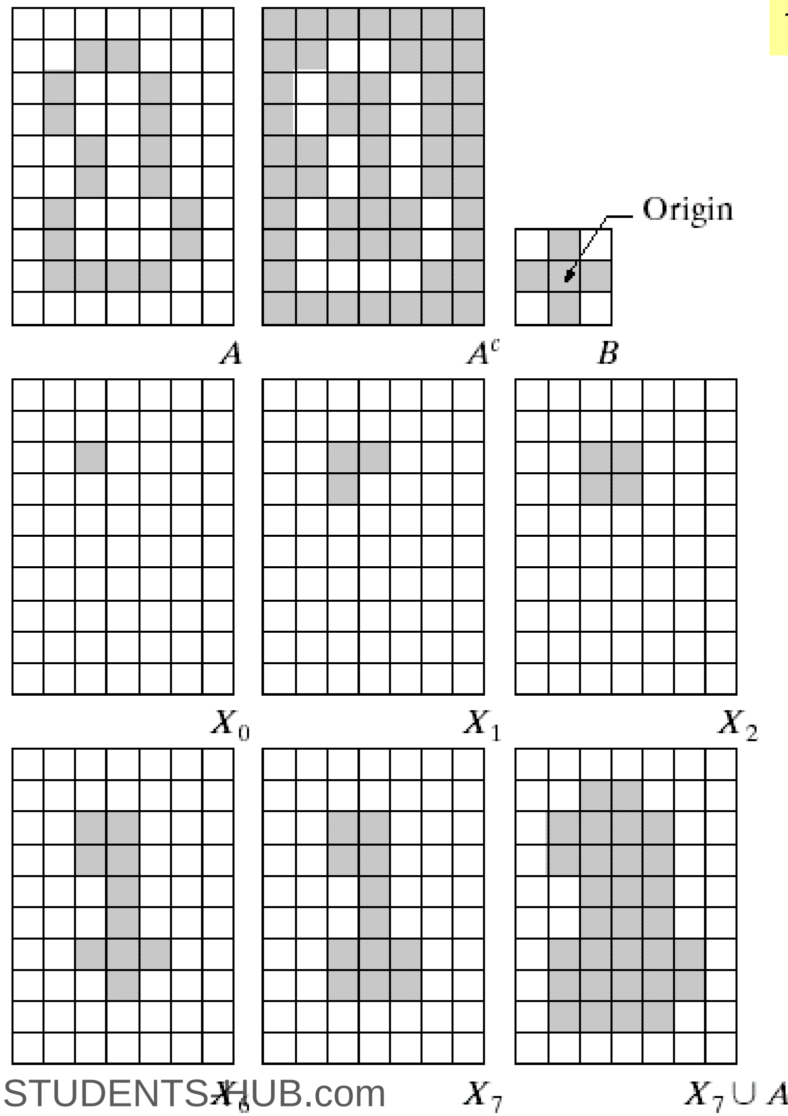
  - The original image with filled holes is found by

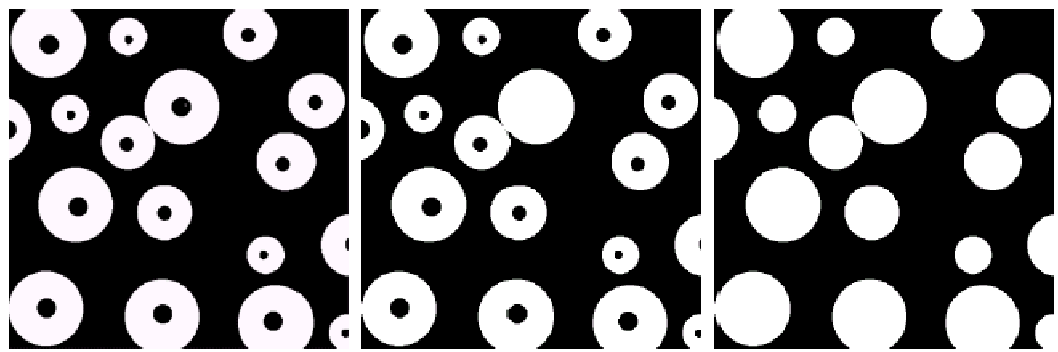
$$A_{filled} = X_k \cup A$$

Hole Filling

$$X_k = \left(X_{k-1} \oplus B\right) \cap A^c$$

where X_0 = seed pixel p





Original image

Results of region filling

Connected Components

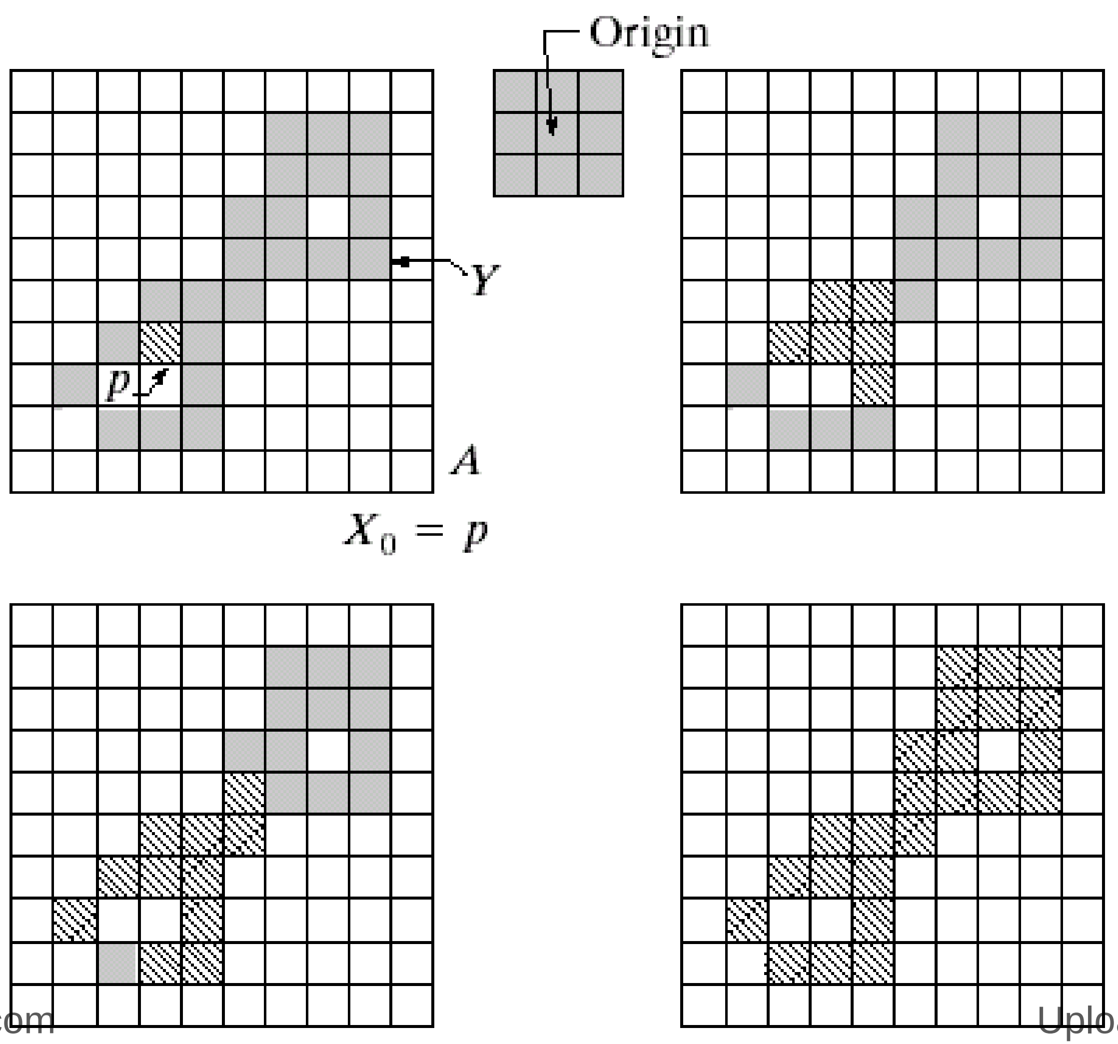
- Let Y represent a connected component contained in a set A
  - Assume that a point p of Y is known
  - Then the following yields all the element of

$$X_k = (X_{k-1} \oplus B) \bigcap A, \quad k = 1,2,3,\ldots$$

    - where X_0=p, B is the suitable structuring element
    - The algorithm terminates if $X_k = X_{k-1}$, and let $Y = X_k$

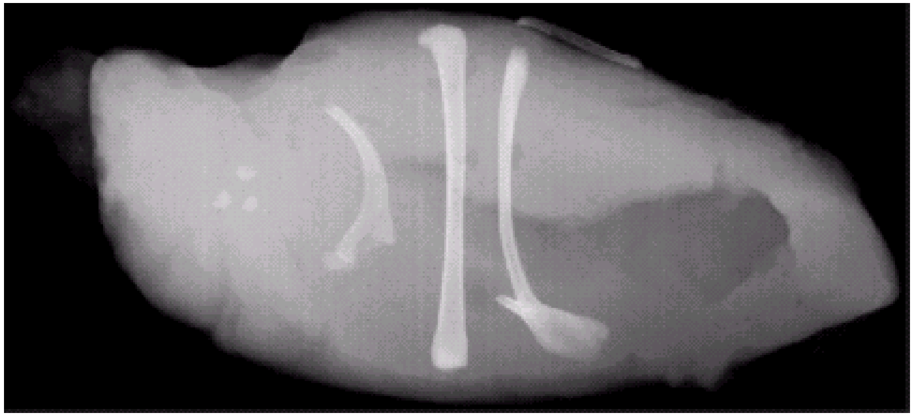
Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A$$ where X_0 = seed pixel p



Connected Components

X-ray image of bones



Thresholded image

Connected components

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

- A set A is said to be convex
  - if the straight line segment joining any two points in A lies entirely within A
- The convex hull H(=C(S)) of an arbitrary set S is the smallest convex set containing S
- The set difference H-S is called the convex deficiency of S
- Morphological algorithm for obtaining the convex hull C(A) of a set A

➡ Let B^i , i=1,2,3,4, representing the four structuring elements

➡ $X_k^i = (X_{k-1}^i \circledast B^i) \cup A, \quad i = 1,2,3,4 \; and \; k = 1,2,3,\ldots$

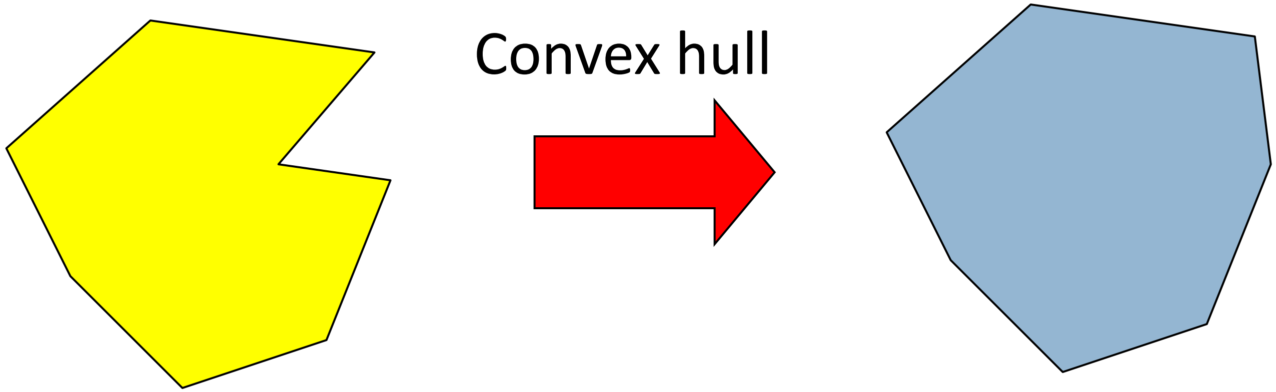
with $X_0^i = A$

➡ When the results converge to D^i , the convex hull of A is given by

$$C(A) = \bigcup_{i=1}^{4} D^i$$

Convex Hull

Convex hull has no concave part.



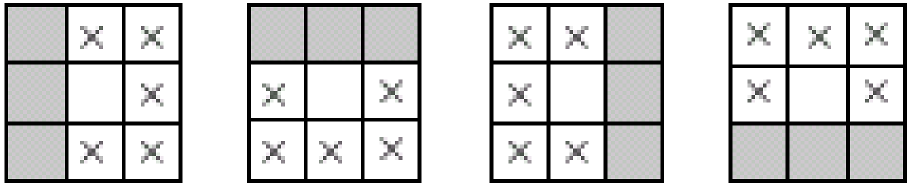
Algorithm:

$$C(A) = \bigcup_{i=1}^{4} D^i$$

where

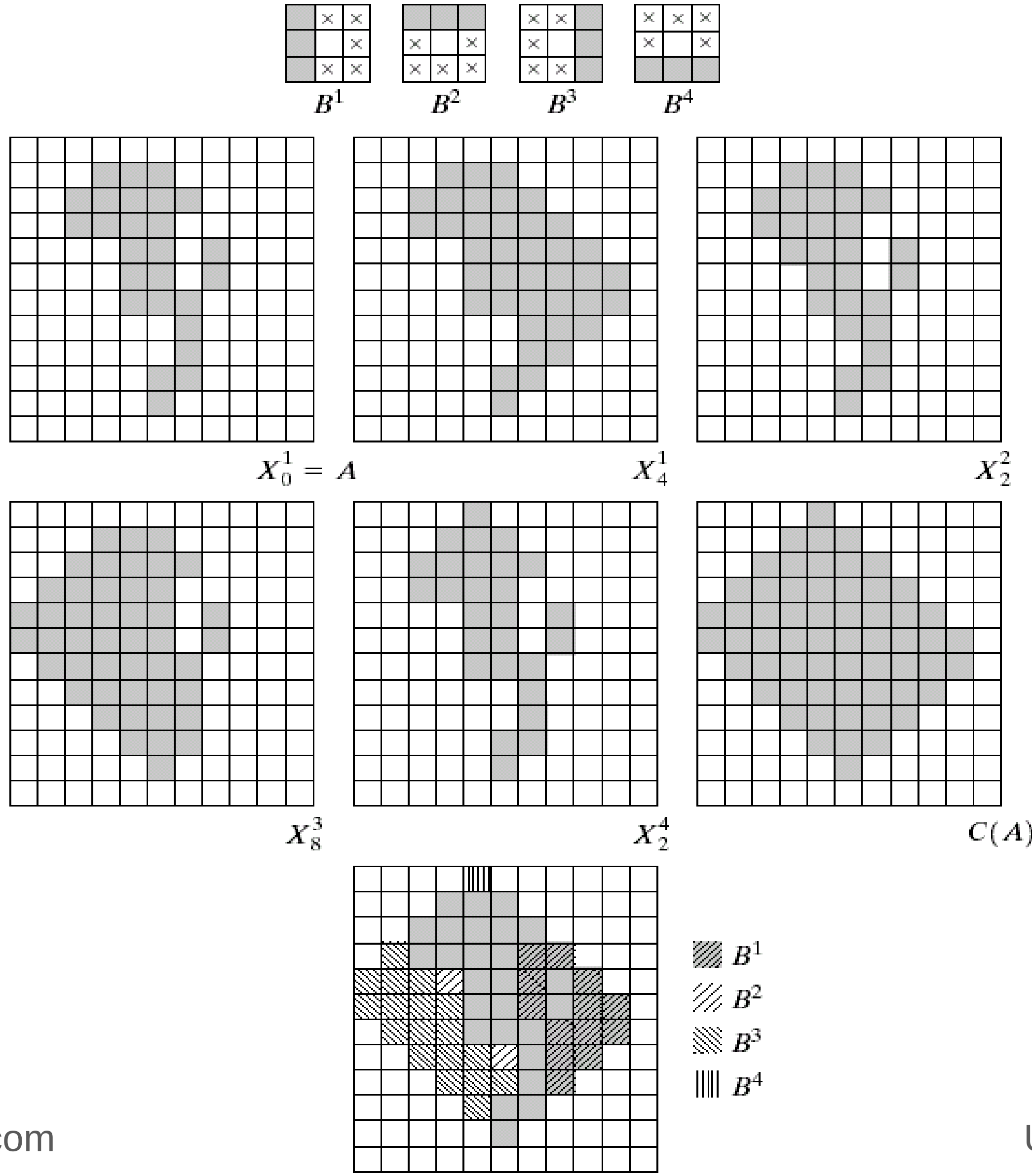
$$D^i = X^i_{conv}$$

$$X^i_k = \left(X_{k-1} \circledast B^i\right) \cup A, \qquad i = 1,2,3,4$$



B^1 B^2 B^3 B^4

Convex Hull



Thinning

- The thinning of a set A by a structuring element B

$$A \otimes B = A - (A \circledast B)$$

$$= A \bigcap (A \circledast B)^c$$

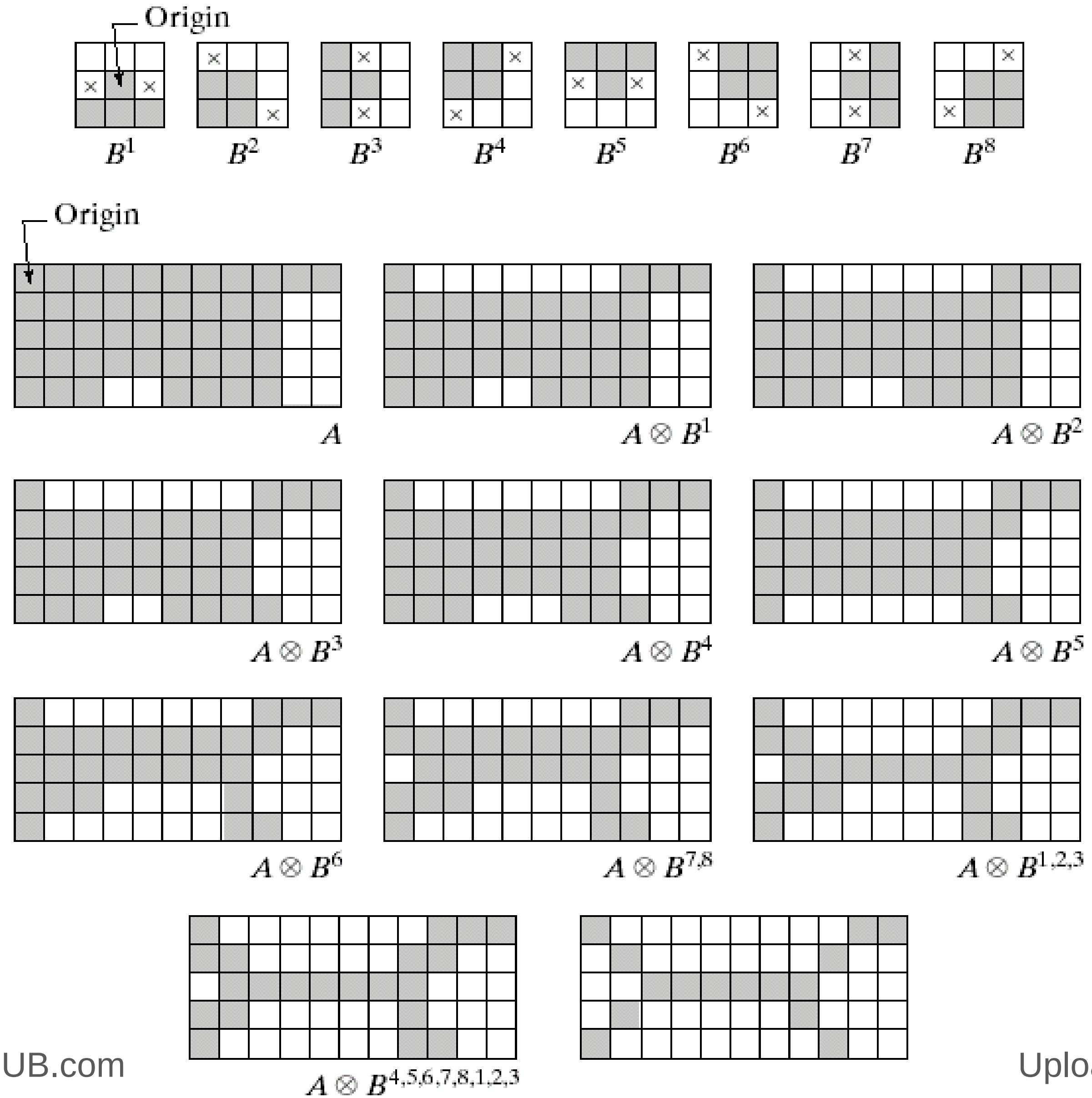
- A more useful expression for thinning based on a sequence of structuring elements

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

- where B^i is a rotated version of B^{i-1}

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

Thinning



Origin
B^1
B^2
B^3
B^4
B^5
B^6
B^7
B^8
Origin
A
$A \otimes B^1$
$A \otimes B^2$
$A \otimes B^3$
$A \otimes B^4$
$A \otimes B^5$
$A \otimes B^6$
$A \otimes B^{7,8}$
$A \otimes B^{1,2,3}$
$A \otimes B^{4,5,6,7,8,1,2,3}$

Thickening

- Thickening is the morphological dual of thinning

$$A \odot B = A \cup (A \circledast B)$$

- As in thinning, thickening can be defined as a sequential operation

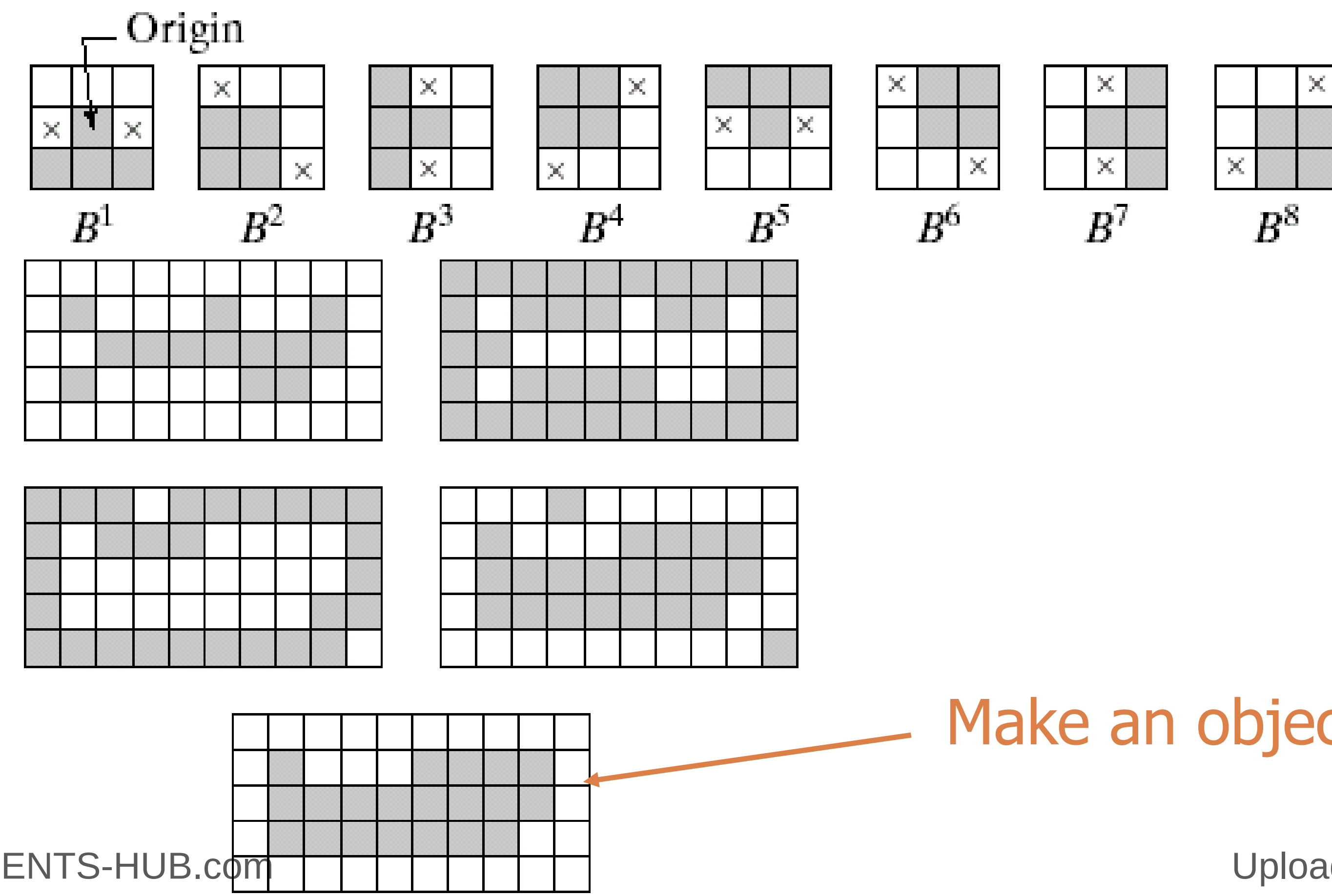
$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

- The structuring elements used for thickening have the same form in thinning, but with all 1's and 0's interchanged
- In general, thickening is accomplished by thinning the background and then taking complement of the result
- The thinned background forms a boundary for the thickening process

Thickening

$$A \odot B = A \cup (A \circledast B)$$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$



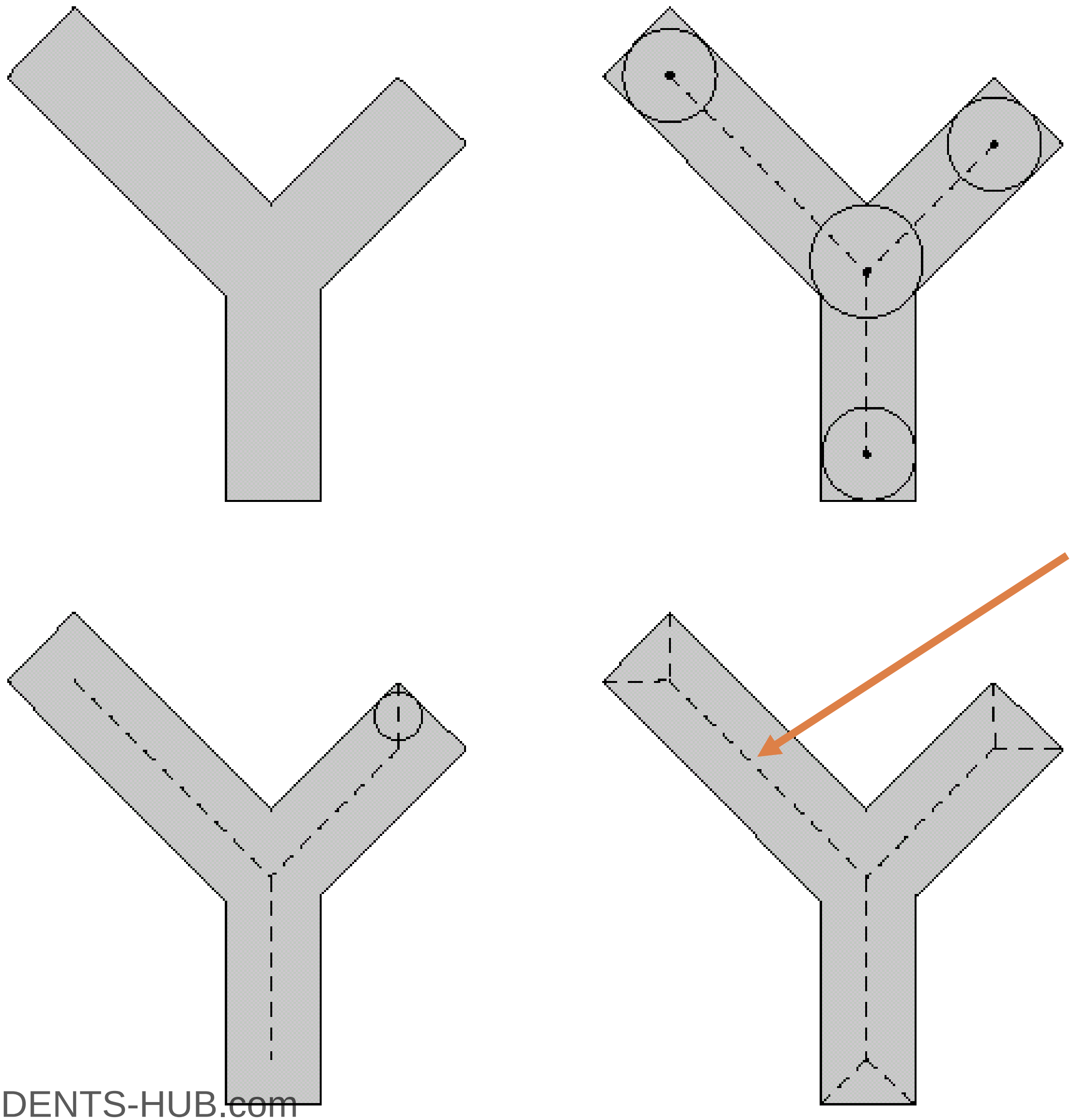
Make an object thicker

Skeletons

- Skeleton, S(A), of a set A
  1. If z is a point of S(A) and (D)z is the largest disk centered at z and contained in A, one cannot find a larger disk containing (D)z and included in A. The disk (D)z is called a maximum disk
  2. The disk (D)z touches the boundary of A at two or more different places

- An inner point belongs to the skeleton if it has at least two closest boundary points

Skeletons



Dot lines are skeletons of this structure

Skeletons

- Morphological Skeleton $$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

  where $$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$
  $$(A \ominus kB) = (\ldots(A \ominus B) \ominus B) \ominus \ldots) \ominus B$$

  k successive erosions
- And K is the last iterative step before A erodes to an empty set
- The set A can be reconstructed by
  $$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

  where k successive dilations

$$(S_k(A) \oplus kB) = ((\ldots(S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B$$