📘 Chapter 7 Summary: Production Analysis and Compensation Policy

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🔑 1. Key Concepts

- **Production Function** (discrete vs continuous)
- **Total Product (TP), Marginal Product (MP), Average Product (AP)**
- **Law of Diminishing Returns**
- **Returns to a Factor vs Returns to Scale**
- **Isoquants & MRTS**
- **Marginal Revenue Product (MRP)**
- **Isocost & Expansion Path**
- **Productivity Measurement**

⚙️ 2. Production Functions

- **Production Function**: Describes output as a function of inputs.
- **Discrete**: Inputs change in lumps.
- **Continuous**: Inputs vary in small increments.

Returns to Scale (RtS): Changes in output from changing *all* inputs.
Returns to a Factor: Changes in output from changing *one* input.

📈 3. Product Measures

- **Total Product (TP)**: Total output.
- **Marginal Product (MP)**:

$$MP_X = \frac{\partial Q}{\partial X}$$

  - If MP > 0 → TP is rising
  - If MP < 0 → TP is falling
- **Average Product (AP)**:

$$AP_X = \frac{Q}{X}$$

🔻 4. Law of Diminishing Returns

- As more of one input is used (holding others fixed), **MP eventually declines**.
- **Three Stages of Production**:
  - **Stage I**: $\varepsilon > 1$ (increasing returns)
  - **Stage II**: $0 \leq \varepsilon \leq 1$ (**economically efficient**)
  - **Stage III**: $\varepsilon < 0$ (output falls)

💡 5. Isoquants & Input Choices

- **Isoquants**: Curves showing efficient combinations of inputs for same output.

Shapes:

- Straight: Perfect substitutes
- Curved (C-shape): Imperfect substitutes
- L-shape: No substitutability
- **Technical Efficiency**: Least-cost input combination.
- **Marginal Rate of Technical Substitution (MRTS)**:

$$MRTS = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$$

💰 6. Marginal Revenue Product (MRP) & Optimal Input Use

MRP of Labor (MRP_L):

$$MRP_L = MP_L \times MR = \frac{dTR}{dL}$$

- **Optimal employment** occurs when:

$$MRP_L = w \quad \text{(wage rate)}$$

- **General Rule:**

$$MRP_X = P_X \quad \text{(Input's price)}$$

Example:

- Price = $10/unit, Wage = $20/labor unit
- Find MRP and compare to wage → Employ labor until MRP = w

⚖️ 7. Optimal Combination of Multiple Inputs

- **Condition:**

$$MRTS = \frac{w}{r}$$

Examples:

- Q = 10: K = 5, L = 5 → Total Cost = $100
- Q = 14: K = 5, L = 10 (w = $5, r = $10) → TC = $100
- **Expansion Path:** All optimal points at same w/r

📊 8. Optimal Levels of Inputs and Profit Maximization

- Maximize profit when:

$$MRP_X = P_X \quad \text{for all inputs}$$

- Requires:
  - (A) Optimal input **proportions**
  - (B) Optimal **output level**
- Leads to **allocative efficiency**.

📏 9. Returns to Scale (RtS)

- Measures output change from proportional increase in *all* inputs.

Output Elasticity:

$$\varepsilon_Q = \frac{\partial Q/Q}{\partial X_i/X_i}$$

- **If** $\varepsilon > 1$ → Increasing RtS (IRS)
- **If** $\varepsilon = 1$ → Constant RtS (CRS)
- **If** $\varepsilon < 1$ → Decreasing RtS (DRS)

Power Production Function:

$$Q = AK^{\alpha}L^{\beta}$$

- IRS if $\alpha + \beta > 1$
- CRS if $\alpha + \beta = 1$
- DRS if $\alpha + \beta < 1$

📈 10. Productivity Measurement

- **Productivity Growth**: Rate of output increase per unit input.
- **Labor Productivity**: Output per worker hour.

Sources of Growth:

- **Efficiency gains**: Better use of inputs
- **Capital Deepening**: More capital per worker

📌 Three Important Relationships

1. A certain output can be made with **various K-L combinations** (substitutability).
2. **Returns to scale** apply when **all** inputs change.
3. **Returns to a factor** analyze one input change (others fixed), often showing **diminishing returns**.