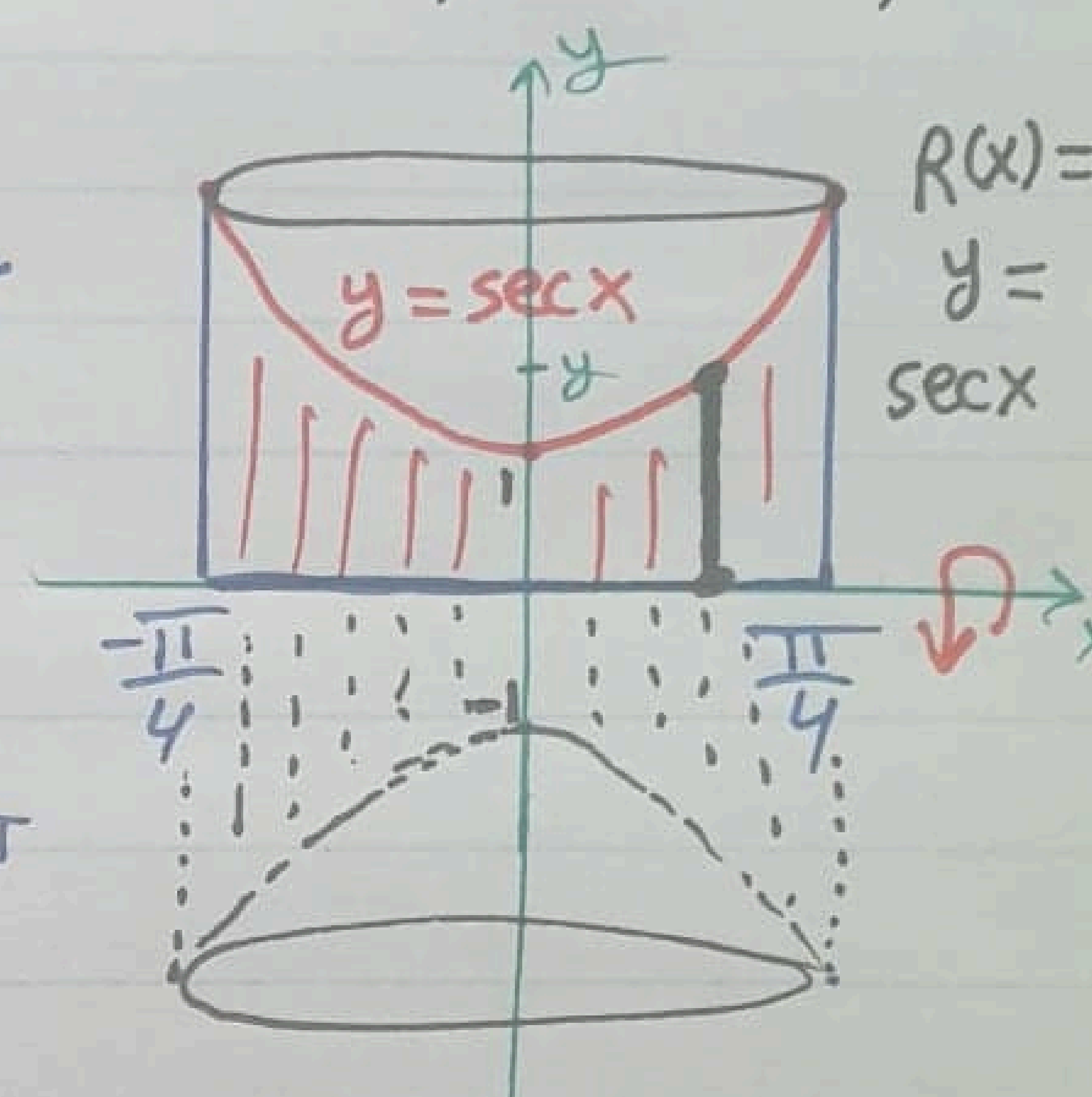
Discussion $\boxed{6.1}$

$\boxed{24}$ Find the volume of the solid generated by revolving the region bounded by the curve

$y = \sec x$ and the lines $y=0$, $x = -\frac{\pi}{4}$, $x = \frac{\pi}{4}$ about x-axis



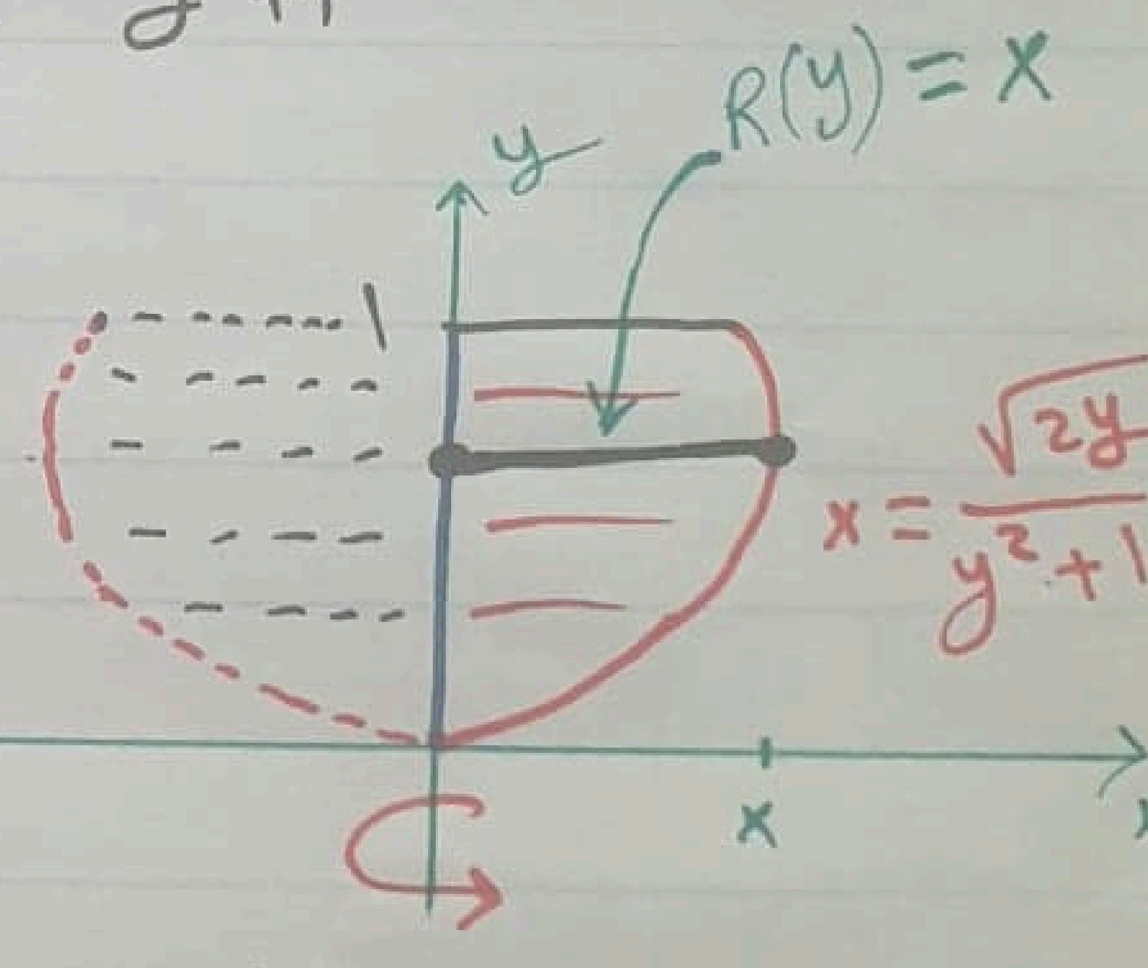
$$V = \int_a^b A(x)\,dx$$

since CS is disk $A(x) = \pi R^2(x)$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi \sec^2 x \, dx$$

$$= \pi \tan x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \pi \left[1 - -1\right] = 2\pi$$

$\boxed{32}$ Find the volume of the solid generated by revolving the region bounded the curve $x = \frac{\sqrt{2y}}{y^2+1}$ and the lines $x=0$, $y=1$ about y-axis



$$V = \int_c^d A(y)\,dy$$

since CS is disk $A(y) = \pi R^2(y)$

$$= \int_0^1 \pi \frac{2y}{(y^2+1)^2}\,dy$$

$u = y^2+1$

$du = 2y\,dy$

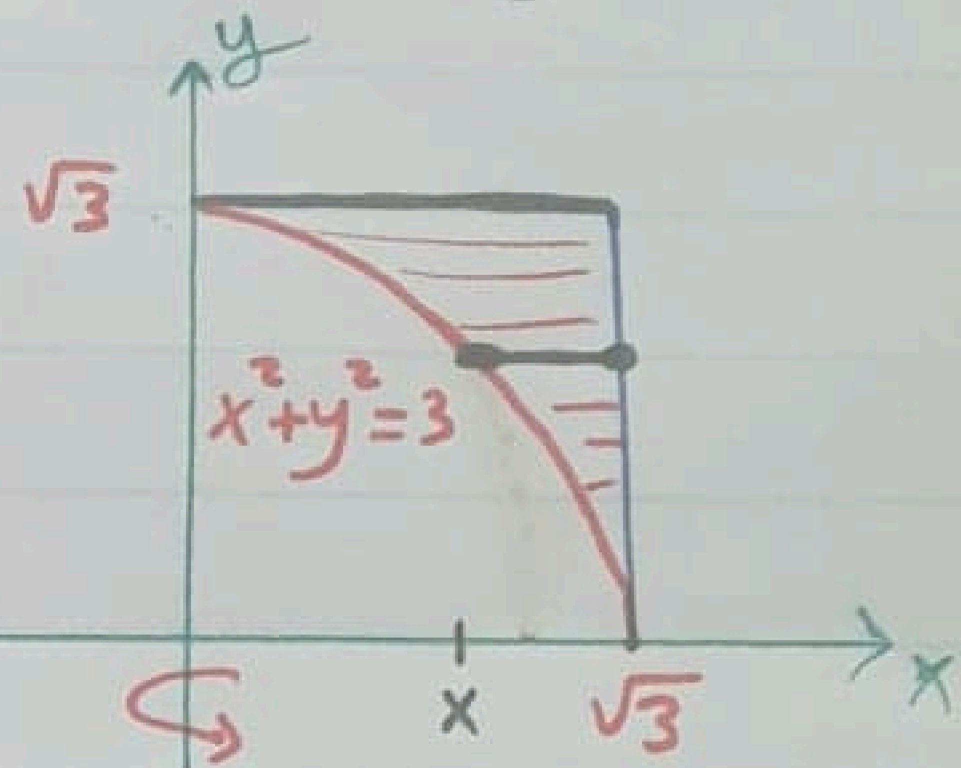
$$= \pi \int_1^2 \frac{du}{u^2}$$

$y = 1 \Rightarrow u = 2$

$y = 0 \Rightarrow u = 1$

$$= \pi \frac{u^{-2+1}}{-2+1}\Big|_1^2 = \pi \frac{-1}{u}\Big|_1^2 = -\pi\left[\frac{1}{2} - 1\right] = \frac{\pi}{2}$$

44 Find the volume of the solid generated by revolving the region in the first quadrant bounded on the left by the circle $x^2+y^2=3$ on the right by the line $x=\sqrt{3}$ and above by $y=\sqrt{3}$ about y-axis.



$$V=\int_c^d A(y)\,dy = \int_0^{\sqrt{3}} \pi\left[R^2(y)-r^2(y)\right]dy$$

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$$= \int_0^{\sqrt{3}} \pi\left[(\sqrt{3})^2-\left(\sqrt{3-y^2}\right)^2\right]dy$$

$$= \int_0^{\sqrt{3}} \pi\left(3-3+y^2\right)dy = \int_0^{\sqrt{3}} \pi y^2\,dy = \pi \frac{y^3}{3}\Big|_0^{\sqrt{3}} = \pi\sqrt{3}$$

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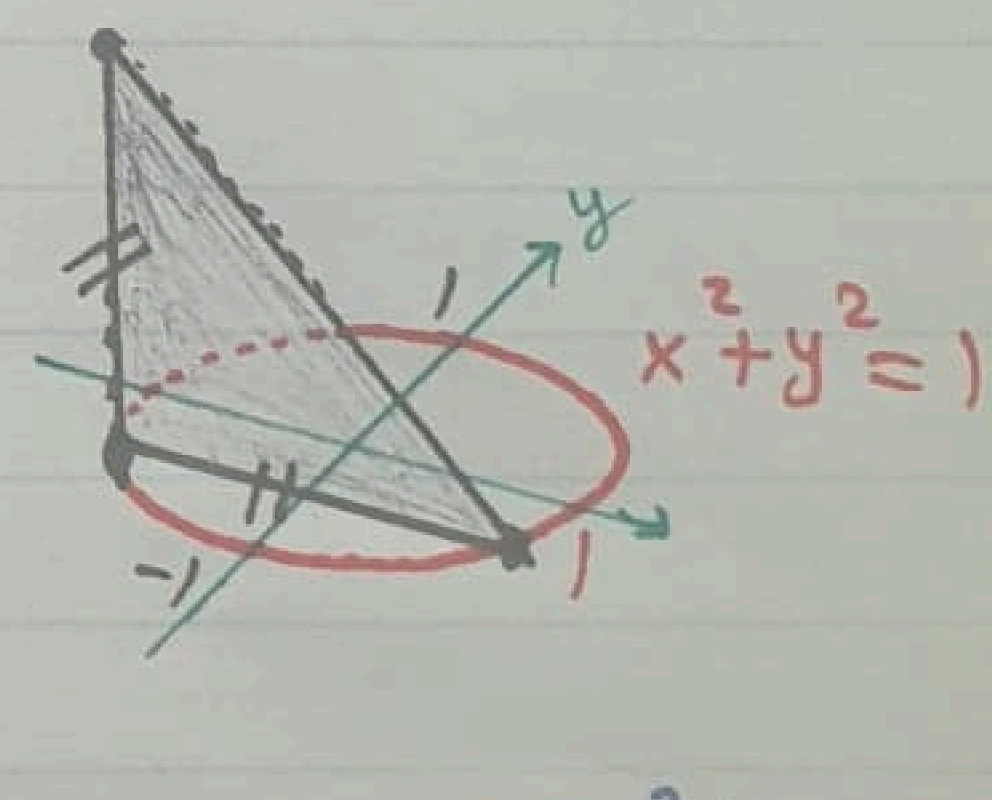
10 Find the volume of the solid whose base is the disk $x^2+y^2\le 1$, CS's are isoscels right triangles with one leg on the disk, CS's $\perp$ y-axis between $y=-1$ and $y=1$.



- $A(y)=\frac{1}{2}(\text{base})(\text{hight}) = \frac{1}{2}(\text{leg})^2$

$$= \frac{1}{2}\left(\sqrt{1-y^2} - -\sqrt{1-y^2}\right)^2$$

$$= \frac{1}{2}\left(2\sqrt{1-y^2}\right)^2 = \frac{1}{2}(4)(1-y^2) = 2(1-y^2)$$

- $V=\int_c^d A(y)\,dy = \int_{-1}^{1} 2(1-y^2)\,dy = 2y-\frac{2y^3}{3}\Big|_{-1}^{1}$

$$=\left(2-\frac{2}{3}\right)-\left(-2-\frac{-2}{3}\right) = 4-\frac{4}{3} = \frac{8}{3}$$