(2.6) exercises

6) $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{1}{8-\frac{6}{x^2}} = \frac{1}{8-0} = \frac{1}{8}$, $\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty} \frac{1}{8-\frac{6}{x^2}} = \frac{1}{8-0} = \frac{1}{8}$

7) $\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{-5+\frac{7}{x}}{3-\frac{1}{x^2}} = \frac{-5+0}{3-0} = \frac{-5}{3}$, $\lim_{x\to-\infty} h(x) = \lim_{x\to-\infty} \frac{-5+\frac{7}{x}}{3-\frac{1}{x^2}} = \frac{-5}{3}$

8) $\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{3-\frac{2}{x}}{4+\frac{\sqrt{2}}{x^2}} = \frac{3-0}{4+0} = \frac{3}{4}$, $\lim_{x\to-\infty} h(x) = \lim_{x\to-\infty} \frac{3-\frac{2}{x^2}}{4+\frac{\sqrt{2}}{x^2}} = \frac{3-0}{4+0} = \frac{3}{4}$

9 → 12

9) $\lim_{x\to\infty} \frac{\sin 2x}{x} =$

$-1 \le \sin 2x \le 1 \quad \forall x \in D$

$\frac{-1}{x} \le \frac{\sin 2x}{x} \le \frac{1}{x}$

$\lim_{x\to\infty} \frac{-1}{x} = 0$, $\lim_{x\to\infty} \frac{1}{x} = 0$

by sandwich theorem

$\lim_{x\to\infty} \frac{\sin 2x}{x} = 0$

10) $\lim_{\theta\to\infty} \frac{\cos\theta}{3\theta}$

$-1 \le \cos\theta \le 1$

$\frac{-1}{3\theta} \le \frac{\cos\theta}{3\theta} \le \frac{1}{3\theta}$

$\lim_{\theta\to\infty} \frac{-1}{3\theta} = 0$, $\lim_{\theta\to\infty} \frac{1}{3\theta} = 0$

by sandwich theorem.

$\lim_{\theta\to\infty} \frac{\cos\theta}{3\theta} = 0$

11) $\lim_{t\to-\infty} \frac{2-t+\sin t}{2t+\cos t}$

$= 1$

12) $\lim_{r\to\infty} \frac{r+\sin r}{2r+7-5\sin r} = \frac{1}{2}$

13)

a. $\lim_{x\to\infty} \frac{2x+3}{5x+7} = \frac{2}{5}$, b. $\lim_{x\to-\infty} \frac{2x+3}{5x+7} = \frac{2}{5}$

14)

a. $\lim_{x\to\infty} \frac{2x^3+7}{x^3-x^2+x+7} = 2$, b. $\lim_{x\to-\infty} \frac{2x^3+7}{x^3-x^2+x+7} = 2$

15) a. $\lim_{x\to\infty} \frac{x+1}{x^2+3} = 0$, b. $\lim_{x\to-\infty} \frac{x+1}{x^2+3} = 0$

16) a. $\lim_{x\to\infty} \frac{3x+7}{x^2-2} = 0$, b. $\lim_{x\to-\infty} \frac{3x+7}{x^2-2} = 0$

17) a. $\lim_{x\to\infty} \frac{7x^3}{x^3-3x+6x} = \frac{7}{1} = 7$, b. $\lim_{x\to\infty} \frac{7x^3}{x^3-3x^2+6x} = 7$

18) a. $\lim_{x\to\infty} \frac{1}{x^3-4x+1} = 0$, b. $\lim_{x\to-\infty} \frac{1}{x^3-4x+1} = 0$

19) a. $\lim_{x\to\infty} \frac{10x^5+x^4+3)}{x^6} = 0$, b. $\lim_{x\to-\infty} \frac{10x^5+x^4+3)}{x^6} = 0$

20) a. $\lim_{x\to\infty} \frac{9x^4+x}{2x^4+5x^2-x+6} = \frac{9}{2}$, b. $\lim_{x\to-\infty} // = \frac{9}{4}$

21) a. $\lim_{x\to\infty} \frac{-2x^3-2x+3}{3x^3+3x^2-5x} = \frac{-2}{3}$, b. $\lim_{x\to-\infty} \frac{-2x^3-2x+3}{3x^3+3x^2-5x} = \frac{-2}{3}$

22) $\lim_{x\to\infty} \frac{-x^4}{x^4-7x^3+7x^2+9} = -1$, b. $\lim_{x\to-\infty} // = -1$

23 → 36

23) $\lim_{x\to\infty} \sqrt{\frac{8x^2-3}{2x^2+x}} = \left[\lim_{x\to\infty} \frac{8x^2-3}{2x^2+x}\right]^{\frac{1}{2}} = \sqrt{\frac{8}{2}} = 2$

24) $\lim_{x\to\infty} \left(\frac{x^2+x-1}{8x^2-3}\right)^{\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2}$

25) $\lim_{x\to-\infty} \left(\frac{1-x^3}{x^2+7x}\right)^5 = \lim_{x\to-\infty} \left(\frac{\frac{1}{x^2}-x}{1+\frac{7}{x}}\right)^5 = (-\infty)^5 = \infty$

26) $\lim_{x\to\infty} \sqrt{\frac{x^2-5x}{x^3+x-2}} = 0^{\frac{1}{2}} = 0$

(2.6) exercises

27) $\lim_{x\to\infty} \frac{2\sqrt{x}+\frac{1}{x}}{3x-7} = \lim_{x\to\infty} \frac{\frac{2}{\sqrt{x}}+\frac{1}{x^2}}{3-\frac{7}{x}} = 0$

28) $\lim_{x\to\infty} \frac{2+\sqrt{x}}{2-\sqrt{x}} = \lim_{x\to\infty} \frac{\frac{2}{\sqrt{x}}+1}{\frac{2}{\sqrt{x}}-1} = \frac{0+1}{0-1} = -1$

29) $\lim_{x\to-\infty} \frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}} = \lim_{x\to-\infty} \frac{1-x^{\frac{2}{15}}}{1+x^{\frac{2}{15}}}$

30) $\lim_{x\to\infty} \frac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}} = \lim_{x\to\infty} \frac{x+x^{-2}}{1-x^{-1}} = \infty$

31) $\lim_{x\to\infty} \frac{2x^{\frac{5}{3}}-x^{\frac{1}{3}}+7}{x^{\frac{8}{5}}+3x+\sqrt{x}} = \lim_{x\to\infty} \frac{x^{\frac{1}{15}}\left(2x^{\frac{24}{15}}-x^{\frac{-19}{5}}+7x^{-\frac{24}{15}}\right)}{x^{\frac{1}{15}}\left(x^{-\frac{3}{5}}+3x^{-\frac{11}{10}}\right)} = 1$

32) $\lim_{x\to\infty} \frac{\sqrt[3]{x-5x+3}}{2x+x^2-4} = \lim_{x\to-\infty} \frac{x^{-\frac{2}{3}}\left|-5+3x^{-1}\right|}{2-x^{-\frac{1}{3}}-4x^{-1}} = 0$

33) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1} \to \frac{\infty}{\infty} = \frac{\left[\lim_{x\to\infty} x^2+1\right]^{\frac{1}{2}}}{\lim_{x\to\infty}(x+1)} = \frac{\infty}{\infty} = 1$

34) $\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x+1} = \frac{\infty}{-\infty} = -1$

35) $\lim_{x\to\infty} \frac{x-3}{\sqrt{4x^2-25}} = \frac{\lim_{x\to\infty} x-3}{\lim_{x\to\infty}\sqrt{4-\frac{25}{x^2}}} = 1 \cdot \lim_{x\to\infty} \frac{x-3}{x} \cdot \lim_{x\to\infty} \frac{1}{\sqrt{4-\frac{25}{x^2}}}$

because $x\to\infty \to 0$, $= 1 \cdot \frac{1}{2} = \frac{1}{2}$

36) $\lim_{x\to-\infty} \frac{4-3x^2}{\sqrt{x^6+9}} = \lim_{x\to-\infty} \frac{4-3x^2}{|x^3|\sqrt{1+\frac{9}{x^6}}} = \lim_{x\to-\infty} \frac{4-3x^2}{-x^3\sqrt{1+9x^{-6}}}$

$\Rightarrow \lim_{x\to-\infty} \frac{4-3x^2}{-x^3} \cdot \lim_{x\to-\infty} \frac{1}{\sqrt{1+9x^{-6}}} = 0 \cdot 1 = 0$

(2.6) exercises

37) $\lim_{x\to0^+} \frac{1}{3x} = \infty$

38) $\lim_{x\to0^-} \frac{5}{2x} = -\infty$

39) $\lim_{x\to2^-} \frac{3}{x-2} = -\infty$

40) $\lim_{x\to3^+} \frac{1}{x-3} = \infty$

41) $\lim_{x\to-8^+} \frac{2x}{x+8} = -\infty$

42) $\lim_{x\to-5^-} \frac{3x}{2x+10} = \infty$

43) $\lim_{x\to7} \frac{4}{(x-7)^2} = \infty$

44) $\lim_{x\to0} \frac{-1}{x^2(x+1)} = -\infty$

45) a. $\lim_{x\to0^+} \frac{2}{3x^{\frac{1}{3}}} = \infty$ b. $\lim_{x\to0^-} \frac{2}{3x^{\frac{1}{3}}} = -\infty$

46) a. $\lim_{x\to0^+} \frac{2}{x^{\frac{1}{5}}} = \infty$ b. $\lim_{x\to0^-} \frac{2}{x^{\frac{1}{5}}} = -\infty$

47) $\lim_{x\to0} \frac{4}{x^{\frac{2}{5}}} = \infty$

48) $\lim_{x\to0} \frac{2x^{-\frac{3}{3}}}{} = \infty$

49) $\lim_{x\to\frac{\pi}{2}^-} \tan x = \infty$

50) $\lim_{x\to\left(\frac{\pi}{2}\right)^+} \sec x = \infty$

51) $\lim_{\theta\to0^-} (1+\csc\theta) = 1+(-\infty) = -\infty$

52) $\lim_{\theta\to0} (2-\cot\theta) = \text{DNE}$

53) a. $\lim_{x\to2^+} \frac{1}{x^2-4} = \infty$, b. $\lim_{x\to2^-} \frac{1}{x^2-4} = -\infty$, c. $\lim_{x\to-2^+} \frac{1}{x^2-4} = \infty$, $\lim_{x\to-2^-} \frac{1}{x^2-4} = -\infty$

54) a. $\lim_{x\to1^+} \frac{x}{x^2-1} = \infty$ b. $\lim_{x\to1^-} \frac{x}{x^2-1} = -\infty$ c. $\lim_{x\to-1^+} \frac{x}{x^2-1} = -\infty$, d. $\lim_{x\to-1^-} \frac{x}{x^2-1} = \infty$

55) a. $\lim_{x\to0^+} \left(\frac{x^2}{2}-\frac{1}{x}\right) = -\infty$ b. $\lim_{x\to0^-} \left(\frac{x^2}{2}-\frac{1}{x}\right) = \infty$ c. $\lim_{x\to\sqrt[3]{2}} \left(\frac{x^2}{2}-\frac{1}{x}\right) = 2^{-\frac{1}{3}} - 2^{-\frac{1}{3}} = 0$

d. $\lim_{x\to-1} \left(\frac{x^2}{2}-\frac{1}{x}\right) = \frac{1}{2}+1 = \frac{3}{2}$

56) a. $\lim_{x\to-2^+} \frac{x^2-1}{2x+4} = -\infty$ b. $\lim_{x\to-2^-} \frac{x^2-1}{2x+4} = \infty$ c. $\lim_{x\to1^+} \frac{x^2-1}{2x+4} = 0$, d. $\lim_{x\to0} \frac{x^2-1}{2x+4} = -\frac{1}{4}$

exercises(2.6)

57) a. $\lim_{x\to 0^+} \frac{x^2-3x+2}{x^3-2x^2} = \infty$, b. $\lim_{x\to 2^+} \frac{x^2-3x+2}{x^3-2x^2} = \lim_{x\to 2^+} \frac{(x-2)(x-1)}{x^2(x-2)} = \frac{1}{4}$, c. $\lim_{x\to 2^-} \frac{x^2-3x+2}{x^3-2x^2} = \frac{1}{4}$ = d. $\lim_{x\to 2}$ =

e. $\lim_{x\to 0} \frac{x-1}{x^2} = -\infty$

58) a. $\lim_{x\to 2^+} \frac{x^2-3x+2}{x^3-4x} = \lim_{x\to 2^+} \frac{(x-2)(x-1)}{x(x-2)(x+2)} = \frac{3}{2(4)} = \frac{3}{8}$, b. $\lim_{x\to -2^+} \frac{(x-2)(x-1)}{x(x-2)(x+2)} = -\infty$

c. $\lim_{x\to 0^-} \frac{x^2-3x+2}{x^3-4x} = \lim_{x\to 0^-} \frac{(x-2)(x-1)}{x(x-2)(x+2)} = \infty$, d. $\lim_{x\to 1^+} \frac{(x-1)}{x(x+2)} = 0$, e. $\lim_{x\to 0} \frac{(x-1)}{x(x+2)}$ = DNE because $\lim_{x\to 0^+} \frac{(x-1)}{x(x+2)} = -\infty \neq \infty$

59) a. $\lim_{t\to 0^+} \left(2 - \frac{3}{t^{\frac{1}{3}}}\right) = -\infty$, b. $\lim_{t\to 0^-} \left(2 - \frac{3}{t^{\frac{1}{3}}}\right) = \infty$

60) a. $\lim_{t\to 0^+} \left(\frac{1}{t^{\frac{2}{3}}} + 7\right) = \infty$, b. $\lim_{t\to 0^-} \left(\frac{1}{t^{\frac{2}{3}}} + 7\right) = -\infty$

61) a. $\lim_{x\to 0^+} \left(\frac{1}{x^{\frac{2}{3}}} + \frac{2}{(x-1)^{\frac{2}{3}}}\right) = \infty$, b. $\lim_{x\to 2^+} \left(\frac{1}{x^{\frac{2}{3}}} + \frac{2}{(x-1)^{\frac{2}{3}}}\right) = \frac{1}{\sqrt[3]{4}} + 2$, c. $\lim_{x\to 1^+} \left(\frac{1}{x^{\frac{2}{3}}} + \frac{2}{(x-1)^{\frac{2}{3}}}\right) = \infty$

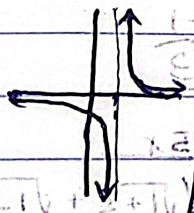
d. $\lim_{x\to 1^-} \left(\frac{1}{x^{\frac{2}{3}}} + \frac{2}{(x-1)^{\frac{2}{3}}}\right) = -\infty$

62) a. $\lim_{x\to 0^+} \left(\frac{1}{x^{\frac{2}{3}}} - \frac{1}{(x-1)^{\frac{4}{3}}}\right) = \infty$, b. $\lim_{x\to 0^-} \left(\frac{1}{x^{\frac{2}{3}}} - \frac{1}{(x-1)^{\frac{4}{3}}}\right) = \infty$, c. $\lim_{x\to 1^+} \left(\frac{1}{x^{\frac{2}{3}}} - \frac{1}{(x-1)^{\frac{4}{3}}}\right) = -\infty$, d. $\lim_{x\to 1^-} \left(\frac{1}{x^{\frac{2}{3}}} - \frac{1}{(x-1)^{\frac{4}{3}}}\right) = -\infty$

63) $y = \frac{1}{x-1}$

$\lim_{x\to 1^+} \frac{1}{x-1} = \infty$, $\lim_{x\to 1^-} \frac{1}{x-1} = -\infty$

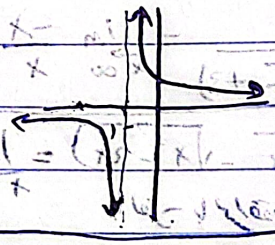
$\lim_{x\to \infty} \frac{1}{x-1} = 0$, $\lim_{x\to -\infty} \frac{1}{x-1} = 0$



64) $y = \frac{1}{x+1}$

$\lim_{x\to -1^+} \frac{1}{x+1} = \infty$, $\lim_{x\to -1^-} \frac{1}{x+1} = -\infty$

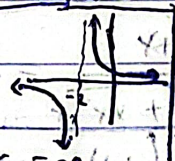
$\lim_{x\to \infty} \frac{1}{x+1} = 0$, $\lim_{x\to -\infty} \frac{1}{x+1} = 0$



65) $y = \frac{1}{2x+4}$

$\lim_{x\to -2^+} \frac{1}{2x+4} = \infty$, $\lim_{x\to -2^-} \frac{1}{2x+4} = -\infty$

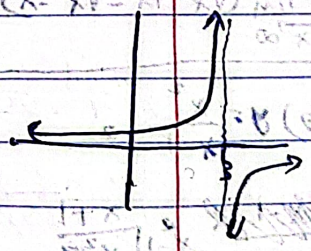
$\lim_{x\to \infty} \frac{1}{2x+4} = 0$, $\lim_{x\to -\infty} \frac{1}{2x+4} = 0$



66) $y = \frac{-3}{x-3}$

$\lim_{x\to 3^+} \frac{-3}{x-3} = -\infty$, $\lim_{x\to 3^-} \frac{-3}{x-3} = \infty$

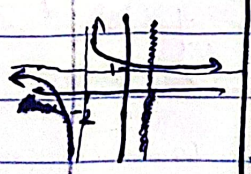
$\lim_{x\to \infty} \frac{-3}{x-3} = 0$, $\lim_{x\to -\infty} \frac{-3}{x-3} = 0$



67) $y = \frac{x+3}{x+2}$

$\lim_{x\to -2^+} \frac{x+3}{x+2} = \infty$, $\lim_{x\to -2^-} \frac{x+3}{x+2} = -\infty$

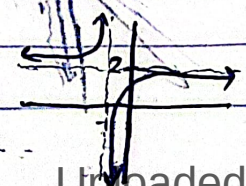
$\lim_{x\to \infty} \frac{x+3}{x+2} = 1$, $\lim_{x\to -\infty} \frac{x+3}{x+2} = 1$



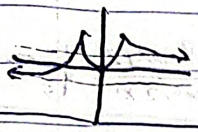
68) $y = \frac{2x}{x+1}$

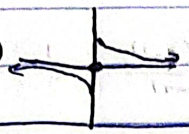
$\lim_{x\to -1^+} \frac{2x}{x+1} = -\infty$, $\lim_{x\to -1^-} \frac{2x}{x+1} = \infty$

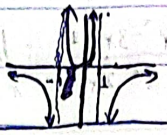
$\lim_{x\to \infty} \frac{2x}{x+1} = 2$, $\lim_{x\to -\infty} \frac{2x}{x+1} = 2$

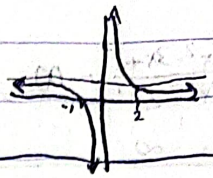


(2.6) exercises

69) 

70) 

71) 

72) 

73) $f(x) = \frac{1}{(x-2)^2}$

74) $g(x) = \frac{1}{x-3}$

75) $h(x) = \begin{cases} -1 & , 0 < x \\ 1 & , x > 0 \end{cases}$

76) $H(x) = \frac{x}{1-x} + 2$

77) $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 2$

78) yes it could, it's a Horizontal asymptote

79) Just 2

80) $\lim_{x\to\infty} (\sqrt{x+9} - \sqrt{x+4}) = 0$

81) $\lim_{x\to\infty} (\sqrt{x^2+25} - \sqrt{x^2-1})$ ← بالضرب في المرافق ⇒ $\lim_{x\to\infty} \frac{26}{\sqrt{x^2+25}+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{\frac{26}{x^n}}{\sqrt{1+\frac{25}{x^2}}+\sqrt{1-\frac{1}{x^2}}}$

82) $\lim_{x\to-\infty} (\sqrt{x^2+3}+x) \cdot \frac{(\sqrt{x^2+3}-x)}{(\sqrt{x^2+3}-x)} = \lim_{x\to-\infty} \frac{x^2+3-x^2}{\sqrt{x^2+3}-x} = \lim_{x\to-\infty} \frac{3}{\sqrt{1+\frac{3}{x^2}}-1} = 0$

83) $\lim_{x\to-\infty} (2x+\sqrt{4x^2+3x-2}) \cdot \frac{(2x-\sqrt{4x^2+3x-2})}{(2x-\sqrt{4x^2+3x-2})} = \lim_{x\to-\infty} \frac{4x^2-4x^2+3x+2}{2x-\sqrt{4x^2+3x-2}} = \lim_{x\to-\infty} \frac{3x+2}{2x-\sqrt{4x^2+3x-2}}$

$= \lim_{x\to-\infty} \frac{3+\frac{2}{x}}{2-\sqrt{4+\frac{3}{x}-\frac{2}{x^2}}} = \frac{3}{2-\sqrt{4}} = \frac{3}{0} = -\frac{3}{4}$

$\lim_{x\to-\infty} \frac{2-3x}{2x-x\sqrt{4+\frac{3}{x}-\frac{2}{x^2}}} = \lim_{x\to-\infty} \frac{2-3x}{x(2+\sqrt{4+\frac{3}{x}-\frac{2}{x^2}})}$

84) $\lim_{x\to\infty} \sqrt{9x^2-x} - 3x \cdot \frac{(\sqrt{9x^2-x}+3x)}{(\sqrt{9x^2-x}+3x)} = \lim_{x\to\infty} \frac{9x^2-x-9x^2}{\sqrt{9x^2-x}+3x} = \lim_{x\to\infty} \frac{-x^0-\frac{1}{x}}{x\sqrt{9-\frac{1}{x}}+3x}$

$\lim_{x\to\infty} \frac{-x}{x(\sqrt{9+\frac{1}{x}}+3)} = \lim_{x\to\infty} \frac{-x}{x} \cdot \lim_{x\to\infty} \frac{1}{(\sqrt{9+\frac{1}{x}}+3)} = -1 \cdot \frac{1}{3} = -\frac{1}{3}$

85) $\lim_{x\to\infty} (\sqrt{x^2+3x} - \sqrt{x^2-2x})$ ← بالضرب في المرافق $= \lim_{x\to\infty} \frac{x^2+3x-x^2+2x}{\sqrt{x^2+3x}+\sqrt{x^2-2x}} = \lim_{x\to\infty} \frac{5x}{x(\sqrt{1+\frac{3}{x}}+\sqrt{1-\frac{2}{x}})} = \frac{5}{2}$

86) $\lim_{x\to\infty} (\sqrt{x^2+x}-\sqrt{x^2-x}) \cdot \frac{(\sqrt{x^2+x}+\sqrt{x^2-x})}{(\sqrt{x^2+x}+\sqrt{x^2-x})} = \lim_{x\to\infty} \frac{x^2+x-x^2+x}{\sqrt{x^2+x}+\sqrt{x^2-x}} = \lim_{x\to\infty} \frac{2x}{x(\sqrt{1+\frac{1}{x}}+\sqrt{1-\frac{1}{x}})} = \frac{2}{2} = 1$

99) $y = \frac{x^2}{x-1}$

$x-1 \overline{)\, x^2 \;\; x+1}$

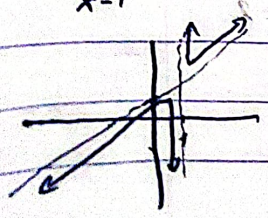
$-(x^2-x)$

x

$-(x-1)$

1

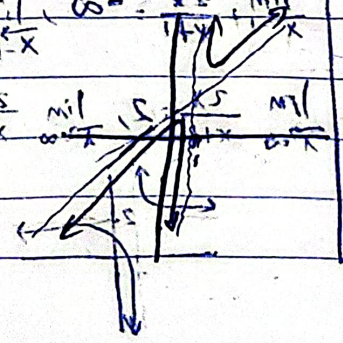
$y = \frac{1}{x-1} + x + 1$



100) $y = x^2+1+2-2$

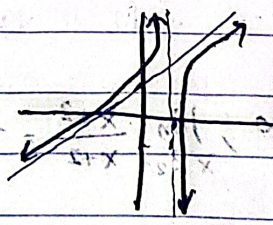
$y = \frac{x^2-1}{x-1} + \frac{2}{x-1}$

$y = x+1 + \frac{2}{x-1}$



101) $y = \frac{x^2-1}{x-1} = \frac{x^2-1-3}{x-1} = \frac{(x-1)(x+1)}{(x-1)} - \frac{3}{x-1}$

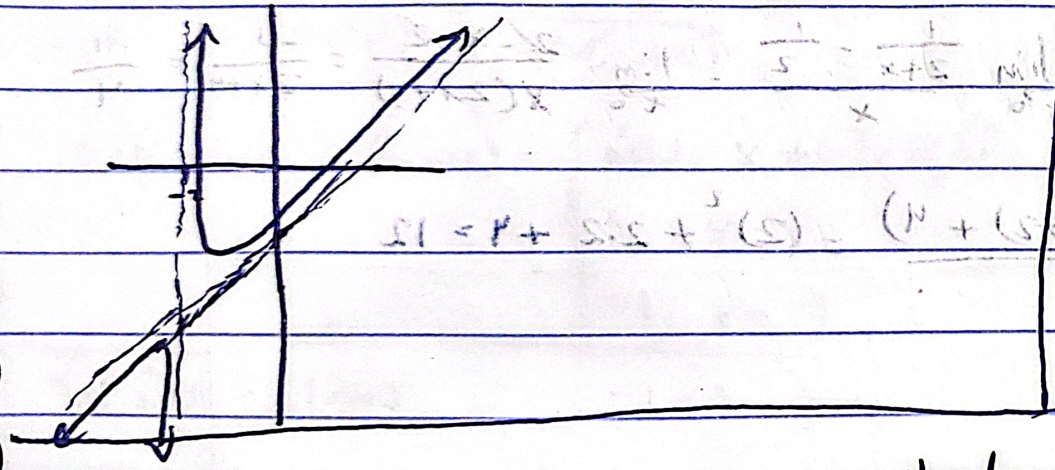
$y = x+1 - \frac{3}{x-1}$



(2.6) exercises

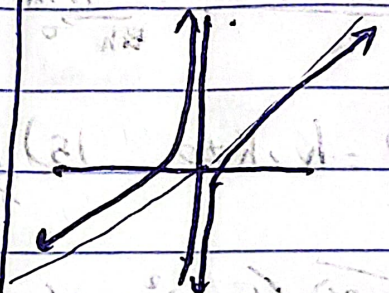
102) $y = \frac{x^2-1}{2x+4} = \frac{x^2-4+3}{2(x+2)} = \frac{(x-2)(x+2)}{2(x+2)} + \frac{3}{2(x+2)}$

$y = \frac{x-2}{2} + \frac{3}{2(x+2)}$



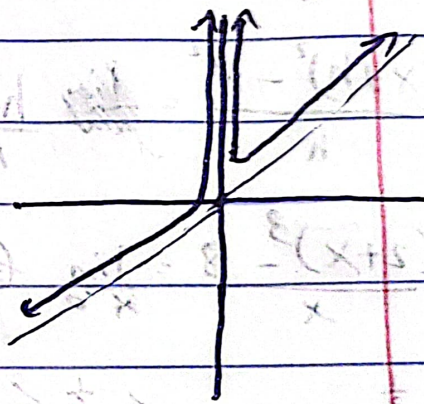
103) $y = \frac{x^2-1}{x}$

$y = x - \frac{1}{x}$



104) $y = \frac{x^3+1}{x^2}$

$y = x + \frac{1}{x^2}$



chapter 2 practice exercises