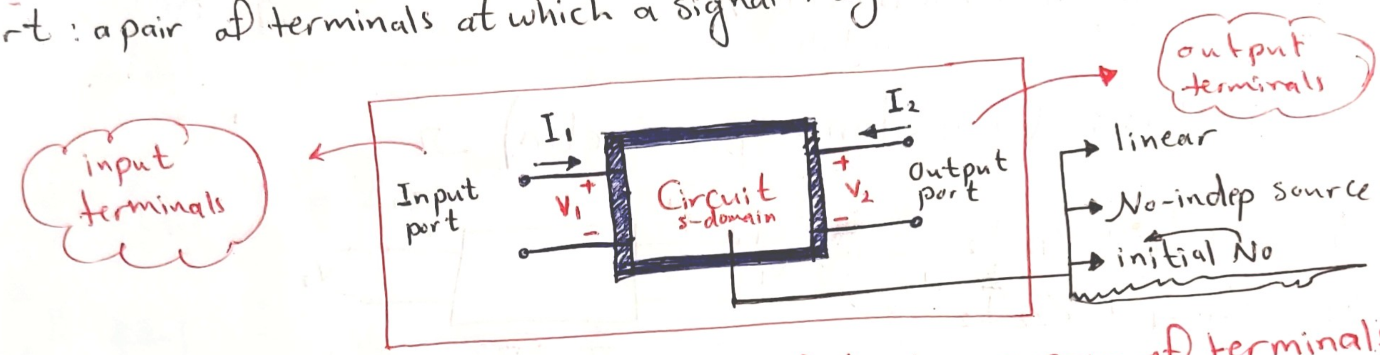
Two-Port Circuits

- two-port model is used to describe the performance of a circuit in terms of the voltage and current at its input and output ports.
- Port : a pair of terminals at which a signal may enter or leave a network.



- In particular, this is helpful when a signal is fed into one pair of terminals and then, after being processed by the system, is extracted at a second pair of terminals.

- Of these four terminal variables $[I_1, V_1, I_2, V_2]$, only two are independent. Thus for any circuit, once we specify two of the variables, we can find the two remaining unknowns.
- We can describe a two-port network with just two simultaneous equations. However, there are six different ways in which to combine the four variables:-

[1] $V_1 = z_{11} I_1 + z_{12} I_2$
$V_2 = z_{21} I_1 + z_{22} I_2$

[2] $I_1 = y_{11} V_1 + y_{12} V_2$
$I_2 = y_{21} V_1 + y_{22} V_2$

[3] $V_1 = a_{11} V_2 - a_{12} I_2$
$I_1 = a_{21} V_2 - a_{22} I_2$

[4] $V_2 = b_{11} V_1 - b_{12} I_1$
$I_2 = b_{21} V_1 - b_{22} I_1$

[5] $V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

[6] $I_1 = g_{11} V_1 + g_{12} I_2$
$V_2 = g_{21} V_1 + g_{22} I_2$

These six sets of equations may also be considered as three pairs of mutually inverse (متبادل) relations

parameters of the two-port circuit.
- → z- parameters. } immittance par.
- → y parameters.
- → a parameters. } transmission par.
- → b parameters.
- → h parameters. } hybrid par.
- → g parameters.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{\Delta z} \begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

y_{11} y_{12} y_{21} $(-z_{21}/\Delta z)$ V_1

» The Two-Port Parameters :

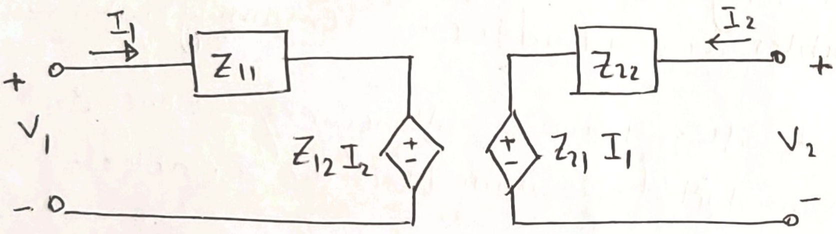
⇓

From The parameter equations
((Computation or measurement))
حساب القياس

1→ Z parameters : (impedance parameters) Ω

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Z- parameter Equivalent CKT :-



$Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} \Omega$, open circuit, input impedance.

Z_{11} is the impedance seen looking into port 1 when port 2 is open.

$Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0} \Omega$, open circuit, reverse transfer impedance.

Z_{12} is a transfer impedance. It is the ratio of the port 1 voltage to the port 2 current when port 1 is open.

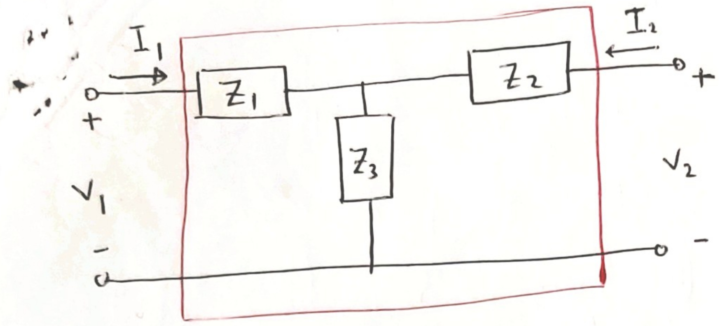
$Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0} \Omega$, open circuit, forward transfer impedance.

Z_{21} is a transfer impedance. It is the ratio of the port 2 voltage to the port 1 current when port 2 is open.

$Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0} \Omega$, open circuit, output impedance.

Z_{22} is the impedance seen looking into port 2 when port 1 is open.

example



$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = Z_1 + Z_3$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$$V_2 = \frac{Z_3}{Z_1+Z_3} V_1 \rightarrow I_1(Z_1+Z_3)$$

$$= Z_3$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$V_1 = V_{Z_3} = \frac{Z_3}{Z_2+Z_3} V_2 \leftarrow (Z_2+Z_3) I_2$$

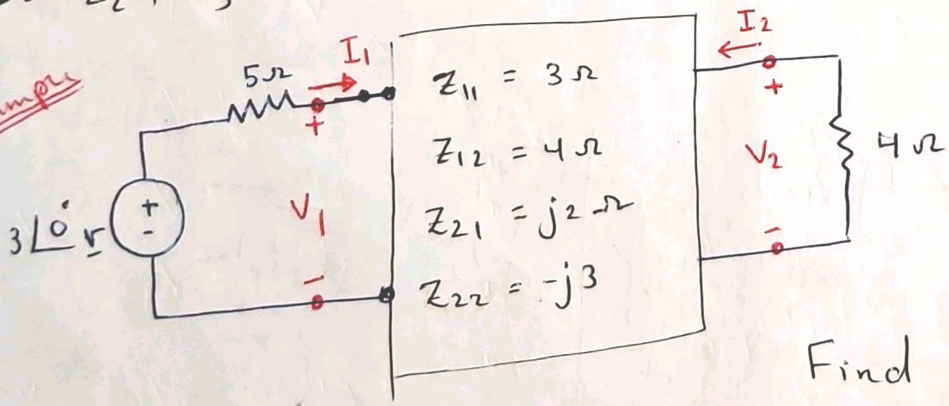
$$= Z_3 = Z_{21}$$

اللهم إنا لا نسألك رد القضاء

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$$= Z_2 + Z_3$$

example



Find V_2 ?

1. $V_1 = Z_{11} I_1 + Z_{12} I_2$
   $V_2 = Z_{21} I_1 + Z_{22} I_2$
   $\Longrightarrow$ $V_1 = 3I_1 + 4I_2$
   $V_2 = j2I_1 - 3jI_2$

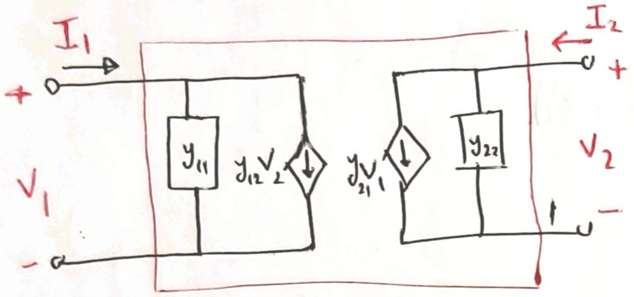
2. $V_1 = -5I_1 + 3\angle 0$
   $V_2 = -4I_2$

$$\Longrightarrow \boxed{V_2 = 2.53 \angle 135° \text{ V}}$$

2 Y- Parameters ((admittance $\mho$))

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

Y- Parameter Equivalent CKT



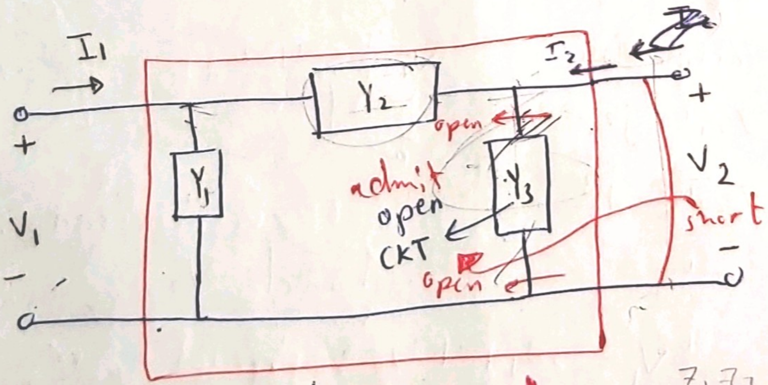
$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$ $\mho$, short circuit, input admittance.

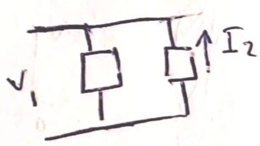
$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$ $\mho$, short circuit, reverse transfer adm.

$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$ $\mho$, short circuit, output adm.

$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$ $\mho$, short circuit, forward transfer admi.

example





$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_1 + Y_2$ (pallel)

$\frac{Z_1 Z_2}{Z_1+Z_2} = \frac{1}{\frac{1}{Z_2}+\frac{1}{Z_1}}$

$= \frac{1}{Y_2+Y_1} = \frac{V_1}{I_1}$

~~$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_3 + Y_2$~~

$\frac{I_1}{V_1} = Y_2 + Y_1$

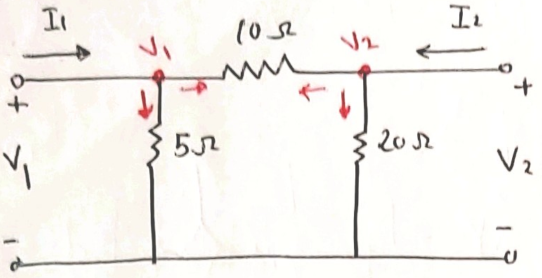
$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \Rightarrow I_2 = -V_1 Y_2$

$\left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_2 = y_{21}$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -Y_2$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_2 + Y_3$$

Example



using par eq.

$$y_{11} = \frac{1}{5} + \frac{1}{10} = 0.3 \, \mho$$

$$y_{12} = -\frac{1}{10} = -0.1 \, \mho$$

$$y_{21} = -0.1 \, \mho$$

$$y_{22} = \frac{1}{20} + \frac{1}{10} = 0.15 \, \mho$$

$$\Rightarrow \boxed{\begin{aligned} I_1 &= 0.3 V_1 - 0.1 V_2 \\ I_2 &= -0.1 V_1 + 0.15 V_2 \end{aligned}}$$

using nodal

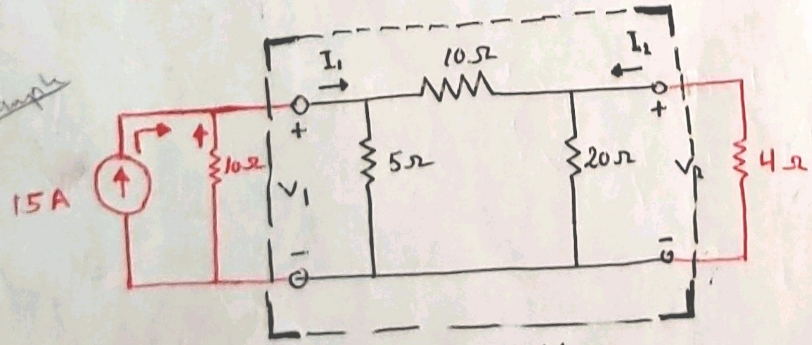
1. $\frac{V_1}{5} + \frac{V_1 - V_2}{10} = I_1$

$I_1 = 0.3 V_1 - 0.1 V_2 \longrightarrow I_1 = 0.3 V_1 - 0.1 V_2$ —

2. $I_2 = \frac{V_2}{20} + \frac{V_2 - V_1}{10}$

$I_2 = -0.1 V_1 + 0.15 V_2 \longrightarrow I_2 = -0.1 V_1 + 0.15 V_2$ —

example



[2] $I_2 = -\frac{V_2}{4}$

$I_1 = 15 - \frac{V_1}{10}$

[1] $I_1 = 0.3 V_1 - 0.1 V_2$

$I_2 = -0.1 V_1 + 0.15 V_2$

[3] $\Rightarrow$ $15 - \frac{V_1}{10} = 0.3 V_1 - 0.1 V_2$

$-\frac{V_2}{4} = -0.1 V_1 + 0.15 V_2$

[4] $\longrightarrow V_1 = 40V, V_2 = 10V \longrightarrow I_2 = -2.5A, I_1 = 11A$ [5]

The Two-Port Parameters :-

منفذ بوابة ((a pair of terminals at which a signal may enter or leave a network))

- » z-parameters
- » y-parameters

Z, Y (immittance) par.

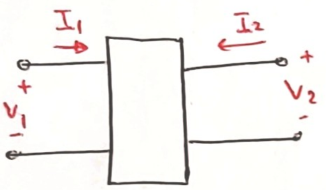
- » h-parameters.
- » g-parameters.

هجين (hybrid) par. ((criss-variables)) ⟸

- » a-parameters
- » b-parameters

انتقال (transmission) par.

هجين (خليط بين متغيرات) hybrid parameters :- (h-parameters)



$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ Ω , $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$

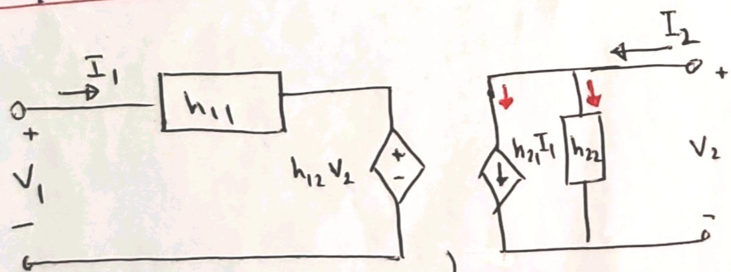
$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$, $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ $\mho$

$h_{11} \equiv$ short circuit, input impedance Ω — h_i

$h_{12} \equiv$ open circuit, reverse voltage ratio — h_r

$h_{21} \equiv$ short circuit, forward current ratio — h_f

$h_{22} \equiv$ open circuit, output admittance $\mho$ — h_o



Common emitter

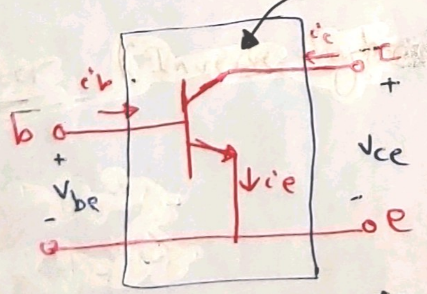
hie, hfe, hoe , hre

Common base

hib , hfb, hob, hrb

Common collector

hic, hfc, hoc, hrc



(Common emitter).

$V_1 = V_{be}$

$V_2 = V_{ce}$

$I_1 = i_b$

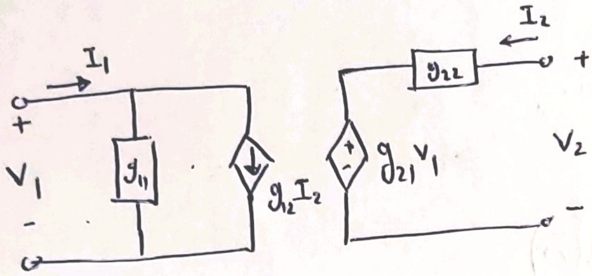
$I_2 = i_c$

$h_{fe} = \beta$ ((typical Values))

g-parameters ((Inverse hybrid)) :-

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$



Transmission parameters :-

$$V_1 = a_{11} V_2 - a_{12} I_2$$

$$I_1 = a_{21} V_2 - a_{22} I_2$$

(a-parameters)

$$V_2 = b_{11} V_1 - b_{12} I_1$$

$$I_2 = b_{21} V_1 - b_{22} I_1$$

(b- parameters)

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega,$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega,$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega,$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega.$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \mathrm{S}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \mathrm{S},$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \mathrm{S}, \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \mathrm{S}.$$

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \qquad a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0} \Omega,$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} \mathrm{S}, \qquad a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}.$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}, \qquad b_{12} = -\left.\frac{V_2}{I_1}\right|_{V_1=0} \Omega,$$

$$b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0} \mathrm{S}, \qquad b_{22} = -\left.\frac{I_2}{I_1}\right|_{V_1=0}.$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \Omega, \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0},$$

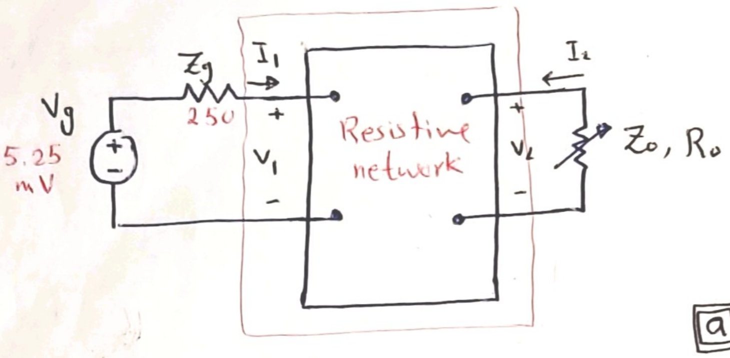
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}, \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \mathrm{S}.$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \mathrm{S}, \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0},$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}, \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \Omega.$$

example :-

The following dc measurement were made on the resistive network shown in Figure below :-



measurement 1

$V_2 = 0\,V$

$V_1 = 4\,V$

$I_1 = 5\,mA$

$I_2 = -200\,mA$

measurement 2

$I_2 = 0\,A$

$V_1 = 20\,mV$

$I_1 = 20\,\mu A$

$V_2 = 40\,V$

a)) find h-par.

$\boxed{a}$ $V_1 = a_{11} V_2 - a_{12} I_2$

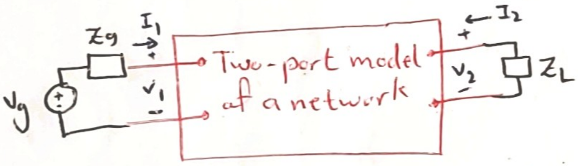
$I_1 = a_{21} V_2 - a_{22} I_2$

$\boxed{h}$ $V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

b)) A variable resistor R_o is connected across port 2 and adjusted for maximum power transfer to R_o.

Find the maximum power ??



⊙ Analysis of this circuit involves expressing the terminal currents and voltages as a function of the two-port parameters, V_g, Z_g, Z_L

⊙ Six characteristics of the terminated two-port circuit define its terminal behavior :-

>> $Z_{in} = \frac{V_1}{I_1}$, $Y_{in} = \frac{I_1}{V_1}$

>> I_2

>> V_{TH}, Z_{TH}, with respect to port 2.

>> The current gain $\frac{I_2}{I_1}$.

>> The voltage gain $\frac{V_2}{V_1}$.

" " gain $\frac{V_2}{V_g}$.

$\Rightarrow$ z-par
h-par
a-par
y-par
b-par
g-par
$+ V_g + Z_g + Z_L$

$\boxed{1}$ The parameter equation ①

$\boxed{2}$ The parameter of Z ②

$\boxed{3}$ $V_1 = -Z_g I_1 + V_g$

$\boxed{4}$ $V_2 = -I_2 Z_L$

① $\boxed{V_2 = 0\,V}$

$V_1 = 4V$

$I_1 = 5mA$

$I_2 = -200mA$

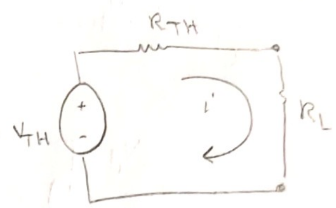
② $\boxed{I_2 = 0\,A}$

$V_1 = 20mV$

$I_1 = 20\mu A$

$V_2 = 40\,V$

$R_L = R_{TH}$



$P(t) = i^2(t) R_L$

$i(t) = \dfrac{V_{th}}{R_L + R_{th}}$

$P(t) = \left(\dfrac{V_{th}}{R_L + R_{th}}\right)^2 \cdot R_L$

$\dfrac{dP(t)}{dR_L} = 0$

$\boxed{R_{th} = R_L}$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$
$$\Rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

① $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{4}{5} * 10^3 = 800\,\Omega$$

h_{11} ✓

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-200m}{5m} = -40$$

h_{21} ✓

② $$I_2 = h_{21} I_1 + h_{22} V_2$$
$$0 = h_{21} I_1 + h_{22} V_2$$
$$40 I_1 = h_{22} V_2 \Longrightarrow h_{22} = 40 \frac{I_1}{V_2} = \frac{40(20 * 10^{-6})}{40}$$
$$= 20\mu\,\mho$$

h_{22} ✓

قاعدة مهمة

$$V_1 = h_{11} I_1 + h_{12} V_2$$

from measurment ②

$$20mV = 800\,(20\mu A) + h_{12}\,(40)$$

$$h_{12} = \frac{20 * 10^{-3} - 800 * 20 * 10^{-6}}{40} = 1 * 10^{-4}$$

$$\boxed{h_{12} = 1 * 10^{-4}}$$

h_{12} ✓

⇒ The Thevenin voltage with respect to port 2 equals V_2 when $I_2=0$.

$$V_2\Big|_{I_2=0}$$

$$\boxed{V_{TH} = V_2\Big|_{I_2=0}}$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$0 = h_{21}I_1 + h_{22}V_2$$

$$\boxed{V_2 = -\frac{h_{21}I_1}{h_{22}}} \rightarrow ?!$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$V_g - Z_g I_1 = h_{11}I_1 + h_{12}V_2$$

$$V_g - h_{12}V_2 = h_{11}I_1 + Z_g I_1$$

$$\boxed{I_1 = \frac{V_g - h_{12}V_2}{h_{11}+Z_g}}$$

$$V_2 = -\frac{h_{21}}{h_{22}}\left(\frac{V_g - h_{12}V_2}{h_{11}+Z_g}\right)$$

$$V_2(h_{22})(h_{11}+Z_g) = -h_{21}(V_g - h_{12}V_2)$$

$$V_2(h_{22})(h_{11}+Z_g) - h_{21}h_{12}V_2 = -h_{21}V_g$$

$$V_2\left[h_{22}h_{11} + h_{22}Z_g - h_{21}h_{12}\right] = -h_{21}V_g$$

$$\boxed{V_2 = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h} = V_{TH}}$$

The Thevenin or output impedance is the ratio $\frac{V_2}{I_2}$ when V_g is replaced by a short circuit :

replaced

$$\boxed{Z_{TH} = \left.\frac{V_2}{I_2}\right|_{V_g=0}}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$
$$V_1 = V_g - Z_g I_1$$

① $\Rightarrow$

$$\cancel{V_g}^{0} - Z_g I_1 = h_{11} I_1 + h_{12} V_2$$

$$-Z_g I_1 = h_{11} I_1 + h_{12} V_2 \Rightarrow I_1 = \left(\frac{-h_{12} V_2}{h_{11} + Z_g}\right)$$

② $\Rightarrow$

$$I_2 = h_{21} \underbrace{\left(\frac{-h_{12} V_2}{h_{11} + Z_g}\right)}_{I_1} + h_{22} V_2$$

$$I_2 = \frac{-h_{21} h_{12} V_2}{h_{11} + Z_g} + \frac{h_{22}(h_{11} + Z_g) V_2}{h_{11} + Z_g}$$

$$I_2 = \frac{[h_{22} h_{11} + Z_g h_{22} - h_{21} h_{12}] V_2}{h_{11} + Z_g}$$

$$\boxed{\frac{V_2}{I_2} = \frac{h_{11} + Z_g}{h_{22} Z_g + \Delta h}}$$

∴ ① $Z_{TH} = \frac{h_{11} + Z_g}{h_{22} Z_g + \Delta h}$, $\Delta h = h_{22} h_{11} - h_{12} h_{21}$

$\Delta h = 20 * 10^{-3}$

$$Z_{TH} = 42k\,\Omega$$

② $V_{TH} = \frac{-h_{21} V_g}{h_{22} Z_g + \Delta h} = 8.4\ V$

To Find the impedance seen looking into port 1, that is, $Z_{in} = \frac{V_1}{I_1}$

$$\boxed{Z_{in} = \frac{V_1}{I_1}} \Rightarrow Z_{in}$$

① $V_1 = h_{11} I_1 + h_{12} V_2$

② $I_2 = h_{21} I_1 + h_{22} V_2$

③ $V_1 = V_g - Z_g I_1$

④ $V_2 = - I_2 Z_L$

$\Rightarrow$ ① $V_1 = h_{11} I_1 + h_{12} V_2$?! in terms of I_1

② $I_2 = h_{21} I_1 + h_{22} V_2$

$-\frac{V_2}{Z_L} = h_{21} I_1 + h_{22} V_2 \Longrightarrow V_2$ ✓

$-h_{21} I_1 = h_{22} V_2 + \frac{V_2}{Z_L}$

$-h_{21} Z_L I_1 = h_{22} Z_L V_2 + V_2$

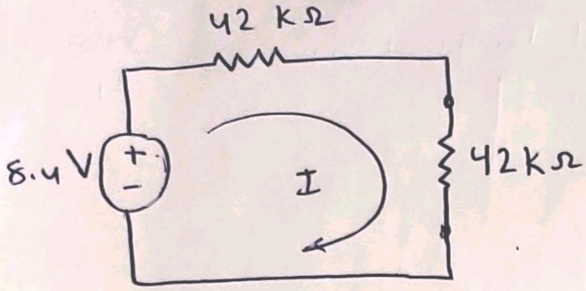
$$\boxed{V_2 = \frac{-h_{21} Z_L I_1}{1 + h_{22} Z_L}}$$

$V_1 = h_{11} I_1 + h_{12} V_2$

$V_1 = h_{11} I_1 + h_{12} \left[\frac{-Z_L h_{21} I_1}{1 + h_{22} Z_L} \right]$

$\frac{V_1}{I_1} = h_{11} - \frac{h_{12} (Z_L h_{21})}{h_{22} Z_L + 1}$

$$\boxed{Z_{in} = h_{11} - \frac{h_{12} h_{21} Z_L}{h_{22} Z_L + 1}}$$



• A variable resistor R_o is connected across port 2 and adjusted for max. power transfer to R_o. Find the max. power ??

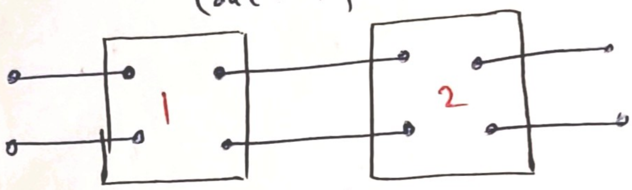
$$i = \frac{8.4}{84{,}000} = 0.1 \text{ mA}$$

$$P = I^2 R = (0.1 * 10^{-3})^2 (42 * 10^{3})$$

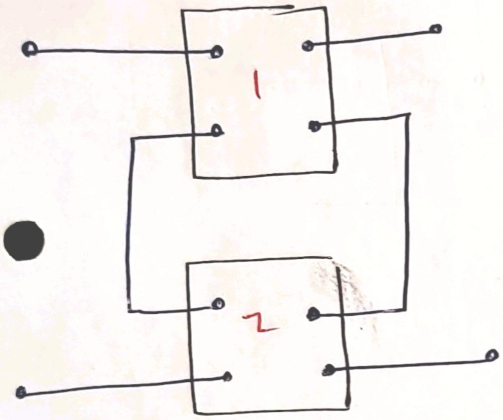
$$= 420 \, \mu W$$

Interconnected Two-Port Circuits :-

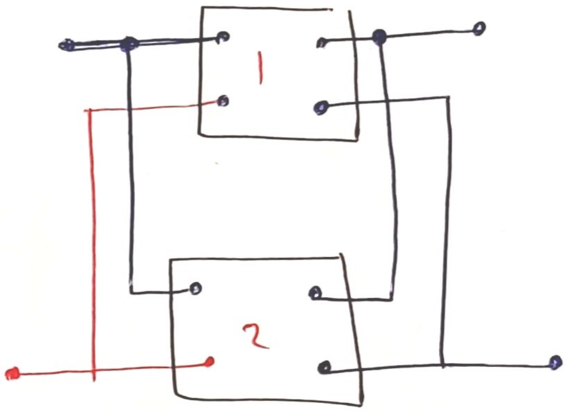
(out - input)



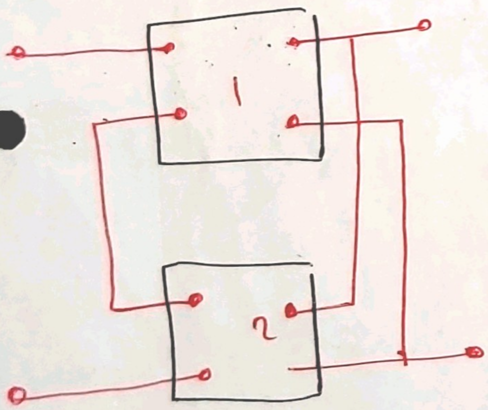
Cascade



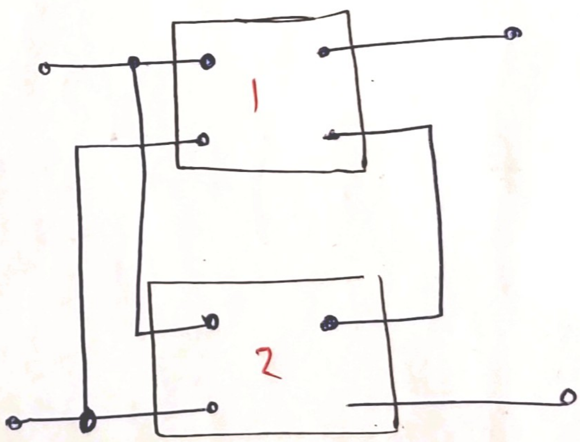
Series



Parallel



Series - Parallel

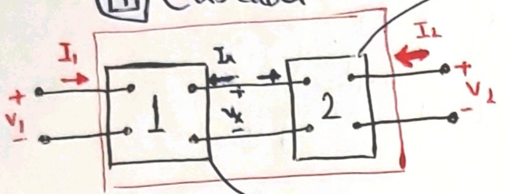


Parallel - Series.

Interconnected Two-Port Circuits :-

• Two-port circuits may be interconnected five ways :-
[1] In Cascade , [2] In series, [3] In parallel, [4] In series-parallel, and [5] in parallel-series.

[1] Cascade



$V_1 = a_{11} V_2 - a_{12} I_2$

$I_1 = a_{21} V_2 - a_{22} I_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix}}_{A} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} a^{a}_{11} & -a^{a}_{12} \\ a^{a}_{21} & -a^{a}_{22} \end{bmatrix}}_{A^{a}} \begin{bmatrix} V_x \\ I_x \end{bmatrix}$$

$$\begin{bmatrix} +V_x \\ -I_x \end{bmatrix} = \underbrace{\begin{bmatrix} a^{b}_{11} & -a^{b}_{12} \\ a^{b}_{21} & -a^{b}_{22} \end{bmatrix}}_{A^{b}} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

V_x ✓

$I_x \rightarrow (-1)$

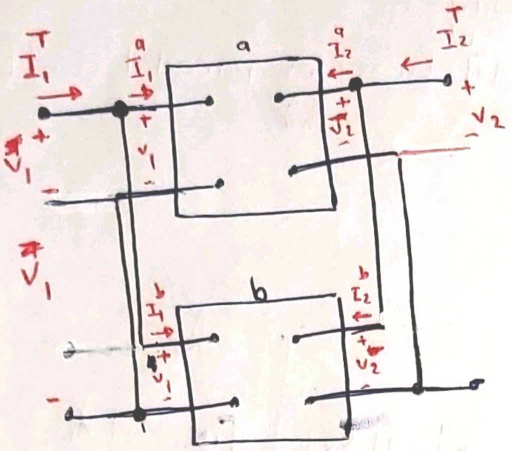
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A^{1} \end{bmatrix} \begin{bmatrix} A^{2} \end{bmatrix}} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} a^{a}_{11} & -a^{a}_{12} \\ a^{a}_{21} & -a^{a}_{22} \end{bmatrix} \begin{bmatrix} +a^{b}_{11} & a^{b}_{12} \\ -a^{b}_{21} & +a^{b}_{22} \end{bmatrix}$$ ← just last row

$$\begin{bmatrix} a^{a}_{11} a^{b}_{11} + a^{a}_{12} a^{b}_{12} & -a^{a}_{11} a^{b}_{12} - a^{a}_{22} a^{b}_{12} \\ a^{b}_{11} a^{a}_{21} + a^{a}_{22} a^{b}_{21} & -a^{a}_{21} a^{b}_{12} - a^{a}_{22} a^{b}_{22} \end{bmatrix}$$

$$\boxed{\begin{aligned} a_{11} &= a'_{11} a''_{11} + a'_{12} a''_{21} \\ a_{12} &= a'_{11} a''_{12} + a'_{12} a''_{22} \\ a_{21} &= a'_{21} a''_{11} + a'_{22} a''_{21} \\ a_{22} &= a'_{21} a''_{12} + a'_{22} a''_{22} \end{aligned}}$$

2 Parallel



$$\boxed{\begin{aligned} I_1^T &= I_1^a + I_1^b \\ I_2^T &= I_2^a + I_2^b \end{aligned}}$$

two two-port network in parallel

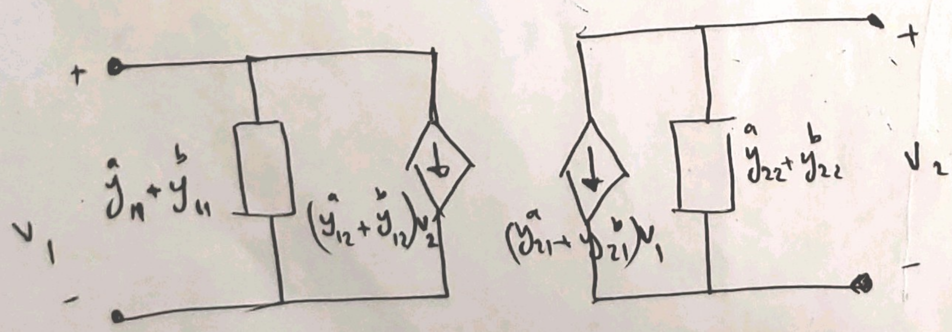
$$\begin{bmatrix} I_1^T \\ I_2^T \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}^a + \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}^b$$

$$\boxed{\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}}$$

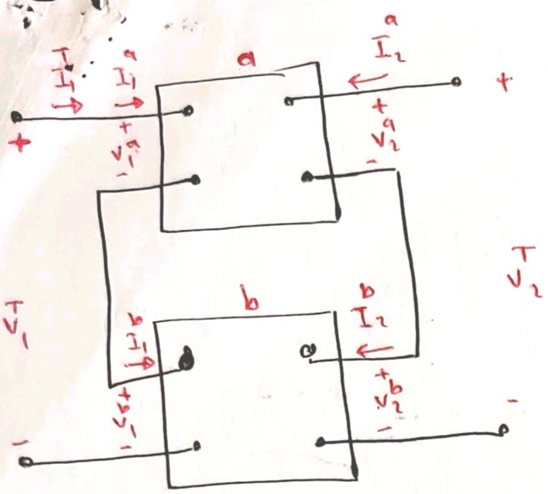
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}^a \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^a + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}^b \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^b$$

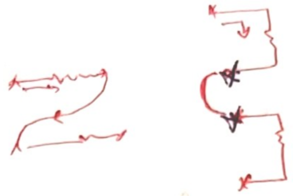
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y'_{11} + y''_{11} & y'_{12} + y''_{12} \\ y'_{21} + y''_{21} & y'_{22} + y''_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$\Downarrow$



3) Series





$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1^T = V_1^a + V_1^b$$
$$V_2^T = V_2^a + V_2^b$$

$$\boxed{\begin{bmatrix} V_1^T \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1^a \\ V_2 \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^b}$$

$$\boxed{\begin{aligned} I_1^a &= I_1^b = I_1^T \\ I_2^a &= I_2^b = I_2^T \end{aligned}}$$ because there are in series

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^T = \left[Z^a + Z^b \right] \begin{bmatrix} I_1^T \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^T = \begin{bmatrix} Z_{11}^a + Z_{11}^b & Z_{12}^a + Z_{12}^b \\ Z_{21}^a + Z_{21}^b & Z_{22}^a + Z_{22}^b \end{bmatrix} \begin{bmatrix} I_1^T \\ I_2 \end{bmatrix}$$

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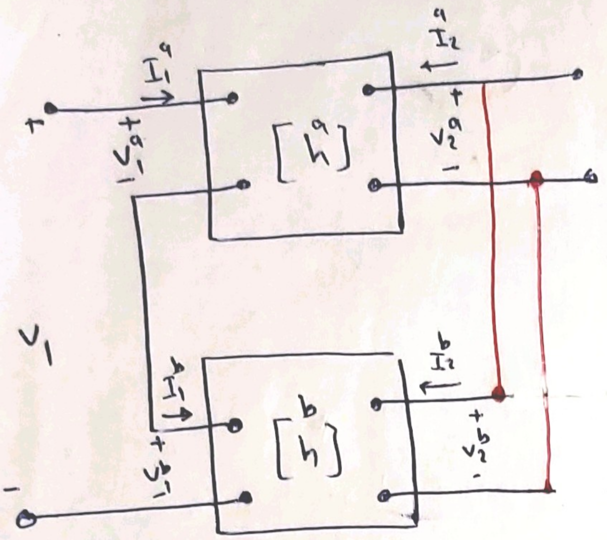
4 Series - Parallel



$$[h^T] = [h^a] + [h^b]$$

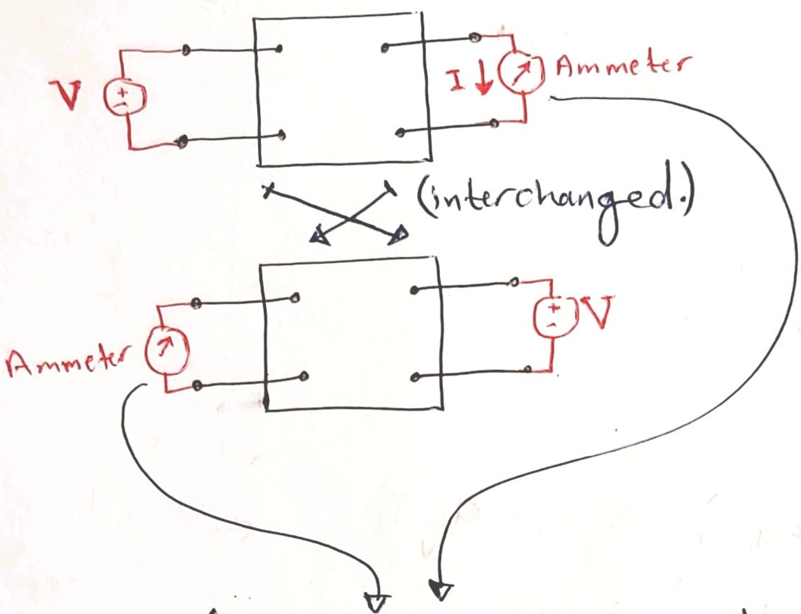
5 Parallel - Series

$$[g^T] = [g^a] + [g^b]$$

~~Reciprocal~~ Reciprocal Two-Port Circuits :-

صفة (متبادل) أ (تبادلي)

A two-port circuit is reciprocal if the interchange of an ideal voltage source at one port with an ideal ammeter at the other port produces the same ammeter reading.

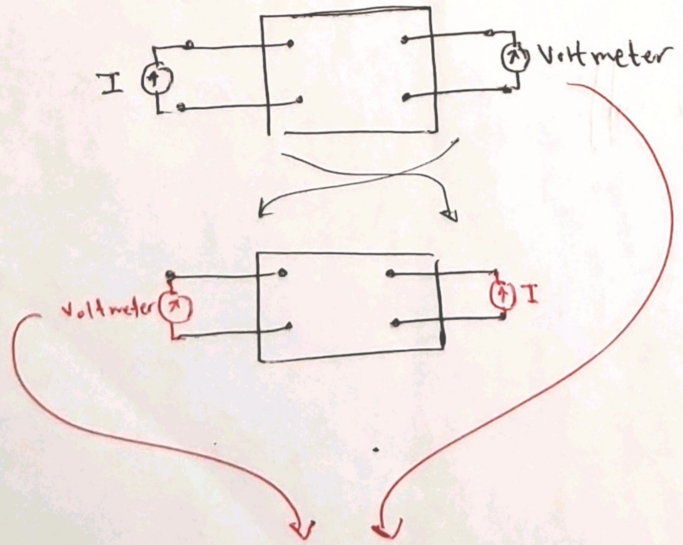


(The same ammeter reading.)

If the two-port circuit is reciprocal, Then :-

$z_{12} = z_{21}$	$h_{12} = -h_{21}$	$\Delta a = a_{11} a_{22} - a_{12} a_{22} = 1$
$y_{12} = y_{21}$	$g_{12} = -g_{21}$	$\Delta b = b_{11} b_{22} - b_{12} b_{21} = 1$

A two-port circuit is also reciprocal if the interchange of an ideal current source at one port with an ideal voltmeter at the other port produces the same voltmeter reading.



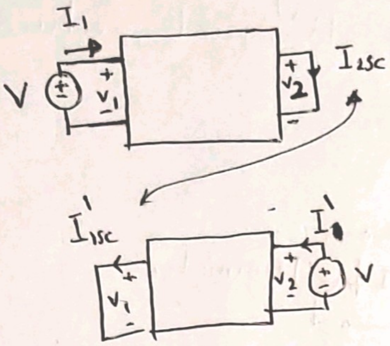
The same voltmeter reading

note

For ~~an~~ a reciprocal two-port circuit, only $\boxed{\text{three}}$ calculations or measurements are needed to determine a set of parameters.

Proof ?!

$$\boxed{a_{11}a_{22} - a_{12}a_{21} = \Delta a = 1}$$



$$\boxed{\begin{aligned} V_1 &= a_{11}V_2 - a_{12}I_2 \\ I_1 &= a_{21}V_2 - a_{22}I_2 \end{aligned}}$$

$V = V_1 = a_{11}(0) + a_{12}I_{2sc} \Rightarrow a_{12} = \frac{V_1}{I_{2sc}} \Rightarrow \boxed{I_{2sc} = \frac{V}{a_{12}}}$

$I_1 = a_{21}(0) + a_{22}I_{2sc} \Rightarrow a_{22} = \frac{I_1}{I_{2sc}}$

$0 = a_{11}V - a_{12}I'_1 \Rightarrow \cancel{a_{11} = a_{12}I_1} \Rightarrow I'_1 = \frac{a_{11}V}{a_{12}}$

$-I'_{1sc} = a_{21}V - a_{22}I'_1 \Rightarrow \cancel{a_{22} =}$

For the two-port to be reciprocals $\boxed{I_{2sc} = I'_{1sc}}$

$I'_{1sc} = -a_{21}V + a_{22}I'_1$

$\frac{V}{a_{12}} = -a_{21}V + a_{22}\left(\frac{a_{11}V}{a_{12}}\right)$

$$\boxed{1 = -a_{21}a_{12} + a_{22}a_{11}}$$

2

A two-port circuit is symmetric if its ports can be interchanged without disturbing the values of the terminal currents and voltages.

مقلق

If the two port circuit is symmetric, Then:

$Z_{11} = Z_{22}$

$y_{11} = y_{22}$

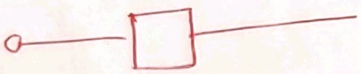
$a_{11} = a_{22}$

$b_{11} = b_{22}$

$h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1$

$g_{11}g_{22} - g_{12}g_{21} = \Delta g = 1$

For a symmetric reciprocal network, only $\boxed{\text{two}}$ calculations or measurements are necessary to determine all the two-port parameters.



From Eq. (18.36), $V_{bd} = 7.5$ V. The current I_{ad} equals

$$I_{ad} = \frac{7.5}{30} + \frac{15}{10} = 1.75\ A. \qquad \textbf{(18.37)}$$

A two-port circuit is also reciprocal if the interchange of an ideal current source at one port with an ideal voltmeter at the other port produces the same voltmeter reading.

For a reciprocal two-port circuit, only three calculations or measurements are needed to determine a set of parameters.

A reciprocal two-port circuit is **symmetric** if its ports can be interchanged without disturbing the values of the terminal currents and voltages. Figure 18.6 shows four examples of symmetric two-port circuits. In such circuits, the following additional relationships exist among the port parameters:

$$z_{11} = z_{22}, \qquad \textbf{(18.38)}$$

$$y_{11} = y_{22}, \qquad \textbf{(18.39)}$$

$$a_{11} = a_{22}, \qquad \textbf{(18.40)}$$

$$b_{11} = b_{22}, \qquad \textbf{(18.41)}$$

$$h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1, \qquad \textbf{(18.42)}$$

$$g_{11}g_{22} - g_{12}g_{21} = \Delta g = 1. \qquad \textbf{(18.43)}$$

For a symmetric reciprocal network, only two calculations or measurements are necessary to determine all the two-port parameters.

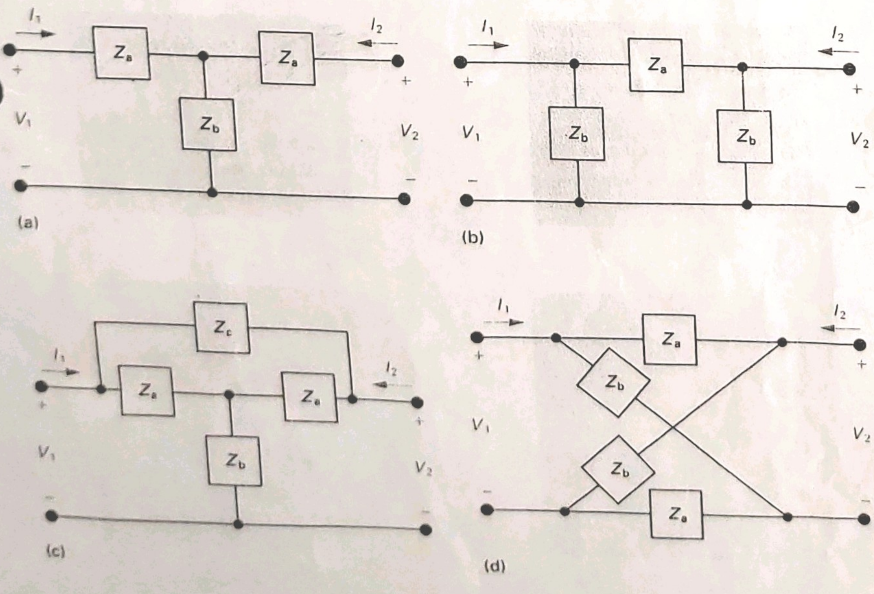


Figure 18.6 Four examples of symmetric two-port circuits. (a) A symmetric tee. (b) A symmetric pi. (c) A symmetric bridged tee. (d) A symmetric lattice.

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