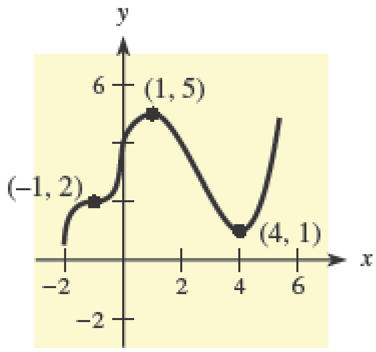
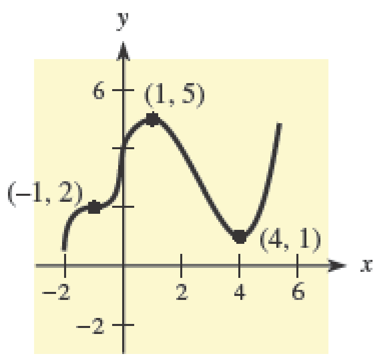
Chapter 10 Applications of Derivatives

1. Use the graph of $y = f(x)$ to identify at which of the indicated points the derivative $f'(x)$ changes from positive to negative.



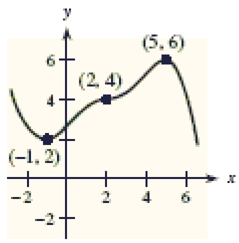
A) (1, 5)
B) (-1,2)
C) (5,6)
D) (-1,2), (5,6)
E) (-1,2), (2,4)
Ans: A

2. Use the graph of $y = f(x)$ to identify at which of the indicated points the derivative $f'(x)$ changes from negative to positive.



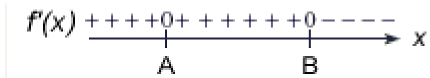
A) (-1,2)
B) (-1, 2)
C) (2,4)
D) (-1,2), (5,6)
E) (2,4), (5,6)
Ans: B

3. Use the graph of $y = f(x)$ to identify at which of the indicated points the derivative $f'(x)$ does not change sign.



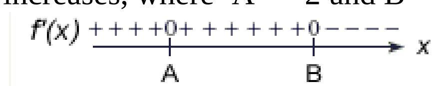
A) (5,6)
B) (-1,2), (5,6)
C) (-1,2), (2,4)
D) (2,4), (5,6)
E) (2,4)
Ans: E

4. Use the sign diagram for $f'(x)$ to determine all critical values of $f(x)$, where $A = -9$ and $B = 8$.



A) $x < 8$
B) $x > 8$
C) $x = -9$ and $x = 8$
D) $x = -9$
E) $x = 8$
Ans: C

5. Use the sign diagram for $f'(x)$ to determine the largest interval on which $f(x)$ increases, where $A = -2$ and $B = 8$.

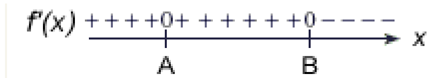


A) $x < 8$
B) $x > 8$
C) $-2 < x < 8$
D) $x > -2$
E) $x < -2$
Ans: A

6. Use the sign diagram for $f'(x)$ to determine the largest interval on which $f(x)$ decreases, where A = 8 and B = 9.

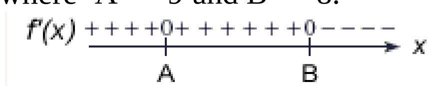
A) $x < 9$
B) $x > 9$
C) $8 < x < 9$
D) $x > 8$
E) $x < 8$
Ans: B

7. Use the sign diagram for $f'(x)$ to determine *x*-values at which relative maxima occur, where A = –6 and B = 4.



A) $x < 4$
B) $x = 4$
C) $x = -6$ and $x = 4$
D) $x = -6$
E) no relative maxima
Ans: B

8. Use the sign diagram for $f'(x)$ to determine *x*-values at which relative minima occur, where A = –9 and B = –8.



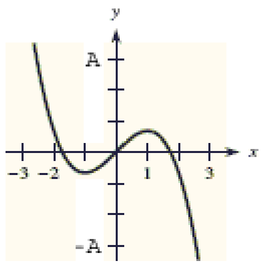
A) $x < -8$
B) $x = -8$
C) $x = -9$ and $x = -8$
D) $x = -9$
E) no relative minima
Ans: E

9. Make a sign diagram for the function and determine all *x*-values at which relative maxima occur.

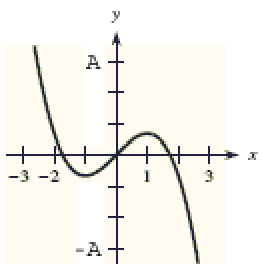
$y = x^3 - 9x^2 - 48x + 4$

A) $x = 0$
B) $x = 4$
C) $x = 8$
D) $x = -2$
E) no relative maxima
Ans: D

10. Make a sign diagram for the function and determine all *x*-values at which relative minima occur.
$y = x^3 - 3x^2 - 72x + 1$
A) $x = 0$
B) $x = 1$
C) $x = 6$
D) $x = -4$
E) no relative minima
Ans: C

11. For the given function and graph, estimate the coordinates of the relative maxima by observing the graph, where $A = 21$.
$y = 7x - \frac{7}{3}x^3$


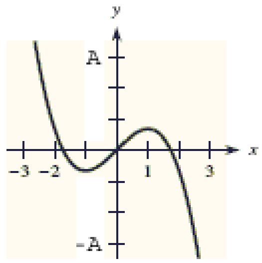
A) $\left(1, \frac{2}{3}\right)$
B) $\left(1, \frac{14}{3}\right)$
C) $\left(-1, -\frac{14}{3}\right)$
D) $\left(-1, -\frac{2}{3}\right)$
E) no relative maxima
Ans: B

12. For the given function and graph, estimate the coordinates of the relative minima by observing the graph, where $A = 15$.
$y = 5x - \frac{5}{3}x^3$


A) $\left(1, \frac{2}{3}\right)$
B) $\left(1, \frac{10}{3}\right)$
C) $\left(-1, -\frac{10}{3}\right)$
D) $\left(-1, -\frac{2}{3}\right)$
E) no relative minima
Ans: C

13. For the given function and graph, determine all critical value(s), where A = 24.

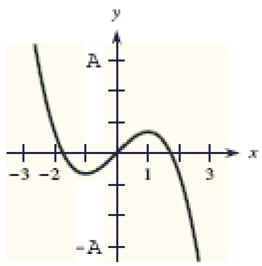
$y = 8x - \frac{8}{3}x^3$



A) $x = 0$
B) $x = 1$
C) $x = -1$
D) $x = -1$ and $x = 1$
E) $x = -1$ and $x = 0$ and $x = 1$
Ans: D

14. For the given function and graph, determine all critical point(s), where A = 21.

$y = 7x - \frac{7}{3}x^3$



A) $(0,0)$
B) $(1, \frac{14}{3})$
C) $(-1, -\frac{14}{3})$
D) $(-1, -\frac{14}{3})$ and $(1, \frac{14}{3})$
E) $(-1, -\frac{14}{3})$ and $(0,0)$ and $(1, \frac{14}{3})$
Ans: D

15. For the given function, find $y' = f'(x)$.

$y = x^3 - 9x^2 - 81x + 1$

A) $3x^2 - 18x - 81$
B) $x^2 - 18x + 81$
C) $x^2 - 36x - 81$
D) $3x^2 - 36x + 81$
E) $3x^2 - 36x - 81$
Ans: A

16. For the given function, find all critical values.
$y = x^3 - 3x^2 - 105x + 3$
A) $x = 0$
B) $x = -7$ and $x = -5$
C) $x = -7$ and $x = 5$
D) $x = -5$ and $x = 7$
E) $x = 5$ and $x = 7$
Ans: D

17. For the given function, find the critical points.
$y = x^3 - 3x^2 - 45x + 6$
A) $(-3, 87)$ and $(5, -169)$
B) $(-3, -169)$ and $(5, 87)$
C) $(-3, -129)$ and $(5, 31)$
D) $(3, -129)$ and $(5, -169)$
E) $(-3, 87)$ and $(-5, 31)$
Ans: A

18. For the given function, find all intervals of *x*-values where the function is increasing.
$y = x^3 - 6x^2 - 96x + 3$
A) $-4 < x < 8$
B) $x > 8$
C) $x < -4$ or $x > 8$
D) $x < -4$
E) $x < 0$
Ans: C

19. For the given function, find all intervals of *x*-values where the function is decreasing.
$y = x^3 - 9x^2 - 81x + 5$
A) $-3 < x < 9$
B) $x > 9$
C) $x < -3$ or $x > 9$
D) $x < -3$
E) $x < 0$
Ans: A

20. For the given function, find $y' = f'(x)$.

$$y = \frac{x^4}{4} - \frac{x^3}{3} - 5$$

A) $x^3 - x$
B) $x^4 - x^2 - 5$
C) $x^3 - x^2 - 5$
D) $x^4 - x^3$
E) $x^3 - x^2$
Ans: E

21. For the given function, find the critical values.

$$y = \frac{x^4}{4} - \frac{x^3}{3} - 3$$

A) $x = 0$ and $x = 1$
B) $x = 0$ and $x = 3$
C) $x = 0$ and $x = -3$
D) $x = 0$ and $x = -1$
E) $x = -1$ and $x = 1$
Ans: A

22. For the given function, find the critical points.

$$y = \frac{x^4}{4} - \frac{x^3}{3} - 4$$

A) $(0,0)$, $\left(1, -\frac{49}{12}\right)$
B) $(0,-4)$, $\left(1, -\frac{49}{12}\right)$
C) $(0,-4)$, $\left(1, \frac{49}{12}\right)$
D) $(0,4)$, $\left(1, \frac{49}{12}\right)$
E) $(0,4)$, $\left(1, -\frac{49}{12}\right)$
Ans: B

23. For the given function, find intervals of *x*-values where the function is increasing.

$$y = \frac{x^4}{4} - \frac{x^3}{3} - 3$$

A) $x > 1$
B) $x < 1$
C) $x > 0$
D) $x < 0$
E) $0 < x < 1$
Ans: A

24. For the given function, find intervals of *x*-values where the function is decreasing.

$y=\frac{x^4}{4}-\frac{x^3}{3}-8$

A) $0<x<1$
B) $x>0$
C) $x<0$
D) $x>1$
E) $x<1$
Ans: E

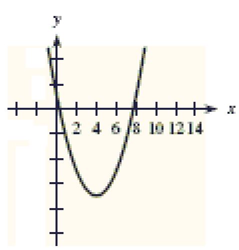
25. For the given function, classify the critical points as relative maxima, relative minima, or points of inflection. In each case, you may check your conclusions with a graphing utility.

$y=\frac{x^4}{4}-\frac{x^3}{3}-3$

A) $(0,-3)$ is a relative minimum and $\left(1,-\frac{37}{12}\right)$ is point of inflection.
B) $(0,-3)$ is a point of inflection and $\left(1,-\frac{37}{12}\right)$ is a relative minimum.
C) $(0,-3)$ is a point of inflection and $\left(1,-\frac{37}{12}\right)$ is a relative maximum.
D) $(0,-3)$ is a relative minimum and $\left(1,-\frac{37}{12}\right)$ is a relative maximum.
E) $(0,-3)$ is a relative maximum and $\left(1,-\frac{37}{12}\right)$ is a relative minimum.
Ans: B

26. For the given function, use the graph to identify *x*-values for which $y'>0$. You may use the derivative to check your conclusion.

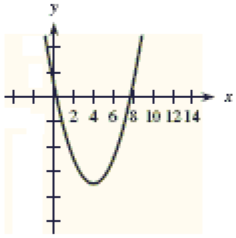
$y=\frac{1}{2}x^2-4x+1$



A) $x>0$
B) $x<0$
C) $x>4$
D) $x<4$
E) $0<x<8$
Ans: C

27. For the given function, use the graph to identify x-values for which $y' < 0$. You may use the derivative to check your conclusion.

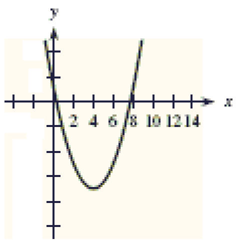
$y = \frac{9}{2}x^2 - 36x + 9$



A) $x > 0$
B) $x < 0$
C) $x > 4$
D) $x < 4$
E) $0 < x < 8$
Ans: D

28. For the given function, use the graph to identify the x-value for which $y' = 0$. You may use the derivative to check your conclusion.

$y = \frac{1}{2}x^2 - 4x + 1$



A) $x = 0$
B) $x = 4$
C) $x = 8$
D) $x = -7$
E) $x = 7$
Ans: B

29. For the given function, find the relative minima.

$y = x^3 - 9x^2 - 48x + 11$

A) $(-2, 63)$
B) $(8, -437)$
C) $(-2, -113)$
D) $(-8, -693)$
E) no relative minima
Ans: B

30. For the given function, find the relative maxima.

$y = x^3 - 3x^2 - 45x + 13$

A) $(-3, 94)$

B) $(5, -162)$

C) $(-3, -122)$

D) $(-5, 38)$

E) no relative maxima

Ans: A

31. For the given function, find the horizontal points of inflection.

$y = x^3 - 6x^2 - 63x + 10$

A) $(-3, 118)$

B) $(7, -382)$

C) $(-3, -206)$

D) $(-7, -186)$

E) no horizontal points of inflection

Ans: E

32. For the given function find the relative maxima, and sketch the graph. You may check your graph with a graphing utility.

$y = \frac{1}{18}x^6 - \frac{1}{3}x^4 + 4$

A) $(0, 4)$

B) $\left(2, \frac{20}{9}\right)$

C) $\left(-2, \frac{20}{9}\right)$

D) $\left(-2, -\frac{20}{9}\right)$

E) $(0, -4)$

Ans: A

33. For the given function find the relative minima, and sketch the graph. You may check your graph with a graphing utility.

$y = \frac{1}{18}x^6 - \frac{1}{3}x^4 + 2$

A) $(0, 2)$

B) $\left(-2, \frac{2}{9}\right)$ and $\left(2, \frac{2}{9}\right)$

C) $\left(-2, -\frac{2}{9}\right)$ and $\left(2, -\frac{2}{9}\right)$

D) $\left(-2, -\frac{2}{9}\right)$

E) $\left(2, \frac{2}{9}\right)$

Ans: B

34. Both a function and its derivative are given. Use them to find all critical values.

$y = \frac{x^2(x-9)^3}{3} \qquad \frac{dy}{dx} = \frac{x(x-9)(5x-18)^2}{3}$

A) $x = 0$

B) $x = 0,\ x = 18/5$

C) $x = 0,\ x = 9$

D) $x = 9,\ x = 18/5$

E) $x = 0,\ x = 9,\ x = 18/5$

Ans: E

35. Both a function and its derivative are given. Use them to find all critical points.

$y = \frac{x^2(x-8)^3}{13} \qquad \frac{dy}{dx} = \frac{x(x-8)(5x-16)^2}{13}$

A) $(0,0)$, $(8,0)$, $(16/5, 0)$

B) $(0,0)$, $(16/5, 0)$, $(8, -87.112)$

C) $(0,0)$, $(8,0)$, $(16/5, -87.112)$

D) $(0,0)$, $(8,0)$, $(16/5, 87.112)$

E) $(0,0)$, $(8,0)$, $(8, 87.112)$

Ans: C

36. Both a function and its derivative are given. Use them to find intervals on which the function is increasing.

$$y = \frac{x^2(x-8)^3}{17} \qquad \frac{dy}{dx} = \frac{x(x-8)(5x-16)^2}{17}$$

A) $x < 0$ or $16/5 < x < 8$ or $x > 8$
B) $0 < x < 8$
C) $x < 0$ or $x > 8$
D) $x < 0$ or $x > 16/5$
E) $16/5 < x < 8$
Ans: A

37. Both a function and its derivative are given. Use them to find intervals on which the function is decreasing.

$$y = \frac{x^2(x-4)^3}{13} \qquad \frac{dy}{dx} = \frac{x(x-4)(5x-8)^2}{13}$$

A) $x > 0$
B) $0 < x < 4$
C) $0 < x < 8/5$
D) $x < 0$ or $x > 8/5$
E) $8/5 < x < 4$
Ans: C

38. Both a function and its derivative are given. Use them to find the relative maxima.

$$y = \frac{x^2(x-8)^3}{17} \qquad \frac{dy}{dx} = \frac{x(x-8)(5x-16)^2}{17}$$

A) $(0,0)\ (8,0)$
B) $(0,0)$
C) $(8,0)$
D) $(16/5, -66.615)$
E) no relative maxima
Ans: B

39. Both a function and its derivative are given. Use them to find the relative minima.

$$y = \frac{x^2(x-9)^3}{19} \qquad \frac{dy}{dx} = \frac{x(x-9)(5x-18)^2}{19}$$

A) $(0,0)$
B) $(18/5, -107.407)$
C) $(9,0)$
D) $(0,0), (9,0)$
E) no relative minima
Ans: B

40. Use the derivative to locate all critical points. Use a graphing utility if desired.

$f(x) = x^3 - 5x^2 - 100x + 100$

A) $(-4.34, -509.93)$, $(7.68, 358.08)$

B) $(-509.93, 7.68)$, $(358.08, -4.34)$

C) $(7.68, -509.93)$, $(-4.34, 358.08)$

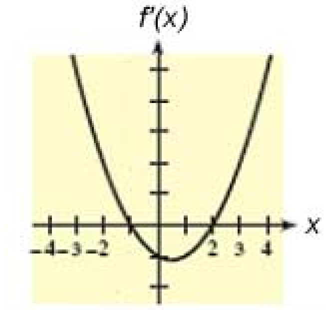
D) $(0,100)$, $(7.68, -4.34)$

E) $(0,100)$, $(358.08, 7.68)$

Ans: C

41. A graph of $f'(x)$ is given. Use the graph to determine all critical values of $f(x)$.

$f'(x) = 6x^2 - 6x - 12$



A) $x = -1$

B) $x = 1$

C) $x = 2$

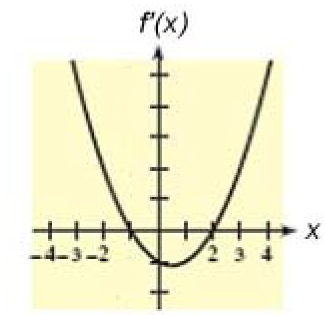
D) $x = -1,\ x = 2$

E) $x = -1,\ x = 0,\ x = 2$

Ans: D

42. A graph of $f'(x)$ is given. Use the graph to determine where $f(x)$ is decreasing.

$f'(x) = 2x^2 - 2x - 4$



A) $x > -1$

B) $x < 2$

C) $x > 2$

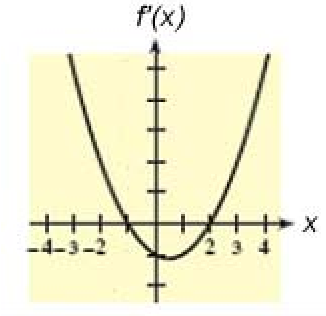
D) $x < -1$ or $x > 2$

E) $-1 < x < 2$

Ans: E

43. A graph of $f'(x)$ is given. Use the graph to determine where the graph of $f(x)$ has a relative maximum.

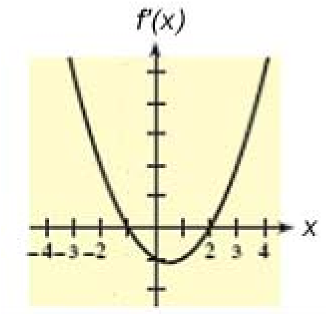
$f'(x) = 3x^2 - 3x - 6$



A) $x = -1$
B) $x = 0$
C) $x = 1$
D) $x = 2$
E) no relative maxima
Ans: A

44. A graph of $f'(x)$ is given. Use the graph to determine where the graph of $f(x)$ has a relative minimum.

$f'(x) = 9x^2 - 9x - 18$



A) $x = -1$
B) $x = 0$
C) $x = 1$
D) $x = 2$
E) no relative minima
Ans: D

45. Suppose that a chain of auto service stations, Quick-Oil, Inc., has found that its monthly sales volume y (in thousands of dollars) is related to the price p (in dollars) of an oil change by $y = \dfrac{160}{\sqrt{p+2}}$, $p > 30$. Is y increasing or decreasing for all values of $p > 30$?

A) increasing
B) decreasing
Ans: B

46. Suppose the average costs of a mining operation depend on the number of machines used, and average costs, in dollars, are given by $\overline{C}(x) = 6x + \dfrac{1944}{x}, \quad x > 0$, where x is the number of machines used. Find the critical values of $\overline{C}(x)$ that lie in the domain of the problem.
A) –18
B) 18
C) –18, 18
D) 18, 28
E) 28
Ans: B

47. Suppose the average costs of a mining operation depend on the number of machines used, and average costs, in dollars, are given by $\overline{C}(x) = 10x + \dfrac{9000}{x}, \quad x > 0$, where x is the number of machines used. Over what interval in the domain do average costs decrease?
A) $x > 0$
B) $x > 30$
C) $0 < x < 30$
D) $x < 30$
E) $x > -30$
Ans: C

48. Suppose the average costs of a mining operation depend on the number of machines used, and average costs, in dollars, are given by $\overline{C}(x) = 7x + \dfrac{3087}{x}, \quad x > 0$, where x is the number of machines used. Over what interval in the domain do average costs increase?
A) $x > 0$
B) $x > 21$
C) $0 < x < 21$
D) $x < 21$
E) $x > -21$
Ans: B

49. Suppose the average costs of a mining operation depend on the number of machines used, and average costs, in dollars, are given by $\overline{C}(x) = 13x + \dfrac{8788}{x}, \quad x > 0$, where x is the number of machines used. How many machines give minimum average costs?
A) Using 26 machines gives the minimum average costs.
B) Using zero machines gives the minimum average costs.
C) Using 36 machines gives the minimum average costs.
D) Using 52 machines gives the minimum average costs.
E) Using 57 machines gives the minimum average costs.
Ans: A

50. Suppose the average costs of a mining operation depend on the number of machines used, and average costs, in dollars, are given by $\overline{C}(x) = 5x + \frac{4500}{x}, \quad x > 0$, where x is the number of machines used. What is the minimum average cost?
A) $0
B) $30
C) $300
D) $150
E) $4505
Ans: B

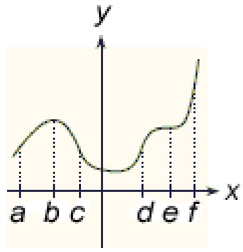
51. The number of milligrams x of a medication in the bloodstream t hours after a dose is taken can be modeled by $x(t) = \frac{1000t}{t^2 + 8} \quad t > 0$. For what t-values is x increasing? Round answers to two decimal places.
A) $t > 0$
B) $t < 2.83$
C) $t > 2.83$
D) $-2.83 < t < 2.83$
E) $0 < t < 2.83$
Ans: E

52. The number of milligrams x of a medication in the bloodstream t hours after a dose is taken can be modeled by $x(t) = \frac{4000t}{t^2 + 18} \quad t > 0$. Find the t-value at which x is maximum. Round your answer to two decimal places.
A) 0 hours
B) 4.24 hours
C) 471.40 hours
D) 6.24 hours
E) 10.35 hours
Ans: B

53. The number of milligrams x of a medication in the bloodstream t hours after a dose is taken can be modeled by $x(t) = \frac{5000t}{t^2 + 14} \quad t > 0$. Find the maximum value of x. Round your answer to two decimal places.
A) 3.74 mg
B) 668.15 mg
C) 1469.94 mg
D) 11.60 mg
E) 1875.50 mg
Ans: B

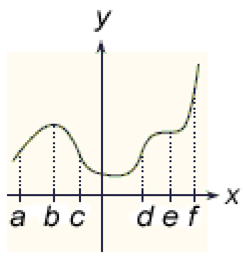
54. Determine whether the given function is concave up or concave down at the indicated point.
$y = 8x^3 - 7x^2 + 4$ at $x = 0$
A) concave down
B) concave up
Ans: A

55. Use the indicated *x*-values on the graph of $y = f(x)$ to determine intervals over which the graph is concave up.



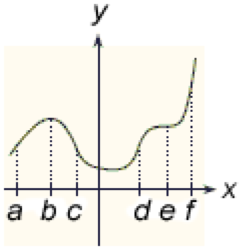
A) (c, d) and (e, f)
B) (c, d)
C) (d, e)
D) (a, c)
E) (e, f) and (b, c)
Ans: A

56. Use the indicated *x*-values on the graph of $y = f(x)$ to find all the intervals where $f''(x) < 0$.



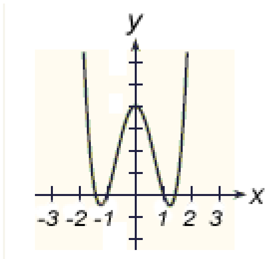
A) (a, c)
B) (a, c) and (d, e)
C) (d, e)
D) (e, f)
E) (a, f) and (c, d)
Ans: B

57. Use the indicated *x*-values on the graph of $y = f(x)$ to find the *x*-coordinate of any horizontal point of inflection.



A) a
B) c
C) b
D) e
E) f
Ans: D

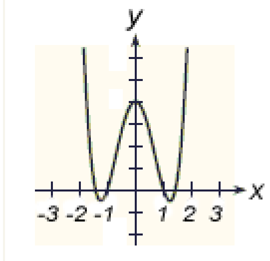
58. A function and its graph are given. Use the second derivative to determine intervals on which the function is concave up. Check these results against the graph shown.
$y = 8x^4 - 24x^2 + 16$



A) $x < -\frac{\sqrt{2}}{2}$
B) $x > \frac{\sqrt{2}}{2}$
C) $-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$
D) $x < -\frac{\sqrt{2}}{2}$ or $x > \frac{\sqrt{2}}{2}$
E) $x > 0$
Ans: D

59. A function and its graph are given. Use the second derivative to determine intervals on which the function is concave down. Check these results against the graph shown.

$y = 14x^4 - 42x^2 + 28$



A) $x < -\frac{\sqrt{2}}{2}$

B) $x > \frac{\sqrt{2}}{2}$

C) $-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$

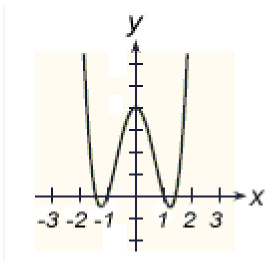
D) $x < -\frac{\sqrt{2}}{2}$ or $x > \frac{\sqrt{2}}{2}$

E) $x > 0$

Ans: C

60. A function and its graph are given. Use the second derivative to locate all *x*-values of points of inflection on the graph of $y = f(x)$. Check these results against the graph shown.

$y = 6x^4 - 18x^2 + 12$



A) $x = -\frac{\sqrt{2}}{2}$

B) $x = 0$

C) $x = \frac{\sqrt{2}}{2}$

D) $x = \frac{\sqrt{2}}{2}$, $x = -\frac{\sqrt{2}}{2}$

E) $x = -\frac{\sqrt{2}}{2}$, $x = 0$, $x = -\frac{\sqrt{2}}{2}$

Ans: D

61. Find all relative maxima of the given function.
$y = x^4 - 8x^3 + 16x^2 + 10$
A) $(0,10)$
B) $(2,26)$
C) $(4,10)$
D) $(0,10),(4,10)$
E) no relative maxima
Ans: B

62. Find all relative minima of the given function.
$y = x^4 - 8x^3 + 16x^2 + 7$
A) $(0,7)$
B) $(2,23)$
C) $(4,7)$
D) $(0,7),(4,7)$
E) no relative minima
Ans: D

63. Find all points of inflection of the given function.
$y = x^4 - 8x^3 + 16x^2 + 10$
A) $(0,10),(4,10)$
B) $(0,10),(2,26)$
C) $(0.8453, 17.1111),(4,10)$
D) $(3.1547, 17.1111),(2,26)$
E) $(0.8453, 17.1111),(3.1547, 17.1111)$
Ans: E

64. A function and its first and second derivatives are given. Use these to find all critical values.
$f(x) = x^5 - 10x^4 + 13$
$f'(x) = 5x^3(x-8)$
$f''(x) = 20x^2(x-6)$
A) $x = 0$
B) $x = 6$
C) $x = 8$
D) $x = 0,\ x = 6$
E) $x = 0,\ x = 8$
Ans: E

65. A function and its first and second derivatives are given. Use these to find the relative maxima.

$f(x) = x^5 - a x^4 + b$

$f'(x) = 5x^3(x - 4/5 * a)$

$f''(x) = 20x^2(x - 3/5 * a)$

A) $(0, b)$

B) $(3/5 * a, inf)$

C) $(minx, miny)$

D) $(minx, -miny)$

E) no relative maxima

Ans: A

66. A function and its first and second derivatives are given. Use these to find all relative minima.

$f(x) = x^5 - a x^4 + b$

$f'(x) = 5x^3(x - 4/5 * a)$

$f''(x) = 20x^2(x - 3/5 * a)$

A) $(0, b)$

B) $(3/5 * a, inf)$

C) $(minx, miny)$

D) $(0, b)$, $(minx, miny)$

E) no relative minima

Ans: C

67. A function and its first and second derivatives are given. Use these to find all points of inflection.

$f(x) = x^5 - a x^4 + b$

$f'(x) = 5x^3(x - 4/5 * a)$

$f''(x) = 20x^2(x - 3/5 * a)$

A) $(0, b)$

B) $(3/5 * a, inf)$

C) $(minx, miny)$

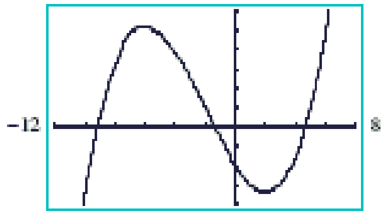
D) $(0, b)$, $(3/5 * a, inf)$

E) no points of inflection

Ans: B

68. A function and its graph are given. From the graph, estimate where $f''(x) > 0$.

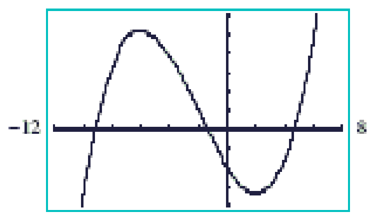
$f(x) = \frac{1}{3}x^3 + 2x^2 - 12x - 20$



A) $x > -2$
B) $x < -2$
C) $x < 2$
D) $x > 2$
E) $x = 2$
Ans: A

69. A function and its graph are given. From the graph, estimate where $f''(x) < 0$.

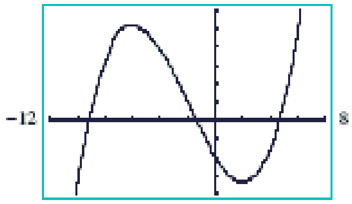
$f(x) = \frac{7}{3}x^3 + 14x^2 - 84x - 140$



A) $x > -2$
B) $x < -2$
C) $x < 2$
D) $x > 2$
E) $x = 2$
Ans: B

70. A function and its graph are given. From the graph estimate where $f'(x)$ has a relative maximum.

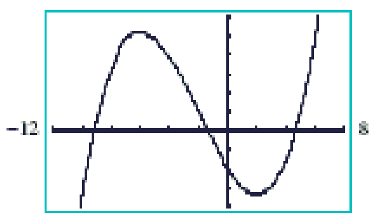
$f(x) = \frac{4}{3}x^3 + 8x^2 - 48x - 80$



A) $x = -3$
B) $x = -2$
C) $x = 1$
D) $x = 2.5$
E) no relative maxima
Ans: E

71. A function and its graph are given. From the graph estimate where $f'(x)$ has a relative minimum.

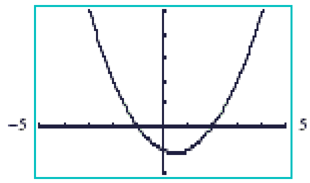
$f(x) = \frac{7}{3}x^3 + 14x^2 - 84x - 140$



A) $x = -3$
B) $x = -2$
C) $x = 1$
D) $x = 2.5$
E) no relative minima
Ans: B

72. In this problem, $f'(x)$ and its graph are given. Use the graph of $f'(x)$ to determine where $f(x)$ is concave up.

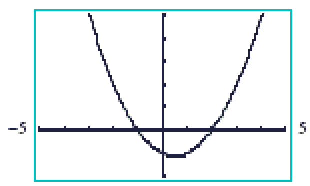
$f'(x) = 3x^2 - 3x - 6$



A) $x < -1$
B) $x > -1$
C) $x < \frac{1}{2}$
D) $x > \frac{1}{2}$
E) $x > 2$
Ans: D

73. In this problem, $f'(x)$ and its graph are given. Use the graph of $f'(x)$ to determine where $f(x)$ is concave down.

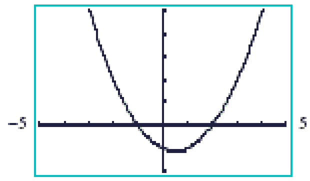
$f'(x) = 9x^2 - 9x - 18$



A) $x < -1$
B) $x > -1$
C) $x < \frac{1}{2}$
D) $x > \frac{1}{2}$
E) $x > 2$
Ans: C

74. In this problem, $f'(x)$ and its graph are given. Use the graph of $f'(x)$ to determine where $f(x)$ has a point of inflection.

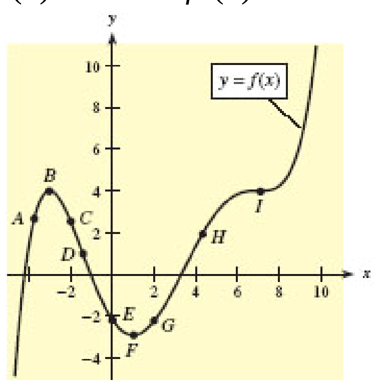
$f'(x) = 8x^2 - 8x - 16$



A) $x = -1$
B) $x = 0$
C) $x = \frac{1}{2}$
D) $x = 2$
E) no point of inflection
Ans: C

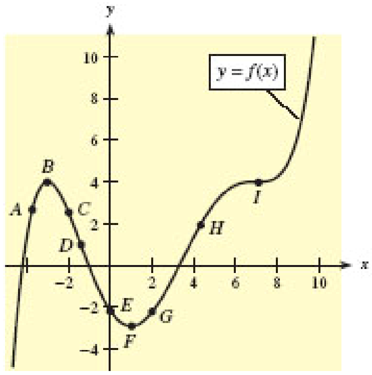
75. Use the graph shown in the figure and identify points from *A* through *I* that satisfy the given condition.

$f'(x) > 0$ and $f''(x) < 0$



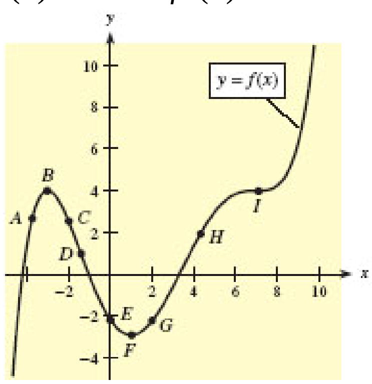
A) *E*
B) *F*
C) *G*
D) *A*
E) *B*
Ans: D

76. Use the graph shown in the figure and identify points from *A* through *I* that satisfy the given condition.
$f'(x) < 0$ and $f''(x) > 0$



A) *B*
B) *H*
C) *I*
D) *D*
E) *E*
Ans: E

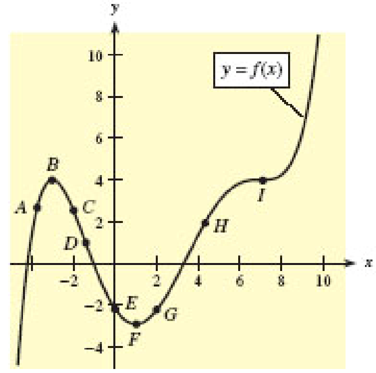
77. Use the graph shown in the figure and identify points from *A* through *I* that satisfy the given condition.
$f'(x) = 0$ and $f''(x) < 0$



A) *B*
B) *C*
C) *I*
D) *G*
E) *A*
Ans: A

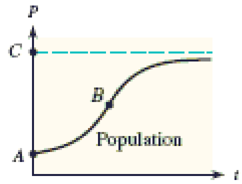
78. Use the graph shown in the figure and identify points from *A* through *I* that satisfy the given condition.

$f'(x) < 0$ and $f''(x) = 0$



A) *C*
B) *B*
C) *H*
D) *D*
E) *E*
Ans: D

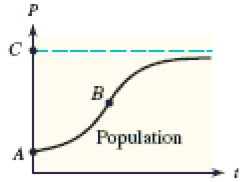
79. The following figure shows the growth of a population as a function of time.



If *P* represents the population and *t* represents the time, write a mathematical symbol that represents the rate of change (growth rate) of the population with respect to time.

A) dy/dt
B) $t'(P)$
C) $t(P)$
D) $P'(t)$
E) $P(t)$
Ans: D

80. The following figure shows the growth of a population as a function of time.

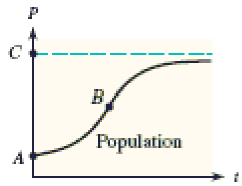


Which of *A*, *B*, and *C* correspond(s) to the point(s) at which the growth *rate* attains its maximum?

A) B
B) C
C) A
D) C and A
E) none of the above

Ans: A

81. The following figure shows the growth of a population as a function of time.



Which of *A*, *B*, and/or *C* correspond(s) to the upper limit of population?

A) B
B) C
C) A
D) B and A
E) none of the above

Ans: B

82. Suppose that the total number of units produced by a worker in *t* hours of an 8-hour shift can be modeled by the production function $P(t): P(t) = 114t + 54t^2 - 2t^3$. Find the number of hours before production is maximized.

A) $t = 0$
B) $t = \inf$
C) $t = 5$
D) $t = 8$
E) $t = 19$

Ans: D

83. Suppose that the total number of units produced by a worker in t hours of an 8-hour shift can be modeled by the production function $P(t): P(t) = 6 * \mathrm{a}\, t + 3 * (\mathrm{a}–1)t^2 - 2t^3$. Find the number of hours before the rate of production is maximized. That is, find the point of diminishing returns.
A) $t = 0$
B) $t = \mathrm{inf}$
C) $t = 5$
D) $t = 8$
E) $t = \mathrm{a}$
Ans: B

84. Suppose that the oxygen level P (for purity) in a body of water t months after an oil spill is given by $P(t) = 900\left[1 - \frac{2}{t+3} + \frac{4}{(t+3)^2}\right]$. Find how long it will be before the oxygen level reaches its minimum.
A) $t = 0$
B) $t = 1$
C) $t = 6$
D) $t = \mathrm{inf}$
E) none of the above
Ans: B

85. Suppose that the oxygen level P (for purity) in a body of water t months after an oil spill is given by $P(t) = 300\left[1 - \frac{5}{t+1} + \frac{10}{(t+1)^2}\right]$. Find how long it will be before the rate of change of P is maximized. That is, find the point of diminishing returns.
A) $t = 0$
B) $t = 3$
C) $t = 2$
D) $t = \mathrm{inf}$
E) none of the above
Ans: D

86. The consumer price data can be modeled by the function $c(x) = –0.01x^3 + 0.40x^2 - 4.47x + 4.02$, where $x = 0$ represents 1945 and $c(x)$ is the consumer price index (CPI) in year $1945 + x$. During what year does the model predict that the rate of change of the CPI reached its maximum?
A) 2003
B) 1956
C) 1965
D) 1973
E) 1958
Ans: E

87. Find the x-value at which the absolute minimum of $f(x)$ occurs on the interval $[a, b]$.
$f(x) = x^3 - 75x + 5,\ [-15,\ 6]$
A) $x = -10$
B) $x = -5$
C) $x = 0$
D) $x = 5$
E) $x = 6$
Ans: A

88. Find the x-value at which the absolute maximum of $f(x)$ occurs on the interval $[a, b]$.
$f(x) = x^3 - 75x + 5,\ [-15,\ 6]$
A) $x = -10$
B) $x = -5$
C) $x = 0$
D) $x = 5$
E) $x = 6$
Ans: B

89. Find the absolute maximum for $f(x)$ on the interval $[a, b]$.
$f(x) = x^3 - 27x + 3,\ [-4,\ 4]$
A) $x = -4$
B) $x = -3$
C) $x = 0$
D) $x = 3$
E) $x = 4$
Ans: B

90. If the total revenue function for a blender is $R(x) = 40x - 0.1x^2$, determine how many units x must be sold to provide the maximum total revenue in dollars.
A) 4,000
B) 3,750
C) 40
D) 150
E) 200
Ans: E

91. If the total revenue function for a blender is $R(x) = 30x - 0.2x^2$, find the maximum revenue.
A) $75
B) $0
C) $30
D) $150
E) $1125
Ans: E

92. If the total revenue function for a blender is $R(x) = 45x - 0.05x^2$, find the maximum revenue if production is limited to at most 200 blenders.
A) $10,125
B) $450
C) $45
D) $7000
E) $200
Ans: D

93. A firm has total revenue given by $R(x) = 2000x - 192.5x^2 - x^3$ dollars for x units of a product. Find the maximum revenue from sales of that product.
A) $4000
B) $5063
C) $505
D) $4400
E) $348
Ans: B

94. A company handles an apartment building with 90 units. Experience has shown that if the rent for each of the units is $\$300$ per month, all the units will be filled, but 1 unit will become vacant for each $10 increase in the monthly rate. What rent should be charged to maximize the total revenue from the building if the upper limit on the rent is $\$600$ per month?
A) $300
B) $600
C) $330
D) $180
E) $450
Ans: C

95. For the revenue function given by $R(x) = 360x + 8x^2 - x^3$ find the maximum average revenue.
A) $360
B) $770
C) $3860
D) $640
E) $376
Ans: E

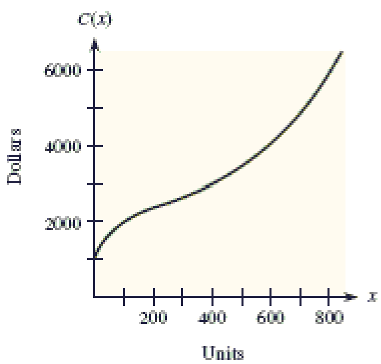
96. For the revenue function given by $R(x) = 2600x + 20x^2 - x^3$, find the x-value where $\overline{R}(x) = \overline{MR}$, that is, where the average revenue equals the marginal revenue.
A) 0
B) 3
C) 10
D) 2
E) 6
Ans: C

97. If the total cost function for a product is $C(x) = 500 + 4x + 0.03x^2$ dollars, determine how many units x should be produced to minimize the average cost per unit?
A) 66 units
B) 500 units
C) 91 units
D) 129 units
E) 97 units
Ans: D

98. If the total cost function for a product is $C(x) = 500 + 5x + 0.02x^2$ dollars. Find the minimum average cost.
A) $11.50
B) $16.00
C) $11.70
D) $11.32
E) $20.00
Ans: D

99. For the cost function $C(x) = 200 + 4x + 0.09x^2$, show that average costs are minimized at the x-value where $\overline{C}(x) = \overline{MC}$ by finding this minimum.
A) $15.00
B) $23.00
C) $12.49
D) $13.03
E) $13.00
Ans: C

100. The graph shows a total cost function. Determine the level of production at which average cost is minimized.



A) 200 units
B) 800 units
C) 0 units
D) 400 units
E) 600 units
Ans: E

101. If the profit function for a commodity is $p = 4600x - 14x^2 - \frac{1}{3}x^3 - 1000$ dollars, determine the number of units x that must be sold to result in maximum profit.
A) 55 units
B) 83 units
C) 52 units
D) 89 units
E) 21 units
Ans: A

102. The profit function for a commodity is $p = 500x - 4x^2 - \frac{1}{3}x^3 - 1500$ dollars. Find the maximum profit. Round your answer to the nearest dollar.
A) $4270
B) $2647
C) $4407
D) $454
E) $8290
Ans: A

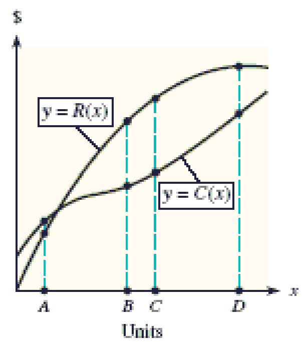
103. A product can be produced at a total cost $C(x) = 800 + 100x^2 + x^3$ dollars, where x is the number produced. If the total revenue is given by $R(x) = 5000x - 70x^2$ dollars, determine the level of production x that will maximize the profit.
A) 13 units
B) 35 units
C) 66 units
D) 70 units
E) 81 units
Ans: A

104. A product can be produced at a total cost $C(x) = 600 + 200x^2 + x^3$ dollars, where x is the number produced. If the total revenue is given by $R(x) = 3000x - 70x^2$ dollars, find the maximum profit.
A) $8374
B) $7525
C) $133
D) $7068
E) $8175
Ans: B

105. A firm can produce 100 units per week. If its total cost function is $C = 900 + 1900x$ dollars, and its total revenue function is $R = 2000x - x^2$ dollars, how many units x should it produce to maximize its profit?
A) 1950 units
B) 1000 units
C) 90 units
D) 50 units
E) 100 units
Ans: D

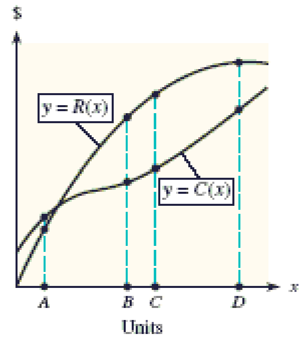
106. A firm can produce 100 units per week. If its total cost function is $C = 400 + 1000x$ dollars, and its total revenue function is $R = 1100x - x^2$ dollars, find the maximum profit.
A) $1744
B) $2100
C) $4677
D) $4179
E) $4380
Ans: B

107. The following figure shows the graph of revenue function $y = R(x)$ and cost function $y = C(x)$. At which of the four x-values shown is the profit largest?



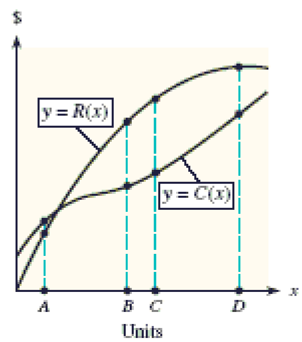
A) A
B) C
C) B
D) D
E) Profit is largest at more than one x-value.
Ans: B

108. The following figure shows the graph of revenue function $y = R(x)$ and cost function $y = C(x)$. At which of the four *x*-values shown is the slope of the tangent to the revenue curve equal to the slope of the tangent to the cost curve?



A) A
B) B
C) C
D) D
E) profit largest at more than one x-value
Ans: C

109. The following figure shows the graph of revenue function $y = R(x)$ and cost function $y = C(x)$. What is the relationship between marginal cost and marginal revenue when profit is at its maximum value?



A) $\overline{MR} \;<>\; \overline{MC}$
B) $\overline{MR} > \overline{MC}$
C) $\overline{MR} = \overline{MC}$
D) $\overline{MR} < \overline{MC}$
E) no relationship exists
Ans: C

110. A travel agency will plan a tour for groups of size 13 or larger. If the group contains exactly 13 people, the price is \$800 per person. However, each person's price is reduced by \$40 for each additional person above the 13. If the travel agency incurs a price of \$200 per person for the tour, what size group will give the agency the maximum profit?

A) 1
B) 15
C) 14
D) 23
E) 7
Ans: C

111. A small business has weekly average costs (in dollars) of $\overline{C} = \frac{180}{x} + 10 + \frac{x}{10}$, where x is the number of units produced each week. The competitive market price for this business's product is \$61 per unit. If production is limited to 465 units per week, find the level of production that yields maximum profit.

A) 233 units
B) 465 units
C) 285 units
D) 255 units
E) 270 units
Ans: D

112. A small business has weekly average costs (in dollars) of $\overline{C} = \frac{170}{x} + 10 + \frac{x}{20}$, where x is the number of units produced each week. The competitive market price for this business's product is \$41 per unit. If production is limited to 361 units per week, find the maximum profit.

A) \$4610
B) \$4504
C) \$4635
D) \$4590
E) \$4623
Ans: C

113. The monthly demand function for x units of a product sold by a monopoly is $p = 7000 - \frac{1}{2}x^2$ dollars per unit, and its average cost is $\overline{C} = 3660 + 3x$ dollars. If production is limited to 131 units, find the number of units that maximizes profit.

A) 45 units
B) 131 units
C) 61 units
D) 85 units
E) 34 units
Ans: A

114. The manufacturer of GRIPPER tires modeled its return to sales from television advertising expenditures in two regions, as follows:

Region 1: $S_1 = 40 + 30x_1 - 0.8x_1^2$

Region 2: $S_2 = 30 + 30x_2 - 0.4x_2^2$

where S_1 and S_2 are the sales revenue in millions of dollars, and x_1 and x_2 are millions of dollars of expenditures for television advertising. How much money will be needed to maximize sales revenue in both districts?

A) $18 million
B) $56 million
C) $37 million
D) $25 million
E) $28 million
Ans: B

115. A ball thrown into the air from a building 110 ft high travels along a path described by $y = \frac{-x^2}{140} + x + 110$ where *y* is the height of the ball in feet and *x* is the horizontal distance of the ball from the building in feet. What is the maximum height the ball will reach? Round your answer to one decimal place.

A) 145.0 feet
B) 180.0 feet
C) 195.0 feet
D) 167.5 feet
E) 150.0 feet
Ans: A

116. The profit from a grove of orange trees is given by $x(230 - x)$ dollars, where *x* is the number of orange trees per acre. How many trees per acre will maximize the profit?

A) 460
B) 120
C) 220
D) 115
E) 230
Ans: D

117. A time study showed that, on average, the productivity of a worker after *t* hours on the job can be modeled by $P = 10t + 8t^2 - t^3, \quad 0 \le t \le 8$, where *P* is the number of units produced per hour. After how many hours will productivity be maximized?

A) 5.90 hours
B) 8.00 hours
C) 5.95 hours
D) 6.03 hours
E) 2.66 hours
Ans: A

118. Suppose that the monthly cost in dollars of mining a certain ore is related to the number of pieces of equipment used, according to $C = 200x + \frac{9800}{x}, \quad x > 0$, where x is the number of pieces of equipment used. Using how many pieces of equipment will minimize the cost?
A) 2
B) 3
C) 8
D) 6
E) 7
Ans: E

119. An inferior product with a large advertising budget sells well when it is introduced, but sales fall as people discontinue use of the product. Suppose that the weekly sales S are given by $S = \frac{140t}{(t+4)^2}$, $t \geq 0$, where S is in millions of dollars and t is in weeks. After how many weeks will sales be maximized?
A) five weeks
B) four weeks
C) one week
D) two weeks
E) three weeks
Ans: B

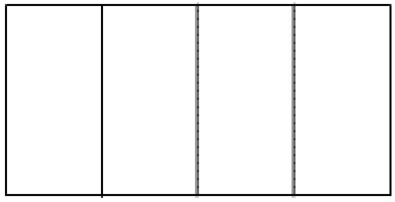
120. Suppose that in an election year, the proportion p of voters who recognize a certain candidate's name t months after the campaign started is given by $p(t) = \frac{6.5t}{t^2+4} + 0.4$. After how many months is the proportion maximized?
A) 5 months
B) 2 months
C) 8 months
D) 4 months
E) 3 months
Ans: B

121. The running yard for a dog kennel must contain at least 900 square feet. If a 15-foot side of the kennel is used as part of one side of a rectangular yard with 900 square feet, what dimensions will require the least amount of fencing?
A) 90 ft by 10 ft
B) 900 ft by 1 ft
C) 30 ft by 30 ft
D) 60 ft by 15 ft
E) 180 ft by 5 ft
Ans: C

122. From a tract of land, a developer plans to fence a rectangular region and then divide it into two identical rectangular lots by putting a fence down the middle. Suppose that the fence for the outside boundary costs $7 per foot and the fence for the middle costs $2 per foot. If each lot contains 2400 square feet, find the dimensions of each lot that yield the minimum cost for the fence.
   A) Dimensions are 42.96 ft for the side parallel to the divider and 55.87 ft for the other side.
   B) Dimensions are 55.87 ft for the side parallel to the divider and 42.96 ft for the other side.
   C) Dimensions are 48.99 ft for the side parallel to the divider and 48.99 ft for the other side.
   D) Dimensions are 45.83 ft for the side parallel to the divider and 52.37 ft for the other side.
   E) Dimensions are 52.37 ft for the side parallel to the divider and 45.83 ft for the other side.
   Ans: D

123. A rectangular area is to be enclosed and divided into thirds. The family has $1000 to spend for the fencing material. The outside fence costs $16 per running foot installed, and the dividers cost $20 per running foot installed. What are the dimensions that will maximize the area enclosed?
   A) Dimensions are 6.00 ft for the side parallel to the dividers and 11.22 ft for the other side.
   B) Dimensions are 11.22 ft for the side parallel to the dividers and 6.00 ft for the other side.
   C) Dimensions are 7.10 ft for the side parallel to the dividers and 10.73 ft for the other side.
   D) Dimensions are 6.94 ft for the side parallel to the dividers and 15.63 ft for the other side.
   E) Dimensions are 15.63 ft for the side parallel to the dividers and 6.94 ft for the other side.
   Ans: D

124. A kennel of 864 square feet is to be constructed as shown. The cost is $15 per running foot for the sides and $4 per running foot for the ends and dividers. What are the dimensions of the kennel that will minimize the cost?



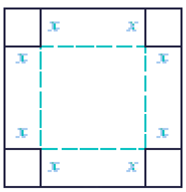
A) The dimensions of the kennel that will minimize the cost are 24 feet for ends and dividers and 36 feet for sides.
B) The dimensions of the kennel that will minimize the cost are 25 feet for ends and dividers and 31 feet for sides.
C) The dimensions of the kennel that will minimize the cost are 31 feet for ends and dividers and 25 feet for sides.
D) The dimensions of the kennel that will minimize the cost are 25 feet for ends and dividers and 36 feet for sides.
E) The dimensions of the kennel that will minimize the cost are 36 feet for ends and dividers and 24 feet for sides.
Ans: E

125. The base of a rectangular box is to be twice as long as it is wide. The volume of the box is 450 cubic inches. The material for the top costs $0.17 per square inch and the material for the sides and bottom costs $0.10 per square inch. Find the dimensions that will make the cost a minimum.
A) Dimensions are 9” by 10” by 12” (high).
B) Dimensions are 13” by 26” by 12” (high).
C) Dimensions are 5” by 10” by 9” (high).
D) Dimensions are 9” by 18” by 5” (high).
E) Dimensions are 12” by 24” by 13” (high).
Ans: C

126. Suppose that a company needs 200 items during a year and that preparation for each production run costs $50. Suppose further that it costs $7 to produce each item and $0.90 to store an item for one year. Use the inventory cost model to find the number of items in each production run that will minimize the total costs of production and storage.
A) 136 items
B) 149 items
C) 178 items
D) 167 items
E) 156 items
Ans: B

127. A company needs 600 items per year. Production costs are $\$90$ to prepare for a production run and $\$9$ for each item produced. Inventory costs are $\$0.60$ per item per year. Find the number of items that should be produced in each run so that the total costs of production and storage are minimized.
A) 424 items
B) 474 items
C) 463 items
D) 452 items
E) 441 items
Ans: A

128. A rectangular box with a square base is to be formed from a square piece of metal with 18-inch sides. If a square piece with side *x* is cut from each corner of the metal and the sides are folded up to form an open box, the volume of the box is $V = (18-2x)^2 x$. What value of *x* will maximize the volume of the box?



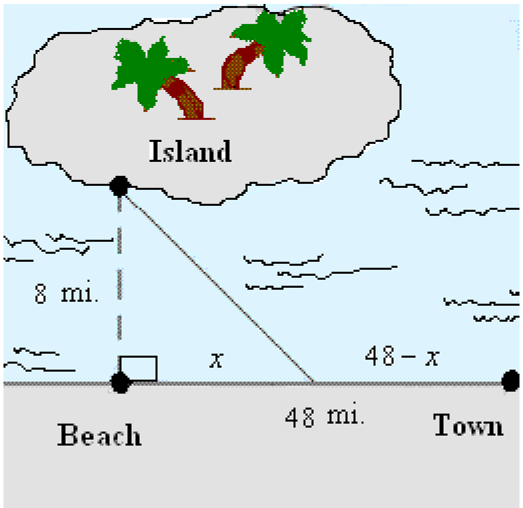
A) 9
B) 7
C) 3
D) 2
E) 4
Ans: C

129. The owner of an orange grove must decide when to pick one variety of oranges. She can sell them for $\$14$ a bushel if she sells them now, with each tree yielding an average of 3 bushels. The yield increases by half a bushel per week for the next 10 weeks, but the price per bushel decreases by $\$0.70$ per bushel each week. When should the oranges be picked for maximum return?
A) 9 weeks from now
B) 5 weeks from now
C) 10 weeks from now
D) 8 weeks from now
E) 7 weeks from now
Ans: E

130. A box with an open top and a square base is to be constructed to contain 13,500 cubic inches. Find the dimensions that will require the minimum amount of material to construct the box.
   A) The dimensions of the box using minimum material are 30 in. by 30 in. by 15 in.
   B) The dimensions of the box using minimum material are 32 in. by 32 in. by 14 in.
   C) The dimensions of the box using minimum material are 15 in. by 15 in. by 30 in.
   D) The dimensions of the box using minimum material are 14 in. by 14 in. by 32 in.
   E) The dimensions of the box using minimum material are 32 in. by 32 in. by 15 in.
   Ans: A

131. A printer has a contract to print 110,000 posters for a political candidate. He can run the posters by using any number of plates from 1 to 30 on his press. If he uses x metal plates, they will produce x copies of the poster with each impression of the press. The metal plates cost $4.00 to prepare, and it costs $6.50 per hour to run the press. If the press can make 1000 impressions per hour, how many metal plates should the printer make to minimize costs? Round your answer to the nearest count of metal plates.
   A) 16
   B) 9
   C) 8
   D) 13
   E) 12
   Ans: D

132. A vacationer on an island 8 miles offshore from a point that is 48 miles from a town to which the vacationer must travel occasionally. (See the figure.) The vacationer has a boat capable of traveling 30 mph and can go by auto along the coast at 55 mph. At what distance *x* from the point on shore closest to the vacationer on the island should the boat be left to minimize the time it takes to get to town? Round your answer to one decimal place.

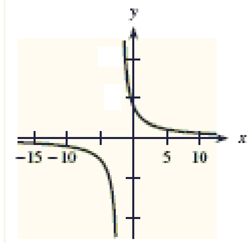


A) 6.1 miles
B) 5.2 miles
C) 5.9 miles
D) 3.8 miles
E) 4.3 miles
Ans: B

133. The millions of worldwide cellular subscribers can be modeled by $C(t) = -0.865t^3 + 28.1t^2 + 2.21t + 49.6$ where *t* is the number of years past 1994. In what year does the model predict the number of worldwide cellular subscribers will reach a maximum?
A) year 2017
B) year 2014
C) year 2018
D) year 2015
E) year 2016
Ans: E

134. A function and its graph are given. Use the graph to find the vertical asymptotes, if they exist. Confirm your results analytically.

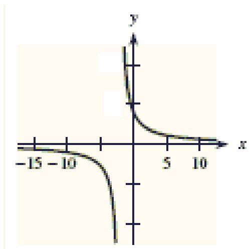
$f(x) = \dfrac{48}{x+2}$



A) $x = 6$
B) $x = 2$
C) $x = 4$
D) $x = -2$
E) no vertical asymptotes
Ans: D

135. A function and its graph are given. Use the graph to find $\lim_{x\to\infty} f(x)$, if it exists. Confirm your results analytically.

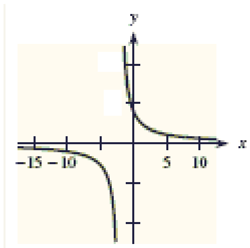
$f(x) = \dfrac{72}{x+2}$



A) 0
B) 2
C) 4
D) –2
E) does not exist
Ans: A

136. A function and its graph are given. Use the graph to find $\lim_{x \to -\infty} f(x)$, if it exists. Confirm your results analytically.

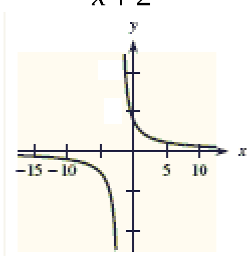
$$f(x) = \frac{56}{x+2}$$



A) 8
B) 2
C) 6
D) 0
E) does not exist
Ans: D

137. A function and its graph are given. Use the graph to find the horizontal asymptotes, if they exist. Confirm your results analytically.

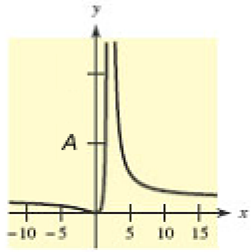
$$f(x) = \frac{16}{x+2}$$



A) $y = 4$
B) $y = 2$
C) $y = 0$
D) $y = 8$
E) no horizontal asymptotes
Ans: C

138. A function and its graph are given. Use the graph to find the vertical asymptotes, if they exist, where $A = 66$. Confirm your results analytically.

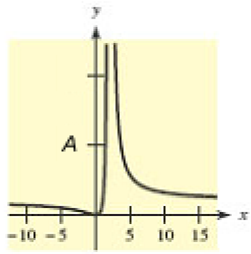
$$f(x) = \frac{22x^2}{(x-2)^2}$$



A) $x = 5$
B) $x = 2$
C) $x = 1$
D) $x = 0$
E) no vertical asymptotes
Ans: B

139. A function and its graph are given. Use the graph to find $\lim_{x\to\infty} f(x)$, if it exists, where $A = 45$. Confirm your results analytically.

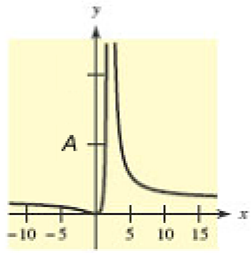
$$f(x) = \frac{15x^2}{(x-2)^2}$$



A) 1
B) 15
C) 2
D) 5
E) does not exist
Ans: B

140. A function and its graph are given. Use the graph to find $\lim_{x \to -\infty} f(x)$, if it exists, where $A = 75$. Confirm your results analytically.

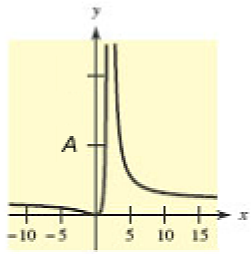
$$f(x) = \frac{25x^2}{(x-2)^2}$$



A) 1
B) 0
C) 25
D) 2
E) does not exist
Ans: C

141. A function and its graph are given. Use the graph to find the horizontal asymptotes, if they exist, where $A = 75$. Confirm your results analytically.

$$f(x) = \frac{25x^2}{(x-2)^2}$$



A) $y = 0$
B) $y = 25$
C) $y = 2$
D) $y = 5$
E) no horizontal asymptotes
Ans: B

142. Find any horizontal asymptotes for the given function.

$$y = \frac{5x-5}{x+9}$$

A) $y = 1$
B) $y = 9$
C) $y = 0$
D) $y = 5$
E) no horizontal asymptotes
Ans: D

143. Find any vertical asymptotes for the given function.

$$y = \frac{3x-3}{x+6}$$

A) $x = 1$
B) $x = 6$
C) $x = -6$
D) $x = 0$
E) no vertical asymptotes
Ans: C

144. Find any horizontal asymptotes for the given function.

$$y = \frac{12x}{4-x^2}$$

A) $y = 3$
B) $y = 12$
C) $y = 4$
D) $y = 0$
E) no horizontal asymptotes
Ans: D

145. Find all vertical asymptotes for the given function.

$$y = \frac{12x}{9-x^2}$$

A) $x = 0$
B) $x = \pm 3$
C) $x = 3$
D) $x = -3$
E) no vertical asymptotes
Ans: B

146. Find any horizontal asymptotes for the given function.

$$y = \frac{6x^3}{3x^2+4}$$

A) $y = 2$
B) $y = 6$
C) $y = 4$
D) $y = 0$
E) no horizontal asymptotes
Ans: E

147. Find any vertical asymptotes for the given function.

$$y = \frac{8x^3}{2x^2 + 2}$$

A) $x = 4$
B) $x = 8$
C) $x = 2$
D) $x = 0$
E) no vertical asymptotes
Ans: E

148. Find any horizontal asymptotes for the given function.

$$y = \frac{4x - 8}{x + 1}$$

A) $y = 2$
B) $y = 0$
C) $y = 1$
D) $y = 4$
E) no horizontal asymptotes
Ans: D

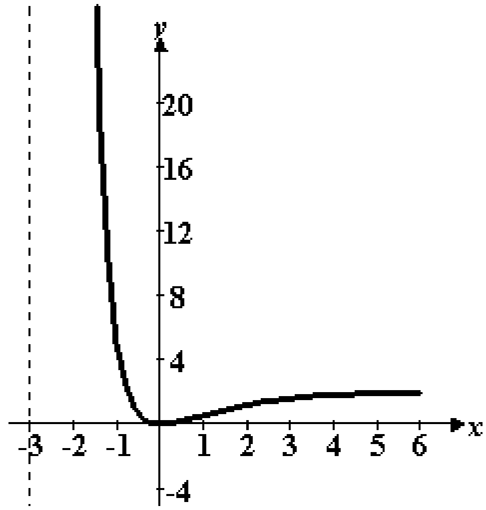
149. Find any vertical asymptotes for the given function.

$$y = \frac{9x - 9}{x + 8}$$

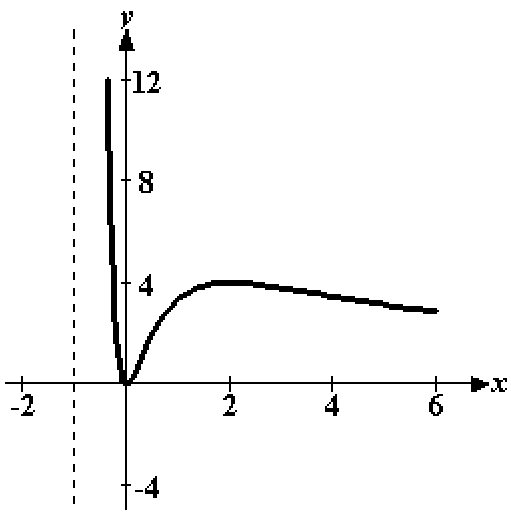
A) $x = -8$
B) $x = 0$
C) $x = 8$
D) $x = 1$
E) no vertical asymptotes
Ans: A

150. Sketch the graph of the function $f(x) = \dfrac{39x^2}{(x+3)^3}$.

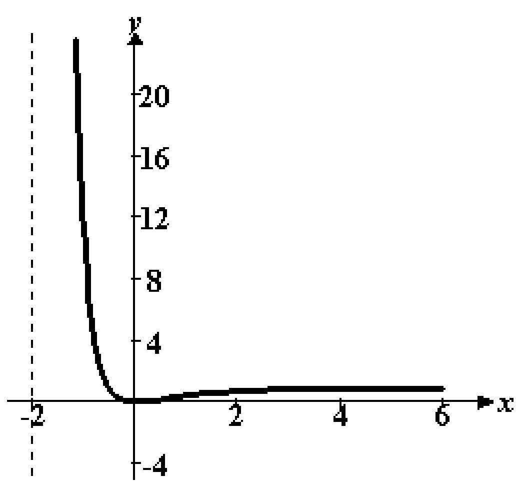
A)



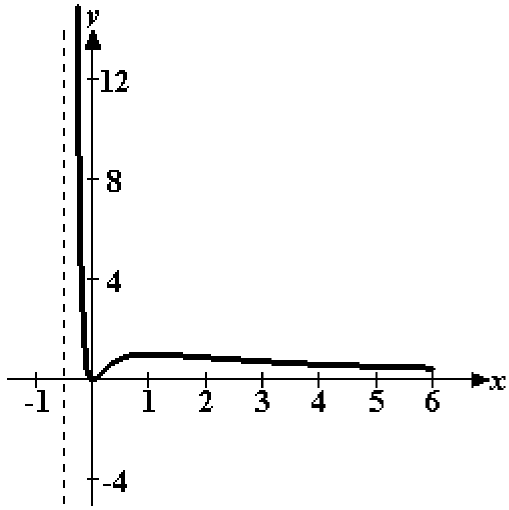
B)



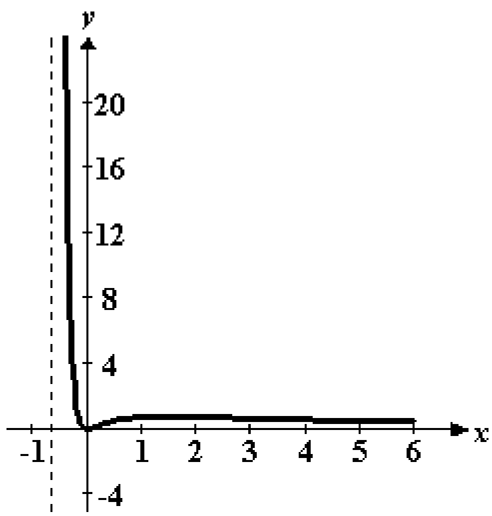
C)



D)



E)



Ans: A

151. Find any vertical asymptotes for the given function.

$f(x) = \left(\frac{x+8}{x-2}\right)^2$

A) $x = 4$
B) $x = 0$
C) $x = 2$
D) $x = 8$
E) no vertical asymptotes
Ans: C

152. Find any horizontal asymptotes for the given function.

$$f(x)=\left(\frac{x+8}{x-2}\right)^2$$

A) $y=4$
B) $y=0$
C) $y=8$
D) $y=1$
E) no horizontal asymptotes
Ans: D

153. A function and its first and second derivatives are given. Use these to find any horizontal asymptotes.

$$y=\frac{(x-6)^2}{x^2}$$
$$y'=\frac{12(x-6)}{x^3}$$
$$y''=\frac{216-24x}{x^4}$$

A) $y=6$
B) $y=0$
C) $y=1$
D) $y=9$
E) no horizontal asymptotes
Ans: C

154. A function and its first and second derivatives are given. Use these to find any vertical asymptotes.

$$y=\frac{(x-6)^2}{x^2}$$
$$y'=\frac{12(x-6)}{x^3}$$
$$y''=\frac{216-24x}{x^4}$$

A) $x=6$
B) $x=1$
C) $x=9$
D) $x=0$
E) no vertical asymptotes
Ans: D

155. A function and its first and second derivatives are given. Use these to find all critical values.

$$y = \frac{(x-2)^2}{x^2}$$

$$y' = \frac{4(x-2)}{x^3}$$

$$y'' = \frac{24-8x}{x^4}$$

A) $x = 1$
B) $x = 0$
C) $x = 2$
D) $x = 3$
E) no critical values
Ans: C

156. A function and its first and second derivatives are given. Use these to find any relative maxima.

$$y = \frac{(x-4)^2}{x^2}$$

$$y' = \frac{8(x-4)}{x^3}$$

$$y'' = \frac{96-16x}{x^4}$$

A) $(4,0)$
B) $(6,0)$
C) $(9,0)$
D) $(1,9)$
E) no relative maxima
Ans: E

157. A function and its first and second derivatives are given. Use these to find any relative minima.

$$y = \frac{(x-6)^2}{x^2}$$

$$y' = \frac{12(x-6)}{x^3}$$

$$y'' = \frac{216-24x}{x^4}$$

A) $(7, 0)$

B) $(8, 0)$

C) $(4, 0)$

D) $(6, 0)$

E) no relative minima

Ans: D

158. A function and its first and second derivatives are given. Use these to find any points of inflection.

$$y = \frac{(x-2)^2}{x^2}$$

$$y' = \frac{4(x-2)}{x^3}$$

$$y'' = \frac{24-8x}{x^4}$$

A) $(3, 0.11)$

B) $(8, 0.56)$

C) $(4, 0.25)$

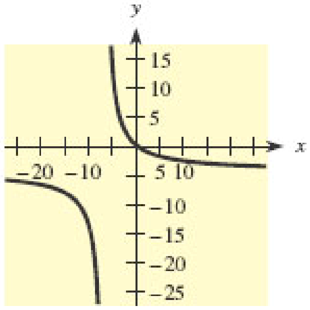
D) $(7, 0.51)$

E) no points of inflection

Ans: A

159. A function and its graph are given. Use the graph to estimate the locations of any horizontal asymptotes.

$f(x)=\frac{5-15x}{3x+21}$



A) $y=-5$

B) $y=5$

C) $y=13$

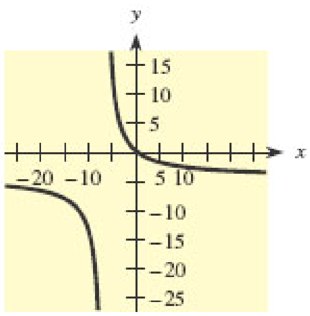
D) $y=-20$

E) no horizontal asymptotes

Ans: A

160. A function and its graph are given. Use the graph to estimate the locations of any vertical asymptotes.

$f(x)=\frac{5-15x}{3x+21}$



A) $x=-7$

B) $x=20$

C) $x=5$

D) $x=0$

E) no vertical asymptotes

Ans: A

161. Analytically determine the location of any vertical asymptotes.

$$f(x)=\frac{x-30}{x^2+1300}$$

A) $x = 5.477226$
B) $x = 0.023077$
C) $x = 36.055513$
D) $x = -36.055513$
E) no vertical asymptotes
Ans: E

162. Analytically determine the location(s) of any horizontal asymptote(s).

$$f(x)=\frac{x-60}{x^2+1800}$$

A) $y = 0$
B) $y = 0.033333$
C) $y = 42.426407$
D) $y = -42.426407$
E) no horizontal asymptotes
Ans: A

163. Analytically determine any relative maxima.

$$f(x)=\frac{x-20}{x^2+100}$$

A) $(-2.36, -0.21)$
B) $(42.36, 0.01)$
C) $(20, 0)$
D) $(0, -0.20)$
E) no relative maxima
Ans: B

164. Analytically determine any relative minima.

$$f(x)=\frac{x-30}{x^2+10}$$

A) $(-0.17, -3.01)$
B) $(60.17, 0.01)$
C) $(30, 0)$
D) $(0, -3.00)$
E) no relative minima
Ans: A

165. Analytically determine the location(s) of any vertical asymptote(s).

$f(x)=\dfrac{400x+3000}{x^2-44x-2495}$

A) $x=76.58$
B) $x=-32.58$
C) $x=76.58$, $x=-32.58$
D) $x=0$
E) no vertical asymptotes
Ans: C

166. Analytically determine the location(s) of any horizontal asymptote(s).

$f(x)=\dfrac{700x+4000}{x^2-38x-2190}$

A) $y=69.51$
B) $y=-31.51$
C) $y=69.51$, $y=-31.51$
D) $y=0$
E) no horizontal asymptotes
Ans: D

167. The percent p of impurities that can be removed from the waste water of a manufacturing process at a cost of C dollars is given by $p=\dfrac{100C}{5400+C}$. Find any C values within the domain of the problem at which the rate of change of p with respect to C does not exist.

A) $C=-5400$
B) $C=0$
C) $C=54$
D) $C=5400$
E) All C values within domain are valid.
Ans: E

168. The percent p of impurities that can be removed from the waste water of a manufacturing process at a cost of C dollars is given by $p=\dfrac{100C}{5600+C}$. Find C values for which p is increasing.

A) $C<0$
B) $C>0$
C) $C>56$
D) $C<5600$
E) $C>5600$
Ans: B

169. The percent p of impurities that can be removed from the waste water of a manufacturing process at a cost of C dollars is given by $p = \dfrac{100C}{4000 + C}$. Find any horizontal asymptotes.

A) $p = -4000$
B) $p = 0$
C) $p = 100$
D) $p = 4000$
E) no horizontal asymptotes
Ans: C

170. The percent p of impurities that can be removed from the waste water of a manufacturing process at a cost of C dollars is given by $p = \dfrac{100C}{4933 + C}$. Can 100% of the pollution be removed?

A) yes
B) no
Ans: B

171. Assume that the total daily cost, in dollars, of producing plastic rafts for swimming pools is given by $C(x) = 338 + 5x + 0.02x^2$ where x is the number of rafts produced per day, then the average cost per raft produced id given by $\overline{C}(x) = C(x)/x$, for $x > 0$. Find the level of production that minimizes average cost.
A) 260 rafts per day
B) 130 rafts per day
C) 125 rafts per day
D) 8450 rafts per day
E) 65 rafts per day
Ans: B

172. An entrepreneur starts new companies and sells them when their growth is maximized. Suppose that the annual profit for a new company is given by $P(x) = 27 - x - \dfrac{36}{x + 2}$, where P is in thousands of dollars and x is the number of years after the company is formed. If she wants to sell the company before profits begin to decline, after how many years should she sell it?

A) sell the company after 3 years
B) sell the company after 4 years
C) sell the company after 5 years
D) sell the company after 6 years
E) sell the company after 7 years
Ans: B

