0.5 Operations with Algebraic Expressions.

- Variable: Letter used to represent real numbers

- Algebraic expressions: additions, subtractions, multiplications, divisions or roots with one or more real numbers or variables (Letters)

Example: ① $3x + 6y^2$ 2 terms

② $\sqrt{x} + 7$ note that $x \geq 0$ 2 terms

③ $\dfrac{x^2 y - x}{x+1}$ note that $x \neq -1$ 4 terms

Term: is any product of a real number (coefficient معامل) and one or more variables to powers.

Polynomial: is the sum of a finite number of ~~terms~~ terms with nonnegative integer powers on the Variables

Example: ① $x^2 - 3y + \frac{1}{2}x + x^2 y^5$ is a polynomial

② $x^2 + y^3 + x^{-1}$ not a polynomial

③ $y^4 - 2\sqrt{x} = y^4 - 2x^{1/2}$ not a polynomial

If a polynomial contains only one variable x, then it is a polynomial in x.

Example: ① $x^3 - x + 6$ a polynomial in x

② $3x^7 - x^2 + x - 1$ a polynomial in x

③ $x^2 + y^3 + 1$ a polynomial of 2 variable x and y

In general, the form of polynomial in x is

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0$$

The degree of a polynomial: is the degree of the term with the highest degree.

Leading coefficient: the coefficient of the term with the highest degree.

Example: ① $x^3 - x + 6$ the degree is 3

the general form: $a_3 x^3 + a_2 x^2 + a_1 x + a_0 x^0$

: $\underline{\underline{1}} \cdot x^3 + 0 \cdot x^2 - 1 \cdot x + 6 \cdot x^0$

Leading coefficient

② $3x^7 - x^2 + x - 1$: the degree is 7

the general form: $a_7 x^7 + a_6 x^6 + a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x^1 + a_0 x^0$

$= 3x^7 + 0 \cdot x^6 + 0 \cdot x^5 + 0 \cdot x^4 + 0 \cdot x^3 - x^2 + x - 1$

- polynomial with one term is called monomial

$3x^2$, xy^3 , $4y^2zx$

- polynomial with two terms is called binomial

x^5+x^2 , $2xy+3$, $-x^3-y^2$

- polynomial with three terms is called trinomial

$2x^5+x^3-2$, $xy-y^2+x$

* Operations with Algebraic Expressions.

Example: compute :

a) $(4xy+3x)+(5xy-2x) = 4xy+3x+5xy-2x$

$= 9xy + x$

b) $(3x^2+4xy+5y^2+1)-(6x^2-2xy+4)$

$= 3x^2+4xy+5y^2+1-6x^2+2xy-4$

$= -3x^2+6xy+5y^2-3$

Example: Perform the indicated operations.

a) $(8xy^3)(2x^3y)(-3xy^2) = 8.2.(-3).x.x^3.x.y^3.y.y^2$

$= -48x^5y^6$

b) $-15x^2y^3 \div (3xy^5) = \dfrac{-15x^2y^3}{3xy^5} = -5x^1y^{-2}$

$= -5\dfrac{x}{y^2}$

Example: Find the following products.

a) $-4ab(3a^2b + 4ab^2 - 1)$

$= -4ab \cdot 3a^2b + (-4ab).(4ab^2) + (-4ab)(-1)$

$= -12a^3b^2 - 16a^2b^3 + 4ab$

b) $(4a + 5b + c)ac$

$= 4a.ac + 5b.ac + c.ac$

$= 4a^2c + 5abc + ac^2$

Example: Multiply the following

a) $(x-4)(x+3)$

$= x.x + x.3 + (-4).x + (-4).3$

$= x^2 + 3x - 4x - 12 \quad = x^2 - x - 12$

b) $(3x+2)(2x+5)$

$= 3x.2x + 3x.5 + 2.2x + 2.5$

$= 6x^2 + 15x + 4x + 10 = 6x^2 + 19x + 10$

Special products:-

* $(x+a)^2 = x^2 + 2ax + a^2$ binomial squared
* $(x-a)^2 = x^2 - 2ax + a^2$ binomial squared
* $(x+a)(x-a) = x^2 - a^2$ difference of 2 squared.

* $(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$ binomial cubed

* $(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$ binomial cubed

Example: Multiply the following:

a) $(x+5)^2 = x^2 + 2.5.x + (5)^2$

$= x^2 + 10x + 25$

b) $(3x^2 - 4y)^2 = (3x^2)^2 - 2.(3x^2)(4y) + (4y)^2$

$= 9x^4 - 24x^2y + 16y^2$

c) $(x+4)^3 = x^3 + 3.4.x^2 + 3.(4)^2.x + (4)^3$

$= x^3 + 12x^2 + 48x + 64$

Example: Divide $(4x^3 - 12x - 22)$ by $(x-3)$, $x \neq 3$

$= \dfrac{4x^3 - 12x - 22}{x - 3}$

الناتج → $4x^2 + 12x + 24$

المقسوم عليه → $x-3 \overline{)\, 4x^3 + 0x^2 - 12x - 22}$

$\mp 4x^3 \pm 12x^2$

$12x^2 - 12x - 22$

$\mp 12x^2 \pm 36x$

$24x - 22$

$\mp 24x \pm 72$

الباقي → 50

$= 4x^2 + 12x + 24 + \dfrac{50}{x-3}$