4.1 Experiments and Counting Rules

* **Experiment** is an process that generates well-define outcomes

- **Sample Space (S)** for an experiment is the set of all experimental outcomes

Example: What is the sample space for the following experiments

[a] Toss a coin $S = \{ \text{Head}, \text{Tail} \} = \{H, T\}$

[b] Roll a die $S = \{1, 2, 3, 4, 5, 6\}$

[c] Play a game $S = \{\text{win}, \text{lose}, \text{tie}\}$

* Experimental outcomes are also called **sample points**

Counting Rules

[1] Multiple-step experiment [2] Combinations [3] Permutations

[1] Multiple step experiment:

If an experiment has K steps with

n_1 possible outcomes for step 1,

n_2 " " " step 2,

⋮

n_K " " " step K

then the total number of experimental outcomes is $n_1 n_2 \cdots n_K$

Example: Consider the experiment of tossing two coins. How many experimental outcomes are possible for this expermen

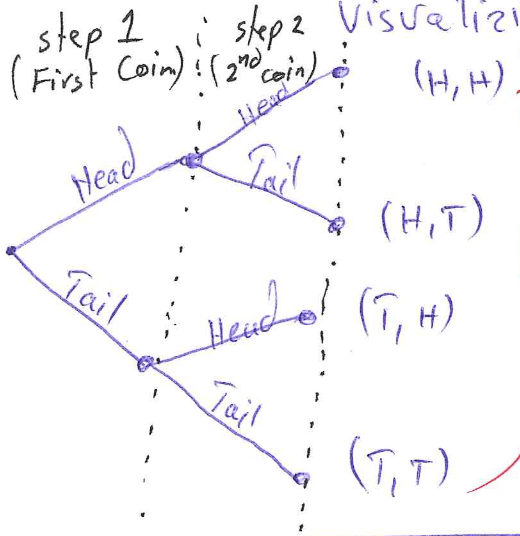
Tossing one coin $(n_1 = 2)$ and tossing the other coin $(n_2 = 2)$

The number of experimental outcomes $= (n_1)(n_2) = (2)(2) = 4$.

$S = \{(H,H), (H,T), (T,H), (T,T)\}$

If we toss 5 coins, then the number of experimental outcomes is $(2)(2)(2)(2)(2) = 32$.

* **Tree diagram:** A graphical representation that helps in visualizing a multiple-step experiment.



Experimental outcom = Sample point

$\boxed{2}$ Combinations (when the experiment involves slecting n objectis from a set of N objectis, $N \geq n$)

(The order is not important)

$$C^N_n = \binom{N}{n} = \frac{N!}{n!\,(N-n)!} \quad \text{where}$$

we use combination to find the number of different samples of size n that can be slected.

$N! = N(N-1)(N-2)\cdots(2)(1).$

$n! = n(n-1)(n-2)\cdots(2)(1)$

$0! = 1$

$4! = 4\times3\times2\times1 = 24$ * see an example on the back?!

$5! = 5\times4\times3\times2\times1 = 120$

Example (Q2 page 150): How many ways can three items be slected from a group of six items? Use the letters A, B, C, D, E, F to identify them, and list each of the different combinations of the three item.

$C^6_3 =$

$$\binom{6}{3} = \frac{6!}{3!\,(6-3)!} = \frac{6!}{3!\,3!}$$

$$= \frac{6\times5\times4\times\cancel{3\times2\times1}}{3\times2\times1\times\cancel{3\times2\times1}} = 20$$

ABC	ACD	ADF	BCF	CDE
ABD	ACE	AEF	BDE	CDF
ABE	ACF	BCD	BDF	CEF
ABF	ADE	BCE	BEF	DEF

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Example: How many ways 3 digit numbers can be formed from the digits 2, 3, 5, 6, 7, 9 which are even without repeating the digits

$$(2)(5)(4) = 40$$

Example: A hotel surveyed 100 guests with the following data:

	satisfied	unsatisfied
Female	42	2
Male	40	16

If two guests are randomly selected, what is the prob. that both are unsatisfied?

$$\frac{18}{100} \times \frac{17}{99} = 0.03 \quad \underline{\text{or}} \quad \frac{\binom{18}{2}}{\binom{100}{2}} = 0.03$$

__Example__ In how many ways can the letters of the word (Formula) be rearranged.

$$P^{7}_{7} = \frac{7!}{(7-7)!} = \frac{7!}{0!} = 7! = 5040$$

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3 Permutations : (when the experiment involves selecting n objects from a set of N objects $N \geq n$, where the order of selection is important.)

$$P_n^N = \frac{N!}{(N-n)!}$$

Example (Q3 page 150) How many permutations of three items can be selected from a group of six?

$$P_3^6 = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 120$$

ABC, ACB, BAC, BCA, CAB, CBA are different outcomes here

~~see the back for one more example:-~~

Assigning Probabilities

- Basic requirements for assigning probabilities:
  1. Each experimental outcome E_i must have
     $$0 \leq P(E_i) \leq 1$$
  2. Considering all experimental outcomes, we must have
     $$P(E_1) + P(E_2) + \cdots + P(E_n) = 1$$

- Three methods for assigining probabilites:

  1. Classical method: when all the experimental outcomes are equally likely. $\frac{1}{n}$

Example ① Toss a fair coin $\Rightarrow S = \{H, T\}$ $n=2$

$P(H) = P(T) = \frac{1}{2}$ "equally likely" $\frac{1}{2}$

② Roll a die $\Rightarrow S = \{1, 2, 3, 4, 5, 6\}$ $n=6$

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$ $\frac{1}{6}$

2 Relative frequency method (when data are available to estimate the proportion of the time the experimental outcome will occure if the experiment is repeated a large number of items.

Example:

Number of patients wating	outcome Number of days
0	2
1	5
2	6
3	4
4	3
	$\boxed{20 \text{ days}}$

number of sample points

2 days were 0 patients wating
5 days wrer one " "
6 days were 2 " "
4 days were 3
3 days were 4 " "

Using the relative frequency method:

the probability of 0 patients were wating $= \frac{2}{20}$

" " " 1 " " " $= \frac{5}{20}$

" " " 2 " " " $= \frac{6}{20}$

" " " 3 " " " $= \frac{4}{20}$

" " " 4 " " " $= \frac{3}{20}$

3 Subjective Method (when we can not assume that the experimental outcomes are equally likely)

- This method express the person's degree of belief (scal 0-1)

Example: Suppose that student A and students B gave an excuse to there teach about their absance.

E_1: the excuse is accepted   student A $\Rightarrow P(E_1) = 0.8$
$\Rightarrow P(E_2) = 0.2$

E_2: the excuse is rejected

student B $\Rightarrow P(E_1) = 0.6$ $P(E_2) = 0.4$