تلخيص فيزياء 141/ مادة First

physics (1)

Mechanics
- → Kinematics (description of motion)
  - * chapters 1, 2, 3, 4
- → Dynamics (cause of motion)
  - * chapters 5, 6, 7, 8, 9, 10, 11, 15

Chapter one
Measurements

(متجهة) vectors — scalar (قياسية)

physical Quantities

Derived Quantities

* coulomb → Amp.s
* speed → m/s
* Force → N → Kg.m/s^2

Basic Quantities

* mass → Kg
* time → S
* length → m
* amount of slabstance → mol
* light Intensity → candela
* current → Amp
* tempreture → K

physics
(D)

$\boxed{\text{ex.1}}$ $X(m) = at^2 + bt + c$ t in sec

$a = \frac{m}{s^2}$ $(at^2 = m \rightarrow a = \frac{m}{t^2})$

$b = \frac{m}{s}$ $(bt = m \rightarrow b = \frac{m}{t})$

$c = m$ $(c = m)$

$\boxed{\text{ex.2}}$ 24 km/h = ? m/s

$24\ km/h = 24 \frac{km}{h} = 24 \frac{(1000)\ m}{(3600)\ s}$

$= 6.7\ m/s$

$\boxed{\text{ex.3}}$ $F = RX$ F = N

X = m

$\therefore R = N/m$

$\boxed{\text{Q.3}}$ ⓐ 1 km = ? micron

* μm = micron = $10^{-6}\ m$

$\therefore$ 1 km = 1000 m

$1\ km = 1 \times 10^3 \times \left(\frac{1}{10^{-6}}\right) \mu m$

$\therefore$ $1\ km = 10^9\ \mu m$

ⓑ $1cm = ?\ \mu m$

$1cm = 10^{-2}\ m$

$1cm = 10^{-2}\left(\frac{1}{10^{-6}}\right)\ \mu m$

$\therefore 1cm = 10^{-2} \times 10^{6}$

$1cm = 10^{4}\ \mu m$

Q 7

$\text{Water Volume} = (26\ Km^2)\ .\ (2in) = ?\ \text{acre-foot}$

area

length

* $1ft = 12\ in$
* $1\ acre = 43560\ ft^2$
* $1m^2 = 10.76\ ft^2$

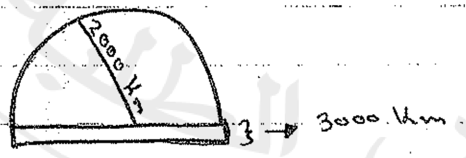
$\therefore \text{Water Volume} = (26 \times 10^{6})\ m^2 \cdot \left(\frac{2}{12}\right) ft$

$= (26 \times 10^{6} \times 10.76)\ ft^2 \cdot \left(\frac{1}{6}\right) ft$

$= \left(26 \times 10^{6} \times 10.76 \times \frac{1}{43560}\right) acre \cdot \frac{1}{6}\ ft$

$= 1070\ acre.ft$

Q. 9



Volume = area × thickness

$= \left(\frac{\pi r^2}{2}\right) \times (D)$

$\therefore = \left(\frac{3.14}{2}\right) \cdot (2 \times 10^{8}) \times (3 \times 10^{5})$

$= 1.8 \times 10^{20}\ cm^3$

* $1m = 10^{2}\ cm$
* $Km = 10^{3}\ m$
* $r = 2000\ Km$
* $r = 2000 \times 10^{3} m$
* $r = 2000 \times 10^{3} \times 10^{2} cm$
* $r = 2 \times 10^{8}\ cm$
* $d = 3000\ Km$
* $d = 3000 \times 10^{2} cm$
* $d = 3 \times 105\ cm$

Physics
(1)

$\boxed{ex.1}$ $36\ m/s = ?\ Km/h$

$* \ Km = 10^3 m \rightarrow \frac{Km}{10^3 m} = 1$ ، $1 = \frac{10^3 m}{Km}$

$* \ 1h = 3600s \rightarrow 1 = \frac{3600s}{1h}$ ، $\frac{1h}{3600s} = 1$

$\therefore 36 m/s = 36m \div s$

$= 36m(1) \div 1s(1)$

$= 36\cancel{m}\left(\frac{Km}{10^3\cancel{m}}\right) \div 1\cancel{s}\left(\frac{1h}{3600\cancel{s}}\right)$

$= 36\left(\frac{Km}{10^3}\right) \div \frac{1h}{3600}$

$= 129.6\ Km/h$

$\boxed{ex.2}$ $1gm/cm^3 = ?\ Kg/m^3$

$* \ 1Kg = 10^3 g \longrightarrow \frac{1Kg}{10^3 g} = 1$ ، $1 = \frac{10^3 g}{Kg}$

$* \ 1m = 10^3 cm \Longrightarrow 1m^3 = 10^6 cm^3 \longrightarrow \frac{1m^3}{10^6 cm^3} = 1$ ، $1 = \frac{10^6 cm^3}{1m^3}$

$\therefore 1gm/cm^3 = 1g \div 1\ cm^3$

$= 1\cancel{g}\left(\frac{1Kg}{10^3\cancel{g}}\right) \div \cancel{cm^3}\left(\frac{1m^3}{10^6\cancel{cm^3}}\right)$

$= 10^{-3} Kg \div 10^{-6} m^3$

$= 10^3 Kg/m^3$

Units for constants in any formula :-

ex.1 $\underset{N}{F} = \underset{m}{[illegible]} \Longrightarrow K = \frac{N}{m}$

ex.2 $[illegible] \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \epsilon_0 = ?$

* $q \rightarrow$ الشحنة

$\therefore \epsilon_0 = \frac{q_1 q_2}{4\pi r^2 F} \longrightarrow C^2/N.m^2$

* $r = m$
* $q = C$
* $F = N$

ex.3 $x(t) = A\cos(wt + Kx)$ ، $x = m$, $t = s$

$w, K, A = ?$

* $A = m$ (لا اتجاه، قيمة موجبة فقط)

لأنها زوايا تقاس بالراد ← * $wt = 1$ (rad)

$w = \frac{rad}{t(s)} \longrightarrow w = rad/s$

* $Kx = rad$

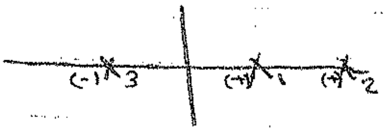
$K = \frac{rad}{x(m)} \longrightarrow K = rad/m$

Chapter two
Motion Along A straight line

* Motion :- relate change in position and time.

* Position :- → reference Point
→ direction

* Position :- X (+) or (−)



- initial Position (x_1)
- final position (x_2)
- Displacement = change in Position
- Displacement = ΔX

* Displacement = $^{+}x_2 - ^{+}x_1$ (+) $\longrightarrow$

* if the Final position X_3

$\therefore$ Displacement = ΔX

$= ^{-}X_3 - ^{+}X_1$ (−) $\longleftarrow$

* average Velocity (متوسط متجه السرعة) = $\frac{\text{Displacement}}{\text{time}}$

(vector) $V_{avg} = \frac{\text{distance}}{\Delta \text{time}}$

(نادرًا ما نستعملها)

* note :-

(~~[illegible]~~ sealer) average speed = $\frac{\text{distance}}{\text{time}}$

$S_{avg} = \frac{\text{distance}}{t}$

example :- the distance between Jerusalem and Birziet = 30 km

* the time " " " " = 2h

* $S_{avg} = 30/2 = 15$ km/h

* Displacement (J-B) = 18 km

* $V_{avg} = 18/2 = 9$ km/h

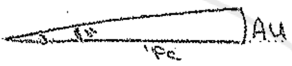
physics (D)

$\boxed{Q.53}$ * AU = Astronomical unit $= (d_{E-S}) = 92.9 \times 10^6$ mi

$\downarrow$

المسافه بين الارض والشمس

$\therefore$ AU $= 92.9 \times 10^6$ mi (mi = mile)

* Parsec (pc) $\longrightarrow$ 

* 1" $\longrightarrow$ زاويه تقاس بالثانيه

* $1^\circ = 60'$

* $1' = 60''$

* 1' $\longrightarrow$ دقيقه

* 1" $\longrightarrow$ ثانيه

* $1^\circ = 3600''$

$\therefore$ يجب ايجاد علاقه بين ال AU وال PC

* $\theta\ \text{rad} = \dfrac{\text{arc (طول القوس)}}{\text{radius (نق)}}$

$\therefore$ نحول ال (1") للراديان

* $1'' = \dfrac{1^\circ}{3600}$

* $1'' = \dfrac{1^\circ}{3600} \times \dfrac{\pi}{180}\ \text{rad} \longrightarrow 4.85 \times 10^{-6}\ \text{rad}$

$\therefore 4.85 \times 10^{-6}\ \text{rad} = \dfrac{AU}{pc} \longrightarrow AU = 4.85 \times 10^{-6}\ pc$

* light year (ly) = the distance travelled by light in 1 year.

$\therefore$ 1y = C.t

(المسافه/السنه الضوئيه) = سرعة الضوء × الزمن

$= \left(186000 \dfrac{mi}{\cancel{s}}\right) \cdot (365 \times 24 \times 60 \times 60\ \cancel{s})$

$\therefore 1y = 5.86 \times 10^{12}$ mi

المطلوب من السؤال هو المسافة بين الارض والشمس بال (pc) وال (ly) :

ⓐ d_{E-S} in pc ?

$1\,AU = d_{E-S}$ (من السؤال)

$\therefore 1\,AU = 4.85 \times 10^{-6}$ pc

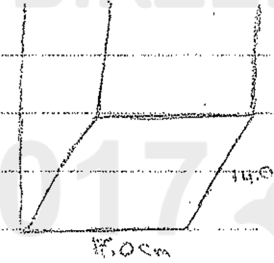
ⓑ d_{E-S} in ly ?

$= 92.9 \times 10^{6}$ mi

$= 92.9 \times 10^{6} \left(\frac{1\,ly}{5.86 \times 10^{12}}\right)$

$= 0.15 \times 10^{-4}$ ly

Q. 31



$m_{candy} = 0.0200$ gm

$V_{candy} = 50.0\ mm^3$

$\rho_{candy} = \frac{m}{V}$ (mass density)

$= \frac{0.02 \times 10^{-3}\ Kg}{50 \times 10^{-3}\ cm^3}$

$= 4 \times 10^{-4}\ cm^3$ (كثافة حجمية)

(معدل زيادة الارتفاع) $\frac{dH}{dt} = 0.250$ cm/s

- 1Kg = 1000 g
- 1 cm = 10 mm
- $1 cm^3 = 10^3 mm^3$

المطلوب ايجاد في الكتلة بالنسبة للزمن (Kg/m) ← $\frac{dm}{dt}$ *

* $M = \rho_c A h$ $\left(\rho_c = \frac{M}{V}\right)$ (الكتلة = مساحة القاعده x الارتفاع)

* $A = 17 \times 14\ cm^2$

* $\frac{dm}{dt} = \rho_c \cdot A \cdot \frac{dH}{dt}$

$= \left(\frac{4 \times 10^{-4}\ Kg}{\cancel{cm^3}}\right) \cdot (17\ \cancel{cm} \times 14\ \cancel{cm}) \cdot \left(\frac{0.250\ \cancel{cm}}{s}\right)$

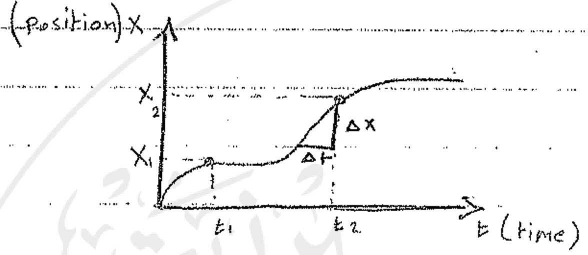
$= 238 \times 10^{-2}$ kg/s $\Rightarrow \frac{238 \times 10^{-2}\ kg}{\frac{m}{60}} \longrightarrow 1.428$ kg/m

physics
(1)

* $V_{\text{intanteous}} \longrightarrow V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

$$\left.\begin{aligned} & V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \\ & V = \frac{dx}{dt} \end{aligned}\right\}$$ instanteous Velocity

eg. 1



* لإيجاد السرعة اللحظية من الرسم ← نرسم مماس ونأخذ ميله .

* هذا الرسم البياني لجسم يسير في خط مستقيم

$\therefore V = \frac{dx}{dt}$

= the $\overset{\text{ميل}}{\text{slope}}$ of the $\overset{\text{المماس}}{\text{tangent}}$ (x-t) at a certain point.

* $a_{avg} = \frac{\Delta V}{\Delta t}$ (average acceleration) (معدل التسارع) (vector)

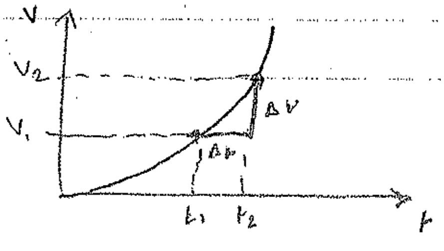
$\therefore a_{avg} = \frac{m/s}{s} = m/s/s = m/s^2$

* $a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$

$= \frac{dV}{dt}$ (instanteous acceleration) (التسارع اللحظي)

= the slope of the tangent (V-t) at a certain point.

eg. 2



$\boxed{\begin{matrix}ex.1\\ Q.5\end{matrix}}$ $x = \overset{m/s}{\overset{\uparrow}{3}}t - \overset{m/s^2}{\overset{\uparrow}{4}}t^2 + \overset{m/s^3}{\overset{\uparrow}{1}}t^3$ $x = m$

$t = sec$

ⓐ $x(1) = 3(1) - 4(1)^2 + 1(1)^3$

$= 0\,m$

ⓑ $x(2) = 3(2) - 4(2)^2 + 1(2)^3$

$= -2\,m$

ⓒ $x(3) = 3(3) - 4(3)^2 + 1(3)^3$

$= 0\,m$

ⓓ $x(4) = 3(4) - 4(4)^2 + (4)^3$

$= 12\,m$

ⓔ $\Delta x = x(4) - x(0)$

$= 12 - 0$

$= 12\,m$

ⓕ V_{avg} between $(t_1 = 2s \ , t_2 = 4s)$

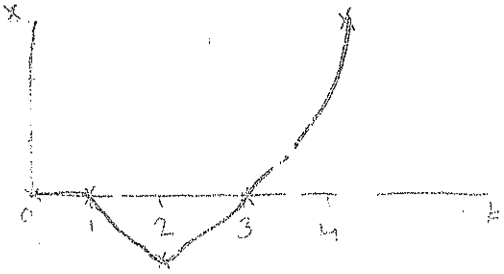
$= \frac{\Delta x}{\Delta t}$

$= \frac{x(4) - x(2)}{4 - 2}$

$= \frac{12 - -2}{2}$

$= 7\,m/s$

ⓖ



ⓗ (addition) إضافة من الأستاذ

$v(2)$? , $v(4)$ (السرعة اللحظية عند 2,4)

(السرعه مشتقة المسافة) $v = \frac{dx}{dt} = 3 - 8t + 3t^2$ (اشتقاق القانون)

$v(2) = 3 - (8 \times 2) + 3(2)^2 \longrightarrow -1 \text{ m/s}$

$v(4) = 3 - (8 \times 4) + 3(4)^2 \longrightarrow 19 \text{ m/s}$

ⓘ Find a_{avg} from $(t_1 = 2s , t_2 = 4s)$

$a_{avg} = \frac{\Delta v}{\Delta t}$

$= \frac{v(4) - v(2)}{4 - 2} \longrightarrow \frac{19 - -1}{2} = 10 \text{ m/s}^2$

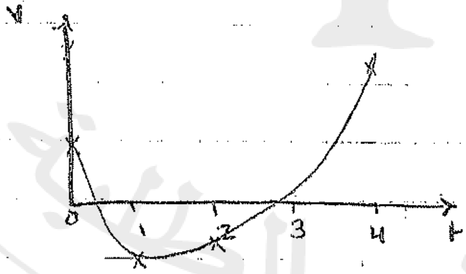
ⓙ Find a at $(t = 2s , t = 4s)$

$a = \frac{dv}{dt} = -8 + 6t$ (التسارع مشتقة السرعة)

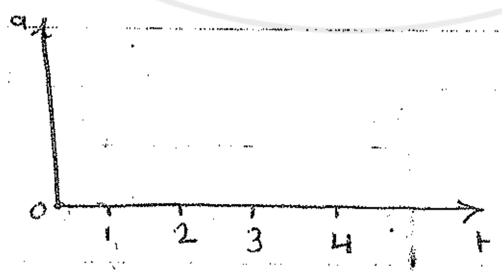
$a(2) = -8 + 12 = 4 \text{ m/s}^2$

$a(4) = -8 + 24 = 16 \text{ m/s}^2$

ⓚ



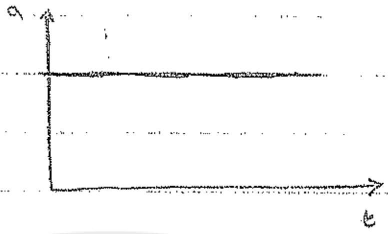
ⓛ

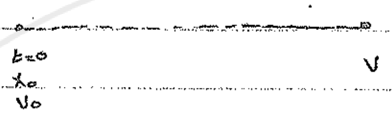


تسارع منتظم

Motion with constant acceleration :

$\vec{a}$ is constant





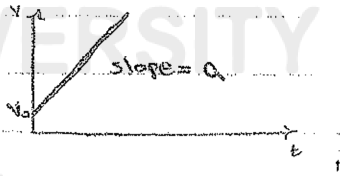
[illegible]

① $a = \frac{\Delta V}{\Delta t}$

$a = \frac{V_2 - V_1}{t_2 - t_1} \longrightarrow a = \frac{V - V_0}{t - 0}$ ([illegible])

③

$\therefore V - V_0 = at \longrightarrow \boxed{V = V_0 + at}$



② $V_{avg} = \frac{\Delta X}{\Delta t}$

①

$V_{avg} = \frac{X - X_0}{t} \longrightarrow \boxed{X - X_0 = V_{avg}\,t} \longleftarrow$ (لكل أنواع الحركة)

②

also $\boxed{V_{avg} = \frac{V_0 + V}{2}}$ only for costant a!

$\therefore$ نعوض من معادلة بمعادلة

⑤

يمكن استخدامها بالحل $\boxed{X - X_0 = \left(\frac{V_0 + V}{2}\right) \times t}$ ② in ①

$\therefore$ نعوض من بدل المعادلة ③ بدل V

$X - X_0 = \left(\frac{V_0 + V_0 + at}{2}\right) \times t$

II

$$\boxed{X - X_0 = V_0 t + \frac{1}{2} a t^2}$$

$\therefore$ التخلص بدل (t) من المعادلة ③ بـ②

III

$X - X_0 = \left(\frac{V_0 + V}{2}\right)\left(\frac{V - V_0}{a}\right) \longrightarrow \boxed{V^2 = V_0^2 + 2a(X - X_0)}$

$\boxed{X - X_0 = Vt - \frac{1}{2} a t^2} \longrightarrow$ ③ من ⑤ في V_0 [illegible]

Physics

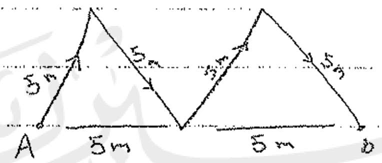
(D)

1.2

$$\vec{V}_{avg} = \frac{displacement}{\Delta t}$$

$$S_{avg} = \frac{distance}{\Delta t}$$

example →

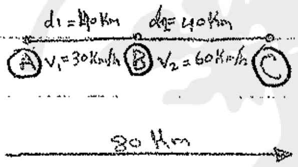


$t = 30s$

(السرعة) $S_{avg} = \frac{20m}{30s} = \frac{2}{3}\ m/s$

(الإزاحة) $\vec{V}_{avg} = \frac{10m}{30s} = \frac{1}{3}\ m/s$

Q.3



* $V = \frac{displacement}{t} \rightarrow t_1 = \frac{40Km}{30Km/h} \rightarrow \frac{4}{3}\ h$

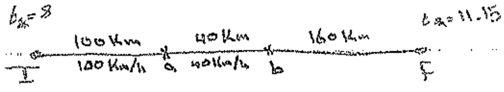
* $t_2 = \frac{40Km}{60Km/h} \rightarrow \frac{2}{3}\ h$

* $t_{a-c} = \frac{4}{3}h + \frac{2}{3}h = 2h$

* $V_{avg} = \frac{80Km}{2h} \rightarrow 40\ Km/h$

* $S_{avg} = 40\ Km/h$ (السرعة = الإزاحة)(لأنه حركة مستقيمة)

Q.11



$d_{I-F} = 300 Km$

$t_1(I-A) = \frac{100 Km}{100 Km/h} \longrightarrow 1 h$

$t_2(A-b) = \frac{40 Km}{40 Km/h} \longrightarrow 1h$

$t_3(b-F) = 3.25 - 2 = 1.25 h$

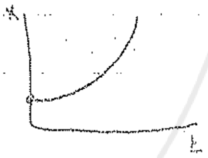
$\therefore d_{(B-F)} = 160 Km$

$V_{B-F} = \frac{160 Km}{1.25 h} = 128 Km/h$

مطلوب بالذات { $V_{avg} = S_{avg}$ for the total trip.

$= V_{avg_{I-F}} = \frac{300 \ Km}{1+1+1.25 h} \longrightarrow 92.3 Km/h$

Q.17 $X = 9.75 + 1.50t^3$, X = cm , t = sec



ⓐ $V_{avg} = \frac{\Delta X}{\Delta t}$ (مثالزيه هي ميل المماس)

$= \frac{X(3) - X(2)}{3-2} \longrightarrow 28.5 cm/s$

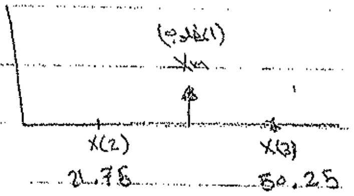
ⓑ $V = \frac{dx}{dt} \longrightarrow 4.5t^2$

$V(2) = 18 cm/s$

ⓒ $V(3) = 40.5 cm/s$

ⓓ $V(2.5) = 28.125 cm/s$

ⓒ V_{inst} at midd way between $t=2s \longrightarrow t=3s$



$$X_{mid} = \frac{X(3)-X(2)}{2} + X(2) = \frac{X(3)-X(2)}{2} \longrightarrow 36\,cm$$

$X = 9.75 + 1.5t^3$

$\therefore 36 = 9.75 + 1.5t^3$

$t_{mid} = 2.6\,s$

$\therefore \quad V = 4.2t^2$

$V_{mid} = 4.2(2.6)^2$

$= 30.4\,cm/s$

$\boxed{Q.22}$ $X = ct^2 - bt^3$ ——— $X = m$, $t = s$

ⓐ $X = ct^2 \longrightarrow c = m/s^2$

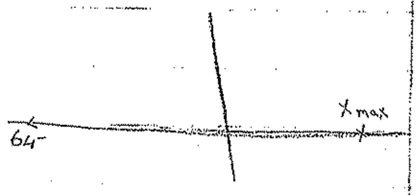
ⓑ $bt^3 = X \longrightarrow b = m/s^3$

ⓒ عند أقصى مسافة , السرعة = صفر

$V = \frac{dx}{dt} \longrightarrow 8t - 6t^2$ (عوضنا بدل b,c)

$V = 0 = 8t - 6t^2 \longrightarrow t = 0, \frac{4}{3}$

$X(0) = 0$ / $X\left(\frac{4}{3}\right) = max = 2.37\,cm$



ⓒ + ⓓ from $t=0$ to $t=4s$

$X(0) = 0$ / $X(4) = -64\,m$

* displacement $= X(4) - X(0) \longrightarrow -64 - 0 = -64\,m$

* distance $= 2.37 + 2.37 + 64 = 68.74\,m$ (الإزاحة الموجبة
[illegible])

physics

(L)

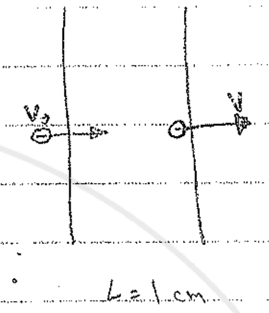
1.2

e.x 1

Q23

$U_0 = 1.5 \times 10^5$ m/s

$V = 5.7 \times 10^6$ m/s



* من المعطيات نختار الاقتران الحركي

$$V^2 = V_0^2 + 2a(X - X_0)$$

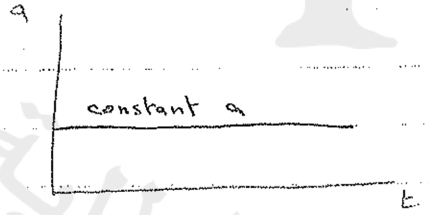
* $L = X - X_0 = 1 \times 10^{-2}$ m

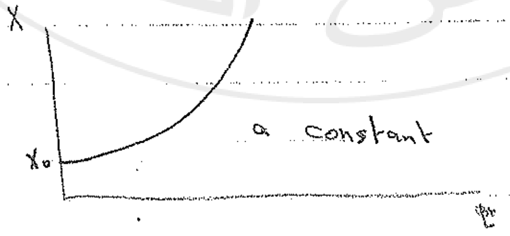
$\therefore (5.7 \times 10^6)^2 = (1.5 \times 10^5) + 2a(1 \times 10^{-2})$

ونوجد (a) (التسارع)

* The best example for moving with constant $\vec{a}$ is: Free Fall:-

a is always downward and equal to 9.8 m/s^2.



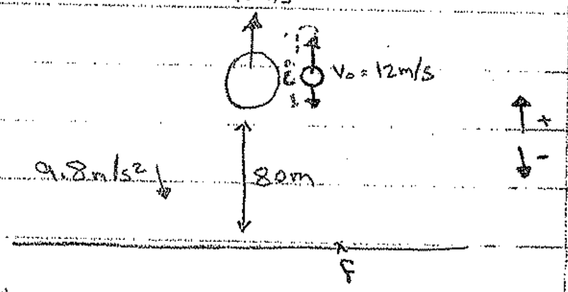


ex. 2
Q. 49

ascend → يصعد

ascends with constant $\vec{V}$

$V = 12\,m/s$



سرعة المقذوف من البالون الابتدائية = سرعة البالون

ⓐ $t(i \to f)$

$$\therefore\; y - y_0 = V_0 t + \frac{1}{2} a t^2$$

$$-80 = +12t + \frac{1}{2} \times -9.8 \times t^2$$

نأخذ الزمن من (i ← f) [illegible] من أعلى مسافة إلى الأرض لأنها إزاحة وليست مسافة (زمن الرحلة كاملة)

ⓑ V on the ground ?

من قانون

$$V = V_0 + at$$

$$V = +12 + -9.8 \times t \quad (\text{نعوض من فرع ⓐ})$$

Graphical integration in motion :-

* $V = \frac{dx}{dt}$ (السرعة اللحظية)

$dx = V \cdot dt$ أخذ تكامل الطرفين

$$\int_{x_0}^{x} dx = \int_{t_1}^{t_2} v \cdot dx \longrightarrow x - x_0 = \int_{t_1}^{t_2} v \cdot dt$$

= Area under the curve of (v-t)

* $a = \frac{dx}{dt}$ (التسارع اللحظي)

$$\int_{v_0}^{v} dv = \int_{t_1}^{t_2} a\, dt \longrightarrow v - v_0 = \int_{t_1}^{t_2} a\, dt$$

= Area under the curve of (a-t)

ex.3
Q.81

$a = 5t$ $\qquad a = m/s^2$

at $t = 2s \rightarrow V = 17\ m/s$ $\qquad t = s$

Find $V = ?$ at $t = 4s$

$a = \frac{dv}{dt} \longrightarrow \int dv = \int a \cdot dt$

$V = \int (5t) \cdot dt \longrightarrow V = \frac{5t^2}{2} + C$

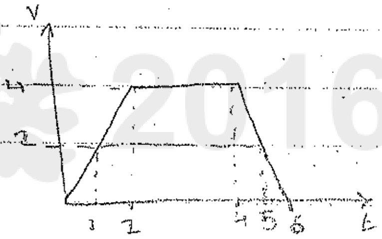
نلاقي قيمة الـ(C):

$17 = \frac{5}{2}(2)^2 + C \longrightarrow C = 7$

$\therefore V = \frac{5t^2}{2} + 7$

$\therefore V(4) = \frac{5}{2}(4)^2 + 7 \longrightarrow 47\ m/s$

ex.4 important
Q.90



ⓐ coordinat at $t = 5s$

X = area (المساحة تحت المنحنى)

(اول ;X1) $X - X_0 = \frac{1}{2} \times 2 \times 4 + 2 \times 4$

ⓑ V at $t = 5s$

$V = 2\ m/s$

ⓒ a at $t = 5s$

$a = $ slope $\longrightarrow \frac{dx}{dt} \longrightarrow \frac{-4}{2}\ m/s^2$ (اعتمادا على المجاور)

ⓓ V avg between (1-5)s

نأخذ المساحة من (1-5) ثم نقسمها على (4) الزمن

$V_{avg} = \frac{area}{4}$

ⓔ a avg (1-5)s

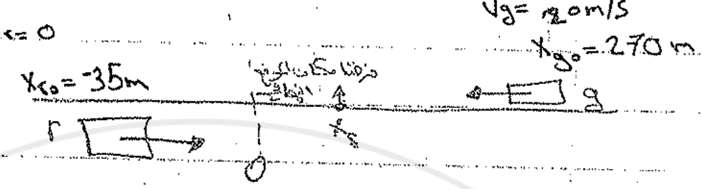
$\frac{\Delta X}{\Delta t} \rightarrow \frac{2-2}{4} = $ zero

physics

(D)

Chapter (2)

Q.35



$a_r = ?$

* for ~~goods~~ car :-

السرعة ثابتة
والتسارع = صفر

$\Delta X = v_x . t$

$(X_f - X_i) = v_x . t$

$(X_f - 270) = -20 (12)$

$X_f = 30 m$

* for red car :-

التسارع ثابت
والسرعة = صفر

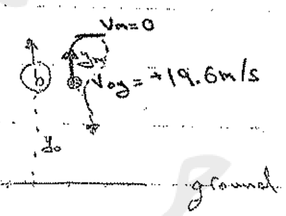
$(X_f - X_i) = v_{r0} . t + \frac{1}{2} a_x t^2$

$(30 = 35) = 0 + \frac{1}{2} a . (12)^2$

$\therefore a = 0.9 m/s^2$

Q. 51

$v_b = 19.6 m/s$



ⓐ المطلوب أقصى ارتفاع (y_m)

$\therefore t_{(o-m)} = 2s$ الزمن من البداية لأقصى ارتفاع

$t_{(o-g)} = 6s$ الزمن من البداية للأرض

$a_y = -9.8 m/s^2$ (للأسفل)

لأن نقطة البداية مشتركة للبالون والهدية عند أقصى ارتفاع

في المرة الأولى ونستغل المعطيات

$\Delta y = V_{oy} \cdot t + \frac{1}{2} g t^2$

$\therefore y_m = (+19.6 \times 2) + (\frac{1}{2} \times -9.8 \times (2)^2)$

$\therefore y_m = 19.6\ m$

* أو يمكن حلها عن طريق مساحة المثلث (تحت المنحنى)

ⓑ من نقطة البداية عند سطح البالون والنهاية عند الأرض

$\Delta y = V_{oy} t + \frac{1}{2} a_y t^2$

$y_o = (+19.6 \times 6) + (\frac{1}{2} \times -9.8 \times (6)^2)$

$\therefore y_o = -58.8\ m$

ⓒ additional

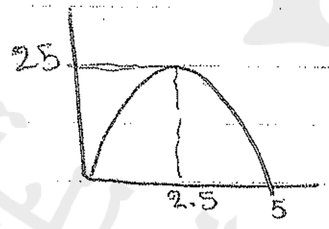
find the $\overset{\text{الاصطدام}}{\text{empact}}$ velocity with the ground

$V_g = V_{oy} + a_y t$

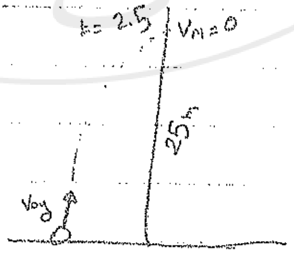
$V_g = +19.6 + (-9.8 \times 6)$

$V_g = -39\ m/s$

Q.64



$a_y, V_{oy} = ?$



⑨ $\therefore \Delta y = \left(\frac{V_{oy} + V_y}{2}\right) \cdot t$ (نستخدمها عندما يكون التسارع ثابت)

$+25 = \left(\frac{V_{oy} + 0}{2}\right) \cdot 2.5$

$\therefore V_{oy} = 20 \text{ m/s}$

$\therefore$ لإيجاد التسارع

$V_y = V_{oy} + a_y t$

$0 = +20 + a_y(2.5)$

$a_y = -8 \text{ m/s}^2$

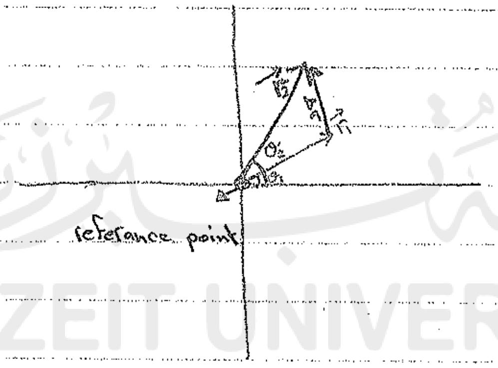
Physics
(L)
1.3 (vectors)

physical quantities
- → scalar (determined by magnitude only)
- → vector (determined by magnitude and direction)
  - eg. (position, displacement, velocity, acceleration, force)

* position :-

$\vec{r_1}$

{ magnitude + direction }



∴ $\vec{r_1}$: first position

$\vec{r_2}$: second position

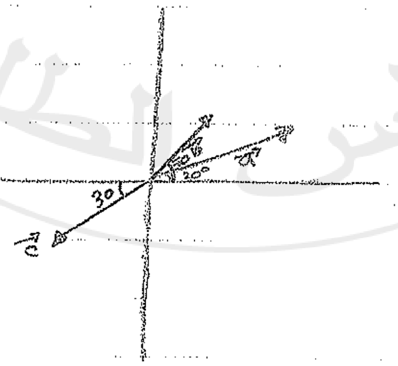
$\vec{\Delta r}$: displacement = change in position (تعتمد على نقطتي البداية والنهاية فقط)

∴ $\vec{\Delta r} = \vec{r_2} - \vec{r_1}$

Adding vectors Geometrically :- (جمع المتجهات هندسياً)

$\vec{r_2} = \vec{\Delta r} + \vec{r_1}$

example :

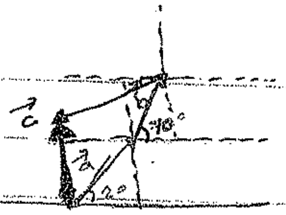


∴ $a = |\vec{a}| = 5, 20°$ with +X

$b = |\vec{b}| = 3, 70°$ with +X

$c = |\vec{c}| = 4, 30°$ with −X

find $\vec{a} + \vec{b} + \vec{c}$



وبالنهاية نصل ما بين ذيل الأول

ورأس الأخير

في حال تم الرسم باستخدام المنقلة والمسطرة = $|\vec{a} + \vec{b} + \vec{c}|$ black

نقيس طول المتجه الناتج من المسطرة

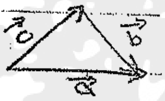
والزاوية من المنقلة

Example :-

$\therefore \vec{a} + \vec{b} + \vec{c} = 0$

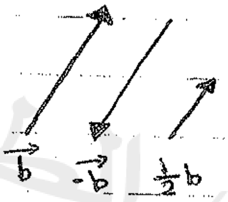
$\vec{a} + \vec{b} = -\vec{c}$

Example :-

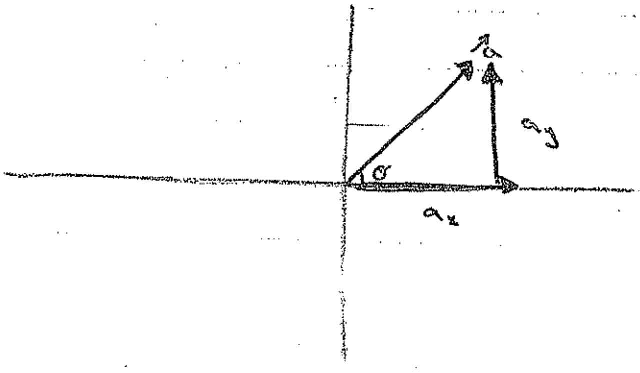


$\therefore \vec{c} + \vec{b} = \vec{a}$

الماشي عكس الباقي هو المحصلة



Component of vectors : (مركبات المتجهات)



∴ a and θ are given :

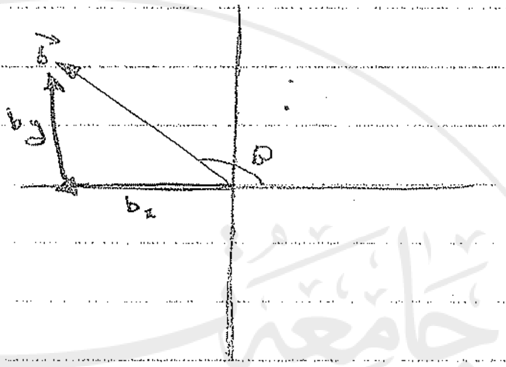
∴ $a_x = a . \cos\theta$

$a_y = a . \sin\theta$

another example :

∴ $b_x = b . \cos B$ (تطلع سالب)

$b_y = b . \sin B$ (″ ″)



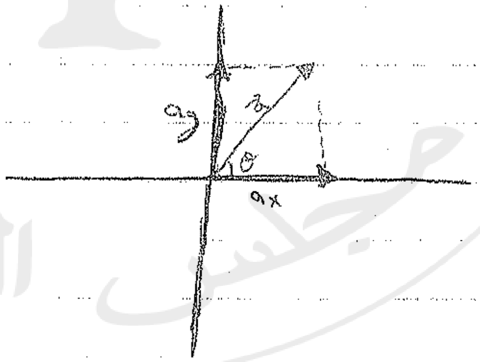
∴ angle : with +x , counter clock wise .

note :

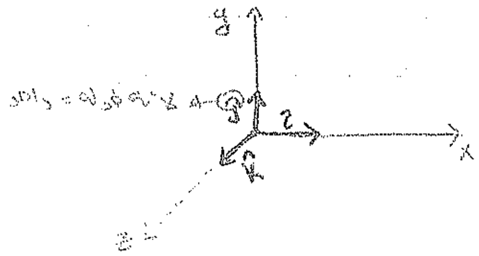
If you are given $\vec{a}_x + \vec{a}_y$, find a and θ ?

∴ $\vec{a} = \sqrt{a_x^2 + a_y^2}$

$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$



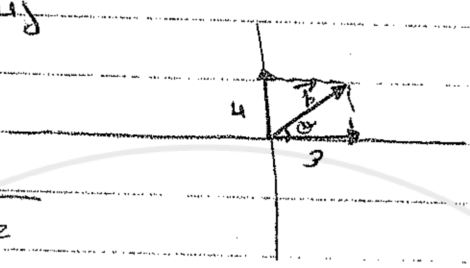
Adding vectors by component :-



∴ $\vec{a} = a_x\hat{i} + a_y\hat{j}$

Example

$\vec{b} = 3\hat{i} + 4\hat{j}$



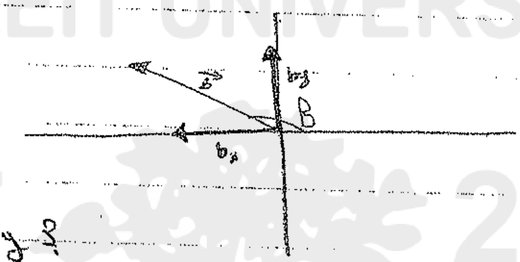
∴ ① $\vec{b} = \sqrt{3^2 + 4^2}$

$\vec{b} = 5$

② $\tan\theta = \frac{4}{3} = 1.3333$

∴ $\theta = 53°$

Example



∴ Find b_x , b_y ?

∴ $b_x = b \cdot \cos B$

$= 15 \cdot \cos 120$

$= -7.5$

$b_y = 15 \sin 120$

$= 13$

∴ $\vec{b} = -7.5\hat{i} + 13\hat{j}$

Physics

(D)

$\boxed{1.2}$

$\boxed{Q.70}$ particle 1, $x = 6t^2 + 3t + 2$ $x = m \,/\, t = s$

particle 2, $a = -8t$, at $t = 0$, $v = 15\ m/s$

when $v_{p.1} = v_{p.2}$?

for p.1 * $v_{p.1} = \frac{dx}{dt} = 12t + 3$

for p.2 * $a = \frac{dv}{dt}$

$\therefore \int dv = \int a\, dt$

$\therefore v_{p.2} = -4t^2 + C$

∴ لإيجاد قيمة C نعوض

$15 = -4 \times 0 + C \implies C = 15$

$\therefore v_{p.2} = -4t^2 + 15$

$\therefore v_{p.1} = v_{p.2}$

$12t + 3 = -4t^2 + 15$

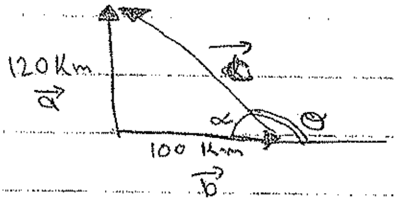
$4t^2 + 12t - 12 = 0$

$t^2 + 3t - 3 = 0 \longrightarrow t = 0.8\,s$

نعوض عن t ، و منها نعوض $v_{p.1}$ أو $v_{p.2}$ فيتساووا .

$\boxed{1.3}$

$\boxed{Q.5}$



$\vec b + \vec c = \vec a$

$\vec c = \vec a - \vec b$

$|C| = \sqrt{(a)^2 + (b)^2}$

$= \sqrt{(120)^2 + (100)^2}$

$= 156\ Km$

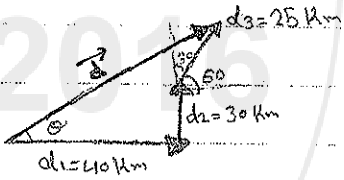
* $\tan \alpha = \frac{100}{120} \Longrightarrow \alpha = 40°$ / $\tan \theta = \frac{120}{100} \Longrightarrow \theta = 50°$

∴ $d_1 = 156$ km , 40° west of north.

in vector notation { * $\vec c = \vec a - \vec b$

$\vec c = 120\hat j - 100\hat i$

$\boxed{Q.12}$



$\vec d = \vec{d_1} + \vec{d_2} + \vec{d_3}$

$\vec d = 40\hat i + 30\hat j + (25 \cos 60\ \hat i + 25 \sin 60\ \hat j)$

$\vec d = 40\hat i + 30\hat j + 12.5\,\hat i + 21.65\,\hat j$

$\vec d = 52.5\,\hat i + 51.65\,\hat j$

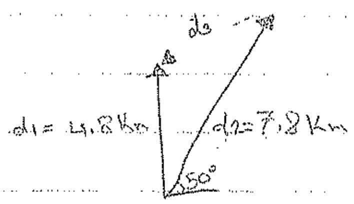
∴ $\vec d = \sqrt{(52.5)^2 + (51.65)^2}$

$= 73.6$ Km

* $\tan \theta = \frac{y}{x} = \frac{51.65}{52.5} \Longrightarrow \theta = 44.5°$

∴ $\vec d = 73.6$ Km in 44.5° North of east.

Q. 20



$\vec{d_2} + \vec{d_3} = \vec{d_1}$

$\vec{d_3} = \vec{d_1} - \vec{d_2}$

$\vec{d_3} = 4.8\hat{j} - (7.8\cos 50\hat{i} + 7.8\sin 50\hat{j})$

$\vec{d_3} = 4.8\hat{j} - 5.01\hat{i} - 5.97\hat{j}$

$\vec{d_3} = -5\hat{i} - 1.2\hat{j}$ Km

$|d_3| = \sqrt{(-5)^2 + (1.2)^2}$

$= 5.14$ Km

$\therefore \tan\theta = \frac{1.2}{5} \Longrightarrow \theta = 13.5^\circ$

$\therefore \vec{d_3} = 5.14$ Km in 13.5° south due west

Physics

(L)

Chapter 3

Q. 26

$\vec{A} = 2\hat{i} + 3\hat{j}$

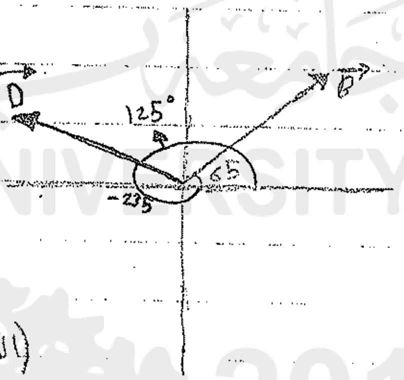
$\vec{B} = 4$ at $+65°$

$\vec{C} = -4\hat{i} - 6\hat{j}$

$\vec{D} = 5$ at $-235°$

$\therefore \vec{S} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

$\therefore \vec{S} = (2\hat{i} + 3\hat{j}) + (4\cos 65\,\hat{i} + 4\sin 65\,\hat{j}) + (4\hat{i} + 6\hat{j}) + (5\cos 125\,\hat{i} + 5\sin 125\,\hat{j})$



$\therefore \vec{S} = -3.2\hat{i} + 4.7\hat{j}$ (الطريقه الاولى)

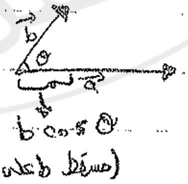
* $|S| = \sqrt{(-3.2)^2 + (4.7)^2} \Longrightarrow 5.68$

* $\theta = \tan^{-1}\left(\frac{4.7}{3.2}\right) = -55.7° \approx -56°$ ($\vec{D}$ المحصوره بين س و)

Vector Maltiplications:

① Dot product (Scalar Product):-

$\therefore \vec{a} \cdot \vec{b} = a\,b\cos\theta$



⊛ if a and b are given by their coponets then:

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ } نضرب القوسين ببعض

$\boxed{\therefore \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z}$

$\boxed{\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1}$

$\boxed{\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0}$

example :

*Note :- $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a} = |\vec{a}|.|\vec{b}|.\cos\theta$

∴ $\vec{A} = 3\hat{i} - 4\hat{j} - 5\hat{k}$

$\vec{B} = 2\hat{i} + 6\hat{j} + 7\hat{k}$

Find the angle (Ø) between $\vec{A}$ + $\vec{B}$?

$\vec{A}\,\vec{B} = |\vec{A}||\vec{B}|\cos\phi$

* $|\vec{A}| = \sqrt{(3)^2+(4)^2+(5)^2} \Longrightarrow 7.1$
* $|\vec{B}| = \sqrt{(2)^2+(6)^2+(7)^2} \Longrightarrow 9.4$
* $\vec{A}\cdot\vec{B} = A_x b_x + A_y B_y + A_z B_z$

$\vec{A}\cdot\vec{B} = 6 + {-24} + {-35} \Longrightarrow -53$

∴ دلوقتي

$$\cos\phi = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

$$= \frac{-53}{(7.1)(9.4)} \Longrightarrow -0.79$$

∴ $\phi = 142^\circ$

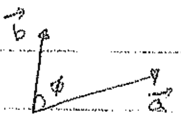
* Exampels of Dot product :-
- Work $= \vec{F}\cdot\vec{d}$ joul
- $\phi_E = \vec{E}\cdot\vec{A}$ (electric flux)
- $\phi_B = \vec{B}\cdot\vec{A}$ (magnetic flux)
- $\phi_{Fluid} = \vec{A}\cdot\vec{V}$ (تدفق الماء)
- Power $= \vec{F}\cdot\vec{V}$ watt

(2) cross product :- (vector product) :-

$\therefore \vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\phi$

$\sin\phi \perp \vec{a}$ and $\vec{b}$

$\vec{b} \times \vec{a} = |\vec{b}||\vec{a}| \sin\phi$



$\vec{a} \times \vec{b}$

outward (خارج الصفح)

$\vec{b} \times \vec{a}$

inward (داخل الصفح)

$\therefore \vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

* if $\vec{a}$ and $\vec{b}$ are given by components then:-

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$\therefore \vec{a} \times \vec{b} = a_x b_y \hat{k} + a_x b_z(-\hat{j})$

$a_y b_x(-\hat{k}) + a_y b_z(\hat{i})$

$a_z b_x \hat{j} + a_z b_y(-\hat{i})$

$= (\quad)\hat{i} + (\quad)\hat{j} + (\quad)\hat{k}$

$\hat{i}\times\hat{j} = \hat{k}$ (حسب قاعدة اليد اليمنى)

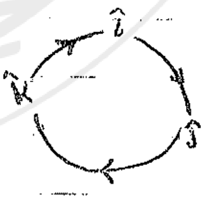
$\hat{i}\times\hat{k} = -\hat{j}$

$\hat{j}\times\hat{i} = -\hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\times\hat{i} = \hat{j}$

$\hat{k}\times\hat{j} = -\hat{i}$



اذا مع الاسهم موجبة

اذا عكس الاسهم سالبة

physics

(D)

1.3

$\boxed{Q.36}$ $\vec{d_1} = 3\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{d_2} = -5\hat{i} + 2\hat{j} - \hat{k}$

$(\vec{d_1} + \vec{d_2}) \cdot (\vec{d_1} \times 4\vec{d_2})$

$4(\vec{d_1} + \vec{d_2}) \cdot (\vec{d_1} \times \vec{d_2})$

* $\vec{S} = \vec{d_1} + \vec{d_2}$

$\vec{S} = 2\hat{i} + 3\hat{k}$

* $\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ -5 & 2 & -1 \end{vmatrix}$

$\vec{C} = \hat{i}(2-8) - \hat{j}(-3-20) + \hat{k}(6-10)$

$= -6\hat{i} - 17\hat{j} - 4k$ $\searrow$ then ✓

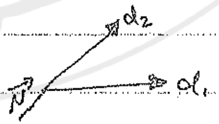
∴ المطلوب

$4\vec{S} \cdot \vec{C}$

$\therefore \vec{S} \cdot \vec{C} = -12 + 12 \Rightarrow$ zero

Note the following :-

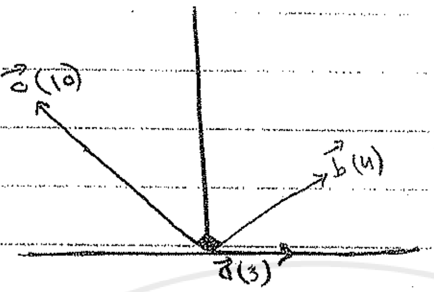
$\vec{d_1} \cdot (\vec{d_1} \times \vec{d_2}) = 0$



$\vec{d_1} \cdot \vec{d_2} = N \perp d_1, d_2$

$\therefore \vec{d_1} \cdot (\vec{d_1} \times \vec{d_2})\ (\cos 90) = 0$

Q. 43



- $a_x = 3$, $a_y = 0$
- $b_x = 4 \cos 30$, $b_y = 4 \sin 30$
- $c_x = 10 \cos 120$, $c_y = 10 \sin 120$

$\therefore \vec{a} = 3\hat{i}$

$\vec{b} = 2\sqrt{3}\,\hat{i} + 2\hat{j}$

$\vec{c} = -5\hat{i} + 5\sqrt{3}\,\hat{j}$

$\vec{c} = p\vec{a} + q\vec{b}$

$-5\hat{i} + 5\sqrt{3}\hat{j} = 3p\hat{i} + 2\sqrt{3}q\hat{i} + 2q\hat{j}$

$\therefore -5 = 3p + 2\sqrt{3}q$ $(\hat{i})$

$\therefore 5\sqrt{3} = 2q$ $(\hat{j})$

$\therefore q = 6.6$

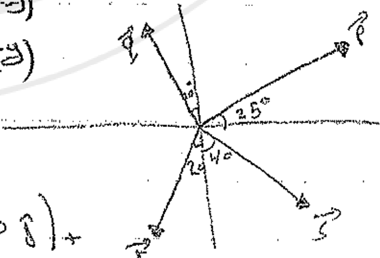
Q. 56

$\vec{p}$: 13 m , at 25° counter clock (عكس دوران الساعة) y → +x

$\vec{q}$: 12 m , at 10° counter clock y → +y

$\vec{r}$: 8 m , at 20° clockwise (−y)

$\vec{s}$: 9 m , at 40° clock wise (−y)



$\vec{N} = \vec{p} + \vec{q} + \vec{r} + \vec{s}$

$\vec{N} = (13\cos 25\,\hat{i} + 13\sin 25\,\hat{j}) + (12\cos 100\,\hat{i} + 12\sin 100\,\hat{j}) + (8\cos 250\,\hat{i} + 8\sin 250\,\hat{j}) + (9\cos 310\,\hat{i} + 9\sin 310\,\hat{j})$

$\boxed{Q.64}$ $\vec{d_1} = 4m$

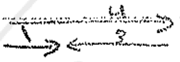
$\vec{d_2} = 3m$

* $\vec{d_1} + \vec{d_2} = \vec{R}$

$\xrightarrow{4}\xrightarrow{3}$

$\xrightarrow{7}\vec{R}$ $\therefore$ ✗

* $\vec{d_1} + \vec{d_2} = \vec{R}$ (X تعاكس بعضها)



$\therefore$ ✓

* $\vec{d_1} + \vec{d_2} = \vec{R}$

$\vec{R} = \sqrt{(3)^2 + (4)^2}$

$= 5$

physics
(L)
chapter 3

$\boxed{ex}$ $\vec{A} = 3\hat{i} - 2\hat{j} + 5\hat{k}$
$\vec{B} = 6\hat{i} + 4\hat{j} - 7\hat{k}$

ⓐ find the angel between $\vec{A}$ + $\vec{B}$

* $\vec{A}.\vec{B} = AB\cos\phi$
* $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$
$\vec{A}.\vec{B} = 18 + -8 + -35$
$\vec{A}.\vec{B} = -25$

نوجد طول A.

$A = \sqrt{3^2 + (-2)^2 + 5^2} \Longrightarrow 6.17$
$B = \sqrt{6^2 + 4^2 + (-7)^2} \Longrightarrow 10$

∴ نعوض لايجاد cos ϕ

$\cos\phi = \frac{A.B}{AB} \Longrightarrow \frac{-25}{6.17 \times 10}$

$\therefore \phi = 114°$

ⓑ find $\hat{B}$ ($\vec{B}$ باتجاه يونت vector)

$\therefore \hat{B} = \frac{\vec{B}}{B}$

$= \frac{6\hat{i} + 4\hat{j} - 7\hat{k}}{10}$

$\therefore \hat{B} = 0.6\hat{i} + 0.4\hat{j} - 0.7\hat{k}$

ⓒ find the angel between $\vec{A}$ + x axis.

- $\vec{A}.\hat{i} = A(\hat{i}) \cos \theta \Longrightarrow A \cos \theta$
- $\vec{A}.\hat{i} = 3$

$\therefore \cos \theta = \frac{\vec{A}.\hat{i}}{A} = \frac{3}{6.17}$

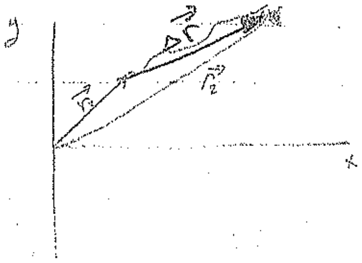
$\therefore \theta = 60^\circ$

ⓓ Find the angel between $\vec{R}$ + y axis

ⓔ find the angel between $\vec{A}$ + z axis

chapter (4)
Motion 2+3 dimensions

Position and displacement :-



$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ (initial position)

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ (final position)

$\Delta r = \vec{r}_2 - \vec{r}_1$

(Displacement)

$\therefore \Delta r = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

⊛ $\Delta r = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$ (نقسم على الزمن)

$\therefore$ avalege Valocity :-

$$\vec{V}_{avg} = \frac{\Delta r}{\Delta t}$$

$$\therefore \vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

(السرعة اللحظية) $\vec{V}_{instantinous} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$

$$\therefore \vec{V}_{inst} = \frac{d\vec{r}}{dt}$$

⊛ $\vec{V}_{inst} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$

⊛ $\vec{a}_{inst} = \frac{dv}{dt} \Rightarrow \frac{d^2x}{dt^2}\cdot\hat{i} + \frac{d^2y}{dt^2}\cdot\hat{j} + \frac{d^2z}{dt^2}\cdot\hat{k}$

$$= \frac{dv_x}{dt}\cdot\hat{i} + \frac{dv_y}{dt}\cdot\hat{j} + \frac{dv_z}{dt}\cdot\hat{k}$$

⊛ Solve the sample problems p.g (59 + 63)

$\boxed{Q.15}$ $\vec{r}_0 = 0$, $\vec{a}_{inst} = -1\hat{i} - \frac{1}{2}\hat{j}$

$\vec{V}_0 = 3\hat{i}$

at X_{max} , Find $\vec{v} + \vec{r}$ ؟

نقسم الحركة إلى اتجاهين (x-motion + y-motion)

- X- motion :

$V_{ox} = 3\ m/s$

$X_o = 0$

$a_x = -1\ m/s^2$

- at x_{max} $V_x = 0$

$V_x = V_{ox} + a_x t$

$0 = 3 - 1 \times t$

$\therefore t = 3\ s$ to reach x_{max}

$\therefore$ x_{max} لإيجاد

$X - X_o = \left(\frac{V_{ox} + V}{2}\right) \cdot t$

$X_{max} - 0 = \left(\frac{3 + 0}{2}\right) \times 3$

$= 4.5\ m$ (x - axis)

- y motion :

$y_o = 0$

$V_{oy} = 0$

$a_y = -0.5\ m/s^2$

* $V_y = V_{oy} + a_y t$ (المرحلة الثانية بال 3 ثواني)

$\therefore V_y = 0 + -0.5 \times 3$

$\therefore V_y = -1.5\ m/s$

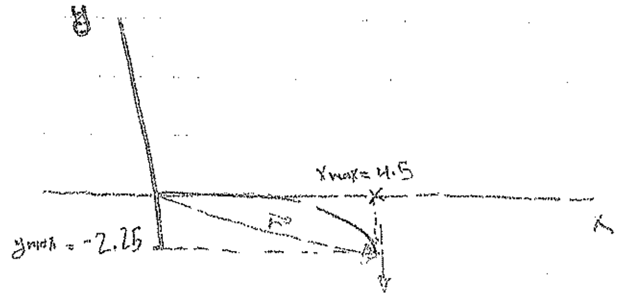
$\therefore$ y_{max} لإيجاد

$y - y_o = \left(\frac{V_{oy} + V_y}{2}\right) \times t$

$y - 0 = \left(\frac{0 + -1.5}{2}\right) \times 3$

$= -2.25\ m$ (y - axis)

$\therefore \vec{r} = 4.5\hat{i} - 2.25\hat{j}$ m



* the direction of $\vec{v}$ is always at the tangent of the path.

physics
(1)

ch. 3

problem

$\vec{A} = 4\hat{i} + 5\hat{j} - 6\hat{k}$

$\vec{B} = -1\hat{i} + 2\hat{j} + 3\hat{k}$

ⓐ find the angle between $\vec{A}$ + $\vec{B}$?

* $\vec{A} \cdot \vec{B} = AB \cos\theta$

* $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$= -4 + 10 - 18 \Longrightarrow -12$

* $|A| = \sqrt{(4)^2 + (5)^2 + (-6)^2}$

$= 8.8$

* $|B| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= 3.75$

$\therefore -12 = (8.8)(3.75) \cos\theta$

$\therefore \theta = 111°$

ⓑ find the angle between $\vec{A}$ + x axis ?

$\vec{A} \cdot \hat{i} = A\hat{i} \cos\theta = 8.8 \cos\theta$

$\vec{A} \cdot \hat{i} = 4$

$\cos\theta = \frac{4}{8.8}$

$\therefore \theta = 63°$

ⓒ find the angle between $\vec{A}$ and y axis ?

$\vec{A} \cdot \hat{j} = 8.8 \cos \beta$

$\vec{A} \cdot \hat{j} = 5$

$\therefore \cos \beta = \frac{5}{8.8}$

$\therefore \beta = 55.4^\circ$

* additional :-

$\cos \alpha = \frac{A_z}{A} \Rightarrow \frac{-6}{8.8}$

$\therefore \alpha = 133^\circ$

ⓓ find $\hat{B} = \frac{\vec{B}}{B}$

$= \frac{-1\hat{i} + 2\hat{j} + 3\hat{k}}{3.75}$

$= -0.27\hat{i} + 0.53\hat{j} + 0.8\hat{k}$

ⓔ find a vector perpendicular to $\vec{A} + \vec{B}$?

$\vec{C} = \vec{A} \times \vec{B}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 4 & 5 & -6 \\ -1 & 2 & 3 \end{vmatrix}$$

$\vec{C} = (15 - -12)\hat{i}\ (12 - 6)\hat{j} + (8 - 5)\hat{k}$

$\vec{C} = 27\hat{i} - 6\hat{j} + 13\hat{k}$

$\vec{C} \perp \vec{A} + \vec{B}$

chapter 4

[redacted] $\vec{r} = 3t\hat{i} - 4t^2\hat{j} + 2\hat{k}$

Q.6

ⓐ $\vec{v}(t) = \frac{dr}{dt}$

$v(t) = 3\hat{i} - 8t\hat{j}$

ⓑ $\vec{v}(3) = 3\hat{i} - 24\hat{j}$ m/s

ⓒ $v(3) = \sqrt{(3)^2 + (-24)^2}$

$= 24.2$ m/s

ⓓ $\tan\phi = \frac{-24}{3}$

$\phi = -83^\circ$

$\therefore$ $v(3) = 24.2$ m/s at 83° clockwise with +x-axis

additional :

ⓔ find the avarege vilocity from $t_1 = 3s \rightarrow t_2 = 10s$

$\therefore V_{avg} = \frac{\Delta r}{\Delta t} \Longrightarrow \frac{\vec{r}(10) - \vec{r}(3)}{7}$

$= \frac{(30\hat{i} - 400\hat{j} + 2\hat{k}) - (9\hat{i} - 36\hat{j} + 2\hat{k})}{7}$

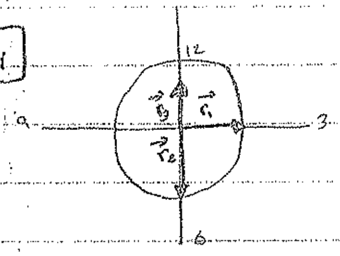
$= \frac{21\hat{i} - 364\hat{j}}{7}$

$= 3\hat{i} - 52\hat{j}$

ⓕ $\vec{v}(10) = 3\hat{i} - 80\hat{j}$

ⓖ $\vec{a}(t) = \frac{\vec{dv}}{dt} = -8\hat{j}$ m/s^2

Q.4



r = 12 Sm

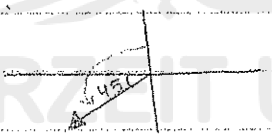
ⓐ $\vec{r_1} = 12\hat{i}$

$\vec{r_2} = -12\hat{j}$

$\vec{\Delta r} = \vec{r_2} - \vec{r_1}$

$\vec{\Delta r} = -12\hat{i} - 12\hat{j}$

$\Delta r = \sqrt{(12)^2+(12)^2} = 17$ sm



∴ $\Delta r = 17$cm at 225° counter clockwise with +x-axis

ⓑ find $\vec{\Delta r}$ after 1/2 h, from $\vec{r_2}$

$\vec{r_3} = 12\hat{j}$

∴ $\vec{\Delta r} = \vec{r_3} - \vec{r_2}$

$= 12\hat{j} - -12\hat{j}$

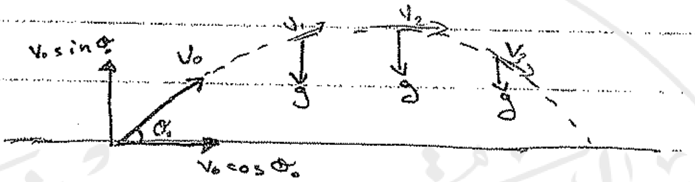
$= 24\hat{j}$

physics
(L)
Ch. 4

المقذوفات

Projectile Motion :-

it is a motion in two dimesions

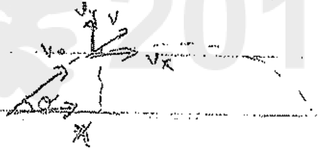


X motion is independent from y motion, could be consider to be 2 linear motion

- X - motion :-

V_x is constant

$\therefore V_x = V_0 \cos\theta_0$



$* \; X - X_0 = (V_0 \cos\theta_0) \times t$ ①

- y - motion :-

$V_{0y} = V_0 \sin\theta_0$

$a_y = -g$

$V_y = V_{0y} + a_y t$

$\therefore V_y(t) = V_0 \sin\theta_0 + -g t$

$y(t) - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$

$\therefore y(t) - y_0 = (V_0 \sin\theta_0) t + \frac{1}{2} \times -g \times t^2$ ②

* from ① $t = \frac{X}{V_0 \cos\theta_0} \Longrightarrow$ ② نعوضها في

$\therefore y(t) - y_0 = (V_0 \sin\theta_0) \frac{X}{V_0 \cos\theta_0} + -\frac{1}{2} g \frac{X^2}{V_0^2 \cos^2\theta_0}$

$$\boxed{\therefore y = (\tan\theta) X - \frac{g \cdot X^2}{2 \cdot V_0^2 \cdot \cos^2\theta_0}}$$

trajectory path

find the time of fly :-

الإزاحة العامودية عند زمن التحليق = صفر

$y - y_o = V_{oy} t + \frac{1}{2} a_y t^2$

$0 = V_o \sin\theta_o t - \frac{1}{2} g t^2$

$\therefore t = \frac{2 V_o \sin\theta_o}{g}$ (tim of fly)

* $\overset{\text{الأفقي}}{\text{horizontal}}$ $\overset{\text{المدى}}{\text{range}}$ $= (V_o \cos\theta_o) t_{flight} = R$

$\therefore R = V_o^2 \left(\frac{2 \sin\theta_o \cos\theta_o}{g} \right)$

$R = \frac{V_o^2}{g} \times \sin 2\theta_o$

* time to reach $y_{max} = \frac{1}{2}$ tim of fly

$\therefore = \frac{V_o \sin\theta_o}{g}$

* at y_{max} $V_y = 0$, find y_{max} ?

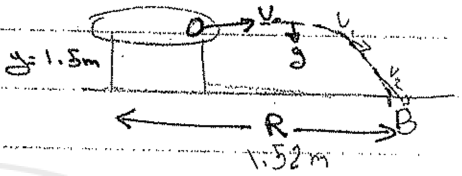
$V_y^2 = V_{oy}^2 + 2 a_y \Delta y$

$0 = (V_o \sin\theta_o)^2 + 2(-g) y_{max}$

$\therefore y_{max} = \frac{V_o^2 \sin^2\theta_o}{2g}$

$\boxed{ex - Q.22}$

$V_{ox} = ?$



ⓐ y motion :-

$V_{oy} = 0$

$a_y = -g$

$y - y_o = -1.5m$

$\therefore y - y_o = V_{oy} t + \frac{1}{2} a t^2$

$\therefore -1.5 = 0 + \frac{1}{2} (-9.8) t^2$

$t = \sqrt{\frac{3}{9.8}} = 0.55s$

ⓑ $X = V_{ox} \times t$ ([illegible])

$1.52 = V_{ox}\ 0.55$

$V_{ox} = 2.76\ m/s$

ⓒ additional :-

find v, v_x, v_y at B ?

- $V_x = 2.76 m/s$ ([illegible])
- $V_y = V_{oy} + a_y t$

$V_y = 0 - 9.8 \times 0.55$

$\therefore V_y = -5.39\ m/s$

- $V = \sqrt{(V_x)^2 + (V_y)^2}$

$V = \sqrt{(2.76)^2 + (-5.39)^2}$

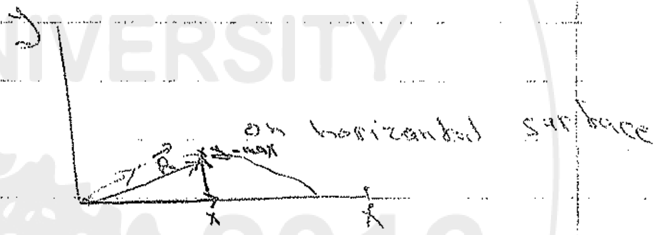
Physics
(D)
ch. 4

$\boxed{Q.17}$ $a_x = 4 m/s^2$, $V_{ox} = 8 m/s$
$a_y = -2 m/s^2$, $V_{oy} = 12 m/s$

ⓐ $\vec{V}$? when y is max
at y max , $V_y = 0$

$\therefore V_y = V_{oy} + a_y t$
$0 = 12 + -2 \times t$
$\therefore t = 6 s$ [to reach y-max]

$\therefore V_x = V_{ox} + a_x t$
$V_x = 8 + 4 \times 6$
$V_x = 32 m/s$
$\vec{V} = 32 \hat{i}$ m/s

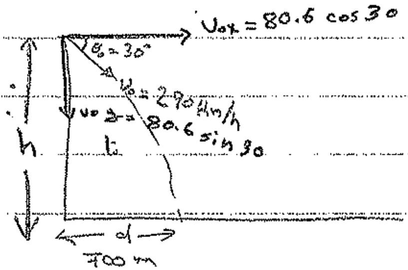


ⓑ additional ?
find x + y coordinate at y-max ?
* $y - y_0 = V_{oy} t + \frac{1}{2} a_y t^2$
$y_{max} = 12 \times 6 + \frac{1}{2} (-2) \times (6)^2$
$y_{max} = 36 m$

* $x - x_0 = V_{ox} t + \frac{1}{2} a_x t^2$
$x = 8 \times 6 + \frac{1}{2} \times 4 \times 36$
$x = 120 m$

$\vec{R} = 120 \hat{i} + 36 \hat{j}$

Q.27



$V_0 = 290\ Km/h$

$V_0 = 80.6\ m/s$

$t, h = ?$

ⓐ $a_y = -9.8\ m/s^2$

$a_x = 0$

$X - X_0 = V_{0x} t$ (لا يوجد للحركة السينية غيرها القانون)

$700 = 80.6 \cos 30 \times t$

$t = 10\ s$

$\therefore y - y_0 = V_{oy} t + \frac{1}{2} a_y t^2$ (يمكن استخدام أي قانون من الثلاث الباقيات)

$h = -(80.6 \sin 30)(10) + \frac{1}{2}(-9.8)(10)^2$

$h = -893\ m$

ⓑ additional:-

find $V_x + V_y$ at point B ?

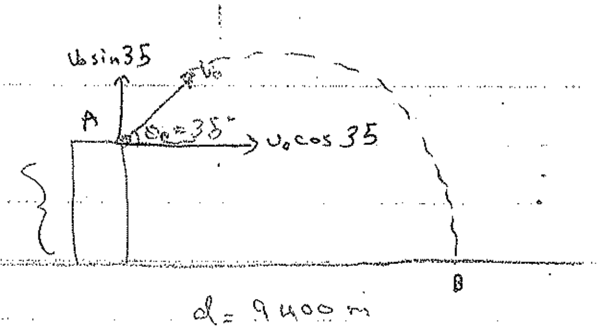
- $V_x = V_{0x} = 80.6 \cos 30 = 70$
- $V_y = V_{oy} + a_y t$

$= -80.6 \sin 30 + (-9.8)(10)$

$= -138.3\ m/s$

$\therefore \vec{V} = 70\hat{i} - 138.3\hat{j}$

$\boxed{Q.91}$



$V_0, t = ?$

- $x - x_0 = V_{0x} t$

$9400 = V_0 \cos 35 \times t$

$V_0 t = \frac{9400}{\cos 35}$ ---- ①

- $y - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$

$-3300 = (V_0 \sin 35) \times t + \frac{1}{2}(-9.8) t^2$

$-3300 = (V_0 t) \sin 35 - 4.9 t^2$ ---- ②

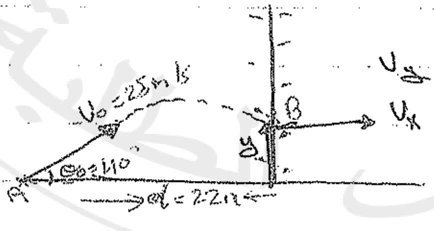
نعوض من ① في ②

$\therefore -3300 = \frac{9400}{\cos 35} \times \sin 35 - 4.9 t^2$

$\therefore t = 45$ s

$V_0 = 255$ m/s

$\boxed{Q.32}$



$\frac{V_y}{V_x} = ?$

هالاحتمالات أن تضرب الكرة الحائط قبل أقصى ارتفاع أو عند أقصى
ارتفاع أو بعد أقصى ارتفاع

ⓐ $x - x_0 = V_{0x} t$

$d = V_0 \, \text{sos} \, 40 \, t$

$22 = 19.15 \, t$

$t = 1.15 \, s$ (الزمن من A ← B)

$\therefore \; y - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$

$y = (+25 \sin 40)(1.15) + \frac{1}{2}(-9.8)(1.15)^2$

$y = +12 \, m$

* at B $V_x = V_{0x} = 25 \cos 40$

$= 19.15 \, m/s$

$\therefore \; V_y = V_{0y} + a_y t$

$V_y = +25 \sin 40 + (-9.8)(1.15)$

$V_y = +4.43 \, m/s$

∴ الجسم طالع أي قبل وصوله الى أعلى ارتفاع

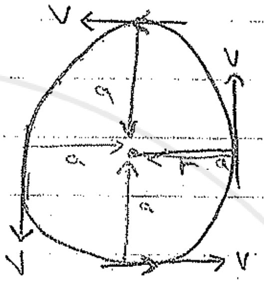
physics
(L)
ch. 4

Uniform Circular :

v is constant in magnitude

$V = \frac{2\pi r}{T}$, (T = periodic time)

- the direction of v is at the tangent of the circle



example :-

$\boxed{Q.56}$

$\vec{a} = \frac{\Delta \vec{V}}{\Delta t}$

$a = \frac{V^2}{r}$ (centripetal acceleration)

* a is always towards the center

* s is at 750 Km above the earth's surface

* T ; 98 m

ⓐ V = ? , ⓑ a = ?

∴ ⓐ $V = \frac{2\pi r}{T}$ (r = earth' radius = 6400 Km)

$\therefore r = r_{earth} + r_s = 6400 + 750 = 7150$ Km

$\therefore V = \frac{2\pi (7150 \times 10^3)}{98 \times 60}$

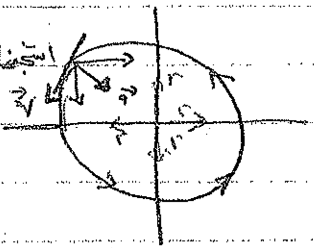
$V = 7640$ m/s

ⓑ $a = \frac{V^2}{r}$

$a = \frac{(7640)^2}{7150 \times 10^3}$ m/s²

$\boxed{Q.60}$

$T = 2s$: استنتجناها لأن $+x, -y$



$r = 3.5\ m$

at $t=1$, $\vec{a} = 6\hat{i} - 4\hat{j}\ m/s^2$

ⓐ $\vec{V} \cdot \vec{a} = ?$

$\vec{V} \cdot \vec{a} = zero$ $(\cos 90)$

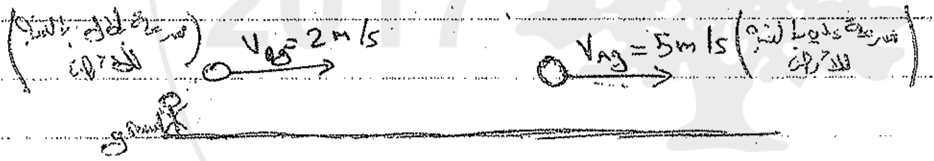
$\vec{V} \perp \vec{a}$
$\vec{V} \perp \vec{r}$
$\vec{a} \parallel \vec{r}$

ⓑ $\vec{r} \times \vec{a} = ?$

$\vec{r} \times \vec{a} = |r||a| \sin 180$
$= zero$

Relative Motion :- (السرعة النسبية)

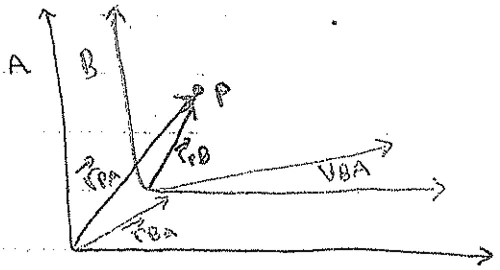
example in one dimention :



* $\vec{V}_{AB} = \vec{V}_{Ag} - \vec{V}_{Bg}$
$= 5 - 2 \Longrightarrow 3 m/s$

* $\vec{V}_{BA} = \vec{V}_{Bg} - \vec{V}_{Ag}$
$= 2 - 5 \Longrightarrow -3 m/s$

Two dimentions :-



- fram: A at rest.
- fram: B is moving at constant $\vec{V}_{BA}$.

- They are observing point p

∴ from the graft :-

$$\vec{r}_{PA} = \vec{r}_{BA} + \vec{r}_{PB}$$

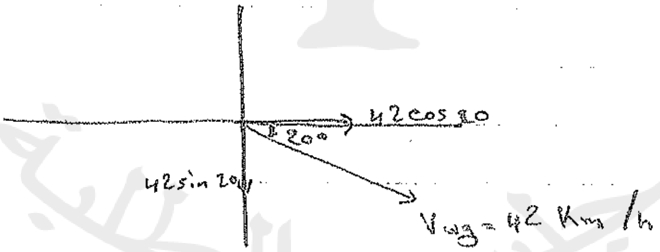
$$\vec{V}_{PA} = \vec{V}_{BA} + \vec{V}_{PB}$$

$$\vec{a}_{PA} = \vec{a}_{BA}(\text{zero}) + \vec{a}_{PB}\left(\begin{smallmatrix}\text{not}\\ \text{zero}\end{smallmatrix}\right)$$ (لأ عرفنا أن V_{BA} ثابت)

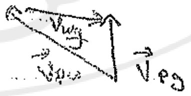
$$\boxed{\therefore \vec{a}_{PA} = \vec{a}_{PB}}$$

example :-

$\boxed{Q.74}$



$V_{pg} = \frac{55\,Km}{(18/60)h} \Longrightarrow 183$ Km/h



$\therefore \vec{V}_{pg} = \vec{V}_{pw} + \vec{V}_{wg}$

$\vec{V}_{pw} = \vec{V}_{pg} - \vec{V}_{wg}$

$\vec{V}_{pw} = 183\hat{j} - [42\cos 20\,\hat{i} - 42\sin 20\,\hat{j}]$

$\therefore \vec{V}_{pw} = 183\hat{j} - 39.5\hat{i} + 14.4\hat{j}$

$= -39.5\hat{i} + 197.4\hat{j}$

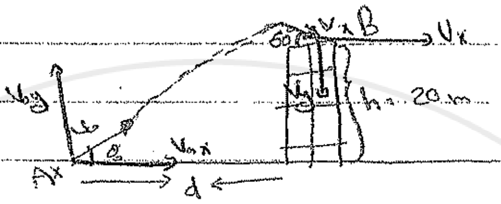
$|V_{pw}| = \sqrt{(-39.5)^2 + (197.4)^2} \Longrightarrow 201$ Km/h $\qquad \theta = \tan^{-1}\left(\frac{197.4}{-39.5}\right)$

physics

(D)

ch.4

$\boxed{Q.48}$



$t_{AB} = 4.5\,s$

$d = ?$

$V_0 = ?$

$\theta_0 = ?$

① $y - y_0 = V_{oy} t + \frac{1}{2} a_y t^2$

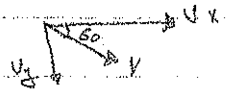
$+20 = V_{oy}(4.5) + \frac{1}{2}(-9.8)(4.5)^2$

$V_{oy} = 26.5\ m/s$

② $V_y = V_{oy} + a_y t$

$V_y = 26.5 - (9.8 \times 4.5)$

$V_y = -17.6\ m/s$



$\therefore 17.6 = V \sin 60$

$\therefore V = 20.3\ m/s$

$V_x = V \cdot \cos 60$

$V_x = 20.3 \times \cos 60$

$V_x = 10.15\ m/s$

③ $V_x = V_{ox} = 10.15$ ($0 = a_x$ لأنه)(لا يوجد تسارع بالسيني)

$\therefore V_0 = \sqrt{(V_{ox})^2 + (V_{oy})^2}$

$V_0 = 28.4\ m/s$

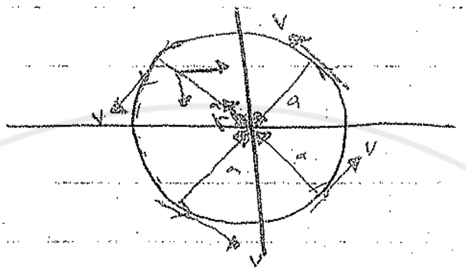
④ $\theta_0 = 69°$

⑤ $d = V_0 \times t_{AB}$

$= 10.15 \times 4.5$

$= 45.7\ m$

Q.68



$T = 2s \ / \ r = 3.5m$

ⓐ $\vec{V} \cdot \vec{a} = 0 = |V||a| \cos 90$

ⓑ $\vec{r} \times \vec{a} = |r||a| \sin 180 = 0$

Q.68 at $t_1 = 2s$, $V_1 = 3\hat{i} + 4\hat{j}$

at $t_2 = 5s$, $V_2 = -3\hat{i} - 4\hat{j}$

ⓐ $\vec{a}_{avg} = \frac{\Delta V}{\Delta t} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t}$

$\therefore \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} \Longrightarrow \frac{-6\hat{i} - 8\hat{j}}{3}$

$\therefore \vec{a}_{avg} = -2\hat{i} - 2.67\hat{j}\ m/s^2$

ⓑ بعد التجربة نفهم أن $\vec{V}_1$ عند الربع الرابع = a

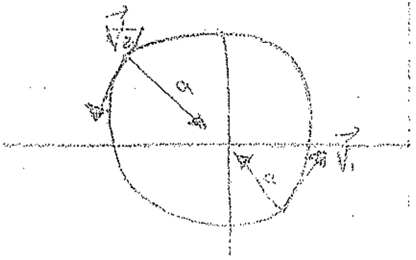
لأن السينية والصادية موجبات

أما $\vec{V}_2$ في الربع الثاني

$t_2 - t_1 = 3s \left(\frac{T}{2}\right)$

$T = 6\ sec$

(السرعة = $\frac{\text{المسافة}}{\text{الزمن}}$) $\therefore V = \frac{2\pi r}{T} \Longrightarrow r = \frac{5(6)}{2(3)}$



$\therefore a = \frac{V^2}{r} = \frac{5^2}{4.8}$

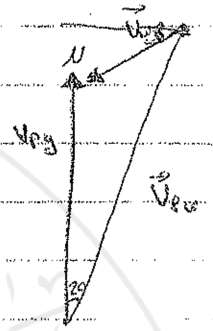
Q. 76

$\vec{V}_{PA} = \vec{V}_{PB} + \vec{V}_{BA}$

P: object

A: rest fram

B: moving fram (V_{BA})



∴ $V_{pw} = 500\ Km/h$ (سرعة الطائرة بالنسبة للريح)

$V_{pg} = \frac{900}{2} = 450\ Km/h$

$\vec{V}_{pg} = \vec{V}_{pw} + \vec{V}_{wg}$

(السرعة بالنسبة للثابت) = (ما نحتسب بالنسبة الأولى) + (ما نحتسب بالنسبة الثاني)

∴ $\vec{V}_{wg} = \vec{V}_{pg} - \vec{V}_{pw}$

$\vec{V}_{wg} = 450\hat{j} - (500 \sin 20 \hat{i} + 500 \cos 20 \hat{j})$

$= -171\hat{i} - 20\hat{j}\ Km/h$

$|\vec{V}_{wg}| = \sqrt{(171)^2 + (20)^2} = 172\ Km/h$

$\theta = \tan^{-1}\left(\frac{20}{171}\right)$ south of west

Physics
(L)
ch.5

Force and Motion:-

* chapters (1+2+3+4) description of motion (Kinematics)
* chapters (5+6) Cause of motion (Dynamics)

Newton's Laws of motion:-

① Newton's first law:-

If no force acts on a body, the body's Velocity cannot change, that is the body has no acceleration. from this law, we understand the following:-

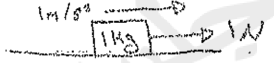
([illegible]) a $\vec{F}_{net} = 0$, on the body, $\vec{V}$ of the body = constant / zero
b $\vec{F}$ is an external agent, try to change the state of the body
(القوه مؤثر خارجي، يحاول أن يغير حالة الجسم).
c Inertial law (القصور الذاتي): mass cannot change it's state alone

② Newton's second law:-

$$\vec{F}_{net} = m.\vec{a}$$
$$\vec{F}_{net\,x} = m.\vec{a}_x$$
$$\vec{F}_{net\,y} = m.\vec{a}_y$$

from this law, we understand the following:-

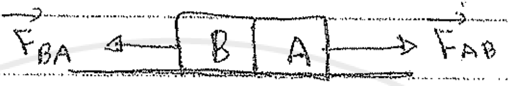
1 $F = kg.\,m/s^2 = 1$ Newton



2 $F = m.a \Longrightarrow 5 = (1)a_1 \,/\, 5 = (2)a_2 \,/\, 5 = (3)a_3$, ∴ the mass try to resist the force (الكتلة تحاول أن تقاوم القوة الخارجية), so the mass is called (inertial mass).

③ Newton's third law:-

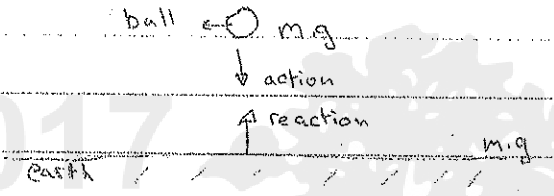
When two bodies interact, each body acts a force on the other body, the two forces are equal in magnitud and opposit in direction.



$\therefore \ \vec{F}_{AB} = (-)\,\vec{F}_{BA}$, $F_{AB} = F_{BA}$

From this law, we understand the following:-

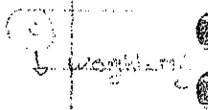
- [a] There is no single force.
- [b] The action acts on body (A), the reaction acts on body (B)
- [c] The action and the reaction acts at the same moment



* الكره تؤثر بالأرض و الأرض تؤثر على الكره.

* Application of Newton's laws:-

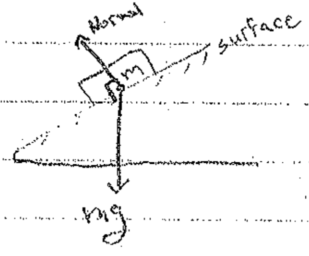
- weight = the gravitational force, acts on the body

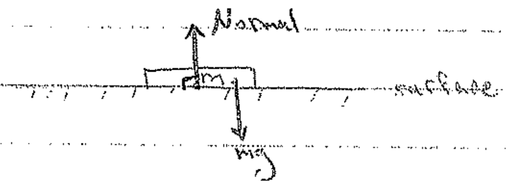


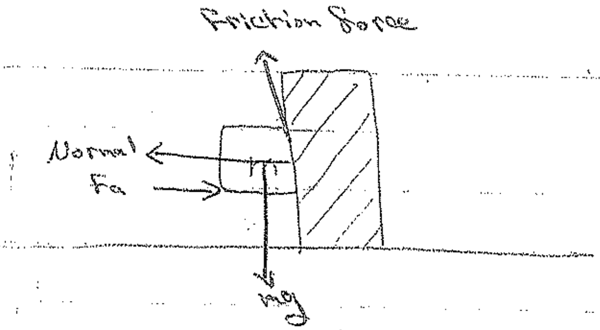
$\vec{F}_{net} = m\vec{a}$

$\vec{F}_{net} = m\vec{g} \longrightarrow \vec{a} = \vec{g} = 9.8\,m/s^2$

- Normal force = it's the force from the $\overset{\text{supporting}}{\uparrow}$surface on the body





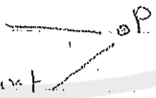


physics

(D)

Q.70 Esam (A) at rest

frame (B) $\vec{v}$ constant



$$\vec{V}_{PA} = \vec{V}_{PB} + \vec{V}_{BA}$$

$\vec{V}_{bw} = 14\hat{i}$ km/h

$\vec{V}_{wg} = 8.2(-\hat{i})$ km/h

(a+b) $\vec{V}_{bg} = ??$

$\therefore \vec{V}_{bg} = \vec{V}_{bw} + \vec{V}_{wg}$

$\vec{V}_{bg} = 14\hat{i} - 8.2\hat{i}$

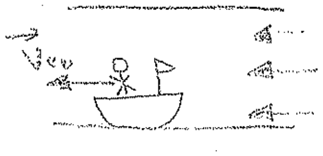
$= 5.8\hat{i}$ km/h

(c,d) $\vec{V}_{cg} = ?$

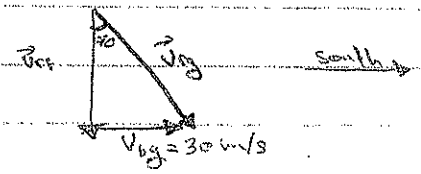
$\vec{V}_{g} = \vec{V}_{cb} + \vec{V}_{bg}$

$= -6\hat{i} + 5.8\hat{i}$

$= -0.2\hat{i}$ km/h



$\boxed{Q.75}$ $\vec{V}_{bg} = 30\ m/s$ (south)



⊛ $\sin 70 = \frac{V_{bg}}{V_{rg}}$

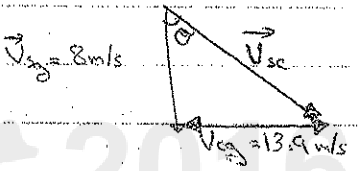
$\sin 70 = \frac{30}{V_{rg}}$

$\therefore \vec{V}_{rg} = \frac{30}{\sin 70} \Longrightarrow 32\ m/s$

⊛ $\cos 70 = \frac{V_{rt}}{V_{rg}}$

$V_{rt} = \frac{\cos 70}{32} \Longrightarrow 10.9\ m/s$

$\boxed{Q.77}$



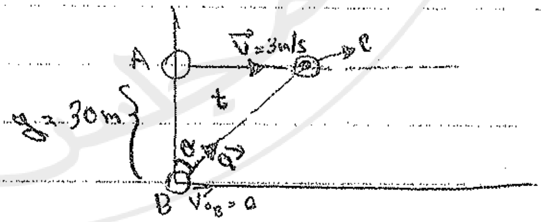
⊛ $\tan \theta = \frac{13.9}{8}$

$\theta = 60°$

$\vec{V}_{sg} = \vec{V}_{se} + \vec{V}_{cg}$

$\boxed{Q.20}$ $\vec{a} = 0.4\ m/s^2$ (تسارع)

$\theta = ?$



$A \to C = X$

* for $\boxed{A} \Longrightarrow X = 3t$ ---- ①
* for $\boxed{B} \Longrightarrow \vec{d}_{B\to C} = \vec{V}_0 t + \frac{1}{2}\vec{a}t^2$

$\vec{d}_{B\to C} = 0 + \frac{1}{2} \times 0.4\,(t)^2$

$\vec{d}_{B\to} = 0.2\,t^2$ ---- ②

* فيثاغورس $\longrightarrow d^2 = X^2 + y^2$ --- ③

$\therefore d^2 = (3t)^2 + (30)^2$

$(0.2t^2)^2 = 9t^2 + 900 \Longrightarrow 0.04t^4 - 9t^2 - 900 = 0$

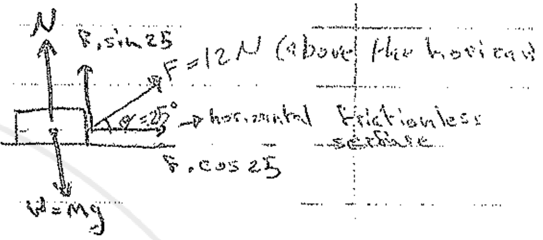
$\therefore t^4 - 225t^2 - 22500 = 0 \Longrightarrow t^2 = \frac{(-)-225 \mp \sqrt{(225)^2 - 4(1)(22500)}}{2}$

Physics
(L)
ch.5

normal force :-

Q.49 $M = 5\,kg$

find the accelaration?



ⓐ $* \; \vec{F}_{net} = m\vec{a}_x$

$(\vec{F}_{net})_x = m.\vec{a}_x$

$\therefore F.\cos 25 = m.a_x$

$a_x = \dfrac{F.\cos 25}{m}$

$= \dfrac{12.\cos 25}{5}$

ⓑ find Normal force

$(\vec{F}_{net})_y = m a_y$ (تساوي المحصلات = صفر)

$(\vec{F}_{net})_y = 0$

$N + F.\sin 25 - mg = 0$

$N = mg - F.\sin 25$

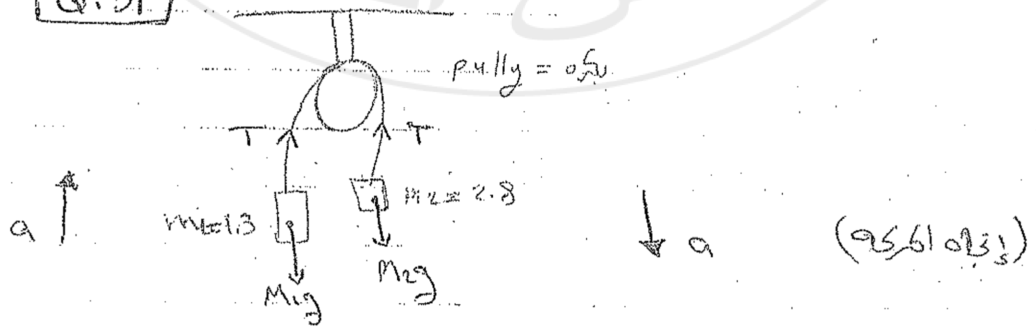
$N = 43.9\,N$

ⓒ $\vec{F}$. ?. just befor $\boxed{m}$ is lifted

⟶ التلاقي بعد

Tension Force :- (T)

Q.51



$\vec{a}, T = ?$

The equation of motion for m_1:-

$m_1 a = (f_{net})_1$

$m_1 a = T - M_1 g$ ------ ①

The equatio of motion for m_2:-

$(-m_2 a = T - m_2 g$ —— ② $) \times$ سالب　　$+\uparrow$ $-\downarrow$

$m_2 a = m_2 g - T$ —— ②'

∴ ① + ②'

$(m_1 + m_2) a = m_2 g - m_1 g$

$a = \frac{(m_2 - m_1) g}{(m_1 + m_2)}$

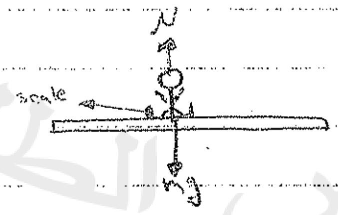
$a = \frac{1.5 \times 9.8}{4.1} \Longrightarrow a = 3.6 \ m/s^2$

from ① $T = m_1 a + m_1 g$

$T = m_1 (a + g)$

$T = 17.42 \ N$

Elevator motion:



ⓐ elevator at rest or Moving with constant $\vec{V}$:-

$ma = N - mg \Longrightarrow 0 = N - mg \Longrightarrow N = mg$

ⓑ elevator moves upward with $\vec{a}$:-　　$\uparrow a$

(you feel heavy) $ma = N - mg \Longrightarrow N = ma + mg$

ⓒ elevator moves downward with $\vec{a}$:-　　$\downarrow a$

(you feel light) $-ma = N - mg \Longrightarrow ma = mg - N \Longrightarrow N = mg - ma$

ⓓ elevator moves downward with deceleration:　　$\uparrow a$ $\downarrow -a$

∴ $N = ma + mg$

Physics
(D)
ch.5

$\boxed{Q.10}$ m=0.150 Kg

$x(t)=-13+2t+4t^2-3t^3$

$\vec{F}_{net}$ at t= 2.6

$\therefore \vec{V}(t)=\frac{dx}{dt}=2+8t-9t^2$

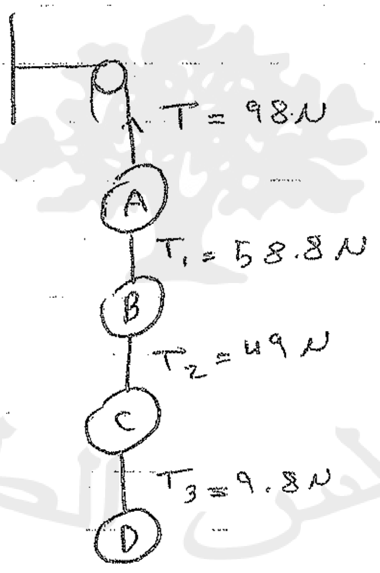
$\vec{a}(t)=\frac{dv}{dt}=8-18t$

$\therefore \vec{a}(2.6)=8-18(2.6)\Longrightarrow -38.8\ m/s^2$

$\therefore \vec{F}_{net}=m\vec{a}\Longrightarrow -38.8\times0.15=-5.8\hat{i}\ N$

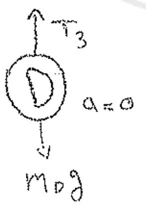
$\boxed{Q..13}$

$m_A, m_B, m_C, m_D = ?$



كل الأجسام في حالة إتزان

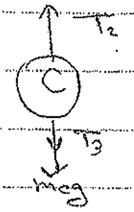
∴ ⊛



$\therefore$ for D : $T_3=m_D g$

$\therefore m_D=1\ Kg$

⊛



$a = 0$

$\therefore$ for C :-

$T_2 = T_3 + m_c g$

$\frac{T_2 - T_3}{g} = m_c$

$\therefore m_c = 4$ Kg

⊛

$a = 0$

for B :-

$T_1 = T_2 + m_B g$

$m_B = \frac{T_1 - T_2}{g}$

$\therefore m_B = 1$ Kg

for A :-

$T = T_1 + m_a g$

$m_a = \frac{T - T_1}{g}$

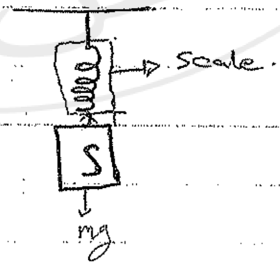
$\therefore m_a = 4$ Kg

Q.15 $m_s = 11$ Kg

(a) scale reading $= mg$

$= (11)(9.8)$

$= 170.8$ N

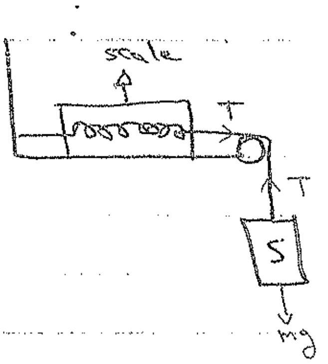


$S \Longrightarrow T = mg$ (ميزان)

ⓑ

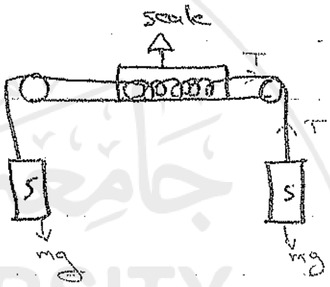
∴ scale reading $= mg$

$= 107.8\ N$



ⓒ

scale reading $= 107.8\ N$

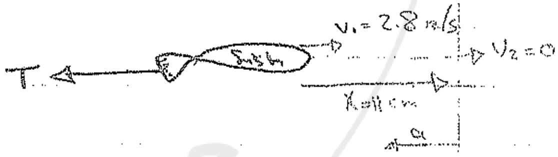


(الميزان يحتاج لتثبيت ليقرأ، في ⓐ ثبتناه بالسقف، في ⓑ ثبتناه بالحائط، و

ⓒ ثبتناه بالكتلة الأخرى لأنه إذا تركه للسقوط مع الكتلة الأولى تكون قراءته $=0$)

$\boxed{Q.26}$

$N = mg \Longrightarrow$ لا تأثير لها



motion is horizontally

$V_2^2 = V_1^2 + 2a\,\Delta X$

$0 = (2.8)^2 + 2a\,.\,0.11$

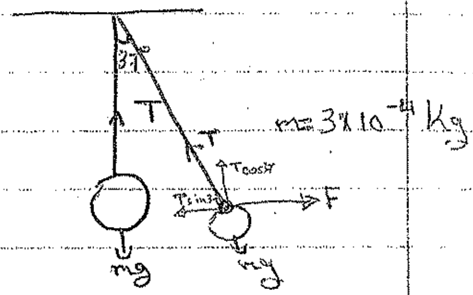
$a = -35.6\ m/s^2$

ⓑ $T = ma$

$W = 90 \Longrightarrow m = 9.2\ Kg$

∴ $T = 9.2 \times -35.6 \Longrightarrow -326\ N$

$\boxed{Q.39}$ $T, F = ?$



$\sum \vec{F} = 0$ (at rest)

$* \sum \vec{F}_x = 0$

$F - T\sin 37 = 0$

$F = T\sin 37$ ---- ①

$* \sum \vec{F}_y = 0$

$T\cos 37 - mg = 0$

$T\cos 37 = mg$ ---- ②

بقسمة ① على ②

$\frac{F}{mg} = \tan 37°$

$\therefore F = mg \tan 37$

$F = 2.2 \times 10^{-3}$ N

$\therefore T = 3.7 \times 10^{-3}$ N

$\boxed{Q.92}$ $\vec{V} = 2\hat{i} - 7\hat{j}$ m/s (unchanging $\vec{v}$)

$F_1 = 2\hat{i} + 3\hat{j} - 2\hat{k}$ N

$F_2 = -5\hat{i} + 8\hat{j} - 2\hat{k}$ N

$F_3 = ??$

$\because$ unchanging $\vec{V}$ means $\vec{a} = 0$

$\vec{F}_{net} = 0$

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

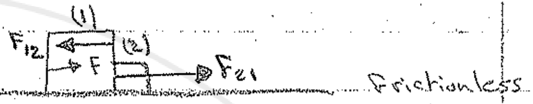
$-3\hat{i} + 11\hat{j} - 4\hat{k} + \vec{F}_3 = 0$

$F_3 = 3\hat{i} - 11\hat{j} + 4\hat{k}$

physics
(L)
Ch.5

⑤ contact force between two bodies:

$\boxed{Q.55}$



$F = (m_1 + m_2) \cdot a$ ---- ① $\qquad F = 3.2\ N$

find the force on ② from ① $\qquad m_1 = 2.3$

∴ equation of motion for m_2 $\qquad m_2 = 1.2$

$m_2 \cdot a = F_{21}$

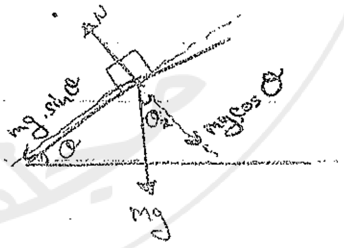
from ① ∴ $a \Longrightarrow \frac{3.2}{2.3 + 1.2} = 0.91\ m/s^2$

from ② : $F_{21} = (1.2)(0.91) = 1.1\ N$

* Try to find F_{12} from the equation of motion for m_1

$m_1 a = F - F_{12}$

⑥ Inclined Frictionless Palne:-



- $\boxed{m}$ on Frictionless surface.
- find $N, a = ???$

(y-axis) $N - mg \cdot \cos\theta = 0$

$N = mg \cdot \cos\theta$ ---- ①

(x-axis) $ma = mg \cdot \sin\theta$

$a = g \cdot \sin\theta$ ---- ②

$\theta + X = 90$ ---- ①

$\theta_2 + X = 90$ ---- ②

$\theta = \theta_2$

(*) Try to solve $\boxed{Q.34}$ ^^

Ch. 6

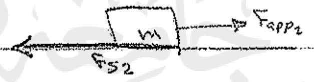
force and motion – II

① force of friction:-

friction force always opposite to the direction of sliding

No motion
$F_{app_1} = F_{s_1}$

- F_s = static friction force acts when $\boxed{m}$ at rest.

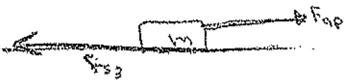


No motion
$F_{s_2} > F_{app_2}$

- $\vec{F_s} \overset{(-)}{=} \vec{F}_{app}$

- $F_{s\,max} = \mu_s . m$

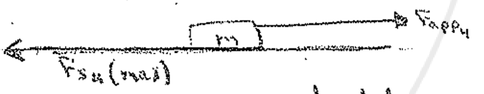
μ_s (معامل الاحتكاك السكوني)
(coefficient of static friction)



No motion

- when $\boxed{m}$ moves, the friction force is called (Kinetic friction force) $= \mu_K . N$

μ_K (معامل الاحتكاك الحركي)
(coefficient of Kinetic friction)

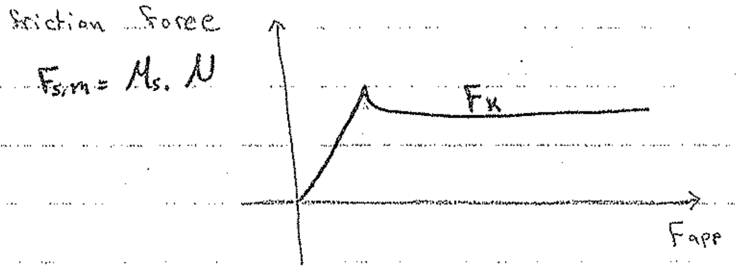


about to move
$F_{app_4} = F_{s\,max}$

$\mu_K < \mu_s$

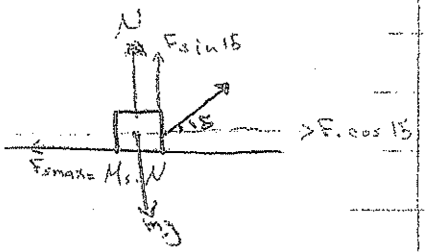
$F_K < f_{s,m}$

it moves



$\boxed{Q.11}$ $\mu_s = 65$

$\theta = 15°$



ⓐ F to start m, moving = ?

m about to move

$F_{net\,x} = 0$

$F.\cos 15 - \mu_s.N = 0$

$F.\cos 15 = \mu_s.N$ ——— ①

$F_{net\,y} = 0$

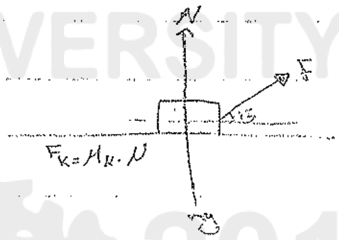
$N + F.\sin 15 - mg = 0$ ——— ②

$F \Leftarrow N$ نعوض

ⓑ $\mu_k = 0.35$, find $a = ?$

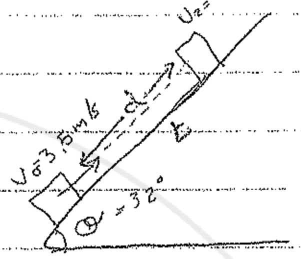
$F_{net\,x} = ma$

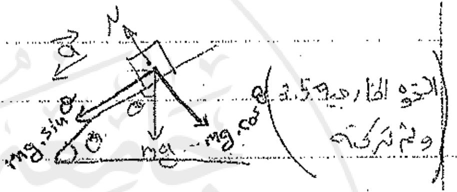
$ma = F.\cos 15 - \mu_k.N$



physics
(D)
ch5

Q.31





$N - mg\cos\theta = 0$

$N = mg\cos\theta$

$ma = mg\sin\theta$

$\therefore a = g\sin\theta$

$\therefore a = -5.2\ m/s^2$

ⓐ $\therefore V_2^2 = V_0^2 + 2a\Delta X$

$0 = (3.5)^2 + 2(-5.2)\,d$

$d = 1.18\ m$

ⓑ $\therefore V_2 = V_0 + at$

$0 = 3.5 + (-5.2)t$

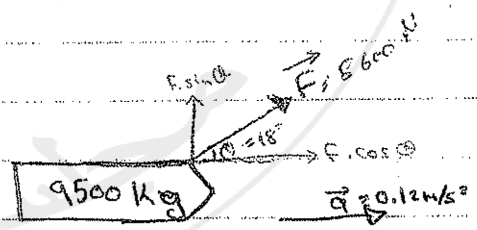
$t = 0.7\ s$

ⓒ -

سرعة الجسم عند نقطة البداية نفس سرعة
عند الصعود ولكن بعكس الاتجاه

$\therefore \vec{V} = -3.5\ m/s$

Q.42



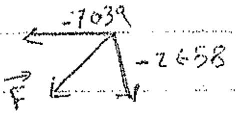
F from the horse + F from the water
ARE HORIZANTAL
mg, N (مش رح نأخذهم)

$\vec{F} + \vec{F}_{bw} = m\vec{a}$

$8600\cos18\,\hat{i} + 8600\sin18\,\hat{j} + \vec{F}_{bw} = (9500)(0.12)\,\hat{i}$

$8179\,\hat{i} + 2658\,\hat{j} + \vec{F}_{bw} = 1140\,\hat{i}$

$\vec{F}_{bw} = -7039\,\hat{i} - 2658\,\hat{i}$



$|\vec{F}| = 7524\ N$

$\theta = 20.7°$

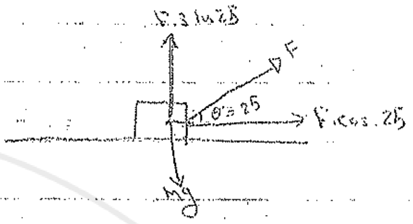
$\therefore |\vec{F}| = 7524N$ at $\theta = (20.7° + 180°)$ with (+x)

تكملة سؤال ٤٩

ⓒ F = ? just befor lifting off the floor

$\vec{F} \sin 25 - mg = 0$

$F = \frac{mg}{\sin 25} = 117 N$



ⓓ $\vec{a} = ?$

$F \cos 25 = m a_x$

$a_x = \frac{117 \cdot \cos 25}{5}$

$a_x = 21 m/s^2$

Q.53 $m_1 = 12 Kg$

$m_2 = 24 Kg$

$m_3 = 31 Kg$

$a, T_1, T_2 = ?$

$(m_1 + m_2 + m_3) a = T_3$

$\vec{a} = \frac{T_3}{\Sigma m} = \frac{65}{67} = 0.97 m/s^2$

$m_1 a = T_1$

$T_1 = 11.6 N$

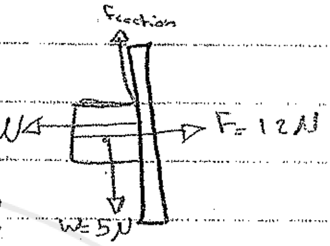
$m_2 a = T_2 - T_1$

$T_2 = 34.8 N$

$m_3 a = T_3 - T_2$

7

physics
ch. 6



Q.19 $\mu_s = 0.6$

$\mu_k = 0.4$

ⓐ the block will move or not ?

$\sum F_x = 0$

$12 - N = 0 \Longrightarrow N = 12\,N$

$\therefore f_{s\,max} = \mu_s N = 0.6 \times 12 = 7.2\,N$

$W < f_{s\,max}$

$\therefore$ it will not move

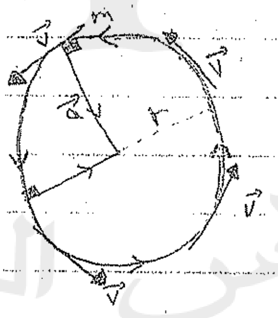
ⓑ لأن الجسم لا يتحرك، قوة الإحتكاك : سكون

$\sum F_y = 0$

$f_s - 5 = 0 \Longrightarrow f_s = 5\,N$

(من الحائط للبلوك) $\therefore \vec{F}_{bw} = 12\hat{i} + 5\hat{j}$

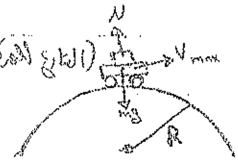
Ⓧ Uniform Circular Motion :



- $V = \frac{2\pi r}{T}$

- $a = \frac{V^2}{r}$ centripetal acceleration

- $F = \frac{mV^2}{r}$ centripetal force { محصلة القوى التي تجعل الجسم يدور }



$\boxed{Q:82}$ $R = 250\,m$

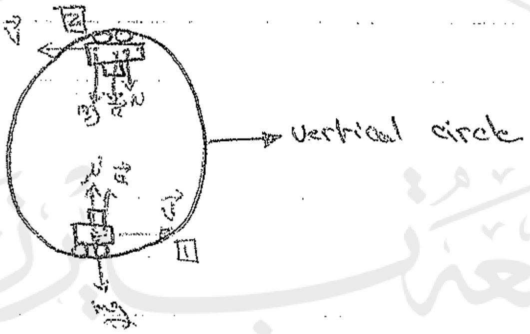
$\therefore (-)\frac{mV^2}{R} = N - mg$

$\therefore V_{max}$ is at $N \rightarrow 0$

$\therefore \frac{mV_{max}^2}{R} = mg$

$V_{max}^2 = Rg \Longrightarrow V_{max} = 50\,m/s$

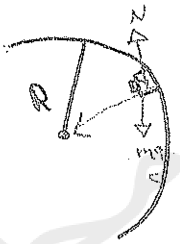
Example :-



position $\boxed{1} \Longrightarrow \frac{mV_1^2}{R} = N - mg$

position $\boxed{2} \Longrightarrow -\left(\frac{mV_2^2}{R}\right) = -(mg + N)$

③ Car in flat circular turn (Unbanked Roadway) :



الرسم أفقي
(لفوق)

$f_{s,m} = \frac{mV^2}{R}$

$\mu_s N = \frac{mV_{max}^2}{R}$ --- ①

$N = mg$ --- ②

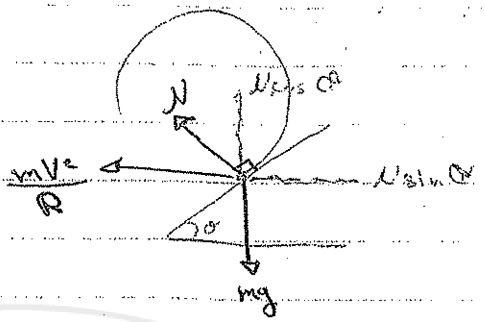
$\therefore \mu_s mg = \frac{mV_{max}^2}{R}$

$V_{max} = \sqrt{\mu_s R g}$

$\therefore V_{max}$ depends on μ_s

Ⓐ Banked Roadway :-

$N \sin\theta = M \frac{V_{max}^2}{R}$ ---- ①



(لا يوجد حركة بالـ y) $N \cos\theta = mg$ ---- ②

$\frac{①}{②} \longrightarrow \tan\theta = \frac{V_{max}^2}{Rg}$

$\therefore V_{max} = \sqrt{R \cdot g \cdot \tan\theta}$

Physics second [illegible]

Chapter 6: Work, energy and power

Dot Product ⇒ "Scalar product"

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\vec{A} \cdot \vec{B} = 0$ "$\vec{A} \perp \vec{B}$"

$\vec{A} \cdot \vec{B}$ could be negative if $\theta > 90^\circ$

in Cartisean coordonath

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\hat{i} \cdot \hat{i} = (1)(1)\cos\theta = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

$\hat{i} \hat{j} = (1)(1)(\cos\theta) = 0$ $\quad \hat{i}\hat{j} = \hat{i}\hat{k} = \hat{k}\hat{j} = 6$

$\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

example :-

$\vec{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

$B = 6\hat{i} + 7\hat{j} - 8\hat{k}$

1) $|\vec{A}| = \sqrt{(3)^2 + (-4)^2 + (5)^2} = 7.1$

2) $|\vec{B}| = \sqrt{(6)^2 + (7)^2 + (8^2)} = 12.2$

$\vec{A} \cdot \vec{B} = 3(6) + (-4)(7) + (5)(-8)$
$18 - 28 - 40$

find the angle between vector A and vector B

4) $\vec{A}+\vec{B}$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$-50 = (7.1)(12.2)\cos\theta$

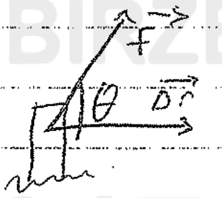
$\cos\theta = \dfrac{-50}{(7.1)(12.2)}$

$\theta = 125$

5) $\hat{A} = \dfrac{\vec{A}}{A} = \dfrac{3\hat{i}-4\hat{j}+5\hat{k}}{7.1} = 0.42\hat{i} - 0.56\hat{j} + 0.7\hat{k}$

6) $\hat{B} = \dfrac{\vec{B}}{B}$

Work done by a constant force $= \vec{F}\cdot\vec{\Delta r}$



$= F(\Delta r)\cos\theta$

$(\Delta r)F\cos\theta$

* where Δr is displacement

Note the following:-

1) Newton $W = N\cdot m = J$

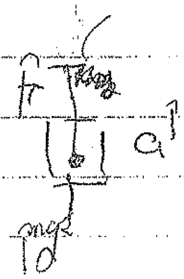
2) New $W = 0$ in $\theta = 90$

or $\Delta r = 0$

3) W is negative when

when $180° \geq \theta > 90$

work done by gravity



$T > mg$

when $\Delta D =$

case 1:

moving my gwall

a) find a:

$ma = T - mg$

$a = \frac{T - mg}{m}$

$W_g = (mg)(\cos\theta)(a)$

word done by creat:ively

$d(-)(11)(g)y$

c) Work done by t.

$W_T = Ty \cos\theta = n$

$T = T \times y$

where done by work

$N = Max$ ~~[illegible]~~

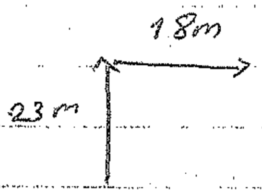
when $0 \geq \theta \geq 90$

or wiked don by

$W_{net\,force} = mgy + Ty$

sol 1) 2) 3)

13) $m = 650$ kg
$d = 23$ m



$23\hat{j} + 18\hat{i}$

$\vec{A}\cdot\vec{B} = A.B\cos\theta$

$\Delta r = 18\hat{i} + 23\hat{j}$

To find W
$W = f_x\Delta x + f_y\Delta y$

by

lifting the beam at constant speed the crane exerts a constant force vertically upward equals in Magnitude to the weight of the beam. During the horizontal swing The force is the same but is perpendicular to the displacement

$F\cdot\Delta r = (mg\hat{j})(\Delta y\hat{j} + \Delta x\hat{i})$

$= (650\,\text{kg})(9.8\,\text{m/s}^2)(23\,\text{m})$

$= 147$ ~~kg~~ kJ

18) $F = 1.8\hat{i} + 2.2\hat{j}$ N $\quad r = 56\hat{i} + 37\hat{j}$
The force is constant

$4\text{km} - x$
3

force ⇒ $W = F\cdot r = \left((1.8\,\text{N})(56\,\text{m}) + (2.2\,\text{N})(37\,\text{m})\right) = 169\,\text{J}$

20) $x = 0$ km $\quad x = 3$ km
$x = 3$ km to $x = 4$ km

$$\underset{0\to 3}{W} = \int_0^{3\text{km}} (f_x)\,dx$$

$$= \int_0^3 \left(\frac{40\,\text{N}}{3\,\text{Km}}\right) x = \frac{40\,\text{N}}{3\,\text{km}}\,\frac{(3\,\text{km})^2}{2} = 60\,\text{kJ}$$

$$b\ \underset{3\to4}{W} = \int_{3km}^{4km} \left(\frac{40N}{3km}\right)(4km - x)\,dx$$

$$= \frac{40N}{3km} \times 4x - \frac{x^2}{2}\Big|_3^4 = 20kJ$$

21) $k = 200\ N/m$

a) 10cm

$W = 1F.n$ $\qquad W = \frac{1}{2}kx^2 = \frac{1}{2}(200N/m)(0.1)^2$

$= 1J$

b) $W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(200N/M)(0.2m^2 - 0.1m^2)$

$= 3J$

$0 \to 20cm = 4J$

$0 \to 10cm = 1J$

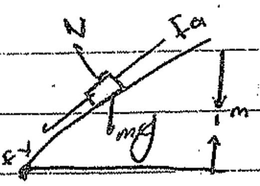
$\boxed{10 \to 20cm = 4J - 1J = 3J}$

24) $k = 70\ mN/m$

$x = 9.6\ cm$

$F = W = \frac{1}{2}kx^2 = \frac{1}{2}(70\times10^{-3})(0.096)^2 =$

41)



$Fa \cdot \Delta r = (200N)(2M) = 400J$

$\Delta r = (1m)/\sin30 = 2m$

$m = Fa/g(\sin\theta + \mu k\cos\theta)$

$m = \frac{200}{(0.18)(\frac{\sqrt{3}}{2}) + 0.5}$ $\qquad m = \frac{200}{1.8 + 0.5}$ $= \frac{20}{}$

$= 87.1g$

Work - Kinetic Energy theorem

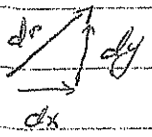
$W_{net} = \Delta K$

$\rightarrow$ Power $= \frac{W}{\Delta t} \left[\frac{d}{s}\right]$ watt $[W]$

$$P = \frac{dw}{dt} = \vec{F} \cdot \vec{v}$$

$$W = \int F_x \, dx$$

$$W = \int \vec{F} \cdot d\vec{r}$$



$$= \int F(x)\,dx + \int F(y)\,dy + \int F(z)\,dz$$

$1\,KW/m^2 \times 75\,m^2$

$75\,KW$

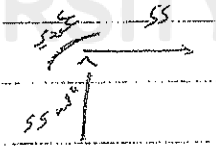
time $\frac{40\,KW \cdot h}{75\,KW}$

$\underline{\underline{2.67\,h}}$

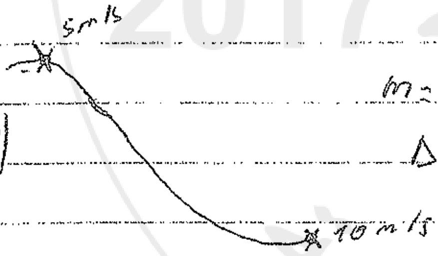
10) Kinetic energy is the same

$K = \frac{1}{2} m v^2$

No Net work bcs $\Delta K = 0$



28)

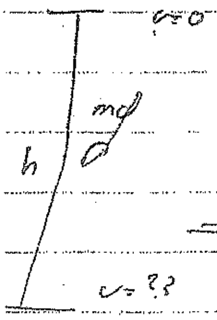


$m = 60\,kg$

$\Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$ ~~α~~ $=$

$\Delta K = W_{net}$

$= \frac{1}{2}(60)(10^2 - 5^2) = 22507\,J$



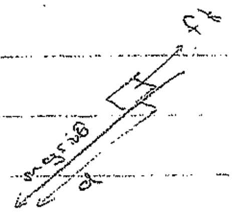
$W = mgh = \frac{1}{2} m (v_f^2 - v_i^2)$

$v_f^2 = v_i^2 + 2gh$

$\Rightarrow \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \underline{mgh}$

$(mg\sin\theta - f_k)\,d$

$= \frac{1}{2} m (v_f^2 - v_i^2)$ $\Leftarrow$



27) $\frac{1}{2} m_f v_c^2 = \frac{1}{2} m_c v_c^2$

32) $w = \vec{F} \cdot \vec{d} = 750(N) \times 200\,m = 150000 = 15 \times 10^4 J$

$\bar{P} = \frac{dW}{\Delta t} = \frac{15 \times 10^4}{5 \times 60} = 500\,Watt$

$P = \frac{500}{746} = 0.67\,hp$

38) $1\,Kw/m^2$

$A = 15\,m^2$

$R = 1\,Kw/m^2 \times 15\,m^2$

$= 15\,Kw$

$time = \frac{40\,KW.h}{15\,Kw}$ $time = 2.67\,h$

$\frac{20 \times 10^3}{900}$

39) $P = \frac{w}{t}$

$t = \frac{w}{P} = \frac{20 \times 10^3 J}{900\,w} = 22\,s$

P.P. 82) c → rate of work

83) c) at 85, 145, 185 ms

84)

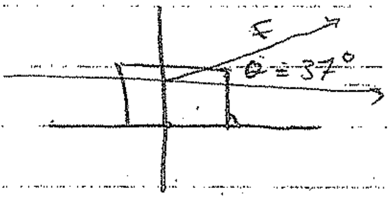
$\Delta k = 550J$ 84) 5.50 J a)

85) $\Delta P = \vec{F} \cdot \Delta \vec{v} = F \Delta v$ $F = \frac{\Delta P}{\Delta v}$ c)

$W = \vec{F} \cdot \vec{\Delta r}$ (work done
$W = F(\Delta r)\cos\theta$ by constant force)

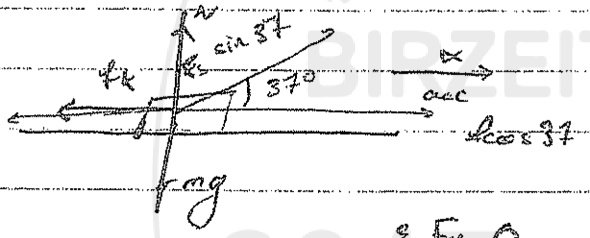
$a = 1.5\ m/s^2$

example:
$m = 50kg$
$\mu_k = 0.4$



Applied force act 37° above the horizontal moves the Mass at constant $a = 1.5\ m/s^2$ for a displacement = 100 m

Find the work done by each force



$\Sigma F_y = 0$

$\Sigma F_y = 0$
$N + F\sin 37 - mg = 0$
$N = mg - F\sin 37 \longrightarrow ①$

$\Sigma F_x = ma$
$ma = F\cos 37 - \mu_k$ ②

① in ②

$ma = F\cos 37 - \mu_k(mg - F\sin 37)$
$ma = F\cos 37 - \mu_k mg + \mu_k F\sin 37$
$ma + \mu_k mg = F(\cos 37 + \mu_k \sin 37)$

$F = \frac{ma + \mu_k mg}{\cos 37 + \mu_k \sin 37} = \cancel{206}\ 264,4\ N$

from ①

$N = mg - F\sin 37$

$= 50 \times 10 - 264,4 \times 0.6$

$= 331,4$

$f_k = \mu_k N$

$= 0.4 \times 331,4$

$= 132,4\ N$

$d = \Delta r$

$W_g = mgd\cos 90 = 0$

$W_N = Nd\cos 90 = 0$

$W_{f_k} = f_k \Delta r \cos 180 = (-1)(132.4)(100)$

$= -13240\ J$

$W_F = F(\Delta r)\cos 37$

$F = (264.4)(100)(0.8)$

$W_F = 21152\ J$ $\underline{ma\cos\theta}$

$W_{net} = W_g + W_{f_k} + W_F + W_N$

$= +7912\ J$

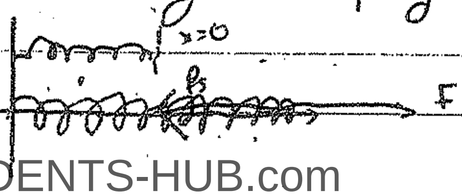
Work done By variable force

$W = \int \vec{F} \cdot d\vec{r}$

in one dimension

$W = \int_{x_i} F_x dx$

stretching a spring:- $F_s = -kx$



$F = kx$ (external force

Hooke's law

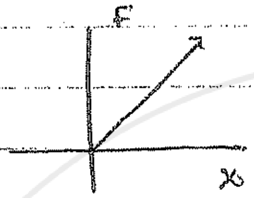
W done in stretching the spring $= \int_{x_i}^{x_f} kx\,dx$

$$W = k\frac{x^2}{2}\Big]_{x_i}^{x_f}$$

$$W = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$x_i = 0$

$$W = \frac{1}{2}kx_f^2$$



work done by spring force

$$W_s = \int_{x_i}^{x_f} -kx\,dx = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2$$

Work done by variable force in 3 Dimensions :-

$$W = \int_{r_i}^{r_f} \vec{F} \cdot \vec{dr} \longrightarrow \vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$dr = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

Example :-

$\vec{F} = 2x\hat{i} + 3\hat{j}$ N

~~at~~ acts on mass $m = 5$ kg

$\downarrow$ moves it from $r_i = 2\hat{i} + 3\hat{j}$ m

to $r_f = -4\hat{i} - 3\hat{j}$ find the work done by this force

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy = \int_{2}^{-4} (2x)\,dx + \int_{3}^{-3} 3\,dy$$

$$\frac{2x^2}{2}\Big]_2^{-4} + 3y\Big]_3^{-3} = (32-8) + (9+9)$$

$$12 + 18 = -6\,J$$

49) $\vec{F} = 67\hat{i} + 23\hat{j} + 55\hat{k}$

$\vec{r}_1 = 16\hat{i} + 31\hat{j}$

$\vec{r}_2 = 21\hat{i} + 10\hat{j} + 14\hat{k}$

$W = F\cdot\Delta\vec{r} = (67\hat{i} + 23\hat{j} + 55\hat{k})\cdot(5\hat{i} - 21\hat{j} + 14\hat{k})$

$W = (67\times5)\hat{i} + (23\times-21)\hat{j} + (55\times14)\hat{k}$

$622\,J$

58) $f(x) = ax^{\frac{3}{2}} = 0.75\,x^{\frac{3}{2}}$

$x = 0 \longrightarrow x = 14\,m$

$W = \int_0^{14} f(x)\,dx$

$= \int_0^{14} 0.75\,x^{\frac{3}{2}\cdot\frac{3x^2}{1\times2}}\,dx$

$= \frac{3}{2}\times 0.75\,x^{\frac{5}{2}}\Big|_0^{14} = \left(\frac{0.75\times(14)^{\frac{5}{3}}}{2.5}\right) + (0) = 22.0\,J$

59) $\vec{F}_1\cdot\vec{F}_2 = \frac{1}{3}|F|^2$

$\theta = \cos^{-1}(\frac{1}{3})$

$\cos\theta = \frac{\vec{F}_1\cdot\vec{F}_2}{|\vec{F}||\vec{F}|} = \frac{\frac{1}{3}|\vec{F}|^2}{|\vec{F}|^2}$ $\quad\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{\frac{1}{3}A^2}{A^2} = \frac{1}{3}$

57) $F = F_0\left(\frac{L_0 - x}{L_0} - \frac{L_0^2}{(L_0 + x)^2}\right)$

$W = \int_0^x F(x)\,dx$

$W = F_0\left(\frac{1}{L_0}\int(L_0 - x)\,dx - L_0^2\int\frac{1}{(L_0+x)^2}\,dx\right)$

$$W = F_0 \left(\left(\frac{1}{l_0} \left(L_0 x - \frac{x^2}{2} \right) - l_0^2 \int u^{-2} du \right. \right.$$

$$W = F_0 \left(\left(\frac{1}{l_0} \left(L_0 x - \frac{x^2}{2} \right) + \left(l_0^2 \frac{1}{(l_0 + x)} \right) \right|_0^x \right.$$

$$W = F_0 \left(x - \frac{x^2}{2 l_0} \right) + \frac{l_0^2}{l_0 + x} - l_0 \Big)$$

73) $P = \dfrac{P_0 t_0^2}{(t + t_0)^2}$

$t = 0 \longrightarrow \infty$

$W = P_0 t_0$ show

$$W = \int_0^\infty P\,dt$$

$$\int_0^\infty \frac{P_0 t_0^2}{(t + t_0)^2} dt$$

$$= P_0 t_0^2 \int_0^\infty \frac{dt}{(t + t_0)^2}$$ let $u = t + t_0$

$du = dt$

$$W = P_0 t_0^2 \int_0^\infty u^{-2} du$$

$$= P_0 t_0^2 \left(\frac{u^{-1}}{-1} \Big|_0^\infty \right)$$

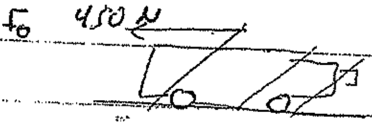
$$P_0 t_0^2 \left(\frac{1}{(t + t_0)} \Big|_0^\infty \right)$$

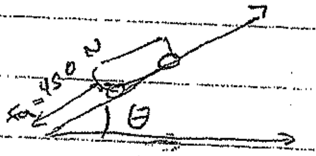
$$= P_0 t_0^2 \left(\frac{1}{\infty} - \frac{1}{t_0} \right)$$

$$= P_0 t_0^2$$

64) 1400 kg car

$v = 60 \, Km/h$





v is constant

$W_{net} = 0$

$W_{net} = 0 \qquad P_{net} = 0$

$P_c - P_d - P_g = 0$

$P_c - \vec{f_a} \cdot \vec{v} - \vec{f_g} \cdot \vec{v} = 0$

$38 \times 10^3 - 450 \times \frac{60}{3.6} - 1400 \times 10 \sin\theta \times \frac{60}{3 \times 6}$ find $\sin\theta$??

$y = -\sec u \qquad u = x^2 + 7x$

$y' = \left(-\sec x^2 + 7x \tan x^2 + 7x\right)\left(2x + 7\right)$

$y = \sec(\tan x)$

$y = \sec u \qquad \tan x = u$

$= \left(\sec \tan x \tan(\tan x)\right) \times \sec^2 x$

$y = 5\cos^{-4} x \qquad y = \frac{5x}{5u^{-4}}$

$y = \cos^{-4} u \qquad +4\cos^{-3} u \times \sin u$

$u = 5x \qquad +4\cos^{-3} 5x \times \sin 5x \,(5)$

$= 20(\cos^{-3} 5x)(\sin 5x)$

$r = (\csc\theta + \cot\theta)^{-1} \qquad \frac{1}{\tan} \quad \frac{\cos}{\sin} \quad \frac{1}{\csc 2}$

$-1(\csc\theta + \cot\theta)^{-2} \times \left(\csc\theta \tan\theta' + \tan\theta\right)$

$$-\frac{\csc\theta \tan\theta + \tan\theta}{(\csc\theta + \cot\theta)^2}$$

$y = -\sec u \qquad u = x^2 + 7x$

$y' = \left(-\sec x^2 + 7x \tan x^2 + 7x\right)\left(2x+7\right)$

$y = \sec(\tan x)$

$y = \sec u \qquad \tan x = u$

$= \left(\sec \tan x \tan(\tan x)\right) \times \sec^2 x$

$y = 5\cos^{-4} x \qquad y = \frac{5x}{5u^{-4}}$

$y = \cos^{-4} u \qquad +4\cos^{-3} u \times \sin u$

$u = 5x \qquad +4\cos^{-3} 5x \times \sin 5x\,(5)$

$= 20\left(\cos^{-3} 5x\right)\left(\sin 5x\right)$

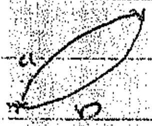
$r = \left(\csc\theta + \cot\theta\right)^{-1} \qquad \frac{1}{\tan} \quad \frac{\cos}{\sin} \quad \frac{1}{\csc x}$

$-1\left(\csc\theta + \cot\theta\right)^{-2} \times \left(\csc\theta\tan\theta + \tan\theta\right)$

$$-\frac{\csc\theta\tan\theta + \tan\theta}{\left(\csc\theta + \cot\theta\right)^2}$$

Ch. 7

1) Cons force & nonconservative

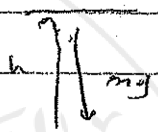


$w_{ab_1} = w_{ab_2}$

$\boxed{w_{round} = 0}$

Potential Energy $\Delta U = \int f_c \, d\vec{r}$

$\Delta U(x) = \int f_c(x)\,dx$



$W_g = mgh$

$\Delta U(x) = -mgh$

$W_s = \int_{x_i}^{x_f} (-kx)\,dx$

$= \frac{1}{2}k(x_f^2 - x_i^2)$

$\Delta K = W_{tot} = W_{con.} + W_{noncons}$

$\Delta U = W_{con.}$

$\Delta K = -\Delta U + W_{noncons}$

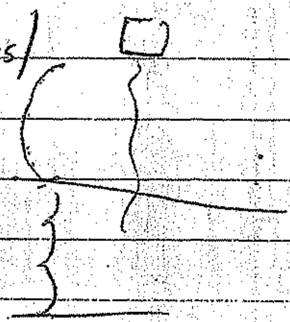
$\Delta K + \Delta U = W_{noncons}$

$\Delta(K + U) = W_{non}$

$\Delta E = W_{noncon}$

$E_i = E_f$ for cons forces

$\Delta E = 0$ (cons)



$\frac{1}{2}kx^2 - mgx - mgh = 0$

$E_i = E_f$

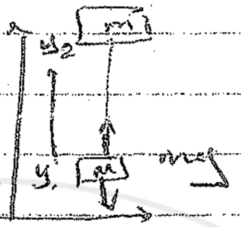
$U_i + 0 = U_f + 0$

$mgh + 0 = \frac{1}{2}kx^2 - mgx$

$W_{cons} = -\Delta U$

In this course there are 2 conservative forces:-

$\boxed{1}$ $\vec{F} = m\vec{g}$



$$\Delta U = W_{cons}$$

$$\Delta U = \int_{y_1}^{y_1} mg\,dy$$

$$U_2 - U_1 = -\int_{y_1}^{y_2} -mg\,dy$$

$$U_2 - U_1 = mgy \Big|_{y_1}^{y_2}$$

$$U_2 - U_1 = mgy_2 - mgy_1$$

take $U_1 = 0$ at $y_1 = 0$

$$U_2 = mgy_2$$

In general $U_g = mgy$ J

$\boxed{2}$ $F_s = -kx$

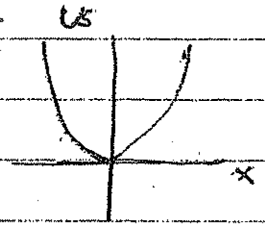
$$\Delta U = -\int_{x_1}^{x_2} F_{cons}\,dx$$

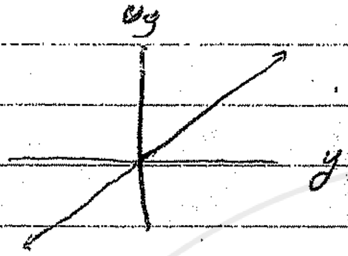
$$U_2 - U_1 = -\int_{x_1}^{x_2} -kx\,dx$$

$$U_2 - U_1 = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

Take $U_1 = 0$ at ~~y~~ $x_1 = 0$

$U_s = \frac{1}{2} k x^2$





Conservation of mechanical energy :-

mechanical → $E = K + U$

In any system there are a set of forces some of them are conservative and the other are nonconservative.

$W_{tot} = W_{con} + W_{non}$

$\Delta K = -\Delta U + W_{non}$

$\Delta K + \Delta U = W_{non}$

$W_{non} = \Delta E$

F non is an friction force

if $F_{non} = 0$

$\Delta E = 0$

$E_1 = E_2$

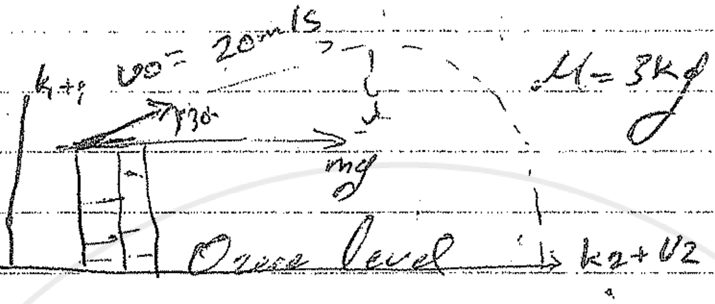
$K_1 + U_1 = K_2 + U_2$

$K_1 + U_1 = K_2 + U_2$

Conservation of Mechanical energy

* $F_{non} = 0$

Example:



find v_2?

$K_1 + K_2$

$K_1 + U_1 = K_2 + U_2$

$\frac{1}{2} m_1 v_1^2 + mgy = \frac{1}{2} m v_2^2 + 0$

$\frac{1}{2}(3)(20)^2 + (3)(10)(100) = \frac{1}{2}(3)(2)^2$

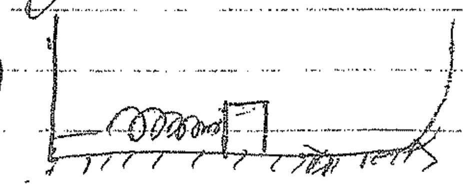
$600 + 300 = 1.5 v_2^2$

$v_2 = 48.9$ m/s

② Find Work done by gravity?

$W_{g,1\to 2} = mgy = 3(10)(100) = 3000$ J

Ex 2)



R = 700 N/m

m = 100g

k = 7 (06. N +1)

$\mu_k = 0.4$ cc

horizontal surfaces

find h on the frictn
on the f.d plane.

the mass compend the spring

30cm

find h

$W_{nonc} = \Delta E$

$W_f = E(2) - E(1)$

$W_f = E_2 - E_1$

$= (K+U)_2 - (K+U)_1$

$= 0 + mgh|_0 - (0 + \frac{1}{2}kx^2)$

$= mgh - \frac{1}{2}k(0.3)^2$

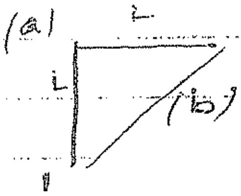
$f_k(20)\cos 180 = mgh - \frac{1}{2}(k)(0.3)^2$

$-\mu_k mg(20) = mgh - \frac{1}{2}k(0.3)$

$-8 = (0.1)(10)h - 31.5$

$23.5 = h$

20) $W_a =$



$W_a = -2L f_k = -2l(M_k mg)$

Razan-T

$W_a = -r_2 L f_k = r_2 l(M_k mg)$

$W = -2l f_k$

$-2L(M_k mg)$

$= -r_2 L f_k = r_2 L(M_k mg)$

17) $U = 210 J$

$K = 1.4 kN$

$x = ??$

$U_s = \frac{1}{2} K x^2$

$x = \sqrt{\frac{2U}{K}} = \sqrt{\frac{2(210)}{1400}} = 0.55 m$

$0.55 m$

$W = -2L f_k$

$= -2L(M_k mg)$

$-2r_2 L(f_k)$

$= 2r_2 L(M_k mg)$

18) $U = ??$

$K = 0.046 kN/pm$

$x = 26 pm$

$U = \frac{1}{2} k x^2$

$= \frac{1}{2}\left(\frac{0.046}{10^{-6}}\right)(10^{-12}) \times (26 \times 10^{6})^2$

$\frac{0.046 \times}{10^{-6}}$

10

$= \quad J$

-12

33) $F = -kx + bx^2 - cx^3$

$b = 4.1 N/m^2$

$c = 3.1 N/m^2$

$K = 223 N/m$

$x_0 = 0 \quad x = 2.26$

$$\mu(x) = \int_{x_0}^{x_0} F(x)\,dx$$

$$v(x) = -\left(-k\int_0^{2.62} x\,dx + b\int_0^{2.62} x^2\,dx - c\int_0^{2.62} x^3\,dx\right.$$

$$u(x) = -\left(\frac{-kx^2}{2}\Big|_0^{2.62} + b\frac{x^3}{3}\Big|_0^{2.62} - c\frac{x}{4}\Big|_0^{2.62}\right.$$

$$v(x) = +777.3$$

20) $m = 10{,}000\,kg$

$K = 40\,kN/m$

$x = 25\,m$

$v = ??$

$E_i = E_f$

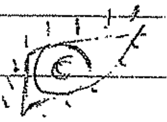
$0 + \frac{1}{2}mv^2 = \frac{1}{2}kx^2 + 0$

$v = \sqrt{\frac{k}{m}}\;x$

$= \sqrt{\frac{40{,}000}{10{,}000}} \times 25$

$50\,m/s = 180\,km/h$

21) $E_i = E_f$



$\frac{1}{2}kx^2 = mgh$

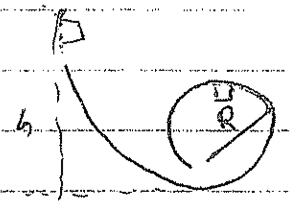
$\frac{1}{2}(930)(0.71)^2 = (0.12)(10)h$

$h = 6.4\,m$

23) $\frac{1}{2}kx^2 = mgh$

$k = \frac{2mgh}{x^2} = \frac{(2)(0.065)(10)(35)}{(0.14)^2} = 232$ N/m

45)



$\frac{mv^2}{R} = mg + N$

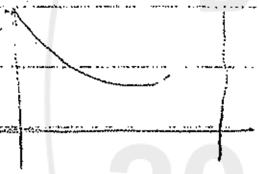
$N = 0$

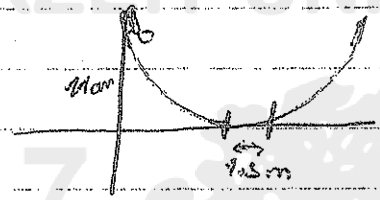
$v = \sqrt{Rg}$

$gh = \frac{1}{2}mv^2 + mg(2R)$

$mgh = \frac{1}{2}mRg + 2mRg = \frac{5}{2}mRg$

54)





$W_{n.c.} = -f_k d$

$= -\mu_k mgd$

$\Delta U = -mgh$

$= mg \times 0.11$

$= -0.61 \times 0.015mg$

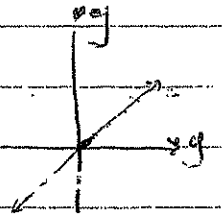
$=$ # of trips $= \frac{\Delta U}{W_{nc}}$

$\frac{mg \times 0.11}{mg \times 0.61 \times 0.015}$

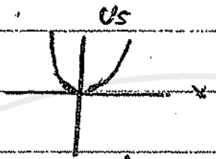
$= 12.022$ un

Ch7.) Potential Energy curves

$\vec{F_g} = \vec{mg} \Rightarrow U_g = mgy$

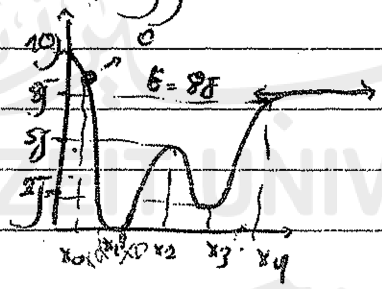


$\vec{F_s} = -kx \quad U_s = \frac{1}{2}kx^2$



for other conservative forces

Potential energy force could be



$k + U = E$

$k = E - U$

let $E = 8J$

at $x = 1 \quad U = 0 \quad k = 8J$

$x_2 = U_2 = 5J \quad k_2 = 3J$

at x_0, $U_0 = 8J \quad k_0 = 0$

x_0 is called turning point

at $\Rightarrow k_4 \Rightarrow x_4$; $U_4 = 8J \quad k_4 = 0$

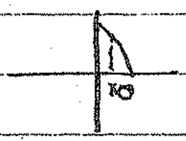
it stays at x_4

$W_{cons} = -\Delta U \qquad F_{cons} = -\frac{\Delta U}{\Delta x} \left(-\frac{dU}{dx}\right)$

$F_{cons} \cdot \Delta x = -\Delta U$

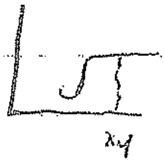
at x_0 F is positive

"to the right"



at x_4

$F = 0$



x_4 is

called equilibrium point

At the turning point $K = 0$ condition

$E = U$ potential energy

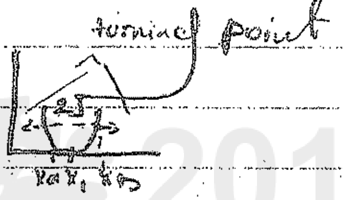
Ex2) Let $E = 2J$

at $x_{1\,left}$ (x_{1a})

at x_{1a}, $U = 2J$

K

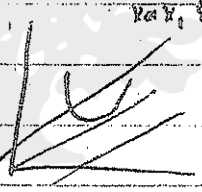
x_{1a} is called a turning point



$F(x_{1a})$ is to the right

at X_{1b} $U = 2J$, $K = 0$

x_{1b} is a turning point



at x_3 $U = 2$ $k = 0$ $F = 0$ equalibrium point

$F(x_4) = 0$, (x_4, x_5) Neutral equalibrium

x_2 unstable equalibrium

$F(x_2) = 0$ unstable equalibrium point

Ex2) $U(x) = \frac{6}{x^2} - \frac{1}{x}$ $x > 0$

Let $E = 15\,J$

1) find the turning points

~~At the turning point~~

$E = U$ // $15J = U$

$$15 = \frac{6}{x^2} - \frac{1}{x}$$

$$15 = \frac{6-x}{x^2}$$

$$15x^2 = 6 - x$$

$$15x^2 - x + 6 = 0$$

~~$(15x - 5)(x -) = 0$~~

$$(5x-3)(3x+2) = 0$$

$x = -\frac{2}{5}$ m − Refused

$\underline{x > 0}$

$x = \frac{3}{5}$ m The turning point

2) find the conservative force associated to U (corresponds)

$F = -\frac{dU}{dx}$ $F(x) = -\left[[illegible] \right]$

$$= -\left[\frac{-12}{x^3} + \frac{1}{x^2} \right]$$

$$= \frac{12}{x^3} - \frac{1}{x^2}$$

$$= \frac{12 - x}{x^3}$$

3) find the equalibrium points

$F=0$

$0 = \frac{12 - x}{x^3}$

$\underline{\underline{x = 12 \text{ m}}}$

$x = 12$ (lossetups $= c$)

Homework

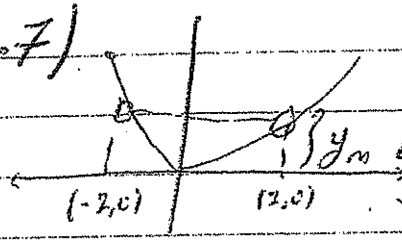
$m = 0.2$ kg moves
under the action of
Force.
where U as a function of x
$U(x) = 8x^2 + 2x^2$ J

at $x = 1$m
$v = 5$m/s

① find the coordinates of the next turning point

② find v at $x = \frac{1}{2}$ m 0.5m

Ch.7)



y_m turning point $E = U$

(stop) instantly

at $x_1 \Rightarrow E = U$

$y = 0.92x^2$
$v = 8.5 \text{ m/s}$

$E = \frac{1}{2}mv_m^2$

$E = mgy_m$

$\frac{1}{2}\cancel{m}v_m^2 = \cancel{m}gy_m$

$\frac{1}{2}v_m^2 = gy_m$

$y_m = \frac{v_m^2}{2g}$

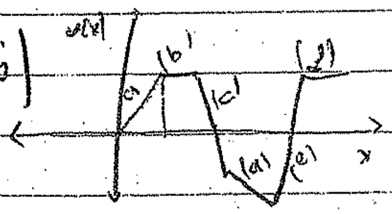
$y_m = \frac{(8.5 m/s)^2}{2(10)}$

$= \underline{\underline{3.5 m}}$

$y_m = 0.92x^2$

$x_m = \sqrt{\frac{y_m}{0.92}} = \sqrt{\frac{3.5m}{0.92}} \approx 2m$

26)



any function U(x) is a cons force

$F(x) = \frac{du}{dx} = \frac{-\Delta U}{\Delta x}$

$F_a = \frac{-(3-0)}{(1.5-0)} = -2N$

49.) $F_x = 5x - 2x^3$

$U = 0$ $x = 0$

$U(x) = -\int F(x)\,dx$

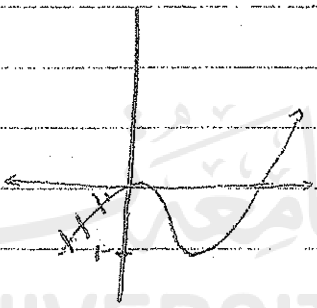
$= -\int (5x - 2x^3)\,dx$

$= -\left(\frac{5x^2}{2} - \frac{x^4}{2}\right) + C$

$U(0) = 0$

$C = 0$

$= -\frac{5x^2}{2} + \frac{x^4}{2}$



$U = E$

$E = -1$

turning point

$-\frac{5}{2}x^2 + \frac{x^4}{2} = -1$ 15 min

$5x^2 - x^4 = 2$ $x^2 = u$

$x^2(5 - x^2) = 2$

$5u - u^2 = 2$

$u^2 - 5u + 2 = 0$

59) $h = 0.18x^2$

$U_{max} = 47$ cm/s

$E = K_{max} = \frac{1}{2} m v_{max}^2$

$E = U = mgh$

$E_f = E_i$

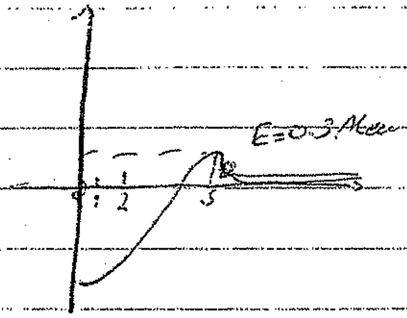
$mgh = \frac{1}{2} m v_{max}^2$

$h = \frac{v^2_{max}}{2g} = \frac{(0.47)^2}{2(10)}$

$\frac{0.18x^2}{0.18} = \frac{0.1}{0.18}$

$\sqrt{x^2} = \sqrt{0.5}$

$= 0.7$ m



68) E=0
at x = 1 , 5fm

69) $\boxed{d}$ k= 0.3 Mev

70) E= 3.3 Mev

8) Gravity (chapter 8)

⇒ Universal Gravitation:-

between any two objects there is an attractive gravitational force

$$F = G \frac{m_1 m_2}{r^2}$$

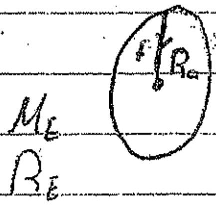
$G = 6.67 \times 10^{-11}$ Nm^2/Kg^2

G: Gravitational Universal force constant

$F = G \frac{m_1 m_2}{r^2}$
- → force between 2 point Masses
- → force between 2 spherical Masses
- → force between 2 any shaph Masses where r is very large

The acceleration of Gravity

1) Near the earth surface



$F = \frac{G m M_E}{R_E^2} = ma$

$a = \frac{G M_E}{R_E^2}$

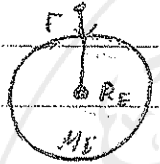
$R_E = 6.37 \times 10^6$ m

$M_E = 5.97 \times 10^{27}$ kg

$g = \frac{GM_E}{R_E^2}$ Near the surface

$9.81 m/s^2$

2) g at 380 km above the earth surface



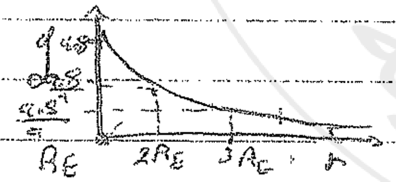
$F_{on\,m} = \frac{GmM_E}{(R_E + R)^2}$

= ~~6.67×~~

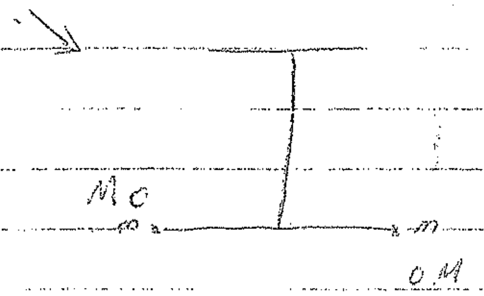
$a = \frac{GM_E}{(R_E + R)^2}$

$= \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6 + 380 \times 10^3)^2}$

$= 8.74 m/s^2$



The Cavendish experment :

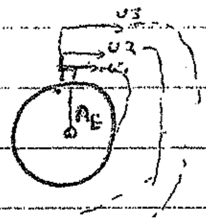


m, M, r

F measure

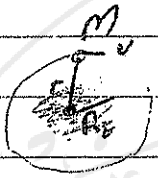
from rotation

Orbital Motion



v1, v2, v3 are not enough for (m) to rotate around the earth

Find |v| for any Mass to Move in a ~~circlea~~ circular Motion around the earth



$$F_{on\,m} = \frac{G m M_E}{r^2}$$

$r = (R_E + \text{height above the surface of the earth})$

this force is a centripetal force

$$\frac{m v^2}{r} = \frac{G m M_E}{r^2}$$

$$v^{\cancel{2}} = \sqrt{\frac{G \cancel{m} M_E \cancel{r}}{\cancel{m} r^{\cancel{2}}}}$$

$$v = \sqrt{\frac{G M_E}{r}} \quad \textcircled{v}$$

$$v = \frac{2\pi r}{T}$$ T Periodic time

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{G M_E}{r} \qquad T^2 = \frac{4\pi^2 r^3}{G M_E}$$

Chapter 8:)

8) M_e, r, G, T

$$T^2 = \frac{4\pi^2 r^3}{G M_e}$$ We can't find the mass from the information above!

$$\frac{G M_e \cancel{m_m}}{r^2} = \frac{\cancel{m_m} v^2}{r}$$

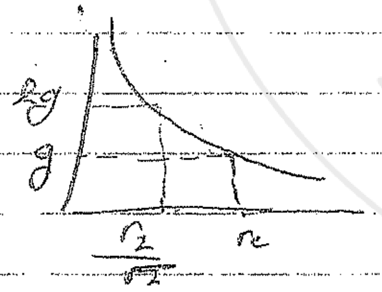
11) $M_u = M_e$

$W_u = 2W_e$

$$a = \frac{GM}{r^2}$$

$$\frac{g_u}{g_e} = 2 = \left(\frac{r_e}{r_u}\right)^2$$

$$r_u = \frac{r_e}{\sqrt{2}}$$

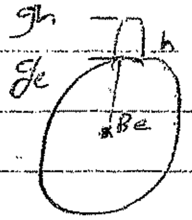


13) $g_n = 3g$

$$\frac{g_n}{g_e} = 3 = \left(\frac{r_e}{r_n}\right)^2$$

$$r_n = \frac{r_e}{\sqrt{3}}$$

17)



$$\frac{g_h}{g_e} = \frac{g - 1.36 \times 10^{-3}}{g}$$

$$= \left(\frac{R_e}{R_h}\right)^2 = \left(\frac{r_e}{r_e + h}\right)^2 \Rightarrow h ??$$

19)
$$T^2 = \frac{4\pi^3 r^3}{GM}$$

$$(24 \times 3600)^2 = \frac{4\pi^2}{G M_e} r^3 \Longrightarrow r ??$$

$$\frac{G M m}{r^2} = \frac{m v^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

$F = G\frac{m_1 m_2}{r^2}$

$U_\infty = 0$ $\quad U = -\frac{G m M_E}{r}$

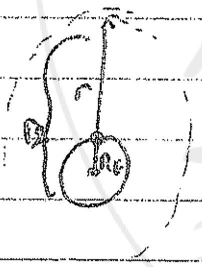
Geosynchronous orbit for a sattelite

$T = 24h$

We calculate $r = 4.22 \times 10^7 m$

Example 1

find the work done ~~by~~ in having sattelite of mass = $m = 11 \times 10^3 Kg$ from the earth surface to the geosyncronous orbit $= 4.22 \times 10^7 m$



$W_g = -\Delta U$

$W_{done\ by\ F_{ext}} = \Delta U$

$\underset{(1) \longrightarrow (2)}{\underset{SE \to Geo}{W_{done}}} = 6.67 \times 10^{-11} \times 11 \times 10^3 \times 5.97 \times 24$

$\underset{\overset{SE \to Geo}{1 \quad\quad 2}}{W_{done}} = U_2 - U_1$

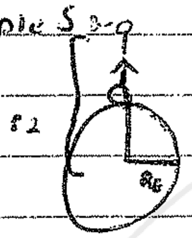
$= -\frac{G m M_E}{r_2} - \left(-\frac{G m M_E}{R_E}\right)$

$G m M_E \left(\frac{1}{R_E} - \frac{1}{r_2}\right)$

$$W_{done\ 1\to 2} = 6.67\times10^{-11}\times 11\times10^{3}\times 5.97\times10^{24}\left[\frac{1}{6.37\times10^{6}} - \frac{1}{4.22\times10^{7}}\right]$$

$$= 5.8\times10^{11}\ \text{J}$$

Examples 5-9



$v_0 = 3.1\times10^{3}$ m/s

$$(K+U)_0 = (K+U)_2$$

$$\frac{1}{2}mv_0^2 - \frac{GmM_E}{R_E} = 0 + -\frac{GmM_E}{r_2}$$

$$\frac{v_0^2}{2} - \frac{GM_E}{R_E} = -\frac{GM_E}{r_2}$$

6Ans:

$$r_2 = 6.9\times10^{6}\ \text{m}$$

$$h = r_2 - R_E$$

$$= 6.9\times10^{6} -$$

ⓐ

→ Escape speed:-

find the escape speed from earth surface?

outside at the gravity

$K_2 = $ zero

$U_2 = $ zero

$$K_0 + U_0 = (K+U)_2$$

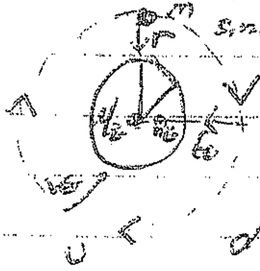
$$\frac{1}{2}mv_0^2 - \frac{GmM_E}{R_E} = 0 + 0$$

$$v_0 = \sqrt{\frac{2GM_E}{R_E}} = 11.2\ \text{km/s}$$

At the earth surface $K_0 = \frac{1}{2}mv_0^2$ escape speed

$$U_0 = \frac{GmM_E}{r_E}$$

Energy in circular orbits:-



you are given (G, M_E, R_E, m, r)

find $U(r)$ at $k(r), E(r)$

$$U(r) = -\frac{GmM_E}{\cancel{R_E}\, r}$$

To find K

$$F = \frac{GmM_E}{r^2}$$

$$\frac{mv^2}{r} = \frac{GmM_E}{r^2} \Rightarrow \text{Multiplay by half}$$

$$\frac{1}{2}mv^2 = \frac{GmM_E}{2r} = K$$

note that $k = -\frac{1}{2}U$, $U = -2K$

When it zero its bind to go of

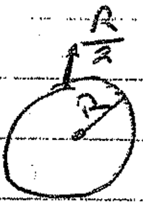
$$E = K + U$$

$$E = \frac{-GmM_E}{2\cancel{r}} , E < 0$$

means m is bounded to the earth

37/ $g = 22.5\ m/s^2$

find g' at $h = \frac{R}{2}$



$$a = \frac{GM}{r^2}$$

$$\frac{a_2}{a_1} = \frac{r_1^2}{r_2^2} = \frac{(\cancel{\tfrac{3}{2}}R)^2}{(\frac{3}{2}R)^2}$$

$$\frac{a_2}{a_1} = \frac{\cancel{9}}{\cancel{4}}\ \frac{4}{9}$$

$$a_2 = \frac{4}{9} a_1 = \frac{4}{9} \times 22.5 = 10\ m/s^2$$

26

$E_i = E_f$

$$\frac{1}{2} m v_0^2 - \frac{GMm}{r_e} = -\frac{GMm}{r_m}$$

find r_m

$h = r_m - r_e$

38/ $v_e = 7.1\ km/s$

$M = 2.9 \times 10^{24}\ Kg$

$r_p = ??$

$$v_e = \sqrt{\frac{2GM}{r_p}}$$

$$7.1 \times 10^3\ m/s = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 2.9 \times 10^{24}}{r_p}}$$

$$r_p = \frac{2 \times 6.67 \times 10^{-11} \times 2.9 \times 10^{24}}{(7.1 \times 10^3)^2}$$

$$= 7.6 \times 10^6\ m = 7660\ Km$$

38) $m = 1\,kg$

$$W = \Delta U$$

$$= U_f - U_i$$

$$= \frac{GMm}{r_s} - \left(-\frac{GMm}{r_e}\right)$$

$$= GMm\left(\frac{1}{r_e} - \frac{1}{r_s}\right)$$

$$= 6.67\times10^{-11} \times 5.97\times10^{24} \times 1\left(\frac{1}{6.37\times10} - \frac{1}{42.2\times10^{6}}\right)$$

49) $\frac{1}{2}mv^2 - \frac{GMm}{r_e} = 0$

$$v = \sqrt{\frac{2GMm}{r_e}}$$

61) $\frac{v_E}{v_A} = \frac{\sqrt{\frac{2G M_E \cancel{r}}{R_E}}}{\sqrt{\frac{2G M_A \cancel{b}}{R_A}}} = \sqrt{\frac{M_E R_A}{M_A R_E}}$

$$= \frac{11.2\times10^{3}}{7.74}$$

$$= \sqrt{\frac{(5.97\times10^{24})R_A}{\left(\frac{4}{3}\pi R_A^2\right)(e)(6.37\times10^{6})}}$$

$$\left(\frac{11.2\times10^{3}}{7.74}\right)^2 = \quad R_A = 6560\,m$$

$$T^2 = \frac{4\pi^2 r^3}{GM} \qquad r = 20\,200\,m$$

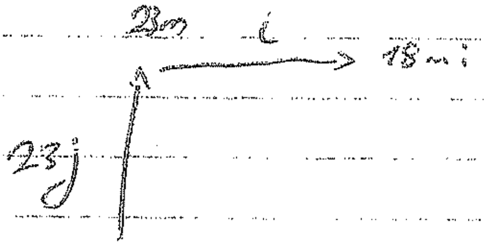
$$v = \frac{2\pi r}{T}$$

$$E = 0 = \frac{1}{2} m v_e^2 - \frac{GMm}{r_s}$$

$$v_e = \sqrt{\frac{2Gm}{r_s}}$$

$$E = \frac{1}{2} v^2 m \rightarrow 8411 kg$$

$m = 650$ kg



$W = \oint F \Delta r$

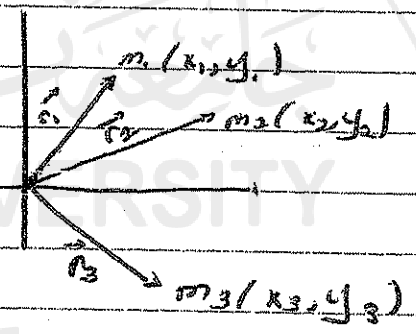
Chapter 9:-

Systems of Particles

⇒ Centre of mass: "CM"

Centre of mass for a system containing many Particles is a point where ①all the masses are position (centre) ② $\Sigma\vec{F}$ are applied at that point

cm for many particles:



$$X_{cm} = \frac{m_1x_1 + m_2x_2 + \dots}{m_1 + m_2 + \dots}$$

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i$$

$$y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_iy_i$$

total Mass

$$Z_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_iz_i$$

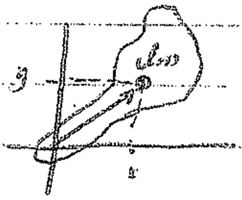
$$\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

mple: $m_1 = 0.5\,kg$ at $(1,2)m$ ⇒
$m_2 = 0.4\,kg$ at $(3,2)m$
$m_3 = 1.1\,kg$ at $(3,-1)m$
find $\vec{r}_{cm}$?

$$x_{cm} = \frac{1}{M}\sum m_ix_i$$
$$= \frac{1}{2}\left[0.5(1) + (0.4)(3) + (1.1\times 3)\right]$$
$$= \frac{5}{2} = 2.5\,m$$
$$y_{cm} = \frac{1}{2}\left[0.5\times 2 + 0.4\times 2 + 1.1\times -1\right]$$
$$= 0.35\,m$$

$r_{cm} = 2.5\hat{i} + 0.35\hat{j} = 2485\,m$

cm for a rigid body?

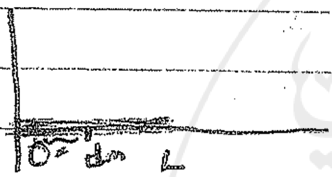


$$X_{cm} = \frac{1}{M}\int x\,dm$$

$$y_{cm} = \frac{1}{M}\int y\,dm$$

Example one:-

Find x for cm for a uniform rod of length = L, mass = L



$X_{cm} = \frac{1}{M}\int x\,dm$, $dm = \lambda\,dx$ λ = linnear massdensity

$$\lambda = \frac{M}{L} \text{ kg/m}$$

$$= \frac{1}{M}\int_0^L x(\lambda\,dx)$$

$$= \cancel{\frac{1}{M}}\int_0^L x\left(\frac{\cancel{M}}{L}\right)$$

$$= \frac{1}{M}\int_0^L x\left(\frac{M}{L}\,dx\right)$$

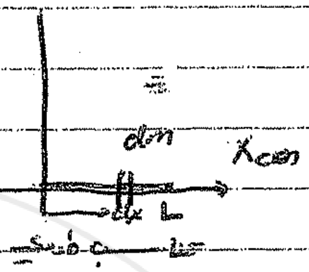
$$X_{cm} = \frac{1}{L}\int x\,dx = \frac{1}{L}\left(\frac{L^2}{2}\right)$$

$$X_{cm} = \frac{L}{2}$$

example 2 :-

find X_{cm} for a nonuniform rod of length (L) $A \cdot \lambda = \alpha x$

$\Uparrow$ any number



Find the mass of the rod?

$$M = \int_0^L dm = \int_0^L \lambda \, dx$$

$$= \int_0^L \alpha x \, dx$$

$$(\alpha)\left(\frac{x^2}{2}\right)\Big|_0^L = \alpha\left(\frac{L^2}{2}\right) \text{ kg}$$

$$X_{cm} = \frac{1}{M}\int x \, dx$$

$$\frac{1}{M}\int_0^L x(\lambda \, dx)$$

$$\frac{1}{M}\int_0^L x(\alpha x) \, dx$$

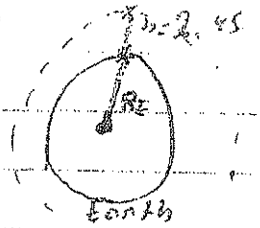
$$= \frac{1}{M}\int_0^L \alpha x^2 \, dx$$

$$= \frac{\alpha}{M}\int_0^L x^2 \, dx$$

$$= \frac{\alpha}{M}\left[\frac{x^3}{3}\Big|_0^L\right] = \frac{\alpha}{M}\left[\frac{L^3}{3}\right] = \frac{\cancel{\alpha} L^{\cancel{3}}}{\cancel{\alpha}\cancel{L^2}/2 \times 3}$$

$$\boxed{X_{cm} = \frac{2L}{3}}$$

ch 8)

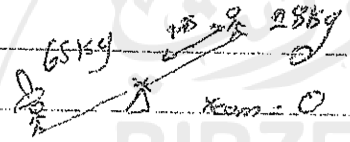


$E_i = E_f$

$$\frac{1}{2}mv_e^2 - \frac{GM_e m}{R_e} = \frac{-GM_e m}{R_e + h}$$

$$\frac{1}{2}mv_0 - \frac{GM_m}{R_m} = \frac{-GM_m m}{R_m + h'}$$

ch 9) 12)

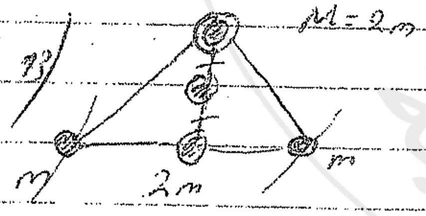


$$x_{cm} = 0 = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 0$$

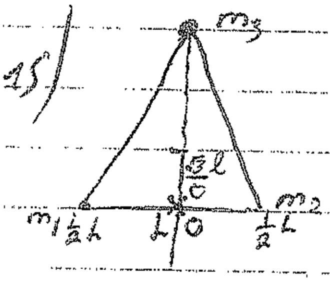
$$\frac{28 \times 1.75 + 65(x_2)}{28 + 65} = 0$$

$$x_2 = \frac{-28 \times 1.75}{65} = 0.75m \boxed{\cancel{F}}$$

13)



15)



$$Y_{cm} = \frac{m_1(0) + m_2(0) + m_3(h)}{3m} = \frac{1}{3}h$$

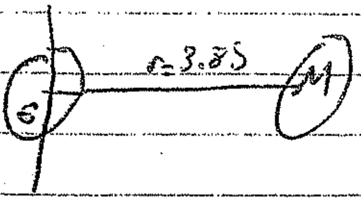
$l \sin 60 = h$

$$\frac{\sqrt{3}}{2}l = h$$

$$Y_{cm} = \frac{1}{3}\left(\frac{\sqrt{3}}{2}l\right)$$

$$C_m = \left(0, \frac{\sqrt{3}}{2}l\right)$$

16)



$$x_{cm} = \frac{M_E x_0 + M_m x_r}{M_E + M_m}$$

$$= \frac{0.6735\times10^{24}}{10^{24}\times(5.97+0.0735)} = 460\,km$$

Chapter 9):-

Newton's second law for a system of many Particles

$$M\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_n\vec{r}_n$$

$$\frac{d}{dt}\left[\;\right].$$

$$m\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_n\vec{v}_n$$

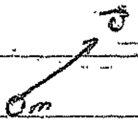
$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \dots + m_n\vec{a}_n$$

$$M\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$$

$$M\vec{a}_{cm} = \vec{F}_{net}$$

⟹ Linear Momentum:-

$\vec{P} = m\vec{v}$ kg m/s



* Linear Momentum for a system of many particles

$M\vec{v}_{cm} = m\vec{v}_1 + m_2\vec{v}_2 + \dots + m_n\vec{v}_n$

$M\vec{v}_{cm} = \vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_n$

$| M\vec{v}_{cm} = \vec{P} |$

$\vec{P} = M\vec{v}_{cm}$

$\vec{F}_{net} = M\vec{a}_{cm}$

The relation between $\vec{F}_{net}$ & $\vec{P}$

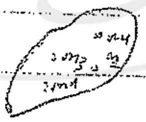
$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots \Rightarrow \frac{d}{dt}[\quad]$

$\frac{d\vec{P}}{dt} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$

$\frac{d\vec{P}}{dt} = \vec{F}_{net}$

The net force is the time rate of changing linear Momentum.

if $\vec{F}_{net} = 0$ on the system



$\frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = $ constant

$\vec{P}_i = \vec{P}_f$ (conservation of linear Momentum)

Condition for Conservation of $\vec{P}$: "System is isolated" "$F_{net}=0$"

Applications on $[\vec{P}_i = \vec{P}_f]$

1) All types of collisions
2) All types of explosions
3) All type of firing missiles
4) Decaying process of mass
انخفاضة

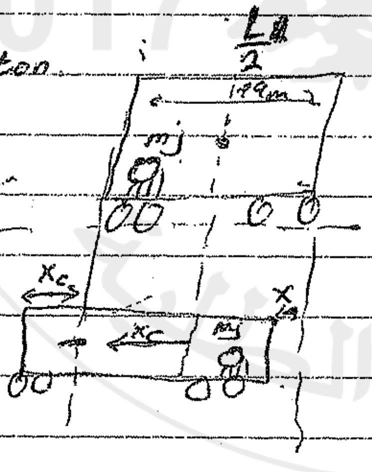
$$\vec{P}_i = \vec{P}_f$$

$$\frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2} = \frac{m_1\vec{v}_{1f} + m_2\vec{v}_{2f}}{m_1 + m_2}$$

$$(\vec{v}_{cm})_i = (v_{cm})_f$$

Ex 4:-

$m_j = 4.8$ ton



$m_c = 95$ ton

$(v_{cm})_i = 0$

$L = 19m$

$\vec{F}_{ext} = 0$ (Jumbo + car) $\qquad (x_{cm})_i = (x_{cm})_f$

$\vec{P}_i = \vec{P}_f \qquad (\vec{v}_{cm})_i = (v_{cm})_f = 0$

$$(X_{cm})_i = \frac{m_j x_{ji} + m_c x_{cd}}{m_j + m_c} = \frac{4.8(0) + M_c(L/2)}{m_j + m_c}$$

$$(X_{cm})_f = \frac{m_j x_{jf} + m_c x_{cf}}{m_j + m_c}$$

$\underline{\underline{m_c = 15}}$

$$(X_{cm})_f = \frac{4.8(19 - x_c) + 15\left(\frac{L}{2} - x_c\right)}{m_j + m_c}$$

$$(X_{cm})_i = (X_{cm})_f$$

$$\cancel{15(L/2)} = 4.8(19) - 4.8x_c + \cancel{15\left(\frac{L}{2}\right)} - 15x_c$$

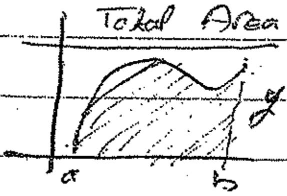
$$0 = 4.8(19) - 19.8x_c$$

$$x_c = 4.6 \text{ m}$$

5.5 , 5.6)

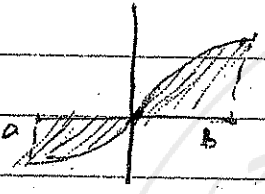
Indefinite Integrals Substitution

Total Area:-



$y=f(x)$

$A=\int_a^b f(x)\,dx$



$= \left|\int_a^0 f(x)\,dx\right| + \int_0^b f(x)\,dx$

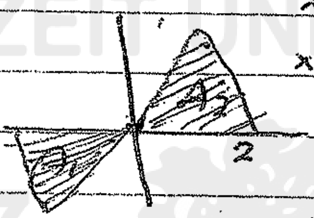
Ex: $f(x)= x^3-x^2-2x$

x-axis $[-1,2]$

$f(x)=0 \Rightarrow x(x^2-x-2)=0$

$x(x-2)(x+1)=0$

$x=0,\ 2,\ -1$



$A=|A_1|+A_2$

maen

wgam

$\int_{-1}^{0}(x^3-x^2-2x)\,dx + \int_0^2 (x^3-x^2-2x)\,dx$

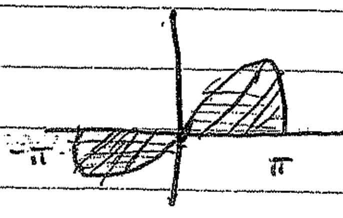
Symmetric functions:-

suppose that $f(x)$ is continuous on a symmetric interval $[-a,a]$

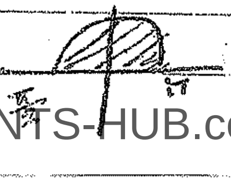
if $f(x)$ is even then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

if $f(x)$ is odd then $\int_{-a}^{a} f(x)\,dx = 0$

Ex: $\int_{-\pi}^{\pi} \sin x\,dx = 0$



$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = 2\int_0^{\pi/2} \cos x\,dx$



$= \int f(g(x))\,g'(x)\,dx = \int f(u)\,du$

$u = g(x)$

$\int_{x=a}^{x=b} f(g(x))\,g'(x) = \int_{g(a)}^{g(b)} f(u)\,du$

Ex:

$\int x\sqrt{2x+1}\,dx$

$u = 2x+1 \Rightarrow \frac{du}{2} = \frac{2}{2}\,dx$

$x = \frac{u-1}{2}$

$I = \int \frac{(u-1)}{2} \cdot u^{1/2} \cdot \frac{du}{2}$

$= \frac{1}{4}\left(u^{3/2} - u^{\frac{1}{2}}\right) du$

$\frac{1}{4} \times \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + c$

$= \frac{1}{4}\left(\frac{2}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2}\right) + c$

② $\frac{1}{3}\int \frac{3x^2}{(x^3+5)^{5/2}}\,dx = \frac{1}{3}\int 3x^2(x^3+5)\,dx$

$= \left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(x^3+5\right)^{-3/2}$

3) $\int \sqrt{\frac{x^4}{x^3-1}}\,dx$ $\qquad u = x^3-1$ $\qquad \sqrt{\frac{x^2}{\sqrt{x^3-1}}}\,dx$

$\frac{du}{3} = \frac{3x^2 dx}{3}$

x+2

$\frac{1}{3}\int u^{-1/2}\,du = \left(\frac{1}{3}\right)(2)\left(u^{\frac{1}{2}}\right) + c$

$= \frac{2}{3}(x^3-1) + c$

④ $\int \sqrt{\frac{x-1}{x^5}}\, dx$

$\int \sqrt{\frac{1}{x^4} - \frac{1}{x^5}}\, dx$

$\int \frac{1}{x^2} \sqrt{1 - \frac{1}{x}}\, dx$

$y = 1 - \frac{1}{x} \qquad dy = \frac{1}{x^2}\, dx$

$\int \sqrt{y}\, dy = \frac{2}{3} y^{3/2} + c = \left(\frac{2}{3}\right)\left(1 - \frac{1}{x}\right)^{3/2} + c$

⑤ $\underline{I} = \int_0^{2\pi} \frac{\cos x}{\sqrt{4 + 3\sin x}}\, dx$

$y = 4 + 3\sin x \qquad \cdot \quad \cancel{dx =} \frac{1}{3} \int_1^4 \frac{dy}{\sqrt{y}}$

$\frac{dy}{3} = \frac{3}{3} \cos x\, dx$

⑥ $\int_0^{\sqrt[3]{\pi^2}} \sqrt{\theta} \cos^2 \theta^{3/2}\, d\theta$

$y = \theta^{3/2}$

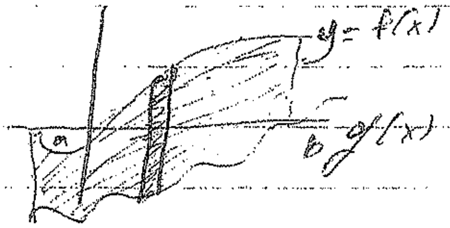
$dy = \frac{3}{2} \theta^{\frac{1}{2}}\, d\theta$

$\frac{2}{3}\, dy = \sqrt{\theta}\, d\theta$

$\underline{I} = \frac{2}{3} \int_0^{\pi} \cos^2 y\, dy \qquad \frac{2}{3} \int_0^{\pi} \left(\frac{1 + \cos(2y)}{2}\right) dy$

$= \frac{2}{3} \left(\frac{y}{2} + \frac{\sin 2y}{4}\right]_0^{\pi}$

Area between 2 curves:



$\sum (f(c_k) - g(c_k))\,\Delta x_k$

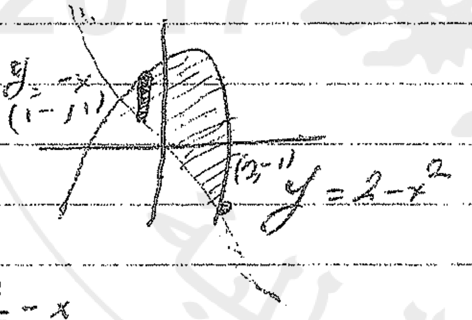
if $f(x)$ $g(x)$ is con $[a, b$

$f(x) \geq g(x)$

$A = \left| \int_a^b (f(x) - g(x))\,dx \right|$

Ex: find Area between

$y = 2 - x^2$ and $y = -x$



$x^2 = -x$

$x^2 - x - 2 = 0$

$x = 2, -1$

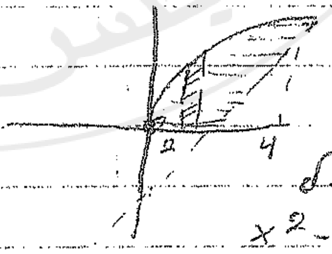
$\int_{-1}^{2} (2 - x^2) - (-x)\,dx$

$\int_{-1}^{2} 2 - x^2 + x\,dx$

Ex: find the area enclosed by $y = \sqrt{x}$

$y = 0$

$y = x - 2$



$\sqrt{x} = x - 2$

$x^2 - 2x + 4 = x$

$x^2 - 5x + 4 = 0$

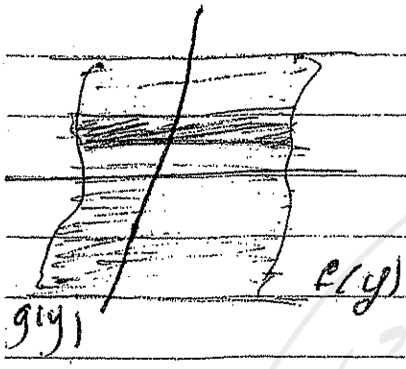
$x = 4$

$x = 1$

$A = \int_0^2 \sqrt{x}\,dx + \int_2^4 \sqrt{x} - x + 2$

integration with respect to y:

$x = f(y)$
$x = g(y)$



$$A = \int_{y=c}^{y=d} (f(y) - g(y))\, dy$$

$$A = \int_{0}^{2} ((y+2) - y^2)\, dy$$

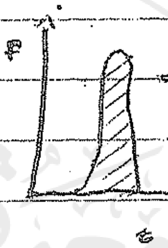
Impulse :- الدفع

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$d\vec{p} = \vec{F}_{net} \cdot dt$$

$$\vec{\Delta P} = \int_{t_i}^{t_f} \vec{F}(t)dt = \vec{J}$$ كمية متجهة

$$\vec{P}_f - \vec{P}_i = \int_{t_i}^{t_f} \vec{F} dt = \vec{J} \quad N.s$$



→ A under the curve

impluse

$\vec{J}$ = area under the curve of F(t) vs. time

Collision :-

"linear momentum is conserved"

external force = 0 "المعزول"

in all types of collisions

$P_i = P_f$

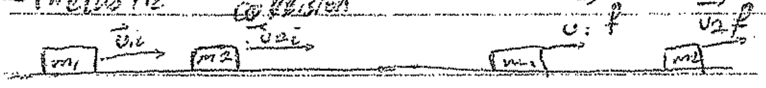
Collision's types:-

Elastic collision | Inelastic collision

طاقة حركية محفوظة | غير محفوظة

$K_i = K_f$ | $K_i \neq K_f$

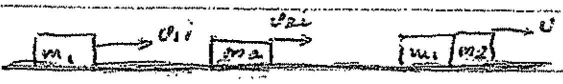
⇒ Inelastic collision



$K_i \neq K_f$

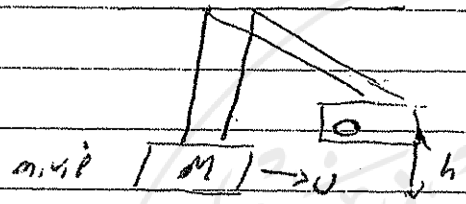
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

* Completely inelastic collision:



$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$ $K_i > K_f$

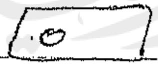
Example (9): Ballistic Pendulum البندول القذفي



in the collision:-

$P_i = P_f$

$= m v_{1i} = (m + M) v$ ①

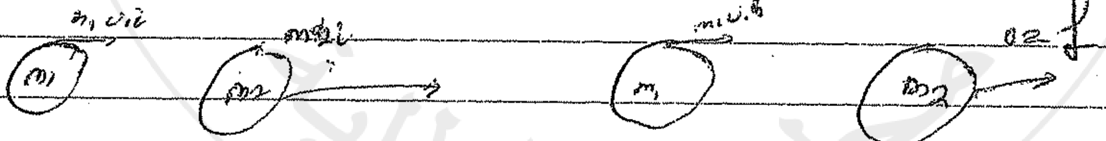
 moves to a height ②

$(K+U)_1 = (K+U)_2$

$\frac{1}{2}(m+M)v^2 = 0 + (M+m)gh$

$v = \sqrt{2gh}$ Sub ① ② find v_1

Elastic collision in One Dimension!



$m_1 v_{1i} + m_2 v_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$ ①

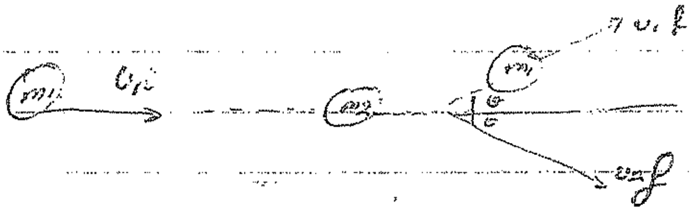
$K_i = K_f$ $\boxed{1 \to 3}$

$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 \vec{v}_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$ ②

from 1 & 2 you can get

$\vec{v}_{1i} - \vec{v}_{2i} = (\vec{v}_{1f} - \vec{v}_{2f})$ ③

Collision 2 Dimensions:-



$\vec{P_i} = \vec{P_f}$

$(P_i)_x = (P_f)_x$

$m_1 v_{1i} = m_1 v_{1f}\cos\theta_1 + m_2 v_{2f}\cos$

$(P_i)_y = (P_f)_y$ ②

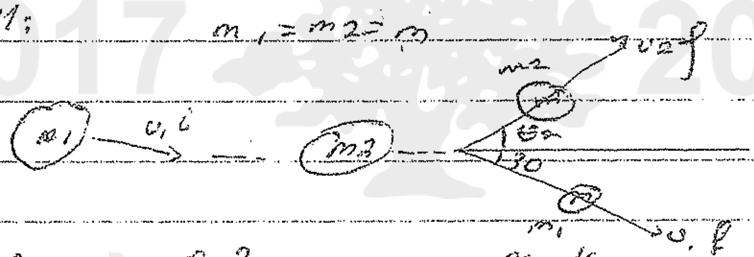
$0 = m_1 v_{1f}\sin\theta_1 + - m_2 v_{2f}\sin\theta_2$

for elastic collisson add

$K_i = K_f$

$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

example 11: $m_1 = m_2 = m$



Find v_{1f}, v_{2f}, θ_2?

$v_{1i} = 50\ m/s$

$v_{2i} = 0$

$(P_i)_x = (P_f)_x$

$m v_{1i} = m v_{2f}\cos\theta_2 + m v_{1f}\cos 30$

$50 = v_{2f}\cos\theta_2 + v_{1f}\cos 30$ —①

$(P_i)_y = (P_f)_y$

$0 = m v_{2f}\sin\theta_2 - m v_{1f}\sin 30$

$v_{2f}\sin\theta_2 = v_{1f}\sin 30 \longrightarrow ②$

② Elastic collision $K_i = K_f$

in One dimension

$\frac{1}{2} m (v_{1i})^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m v_{2f}^2$

$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$ ③

$\vec{P_i} = \vec{P_f}$

$m\vec{v_{1i}} = m\vec{v_{1f}} + m\vec{v_{2f}}$

$\vec{v_{1i}} = \vec{v_{1f}} + \vec{v_{2f}}$

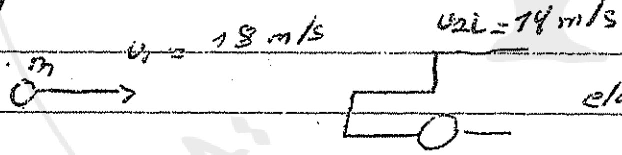
$v_{1i} \cdot v_{1i} = (\vec{v_{1f}} + \vec{v_{2f}}) \cdot (\vec{v_{1f}} + \vec{v_{2f}})$

$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2\vec{v_{1f}} \cdot \vec{v_{2f}}$

$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f}\cos(\theta_2 + 30)$

$\cos(\theta_2 + 30) = 0$

Chapter 9) :



elastic collision

$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{if}$

$m_1 << m_2 = -v_{1i} + 2v_{2i}$

$= -18 + 2(-14)$

$= -46$ m/s

* Chapter 9 *

Faiml

*26-11-2014 (physics 141 - chapter 9)

*Systems of Particles → center of mass → (مركز الكتلة)

$\underbrace{\text{center of mass}}_{(cm)}$

* center of mass for a system containing many Particles is a point where $\boxed{1}$ All the mases are Positioned (centered)

$\boxed{2}$ $\sum \vec{F}$ are applied at that Point

*First ⟹ center of mass for many Particles → ①

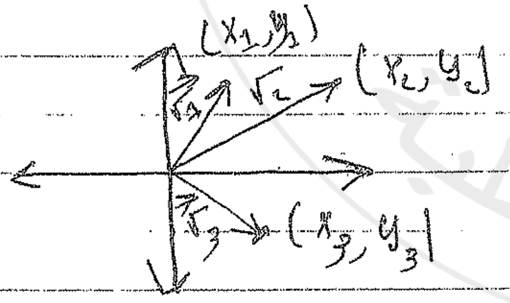
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + m_3} \quad * \; y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \dots}{\sum m}$$

ص م = (ك١ ص١ + ك٢ ص٢ ...) / مج ك

$$x_{cm} = \frac{1}{M} * \sum_{i=1}^{n} m_i x_i \quad * \; z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \dots}{\sum m}$$

*إحداثيات مركز الكتلة (x_{cm}, y_{cm}) or (x_{cm}, y_{cm}, z_{cm})

M ↙ total mass



$$* \; \vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{\sum m} \quad ①$$

$$* \; \vec{r}_{cm} = x_{cm} \hat{i} + y_{cm} \hat{j} + z_{cm} \hat{k} \quad ②$$

* Ex. $m_1 = .5$ Kg at (1, 2) $m_2 = .4$ Kg (3, 2) $m_3 = 1.1$ kg (3, -1)

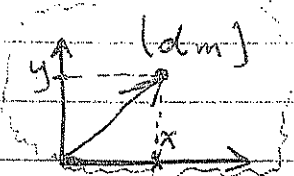
Find $\vec{r}_{cm}$? → $x_{cm} = 2.5$ m $y_{cm} = .35$ m

$\vec{r}_{cm} = 2.5 \hat{i} + .35 \hat{j}$

*26-11-2014 (physics 141 chapter 9)

صلب

*Second ⟹ Cm for a rigid body → ②



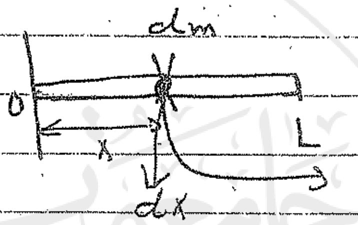
$X_{cm} = \frac{1}{M} * \int x\,dm$ $\quad * Y_{cm} = \frac{1}{M}\int y\,dm$ → M → ~~total~~ mass

→ dm → a dx, a dy → a ↓ constant given in question

منتظم

*Ex. Find X_{cm} for a uniform rod of length = L, mass = M

$X_{cm} = \frac{1}{M}\int x\,dm$



مركز الكتلة

$\boxed{dm = \lambda\,dx}$ → $\lambda = \frac{M}{L}$ → $\frac{kg}{m}$ → Linear mass density

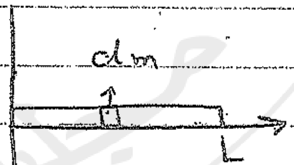
* dm ← بحسب أن نربطها مع dx, dy حسب التطبيق على x-axis or y-axis

$X_{cm} = \frac{1}{\cancel{M}} * \int_0^L x * \frac{\cancel{M}}{L}\,dx = \frac{1}{L} * \frac{L^2}{2} = \frac{L}{2}$

$\boxed{X_{cm} = \frac{L}{2}}$

عصا

*Ex. Find X_{cm} for a non uniform rod of length (L) and $\lambda = \alpha x$ and find the mass of the rod?



$M = \int_0^L dm = \int_0^L \lambda\,dx = \int_0^L \alpha x\,dx$

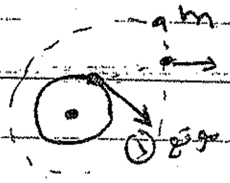
$\boxed{M = \frac{\alpha L^2}{2}}$ $\quad X_{cm} = \frac{1}{M}\int_0^L x\,\lambda\,dx \rightarrow \frac{\alpha}{M} * \int_0^L x^2\,dx$

$X_{cm} = \frac{\alpha}{M} * \frac{L^3}{3}$ but $M = \frac{\alpha L^2}{2}$ $\quad \frac{\cancel{\alpha}}{\frac{\cancel{\alpha}\cancel{L^2}}{2}} \times \frac{L^{\cancel{3}}}{3} = \frac{2L}{3}$

↓ بعيد منتصف الجسم

*1-12-2014 (physics 141 → chapter 9)

*$\boxed{28}$ $T = 24h \rightarrow T^2 = \frac{4\pi^2 r^3}{GM_E} \rightarrow r = 4.2*10^7$ $V = \frac{2\pi r}{T} = 3070 m/s$

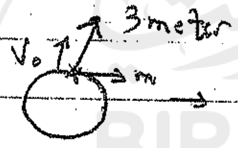


*Work done by this case $W = \Delta E$ $\boxed{W = \Delta E}$ *

$W = \Delta U_g + \Delta K$ $U_2 - U_1 + \frac{1}{2}KV_0^2 \rightarrow V_0 = 0$ →

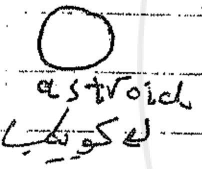
$W = \frac{-GM_E}{r} + \frac{GM_E}{R_E} + \frac{1}{2}KV_1^2 =$

$\boxed{*66}$



$\rightarrow mgh = \frac{1}{2}mV_1^2$ $V_1 = 6g$ $\rightarrow \frac{\rho}{a} \rightarrow$ Roo of astronik $= 2500 \ Kg/m^3$

*Find V_2 on astroid → we suppose $V_1 = V_2$



$V_2 = \frac{2GM_a}{R_a} \rightarrow 6*g = 2\frac{GM_a}{\cancel{R_a}}$ but $M_a = \rho_a * \frac{4}{3} R_a^{\cancel{3}\,2} \pi$

$R_a = \sqrt{\frac{18}{8\pi G * \rho}} = 6500 m$

*

- 1-2-2014 (physics 141 → chapter 9)

① - Newton's second law for a system of many Particles

- $\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{\sum m} \longrightarrow M * \vec{r}_{cm} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots$

then we make the derivative → $M * \vec{V}_{cm} = m_1 V_1 + m_2 V_2 + \dots$

بالنسبة للزمن

→ then the second derivative → $M * \vec{a}_{cm} = m_1 \vec{a}_1 + m_2 a_2 + \dots$

* $M * \vec{a}_{cm} = F_1 + F_2 + F_3 + \dots$ * $\boxed{M * \vec{a}_{cm} = \vec{F}_{net}}$ → التسارع المركزي

② * Linear Momentum (كمية التحرك الخطي)

$\vec{P} = m\vec{v}$ → Kgm/s

* $M\vec{V}_{cm} = m_1 v_1 + m_2 v_2 + \dots \rightarrow M * \vec{V}_{cm} = P_1 + P_2 + \dots$ ① ②

$\boxed{M * \vec{V}_{cm} = \sum \vec{P}}$ $\boxed{* \vec{P} = M * \vec{V}_{cm}}$ $\boxed{* \vec{F}_{net} = M * \vec{a}_{cm}}$

مجموع كميات التحرك

* $\boxed{3}$ The relation between $\vec{F}_{net}$ and $\vec{P}$

$\vec{P} = m_1 V_1 + m_2 V_2 + m_3 V_3 + \dots$ $\frac{dP}{dt} = \vec{F}_{net}$ $\left[\vec{F}_{net} = \vec{F}_{ext} \right.$ خارجية

$\frac{dP}{dt} = m_1 a_1 + m_2 a_2 + \dots$

* The net Force is the time rate of changing Linear momentum

*2-12-2014 (physics 141 → chapter 9)

* if $\vec{F}_{net} = 0$ on the system → $\frac{dP}{dt} = 0$ → $\vec{P}$ = constant

* in this case $\vec{P}_i = \vec{P}_f$ (we called it → conservation of linear momentum
Inital ← ↳ Final

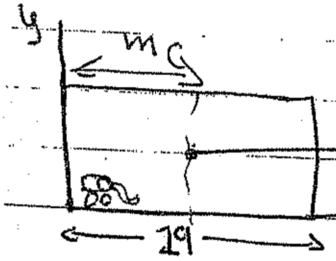
* Condition for conservation of $\vec{P}$ system is isolated ($\vec{F}_{ext} = 0$) هون خارجية

* Application on conservation ① All types of collisions
② All types of explosions ③ All types firing missiles
④ Decaying (انحلال) Process of mass

* $P_i = P_f \longrightarrow m_1\vec{V}_{1i} + m_2\vec{V}_{2i} = m_1\vec{V}_{1f} + m_2\vec{V}_{2f}$ then divided by M

* $$\boxed{(\vec{V}_{cm})_i = (\vec{V}_{cm})_f} \rightarrow \boxed{3}$$

* Example 4 ⟹ * mass of elephant = 4.8 ton * car's tall
* mass of car = 15 ton equal = 19



* $(\vec{V}_{cm})_i = 0 = (\vec{V}_{cm})_f$

$$(X_{cm})_i = \frac{4.8 * 0 + 15 * \overset{L/2}{9.5}}{\Sigma M}$$

$$\boxed{(X_{cm})_f = (X_{cm})_i}$$

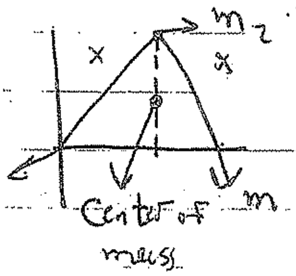
$$(X_{cm})_f = \frac{48 * (19 - X_c) + 15 * (L/2 - X_c)}{\Sigma M}$$

* discussion *

[illegible]

3-12-2014 (physics 141 → chapter 9)

*13

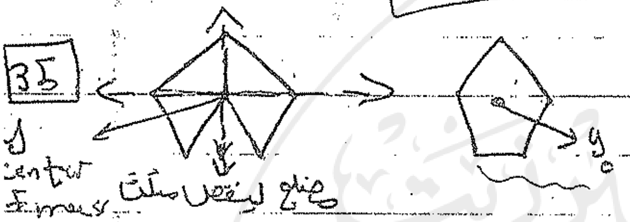


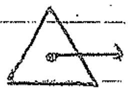
Final $m_2 \rightarrow y_c = \frac{0 + 0 + mL}{2m + m_2}$

$$\frac{\cancel{L}}{2} = \frac{m_2 * \cancel{L}}{2m + m_2}$$

$2m + m_2 = 2m_2 \rightarrow \boxed{m_2 = 2m}$

35





y_t → traynegel

$* \; y_0 = \frac{y_t * m + 4m * y_c}{5m}$ but $y_0 = 0$

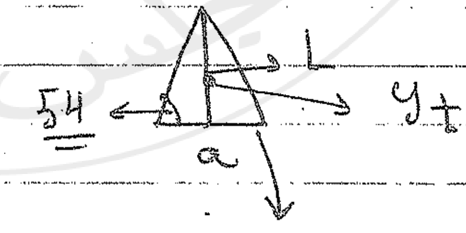
$0 = y_t m + 4m \, y_c \rightarrow \boxed{y_c = -\frac{1}{4} y_t}$

Note → ① △ → $\theta = \frac{180}{3}$ ② $\frac{360}{4}$ ← □

⬠ → $\frac{540}{5}$ → لمعرفة الزاوية الداخلية

* هناك قانون لمعرفة الزاوية الداخلية ← ~~الثلث~~ للمضلع

then We return to example 3



$\tan 54 = \frac{L}{\frac{1}{2}a} \rightarrow \boxed{L = \frac{1}{2} a \tan 54}$

$y_t = \int x \, dm = \frac{2}{3} L$ $\boxed{y_t = -\frac{2}{3} \tan 54 * \frac{a}{2}}$

??

$y_c = \frac{1}{4} \left(\frac{2}{3} \tan 54 * \frac{a}{2} \right) = \frac{a}{12} \tan 54$

Lecture

*4-12-2014 (physics 141 - chapter 9)

*Impulse (قوة الدفع) * $\vec{F}_{net} = \frac{d\vec{p}}{dt} \rightarrow dp = F_{net}\,dt$ then take the Integration $\rightarrow \Delta p = \int_{t_1}^{t_f} F(t)\,dt = \vec{J} \rightarrow \boxed{\vec{P}_f - \vec{P}_i = \vec{J}}$ N.S

*Impulse = Area under the curve of F(t) VS time

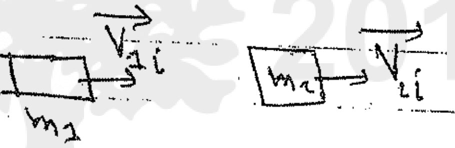
*Collision (التصادمات)

↳ In All types of collisions $\rightarrow \vec{P}_i = \vec{P}_f$

*Types of Collision → $\boxed{1}$ Elastic $(K_i = K_f) \rightarrow$ (التصادم المرن)

$\boxed{2}$ Inelastic $\rightarrow (K_i \neq K_f) \rightarrow$ (التصادم غير المرن)

*Inelastic Collision →



$m_1\vec{V}_{1i} + m_2\vec{V}_{2i} = m_1\vec{V}_{1f} + m_2\vec{V}_{2f}$

*Completly Inelastic Collision (التصادم عديم المرونة)

↳ $m_1\vec{V}_{1i} + m_2\vec{V}_{2i} = (m_1+m_2)\vec{V} \rightarrow K_i > K_f$

*Example $\boxed{9}$ → Ballistic Pendulum (البندول القذفي)

↳ هو بندول مربوط بكتلة خشبية ساكنة والهدف منه قياس سرعة الرصاصة

→ $P_i = P_f \rightarrow m_1 V_i = (m_1+m_2)\,V$

$\frac{1}{2}(m_1+m_2)V^2 = (m_1+m_2)gh$

$\boxed{V = \frac{V^2}{2g}}$

1-12-2014 (physics 141 → chapter 9)

* Elastic collision in one Dimention

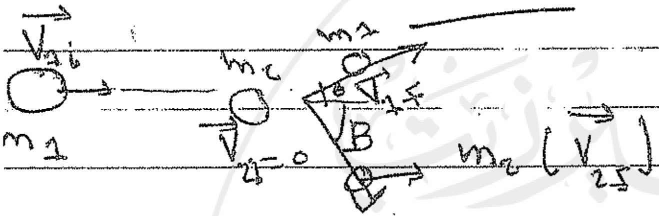
$$m_1\vec{V}_{1i} + m_2\vec{V}_{2i} = m_1\vec{V}_{1f} + m_2\vec{V}_{2f} \rightarrow ①$$

$$\frac{1}{2}m_1\vec{V}_{1i}^2 + \frac{1}{2}m_2\vec{V}_{2i}^2 = \frac{1}{2}m_1\vec{V}_{1f}^2 + \frac{1}{2}m_2\vec{V}_{2f}^2 \rightarrow ②$$

③ $\vec{V}_{1i} - \vec{V}_{2i} = -(\vec{V}_{1f} - \vec{V}_{2f})$ → One dimention فقط

* Collision in two Diminsion

$\boxed{(V_{cm})_i = (V_{cm})_f}$ * In any type of Collision



$* (P_i)_x = (P_f)_x \rightarrow \boxed{m_1V_{1i}\underset{\underset{1}{\cos 0}}{\cos\theta} + 0 = m_1V_{1f}\cos\theta + m_2V_{2f}\cos\theta}$

$* (P_i)_y = (P_f)_y \rightarrow 0 = m_1V_{1f}\sin\theta_1 + m_2V_{2f}\sin\theta_2$

*مراعاة إشارة الاقترانات المثلثية في التصادم في بعدين

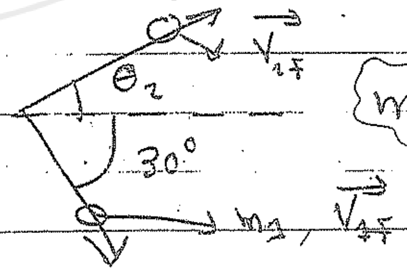
* For elastic collision add → $K_i = K_f$

$$\frac{1}{2}m_1V_{1i}^2 = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}V_{2f}^2m_2$$

* Example 11 → $V_{1i} = 50\,m/s$ m_1 □ m_2 $V_{2i}=0$

Find V_{1f}, V_{2f}, θ_2 ?

$\boxed{m_1 = m_2 = m}$



$50 = \cancel{m}_1V_{1f}\cos30 + \cancel{m}V_{2f}\cos\theta$

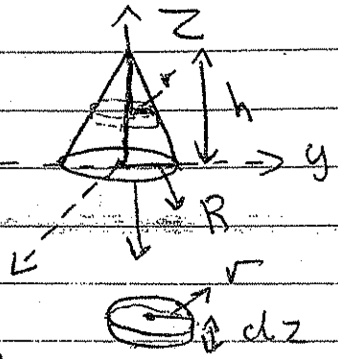
$\boxed{V_{2f}\sin\theta_2 = V_{1f}\sin30}$

$\boxed{V_{1i}^2 = V_{1f}^2 + V_{2f}^2}$

* discusion *

-12-2014 (physics 141 → chapter 9)

$\boxed{1}$



$* \; Y_{cm} = 0 \; , \; X_{cm} = 0 \quad Z_{cm} = \int \overline{Z} \, dm * \frac{1}{M}$

$* \; Z_{cm} = \frac{\int Z \, dm}{M} \qquad * \; \underset{\sim}{dm} = \rho * dV$

$* \; \underset{\sim}{dm} = \rho * r^2 \pi dZ$

$= \frac{\int_0^h Z * \rho r^2 \pi dZ}{\frac{1}{3} R^2 \pi * h * \rho}$

mass = of مخروط

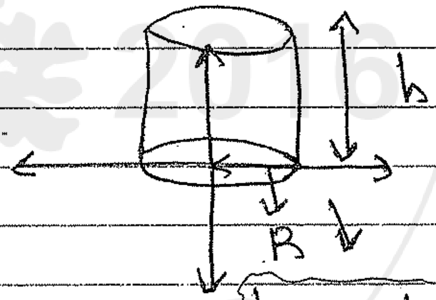
$m = V * \rho$ or $\boxed{m = \int dm}$

$* \; Z_{cm} = \frac{\int_0^h Z * \cancel{\rho} r^2 \cancel{\pi} dZ}{\int_0^h \cancel{\rho} r^2 \cancel{\pi} dZ}$ but $r \rightarrow$ يتغير

$\frac{h-Z}{h} = \frac{r}{R} \rightarrow \boxed{r = \frac{R}{h}(h-Z)}$

$\frac{\int_0^h Z (h-Z)^2 dZ}{\int_0^h (h-Z)^2 dZ} = Z_{cm}$

*



homwork

find the center of mass

$\rho = c \, Z$

constant

* chapter 10 *

*9-12-2014 (phisics 141 → chapter 10)

* Rotational Motion (الحركة الدورانية)

الكميات الزاوية

*Angular quantities

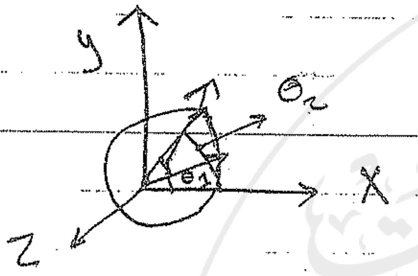
Position

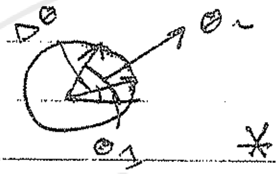
*initial angular = θ_1 rad

*Final Angular Position = θ_2 rad

*Angular displacment = $\Delta\theta$

$= \theta_2 - \theta_1$





*Average Angular Velocity

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t} \text{ rad | second}$$

* $\underline{\omega}$ is positive if $\underline{\omega}$ is counter clock wise → الزوية عكس عقارب الساعة

* 〃 〃 negative 〃 〃 〃 〃 〃 الزاوية مع عقارب

$-6+6t$ —— $\alpha(2) = 6$

$\alpha(4) = \underline{18}$

-12-2014 (physics 141 → chapter 10)

· example → $\theta(t) = 4t - 3t^2 + t^3$

*9-12-2014 (physics 141 → chapter 10)

example 2, $\alpha(t) = 6t^4 - 4t^2$ at $t=0$ → $\theta_0 = +1.5$ rad, $W_0 = +2.5$ rad/s

$$\alpha = \frac{dw}{dt} \rightarrow \int_{W_0}^{w} dw = \int_{t_0}^{t} \alpha\, dt \rightarrow W - W_0 = \int_0^t 6t^4 - 4t^2$$

$$W = \frac{6}{5}t^5 - \frac{4}{3}t^3 + W_0 \rightarrow \boxed{1}\ W = +2.5 + \frac{6}{5}t^5 - \frac{4}{3}t^3$$

$\boxed{2}$

$$W = \frac{d\theta}{dt} = \int_{\theta_0}^{\theta} d\theta = \int_{t_0}^{t} w\, dt = \theta - \theta_0 = \int_0^t 2.5 + \frac{6}{5}t^5 - \frac{4}{3}t^3\, dt$$

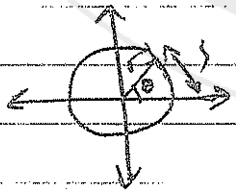
$$\theta(t) = 1.5 + 2.5t + \frac{t^6}{5} - \frac{t^4}{3}$$

* Rotational Motion with Constant α

① $W = W_0 + \alpha t$ * $\boxed{2}$ $W^2 = W_0^2 + 2\alpha(\theta - \theta_0)$

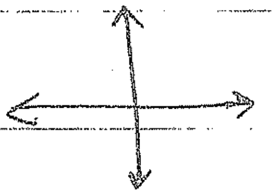
$\boxed{3}$ $\theta - \theta_0 = W_0 t + \frac{1}{2}\alpha t^2$ * $\boxed{4}$ $\overline{W} = \frac{W + W_0}{2}$

* Relation between Linear quantities and Rotational quantities



$s = r\theta$

arc length   costan (radius)



$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow \boxed{V = r\, W}$$

*discusion *

10-12-2014 (physics 141 → chapter 9)

$\boxed{19}$ $^{235}_{92}U \rightarrow {}^{4}_{2}He \rightarrow$ اليورانيوم تحلل إلى هيليوم $\rightarrow P_i = 0 = P_f$

* Kinetic Energy for He = 5.15 $\overset{\text{ميغا}}{\underline{\underline{M}}}$ $\underline{\underline{eV}}$ Find the Velocity of (U)

$m_u V_u + \underset{\downarrow}{m_{He}} V_{He} = 0 \rightarrow$ $5.15 * 10^{6} * 1.6 * 10^{-19} = \frac{1}{2} * 4 * 1.6 * 10^{-27} \times V^2$

كتلة البروتون X عدد البروتونات

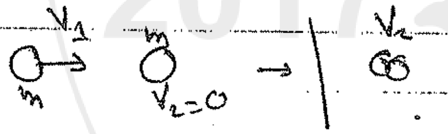
$V = 1.575 * 10^{7}$ m/s

$V_u = -\frac{m_{He} V_{He}}{m_u} = -2.7 * 10^{5}$ m/s

*$\boxed{25}$ Impulus = F * Δt → J = 5.64 N.S → F = 135 mN

$\Delta t = \frac{I}{F} = \frac{5.64}{135 * 10^{-3}} = 42$ $\underline{\text{second}}$

*$\boxed{27}$

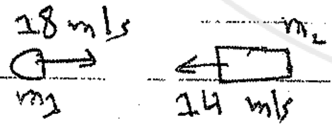


$\rightarrow mV_{1i} = 2m V_{2f} \rightarrow \boxed{V_{2f} = \frac{V_{1i}}{2}}$

$\Delta K = \frac{1}{2} m V_{2f}^2 - \frac{1}{2} m V_{1i}^2 = \Delta K = \frac{1}{2} * 2m * \frac{V_{1i}^2}{4} - \frac{1}{2} m V_{1i}^2$

$\Delta K = \frac{1}{4} m V_{1i}^2 - \frac{1}{2} m V_{1i}^2 = -\frac{1}{4} m V_{1i}^2 = -\frac{1}{2} * K_1$ * النسبة $= \frac{-\frac{1}{2} K_1}{K_1} = -50\%$

*$\boxed{31}$



$1 = \frac{14 - V_{1f}}{-18 - 14} \rightarrow -32 = 14 - V_{1f}$

$V_{1f} = 46$ m/s

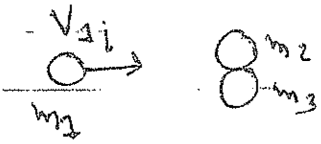
↓

We suppose that $V_{2i} = V_{2f}$

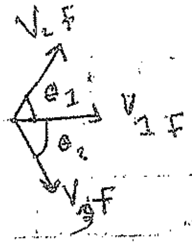
*$\boxed{48}$ → Collision in $\underline{\underline{2}}$ Dimintuon

10-12-2014 (physics 141 → chapter 9)

$\boxed{12}$



$V_{2i} = V_{3i} = 0$ $m_1 = m_2 = m$



$$\cancel{m}V_{1i} = \cancel{m}V_{1f} + \cancel{m}V_{2f}\cos\theta_1 + \cancel{m}V_{3f}\cos\theta_2$$

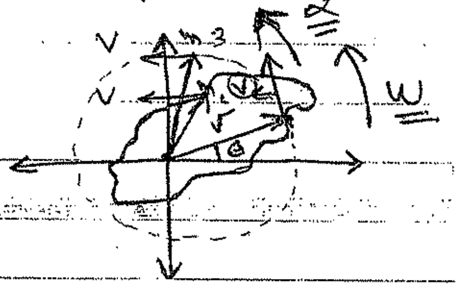
$$\boxed{V_{1i} = V_{1f} + V_{2f}\cos\theta_1 + V_{3f}\cos\theta_2}$$

*In y direction → $\boxed{V_2 \sin\theta_1 = V_3 \sin\theta_2}$

Lecture

*16-12-2014 (physics 141 → chapter 10)

* $\omega = \frac{\Delta\theta}{\Delta t}$, $\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$

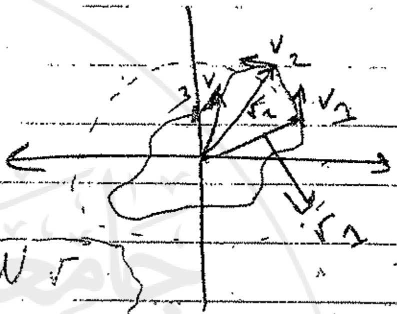


*Rotational Kinetic Energy

$$K_{rot} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \ldots$$

$$= \frac{1}{2} m_1 (\omega r_1)^2 + \frac{1}{2} m_2 (\omega r_2)^2 + \frac{1}{2} m_3 (\omega r_3)^2 + \ldots$$

$$= \frac{1}{2} \omega^2 \left(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^3 \right)$$



* $V = \omega r$
* $a_t = \alpha r$

r_1, r_2, r_3 بعد الجسم عن نقطة الدوران

$\downarrow$

($I \rightarrow$ moment of enertia) → the unite → $Kg \cdot m^2$

$I = \sum m_i r_i^2$ → للأجسام لها كتل وتبعد مسافة عن نقطة الدوران

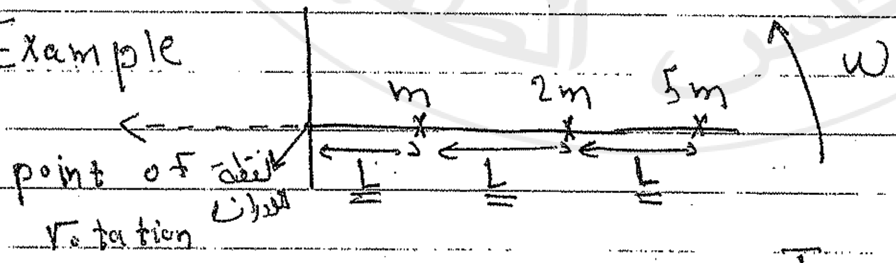
$$K_{rot} = \frac{1}{2} I \omega^2$$

Very important

moment of enertia

$$* \text{ Kinatic Energy of Rotation} = \frac{1}{2} * \underline{\underline{I}} * \omega^2$$

* Example



$* I = m L^2 + 2m (2L)^2 + 5m (3L)^2$

$* I = 54 m L^2$

$* K = \frac{1}{2} 54 m L^2 * \omega^2$

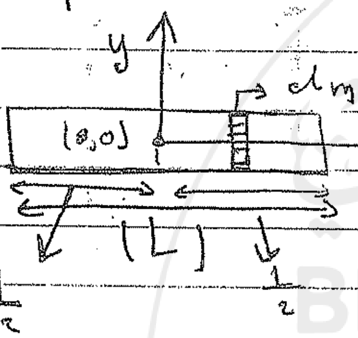
16-12-2014 (physics 141 → chapter 10)

Moment of Inertia for a rigid body (جسم صلب)

$\boxed{I = \int r^2 \, dm}$

$$\boxed{I = \int r^2 \, dm}$$

Example → Find I_{cm} for a rod of length = L and mass = m

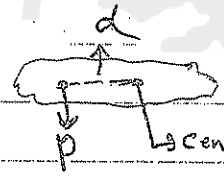


$\rightarrow * \; dm = \lambda \, dx \rightarrow \boxed{\lambda = \frac{M}{L}}$

$$I = \int_{-L/2}^{L/2} x^2 * \frac{M}{L} \, dx = \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx = \frac{1}{12} M L^2$$

Parallel axis Theorem (حساب I حول نقطة)

m is given



$\boxed{??}$

ind I_p at a point p at a distance d From cm

$$\downarrow$$

$$\boxed{I_p = I_{cm} + m d^2} \rightarrow$$ قانون حساب I الدوران حول نقطة

I_{cm} → لمركز الكتلة

d → distemce betwen cm and the point

* 16-12-2014 (physics 141 → chapter 10)

* Torque (العزوم) ⟹ Torque is the Cause of Rotation

$\vec{\tau} = \underset{\text{distance}}{\vec{r}} * \underset{\text{Force}}{\vec{F}} \rightarrow \tau \rightarrow \text{Torque} \rightarrow N.m$

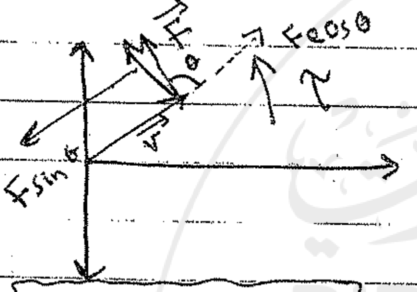
$* \boxed{\vec{\tau} = A_x B_x \hat{i} + A_y B_y \hat{j} + A_z B_z \hat{k}}$ ①

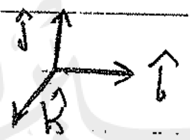
$\vec{\tau} = |\vec{r}| * |\vec{F}| * \sin\theta$

$* \boxed{\vec{\tau} = |\vec{r}| * |\vec{F}| * \sin\theta}$ ②

$* \vec{\tau} \perp (\vec{r} \text{ and } \vec{F})$

*العزوم يكون عموديا على المسافة و القوة





$* \hat{i} \times \hat{j} = 1 * 1 * \sin 90 = 1 \rightarrow \hat{k}$

المتجه الاول مع الهسم
الاصابع مع المتجه الثاني

قاعدة اليد اليمين ⟵ $* \hat{j} \times \hat{k} = 1 * 1 * \sin 90 = 1 \rightarrow \hat{i}$

$* \hat{i} \times \hat{k} = -\hat{j} = \hat{k} \times \hat{i}$ يساوي بالمقدار
ويعاكس بالاتجاه

* Example ⟹ $\vec{F} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ N acts on a body

at point $\vec{r} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ m Find $\tau = (3\hat{i} - 4\hat{j} + 5\hat{k}) \times (3\hat{i} - 2\hat{j} + 4\hat{k})$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ 3 & -2 & 4 \end{vmatrix}$$

محدات

$$= \hat{i}(-10 + 10) - \hat{j}(15 - 12) + \hat{k}(-12 + 6)$$

*Lecture *

* 18-12-2014 (physics 141 → chapter 10)

* Newtons second Law for rotation

* $\tau_{net} = I * \alpha$ → angular accelaration
  - I ↳ momentum of Inartia

* Work done by $\tau = \boxed{\int_{\theta_i}^{\theta_f} \tau \, d\theta}$ → (For variable τ)

* Work done by net $\tau = \Delta K = \boxed{\frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2}$

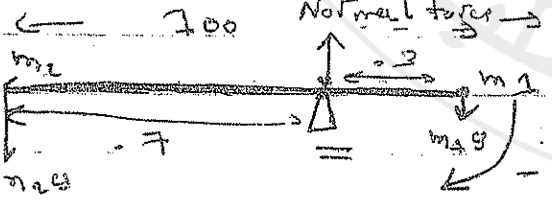
* Work done by $\tau = \boxed{\tau * \Delta\theta}$ (for constant τ)

* Power $= \frac{d\,\text{Work}}{dt} = \tau * \frac{d\theta}{dt} = \tau \omega$

$\boxed{P = \tau \omega}$ Watt

* example → masiles rod of length

$L = 100$ cm , $m_1 = .3$ Kg
$L_1 = .3$ m , $m_2 = .4$ Kg
$L_2 = .7$ m



* مع عقارب الساعة ← سالب (العزم)
* عكس عقارب الساعة ← موجب العزم

↷ *Find τ net around 0

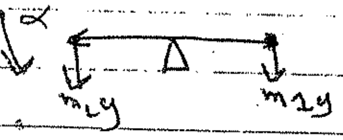
$\tau_{net} = \tau_1 + \tau_2 = -\underbrace{m_1 g}_{F_1} \overset{r \sin 90}{L_1} + m_2 g L_2$

* الاصبع مع مركز الدوران وأصابع اليمين
اليمين مع المسافة وباطن اليد مع القوة
فينتج الاتجاه

$= 2.2$ N.m

* 18-12-2014 (physics 141 → chapter 10)

* $\boxed{2}$ find the initial angular acceleration?

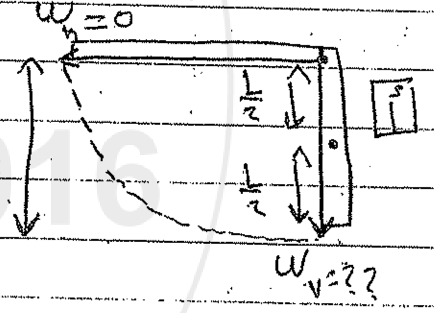


$\tau_{net} = I * \alpha \rightarrow I = m_1 r_1^2 + m_2 r_2^2 = .214$ $\alpha = \frac{2.2}{.214} = 10.3\ rad/s^2$

* $\boxed{3}$ find tangential accelaration at the initial moment for m_1, m_2

* $(a_t)_1 = \alpha r_1 = 3.1\ m/s^2$ * $(a_t)_2 = \alpha r_2 = 7.2\ m/s^2$

* Example $\boxed{2}$ rod of Length = 1m of mass = .8 Kg in horizontal Position finally the rod is in vertical Position?



* We can make $E_1 = E_2$ → mg → conservative

$(K_1 + U_1) = (K_2 + U_2) = 0 + mgL = \frac{1}{2} I w_v^2 + mg\frac{L}{2}$

= العمود I كل نقطة لها طاقة وضع مختلفة ففي هذه الحالة نأخذ ←

U للوضع لـ centerofmass

$mgL = \frac{1}{2}\left(\frac{1}{3}mL^2\right) w_v^2 + mg\frac{L}{2}$

يوجد رسمك توضح ←

I ↓

الدوران حول الطرف

$I = \frac{1}{3}mL^2$

$\boxed{2}$ When the rod is vertical find V_{cm} and $V_{free\ end}$

* $V_{cm} = w\frac{L}{2}$ * $V_{free\ end} = w * L$

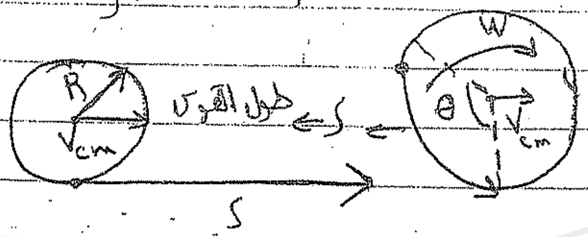
$\boxed{3}$ Find τ at horizontal Position and vertical Position

$\tau_h = mgL\sin 90$ $\tau_v = mg * L * \cos 90 = 0$

18-12-2014 (physics 141 → chapter 10)

قانون طول قوس الدائرة

Rolling Motion



$$* V_{cm} = \frac{S}{t} \rightarrow S = R * \theta$$

$$\frac{dS}{dt} = R\frac{d\theta}{dt} \rightarrow \boxed{V_{cm} = R W}$$

Rolling Motion = Traslational motion of the center f mass + Rolational motion around the (cm) هنا يوجد * حركة انتقالية ودورانية

$_{Rolling}$ = K for traslation + K for rotational

$$\boxed{K_{Rolling} = \underbrace{\frac{1}{2} M V^2_{cm}}_{\text{للحركة الانتقالية}} + \underbrace{\frac{1}{2} I_{cm} W^2}_{\text{(الدورانية لدوران العجل حول نفسه}}}$$

*22-12-2014 (physics 141 → chapter 10)

$\boxed{41}$ $\omega_0 = 0$ * 2π of revol * 18 rpm/s $= \frac{18 * 2\pi}{60} = 1.8$

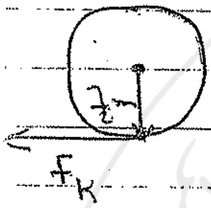
$\underset{2\pi}{\downarrow}$ r p $\underset{\text{minute}}{\downarrow}$ m / s ↳

$\Theta = 2 * 2\pi = 4\pi$

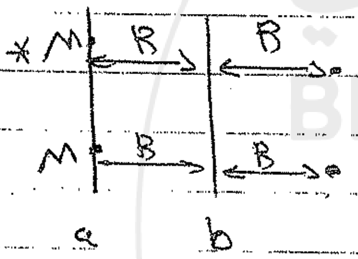
* $\boxed{\alpha = 1.88}$

$\omega^2 = \omega_0 + 2\alpha\theta = \omega^2 = 0 + 2 * 1.88 * 4\pi = 6.9$

* $\boxed{20}$ diameter = 1m Find the turqe around the center



$\tau = \frac{1}{2} * 320 * \sin 90 = 160$ N.m *الوضع ع مركز الدوران الصحيح و التعامد والمسافة



$I = \sum m_i r_i^2$ about $\underline{a}$

$= M * (2B)^2 + m (2B)^2 + 0 * 0 = 8B^2 M$

* about $\underline{b} \longrightarrow MB^2 + MB^2 + MB^2 + MB^2 = 4MB^2$

$\boxed{30}$ $\boxed{a}$ Solid sphere $\rightarrow I = \frac{2}{5} M_E R_E^2 = \frac{2}{5} * 5.97 * 10^{24} * (6.37 * 10^6)^2 = 9.7 * 10^{37}$

$\boxed{30} \rightarrow \boxed{b} \rightarrow$ 1 per century $\Longrightarrow$ dT تغير الدورة Find the τ and F

$$\frac{dT}{dt} = \frac{1}{1 * 100 * 365.25 * 24 * 3600} = 3.2 * 10^{-10}$$

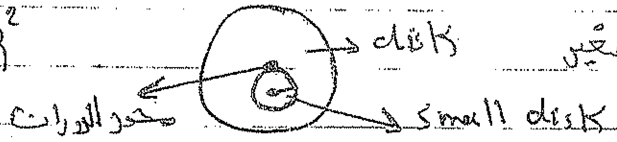
$$\omega = \frac{2\pi}{T} \rightarrow \frac{d\omega}{dt} = 2\pi * \frac{1}{\underset{\substack{\downarrow \\ (24 * 60 * 60)^2}}{T^2}} * \frac{dT}{dt} = \alpha$$

$\tau = \alpha * I$

$\tau = F * R_E \longrightarrow F = \frac{\tau}{R_E} = 10^{13}$ N

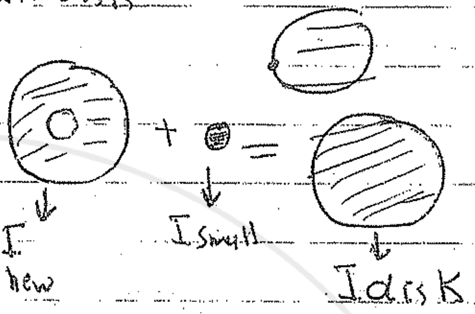
*22-12-2014 (physics 141 → chapter 10)

$\boxed{65}$ $I_{disk} = \frac{1}{2} M R^2$



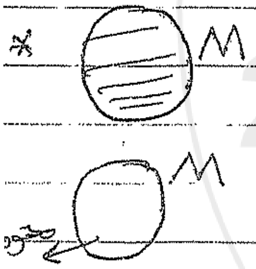
بدي I_{cm} بعد قطع القرص الصغير

$I_{small\,disk} = \frac{1}{2} * M * \left(\frac{R}{4}\right)^2$



$I_{small\,disk} = I_{cm-small} + M'\left(\frac{1}{4}R\right)^2$

$I_{sma} = 1$

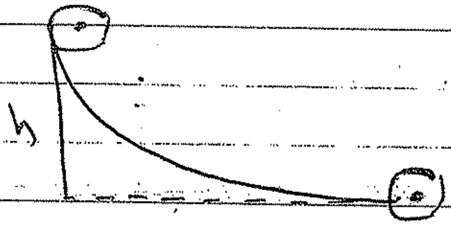


$\rightarrow K_{t_1} = K_{t_2} \rightarrow$ total

* Lecture *

*23-12-2014 (physics 141 → chapter 10)

* Example $\boxed{12}$



* find V_{cm} of the ball

$E_1 = E_2$

Rolling motion

$(U+K)_1 = (K+U)_2 = mgh = \left(\frac{1}{2}mV_{cm}^2 + \frac{1}{2}I\omega^2\right) + 0$

I spherical (كرة مصمتة

$I = \frac{2}{5}MR^2$

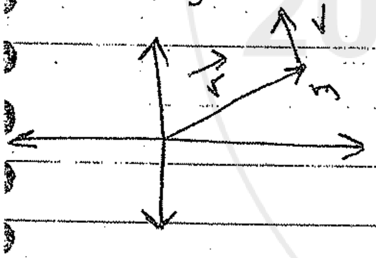
$\cancel{m}gh = \frac{1}{2}\cancel{m}V_{cm}^2 + \frac{1}{2} * \frac{2}{5}\cancel{M} * \cancel{R^2} * \frac{V_{cm}^2}{\cancel{R^2}}$ $V_{cm} = \omega r$

$$\boxed{V_{cm} = \sqrt{\frac{10}{7}gh}}$$

---

*23-12-2014 (physics 141 → chapter 11)

* Angular Momentum (كمية التحرك الزاوي)



* $\vec{L} = \vec{r} \times \vec{P}$ but $\vec{P} = m\vec{V}$

angular momentum * $L = |\vec{r}| * |\vec{P}| * \sin\theta$

but IF $\theta = 90$ ④ → ① $L = r * p$ ⑤ but ② $\vec{P} = m\vec{V}$ and ③ $V = \omega r$

$L = r * mV$ but $V = \omega r$

⑥ $L = \underbrace{mr^2}_{I}\omega$ ⑦

⑧ * $\boxed{\vec{L} = m * \vec{r}^2 * \vec{\omega}}$ * angular Velocity

angular momentum

mass of body

but $I = mr^2$

$\boxed{L = I\omega}$ → The new law of Angular Momentum

⑨

*23-12-2014 (physics 141 → chapter 11)

* $\overset{①}{\boxed{\vec{L} = I * \vec{W}}}$ → Newtons second law for rotational motion → $\overset{②}{\boxed{\vec{\tau}_{net} = I\,\vec{\alpha}}}$ then take the dirivative to the $\overset{①}{\text{equation}}$ → $\vec{\tau}_{net} = I * \frac{d\alpha}{dt} = \vec{\tau}_{net} = \frac{dL}{dt}$ * $\boxed{I * dw}\,/\,dt = dL$

$$\boxed{\tau_{net} = \frac{dL}{dt}}$$ → we take f'(x) to first equation by time → $\frac{dL}{dt} = I * \frac{dw}{dt}$ → $I * \alpha = \frac{dL}{dt}$

$$\tau_{net} = \alpha * I = \boxed{\tau_{net} = \frac{dL}{dt}}$$ Very important

* Example $\boxed{1}$

*23-12-2014 (physics 141 → chapter 11)

*Example $\boxed{2}$ mass = 3 Kg and its Postion given by ~~$\vec{r} = 4t^2\hat{i}$~~

$\vec{r} = 4t^2\hat{i} - (2t + 6t^2)\hat{j}$ m find $\vec{L}(t)$ and $\vec{\tau}(t)$

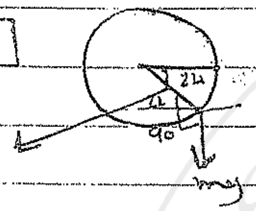
discusion

5-1-2014 (physics 141 → chapter 10)

39 $\frac{K_r}{K_r + K_t} = \frac{\frac{1}{2} I w^2}{\frac{1}{2} I w^2 + \frac{1}{2} m v^2}$ but $w = \frac{v}{r}$

$\frac{\frac{1}{2} I w^2}{\frac{1}{2} I w^2 + \frac{1}{2} m w^2 r^2} = \frac{I \; \frac{1}{2} m r^2}{\frac{1}{2} m r^2 + m r^2} = \frac{\frac{1}{2}}{\frac{3}{2}} = \underline{\underline{\frac{1}{3}}}$

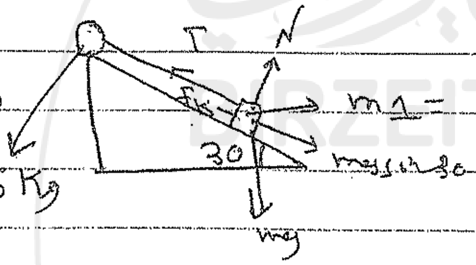
24



$\tau = F * r * \sin\theta$

r $\qquad = mg * r * \sin(90 + 24) =$

57



$m_1 = 2.4$ Find f_K, M_K

K = .85 kg

$\underline{\underline{a}} = 1.6 \; m/s^2$

$* \; F_{net} = ma \rightarrow mg \sin 30 - T - mg \cos 30 = ma$

$* \; r * \alpha = a \rightarrow \alpha = \frac{a}{r} = \frac{1.6}{.05} = 32 \; s^{-2}$ → في البكرة (ثابتة)

$\alpha = \frac{\tau}{I} - \tau = \alpha * \frac{1}{2} M R^2 \quad \tau = .034$

$* \; \tau = F * r * \sin 90 \Rightarrow F = \frac{\tau}{r} = .68 \; N$ → Tension $\underline{\underline{\text{force}}}$

Passage Problem page.. chapter $\underline{4}$

$ma = N - mg \qquad ma = \left(\frac{m' - m}{m}\right) g = \frac{.5}{5} * 10 = 1 \; m/s^2$

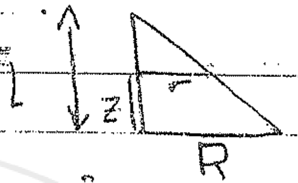
$m' g$

apparant mass

* 5-11-2014 (physics 141 → chapter 10)

* Chapter $\boxed{9}$

$$Z_{cm} = \frac{1}{M} * \int_{Z_i}^{Z_f} Z\,dm$$



$$\frac{r}{R} = \frac{L-Z}{L}$$

$dm = \rho * dv \rightarrow dv = (\pi r^2)\,dz$

$r = \left(\frac{L-Z}{L}\right) * R \rightarrow dv = \pi * \left(\frac{R}{L}\right)^2 * (L-Z)^2\,dZ$

$$= \frac{\int Z * \cancel{\pi * \left(\frac{R}{L}\right)^2} * (L-Z)^2\,dZ * \cancel{\rho}}{\int \cancel{\rho} * \cancel{\pi * \left(\frac{R}{L}\right)^2} * (L-Z)^2\,dZ} = \frac{\int Z(L-Z)^2\,dZ}{\int (L-Z)^2\,dZ}$$

$y = L - Z$ $\cancel{Z = \frac{1}{L} y}$ $\cancel{dy = \frac{1}{L}}$

$$= \int_0^L y^2 ($$