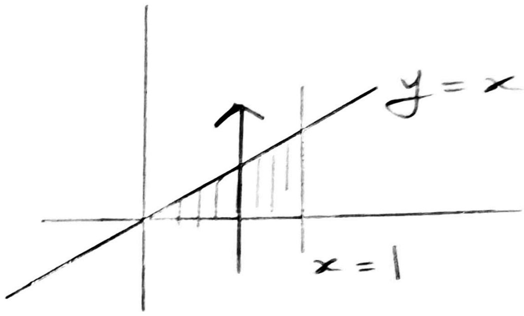
Example 1. Find the Volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane

$$Z = f(x,y) = 3 - x - y.$$

Sol: 1) sketch the region of Integration.

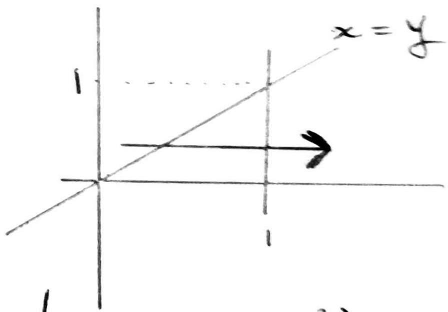


$$\text{Volume} = \iint_R z \, dA$$

$$= \int_0^1 \int_0^x (3 - x - y) \, dy \, dx$$

$$= \int_0^1 \left(3y - xy - \frac{y^2}{2} \Big|_0^x \right) dx = \int_0^1 \left(3x - \frac{3}{2}x^2\right) dx = \boxed{1}$$

OR: $$\int_0^1 \int_y^1 (3 - x - y) \, dx \, dy$$



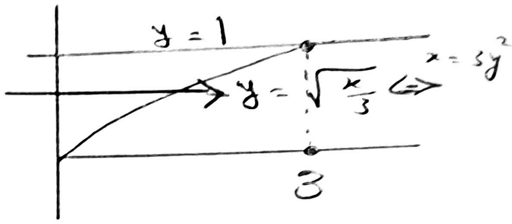
$$= \int_0^1 \left(3x - \frac{x^2}{2} - yx \Big|_y^1 \right) dy = \int_0^1 \left(\frac{5}{2} - y\right) - \left(3y - \frac{y^2}{2} - y^2\right) dy$$

$$= \int_0^1 \left(\frac{5}{2} - 4y + \frac{3}{2} y^2 \right) dy = \boxed{1}$$

(3/4) (106)

Example: Evaluate $\displaystyle\int_0^3 \int_{\sqrt{\frac{x}{3}}}^1 e^{y^3}\, dy\, dx$

sol: Sketch the region of Integration:



$0 \le x \le 3$ & $\sqrt{\frac{x}{3}} \le y \le 1$

$y = \sqrt{\frac{x}{3}} \iff x = 3y^2$

$$\int_0^1 \left(\int_0^{3y^2} \left(e^{y^3} \right) dx \right) dy = \int_0^1 x e^{y^3} \Big]_0^{3y^2} dy$$

$$= \int_0^1 3y^2 e^{y^3} dy =$$

Let $u = y^3$
$du = 3y^2 dy$

$$= \int_0^1 e^u \, du = e^u \Big]_0^1$$

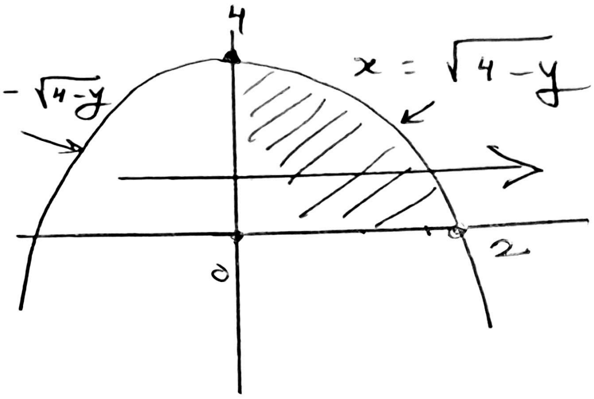
$$= e - 1$$

$\left(\frac{4}{4}\right)(106)$

15.2

(Q50) $I = \int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y}\, dy\, dx$

$I = \int_0^4 \int_0^{\sqrt{4-y}} \frac{x e^{2y}}{4-y}\, dx\, dy$



$= \int_0^4 \left(\left(\frac{e^{2y}}{4-y}\right)\left(\frac{x^2}{2}\right) \Big]_0^{\sqrt{4-y}} \right) dy$

$= \int_0^4 \left(\frac{\cancel{4-y}}{2}\right)\left(\frac{e^{2y}}{\cancel{4-y}}\right) dy = \frac{1}{2} \int_0^4 e^{2y}\, dy$

$= \frac{1}{4} e^{2y} \Big]_0^4 = \frac{1}{4}\left(e^8 - 1\right)$

$\left(\frac{4}{4}\right)(107)$