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FINN3302

النمذجة المالية

CHAPTER 4: Further development and analysis of the classical linear regression model

Chapter 4

Multiple regression model

$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + B_4 X_{4t} + \cdots + B_K X_{Kt} + U_t$$

interest constant term

(Cetris Paribus effect)

slope prameters → represnt the marginal impact of changing one of the explanatory variables assocated with the parameter while holding other variables constant.

$$Y_1 = B_1 + B_2 X_{21} + B_3 X_{31} + \cdots + B_K X_{K1} + U_1$$
$$Y_2 = B_1 + B_2 X_{22} + B_3 X_{32} + \cdots + B_K X_{K2} + U_2$$
$$\vdots$$
$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + \cdots + B_K X_{Kt} + U_t$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_T \end{bmatrix}_{T*1} = \begin{bmatrix} 1 & X_{21} & X_{31} & \cdots & X_{K1} \\ 1 & X_{22} & X_{32} & \cdots & \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & X_{2t} & X_{3t} & \cdots & X_{Kt} \end{bmatrix}_{T*K} * \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_K \end{bmatrix}_{K*1} + \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_t \end{bmatrix}_{T*1}$$

Matrix dimensions number of rows * number of columns

$$\hat{u} = \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \vdots \\ \hat{u}_T \end{bmatrix}_{T*1}$$

transpose ← $\hat{u}^{(t)} = \begin{bmatrix} \hat{u}_1 & \hat{u}_2 & \cdots & \hat{u}_T \end{bmatrix}_{1*T}$

$$\hat{u}^{*} * \hat{u} = \begin{bmatrix} \hat{u}_1 & \hat{u}_2 & \cdots & \hat{u}_T \end{bmatrix}_{1*T} * \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \vdots \\ u_T \end{bmatrix}_{T*1} = \sum \hat{u}^2 = RSS$$

→

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$$\hat{B} = (X'X)^{-1} X'y \qquad \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \\ \vdots \\ \hat{B}_k \end{bmatrix}_{k*1}$$

OLS estimator in matrix notation.

$$S^2 = \frac{\hat{u}'\hat{u}}{T-K}$$

$K \rightarrow$ number of parameters to be estimated in the multiple regression case including the constant term.

$$S^2 (X'X)^{-1}$$

$[\ \ ]$ var of the estimate

$\rightarrow$ variance covariance matrix

Example:

$K=3 \quad T=15$

$y_t = B_1 + B_2 x_{2t} + B_3 x_{3t} + u_t$

$$(X'X)^{-1} = \begin{bmatrix} 2 & 3.5 & -1 \\ 3.5 & 1 & 6.5 \\ -1 & 6.5 & 4.3 \end{bmatrix}_{3\times3} \qquad X'y = \begin{bmatrix} -3 \\ 2.2 \\ 0.6 \end{bmatrix}_{3\times1} \qquad \hat{u}'\hat{u} = RSS = 10.96$$

$$\hat{B} = (X'X)^{-1} X'y = \begin{bmatrix} 2 & 3.5 & -1 \\ 3.5 & 1 & 6.5 \\ -1 & 6.5 & 4.3 \end{bmatrix}_{3\times3} \begin{bmatrix} -3 \\ 2.2 \\ 0.6 \end{bmatrix}_{3\times1}$$

$$= \begin{bmatrix} 2*-3 + 3.5*2.2 + -1*0.6 \\ 3.5*-3 + 1*2.2 + 6.5*0.6 \\ -1*-3 + 6.5*2.2 + 4.3*0.6 \end{bmatrix}_{3*1} = \begin{bmatrix} 1.1 \\ -4.4 \\ 19.88 \end{bmatrix}_{3*1} \begin{matrix} \rightarrow \hat{B}_1 \\ \rightarrow \hat{B}_2 \\ \rightarrow \hat{B}_3 \end{matrix}$$

$$\hat{Y}_t = 1.1 - 4.4\, X_{2t} + 19.88\, X_{3t}$$
$$\quad\ (1.35)\quad (0.96) \quad\quad (1.98)$$

$$\hat{S}^2 = \frac{\hat{u}'\hat{u}}{T-K} = \frac{RSS}{T-K} = \frac{10.96}{15-3} = 0.91$$

$$(X'X)^{-1} = \begin{bmatrix} 2 & 3.5 & -1 \\ 3.5 & 1 & 6.5 \\ -1 & 6.5 & 4.3 \end{bmatrix}$$

$$0.91 * \begin{bmatrix} 2 & 3.5 & -1 \\ 3.5 & 1 & 6.5 \\ -1 & 6.5 & 4.3 \end{bmatrix} = \begin{bmatrix} 1.83 & 3.2 & -0.91 \\ 3.2 & 0.91 & 5.91 \\ -0.91 & 5.91 & 3.91 \end{bmatrix}$$

$$\left.\begin{aligned} var(\hat{B}_1) &= 1.83 \rightarrow SE(\hat{B}_1) = \sqrt{1.83} = 1.35 \\ var(\hat{B}_2) &= 0.91 \rightarrow SE(\hat{B}_2) = \sqrt{0.91} = 0.96 \\ var(\hat{B}_3) &= 3.91 \rightarrow SE(\hat{B}_3) = \sqrt{3.91} = 1.98 \end{aligned}\right]$$

نرجع نكتبهم في المعادلة

$0 \leq R^2 \leq 100\%$

$\downarrow$ how well our model (SRF) fits the data.

$$TSS = \sum (Y_t - \bar{Y}_t)^2$$

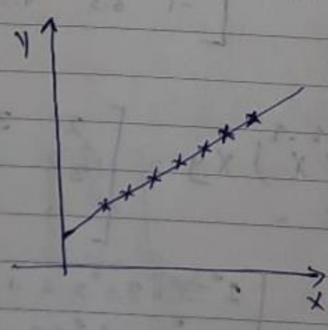
explained sum of squares — Residual sum of squares

$$TSS = ESS + RSS$$

$$\sum (Y_t - \bar{Y})^2 = \sum (\hat{Y}_t - \bar{Y})^2 + \sum (Y_t - \hat{Y}_t)^2$$

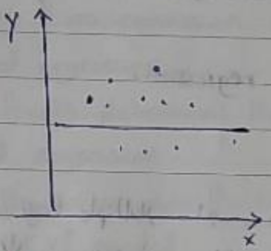
$RSS = 0$ يعني $R^2 = 1$ إذا

$TSS = ESS$



$\rightarrow$

اذا $R^2=0$ يعني ما بقدر اوصف العلاقة



$ESS = 0$

$TSS = RSS$

$$\boxed{R^2 = \frac{ESS}{TSS}}$$

$$\frac{TSS}{TSS} = \frac{ESS}{TSS} + \frac{RSS}{TSS} \qquad \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

$$1 = \frac{ESS}{TSS} + \frac{RSS}{TSS} \qquad \boxed{R^2 = 1 - \frac{RSS}{TSS}}$$

problems with R^2:

هاي المشكلة مالها حل

(1) we cannot use R^2 to compare between two models if the dependent variable was different.

Example:

model 1

$Y_t = B_1 + B_2 X_{2t} + U_t$

$R^2 = 0.8$

بس بقدر افسر R^2

80% of the variation in Y was explained by the variation in X.

model 2

$Z_t = B_1 + B_2 X_{2t} + U_t$

$R^2 = 0.6$

60% of the variation in Z was explained by the variation in X

ما بقدر احكي انه model 1 افضل من model 2

→

② R^2 will never fall if we add explanatory variables → وجدوا انه حلها باستخدام adjusted R^2

③ R^2 takes high values in time Series regression.

Data mining
is searching many series for statistical relationships without theoretical justification

يعني ال Multiple Regression احسن من ال Simple Regression to add variables
↓ حسب ما نزيد

ممكن يكون علاقات بدون Theory وما يكون إلها معنى

في Multiple Regression لما افسر ال slop بحط هاي الجملة
holding other variables constant.

F-test

Cost of shipment $= B_1 + B_2 *$ Package weight$_t$ $+ B_3 *$ distance shipped$_t$ $+ u_t$

Example:

لما الوزن effect package weight على Cost of shipment هل بيساوي distance shipped effect على Cost of shipment

$$H_0 : B_2 = B_3$$
$$H_1 : B_2 \neq B_3$$

two sided test طاق

unrestricted regression → بنفس ols بحدد prameter

restricted regression → القيود مفروضة على Cofficant

→

F-test

1. We estimate two regression equations which are:
   (a) unrestricted regression
   ↳ OLS freely determines the value of the parameters.
   (b) restricted regression
   ↳ take the values under the null hypothesis, impose them and then rearrange the model if needed.

Example:

$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + B_4 X_{4t} + u_t$$

$H_0 : B_3 + B_4 = 1$ F-test

$H_1 : B_3 + B_4 \neq 1$

- unrestricted regression:

$$Y_t = B_1 + B_2 X_{2t} + B_2 X_{3t} + B_4 X_{4t} + u_t$$

- restricted regression:

معظم النظر شو بتخدم بدفع نفس النتيجة.

$$\begin{cases} B_3 = 1 - B_4 \\ B_4 = 1 - B_3 \end{cases}$$

$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + (1 - B_3) X_{4t} + u_t$$

$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + \underline{X_{4t}} - B_3 X_{4t} + u_t$$

بدون coefficient

لازم انقلها للجانب الآخر.

$$Y_t - X_{4t} = B_1 + B_2 X_{2t} + B_3 X_{3t} - B_3 X_{4t} + u_t$$

$$Y_t - X_{4t} = B_1 + B_2 X_{2t} + B_3 (X_{3t} - X_{4t}) + u_t$$

let $P_t = Y_t - X_{4t}$

let $Q_t = X_{3t} - X_{4t}$

$$\boxed{P_t = B_1 + B_2 X_{2t} + B_3 Q_t + u_t} \rightarrow$$

[2] Calculate F-statistic:

$$F\text{-stat} = \frac{RRSS - URSS}{URSS} \times \frac{T-K}{m}$$

RRSS: RSS from the restricted regression

URSS: RSS from the unrestricted regression

T − K: number of prameters to be estimated in the unrestricted regression including the costant term

m: number of restriction

$H_0: B_3 + B_4 = 1$ نعوض عن الإشارات (=) في H_0

$H_1: B_3 + B_4 \neq 1$, $\boxed{B_4 = 1 - B_3}$ عشان نطلع عدد restriction.

F-stat takes positive values

F-stat ~ F-distribution.

[3] get Critical values

[4] perform the test: if F-stat > F-critical then Reject H_0

Hypothesis of interst:

Example:

$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + B_4 X_{4t} + B_5 X_{5t} + u_t$$

joint significance:

$H_0: B_2 = 0$ and $B_3 = 0$ and $B_4 = 0$ and $B_5 = 0$

$H_1: B_2 \neq 0$ or $B_3 \neq 0$ or $B_4 \neq 0$ or $B_5 \neq 0$.

أي Hypothesis بنستخدمها t-test بقدر استخدمها F-test (والعكس غير صحيح)

→

$H_0: \beta_2\beta_3 = 2$ or $H_0: \beta_2^2 = 1$ * ما بقدر استخدملهم لا t-test و F-test

Cannot be tested.

Example:

$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + B_4 X_{4t} + u_t \qquad T = 144$$

$H_0: B_2 = 1$ and $B_3 = 1$

$H_1: B_2 \neq 1$ or $B_3 \neq 1$

Q.1

unrestricted regression:

$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + B_4 X_{4t} + u_t$

Restricted regression:

$Y_t = B_1 + X_2 + X_3 + B_4 X_{4t} + u_t$

$Y_t - X_{2t} - X_{3t} = B_1 + B_4 X_{4t} + u_t$

let $Z_t = Y_t - X_{2t} - X_{3t}$

$- \; Z_t = B_1 + B_4 X_{4t} + u_t$

الاكبر دايما RRSS

Q 2 If two RSS are 436.1 and 397.2 respectively, perform the test.

RRSS = 436.1   URSS = 397.2

$$F\text{-stat} = \frac{RRSS - URSS}{URSS} \times \frac{T - K}{m}$$

$$\frac{436.1 - 397.2}{397.2} \times \frac{144 - 4}{2} = \boxed{6.68}$$

$$F\text{-critical} = \boxed{3.07}$$

F-stat > F-critical then Reject H_0.

$\Rightarrow$

problem 2, 3, 5, 6, 8

* problem 2 page 176

ⓐ $H_0: B_3 = 2$

t-test / F-test

$m = 1$

ⓑ $H_0: B_3 + B_4 = 1$

F-test

$m = 1$

ⓒ $H_0: B_3 + B_4 = 1$ and $B_5 = 1$

F-test

$m = 2$

ⓓ $H_0: B_2 = 0$ and $B_3 = 0$ and $B_4 = 0$ and $B_5 = 0$

Joint significant

F-test

$m = 4$

ⓔ $H_0: B_2 B_3 = 1$

neither t-test nor F-test

* problem 3 Page 176.

$$y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + B_4 X_{4t} + B_5 X_{5t} + U_t$$

اي وحدة من هدول تتغير Joint significant

Hypothesis d

$H_0: B_2 = 0$ and $B_3 = 0$ and $B_4 = 0$ and $B_5 = 0$

$H_1: B_2 \neq 0$ or $B_3 \neq 0$ or $B_4 \neq 0$ or $B_5 \neq 0$

Joint significance test.

* problem 4 page 177

RRSS $\geq$ URSS

OLS method Choose the Coefficients when RSS is at it's minimum.

* problem 5 page 177

$H_0: B_3 + B_4 = 1$ and $\boxed{B_5 = 1}$

$\boxed{B_4 = 1 - B_3}$

$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + (1 - B_3) X_{4t} + X_{5t} + U_t$$

$$Y_t = B_1 + B_2 X_{2t} + B_3 X_{3t} + X_{4t} - B_3 X_{4t} + X_{5t} + U_t$$

$\Rightarrow$

$$Y_t - X_{4t} - X_{5t} = B_1 + B_2 X_{2t} + B_3 X_{3t} - B_3 X_{4t} + u_t$$
$$Y_t - X_{4t} - X_{5t} = B_1 + B_2 X_{2t} + B_3 (X_{3t} - X_{4t}) + u_t$$

let $Z_t = Y_t - X_{4t} - X_{5t}$
let $Q_t = X_{3t} - X_{4t}$

restricted model

$$Z_t = B_1 + B_2 X_{2t} + B_3 Q + u_t$$

$$F\text{-stat} = \frac{RRSS - URSS}{URSS} * \frac{T-k}{m}$$

$$\frac{102.87 - 91.41}{91.41} * \frac{96-5}{2} = \boxed{5.7}$$

F-critical = 3.09

using excel

5.7 > 3.09 then Reject H_0

* problem 6 page 177

$$t\text{-stat} = \frac{\hat{B}}{SE(\hat{B})}$$

T = 200
d.f. = 195
α = 5%

* $\hat{B}_1$ t-ratio = $\frac{0.08}{0.064} = 1.25$

$|1.25| < 1.97$ then fail the Reject H_0

* $\hat{B}_2$ t-ratio = $\frac{0.801}{0.147} = 5.4$

t-critical = 1.97

$|5.4| > 1.97$ then Reject H_0

There is a relationship between size of the firm and the stock Return.

* $\hat{B}_3$ t-ratio = $\frac{0.321}{0.136} = 2.3$

$|2.3| > 1.97$ then Reject H_0

There is a relationship between MB ratio and stock Return.

market to book ratio = $\frac{\text{market price / share}}{\text{BV / share}}$

BV / share = $\frac{\text{Total common equity}}{\text{number of common shares outstanding}}$

P/E ratio = $\frac{\text{market price / share}}{\text{EPS}}$

→

* $\hat{B}_4$ t-ratio $= \frac{0.164}{0.42} = 0.39$

$|0.39| < 1.97$ then fail to Reject H_0

There is no a relationship between P/E ratio and stock Return.

* $\hat{B}_5$ t-ratio $= \frac{-0.084}{0.120} = -0.7$

$|-0.7| = 0.7 < 1.97$ then fail to Reject H_0

There is no a relationship between beta and stock Return.

* Delete P/E , Beta.

$B = \frac{DY}{DX}$ $\quad -0.084 = \frac{DY}{1.2-1} \quad = -0.084(0.2) = DY$

$\boxed{DY = -1.68\%}$

Problem 8 page 177

Second model → R^2 is higher (extra explanatory variable)

second model → adjusted R^2 is expected to be higher but we should consider degrees of freedom.