11.1 Derivatives of Logarithmic Functions

Saturday, June 18, 2022 6:27 AM

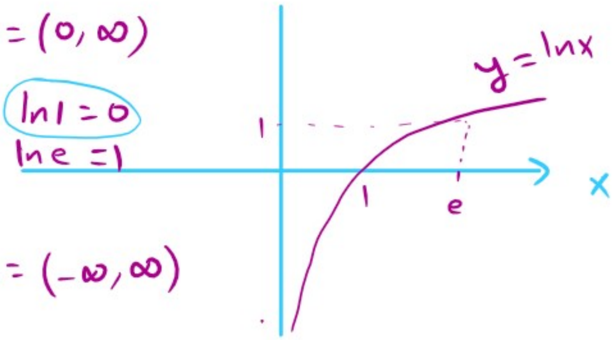
→ Natural logarithmic function $y = \ln \boxed{x}$, $x > 0$

$$y' = \frac{1}{\boxed{x}}$$

$D = (0, \infty)$

$\ln 1 = 0$

$\ln e = 1$



$R = (-\infty, \infty)$

→ logarithmic function $y = \log_a x = \dfrac{\ln \boxed{x}}{\ln a}$

$x > 0$

base
الأساس

$a \neq 1$

$a > 0$

$$y' = \frac{1}{\ln a} \, \frac{1}{\boxed{x}}$$

Exp $y = \log_e x = \dfrac{\ln x}{\ln e} = \dfrac{\ln x}{1} = \ln x$

$a = e$

Natural log. function

Exp Find y' if ① $y = x^2 \ln x$

الأول الثاني

$$y' = \cancel{x^2}\,^{x}\left(\frac{1}{\cancel{x}}\right) + (\ln x)(2x)$$

$= x + 2x \ln x$

$y'(1) = 1 + 2x \underbrace{\ln 1}_{0}$

$= 1 + 0$

$= 1$

② $y = x^3 + \ln \boxed{\sqrt{x}}$

$y' = 3x^2 + \dfrac{1}{2\sqrt{x}} \div \boxed{\sqrt{x}}$
$\phantom{y' = 3x^2 + \dfrac{1}{2\sqrt{x}} \div} 1$

$= 3x^2 + \dfrac{1}{2\sqrt{x}} \cdot \dfrac{1}{\sqrt{x}}$

$= 3x^2 + \dfrac{1}{2x}$

$\sqrt{x} = x^{\frac{1}{2}}$

$(\sqrt{x})' = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}}$

$= \dfrac{1}{2\sqrt{x}}$

$\sqrt{x}\sqrt{x} = x^{\frac{1}{2}} x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^1 = x$

③ $f(x) = \ln(x^4 - 2x + 7)$

$f'(x) = \dfrac{4x^3 - 2}{x^4 - 2x + 7}$

④ $g(x) = \frac{1}{3} \ln(2x^6 - 3x + 2)$

$g'(x) = \frac{1}{3} \dfrac{12x^5 - 3}{}$

$$g'(x) = \frac{1}{3}\left(\frac{12x^{5}-3}{2x^6-3x+2}\right)$$

$$= \frac{1}{\cancel{3}} \frac{\cancel{3}(4x^5-1)}{2x^6-3x+2} = \frac{4x^5-1}{2x^6-3x+2}$$

(5) $h(x) = \dfrac{\ln(3x+1)}{3x+1}$ بسط

مقام

مشتقة البسط

مقام

$$h'(x) = \frac{(3x+1)\left(\frac{3}{3x+1}\right) - \ln(3x+1)\,[3]}{(3x+1)^2}$$

$$= \frac{3 - 3\ln(3x+1)}{(3x+1)^2}$$

$$h'(0) = \frac{3-3\ln(0+1)}{(0+1)^2} = \frac{3-3\overset{0}{\ln 1}}{1} = \frac{3-0}{1} = 3$$

Properties of $y = \ln x$ $\qquad x>0,\ y>0$

① $\ln xy = \ln x + \ln y$ | ④ $\ln e^x = x(\ln e) = x \quad \forall x$

① $\ln xy = \ln x + \ln y$

② $\ln \frac{x}{y} = \ln x - \ln y$

③ $\ln x^{y} = y \ln x$

④ $\ln e^{x} = x \ln e = x$

⑤ $e^{\ln x} = x$, $x > 0$

properties for $y = \log_a x$, $x > 0$, $a > 0$, $a \neq 0$

① $\log_a xy = \log_a x + \log_a y$, $y > 0$

② $\log_a \frac{x}{y} = \log_a x - \log_a y$

③ $\log_a x^{y} = y \log_a x$

④ $\cancel{\log_a} a^{x} = x \log_a a = x$ for all x

⑤ $a^{\log_a x} = x$ for all $x > 0$

$\log_2 x$ 7 7

logarithmic form

$$\boxed{y = \log_a x}$$

Exponensial

$$2^{\log_2 x} = x^7$$

$$a^y = x$$ Exponential form

Exp Use logarithmic properties to find y' if

① $y = \ln\left[x\,(x^5-2)^{10}\right]$

$= \ln x + \ln (x^5-2)^{10}$

$= \ln x + 10 \ln(x^5-2)$

$$y' = \frac{1}{x} + \frac{(10)\ 5x^4}{x^5-2}$$

$$= \frac{1}{x} + \frac{50x^4}{x^5-2}$$

② $y = \ln\left(\frac{\sqrt[3]{3x+5}}{x^2+11}\right)^4$

$$= 4\ln\left(\frac{(3x+5)^{\frac{1}{3}}}{x^2+11}\right)$$

$$= 4\left[\ln(3x+5)^{\frac{1}{3}} - \ln(x^2+11)\right]$$

$$y = 4\left[\frac{1}{3}\ln(3x+5) - \ln(x^2+11)\right]$$

$$y' = 4\left[\cancel{\frac{1}{3}}\ \frac{\cancel{3}}{3x+5} - \frac{2x}{x^2+11}\right]$$

$$= \frac{4}{3x+5} - \frac{8x}{x^2+11}$$

Exp $f(x) = \dfrac{\overset{\text{عل}}{\ln x}}{\underset{\text{سام}}{x}}$, $x > 0$ Find

① intervals where f is increasing / decreasing

② maximum / min values and point

① $$f'(x) = \frac{\cancel{x}\ \frac{1}{\cancel{x}} - \ln x\ (1)}{x^2} = 0$$

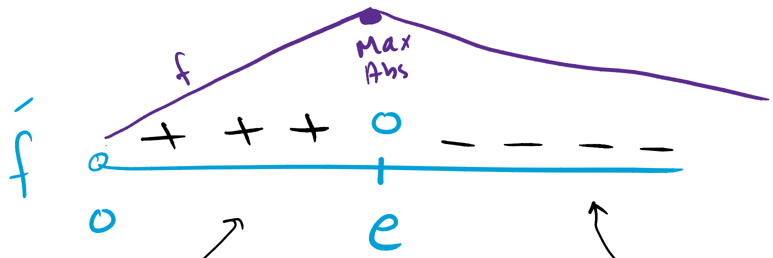
$$f'(x) = \frac{1 - \ln x}{x^2} = 0 \Rightarrow 1 - \ln x = 0$$

$$1 = \ln x \rightarrow x = e$$

$f'(x) = \dfrac{\qquad}{x^2} = 0 \Rightarrow$

$1 = \ln x \to x = e$

$e^1 = e^{\ln x} = x$



$x = \sqrt{e}$

$x = e^2 \Rightarrow f'(e^2) = \dfrac{1 - \ln e^2}{(e^2)^2} = \dfrac{1-2}{e^4} < 0$

$f'(\sqrt{e}) = \dfrac{1 - \ln\sqrt{e}}{(\sqrt{e})^2} = \dfrac{1 - \ln e^{\frac{1}{2}}}{e} = \dfrac{1 - \frac{1}{2}}{e} > 0$

f is increasing on $(0, e)$

f is decreasing on (e, ∞)

(2) $x = e$ is critical value

$f(x) = \dfrac{\ln x}{x}$

$f(e) = \dfrac{\ln e}{e} = \dfrac{1}{e}$

$f(x)$ has Abs. Max of $\frac{1}{e}$ occurs at $x = e$

$\left(e, \frac{1}{e}\right)$

Exp Find tangent line for $f(x) = x \ln x$

Exp Find tangent line for $f(x) = x\ln x$ at $x = 1$

$$m = f'(x_0) = f'(1)$$

$x_0 = 1$

$y_0 = f(x_0) = f(1) = \boxed{1}\ln\boxed{1} = 0$

$$f'(x) = \cancel{x}\,\frac{1}{\cancel{x}} + \ln x\,(1)$$

$$\boxed{f'(x) = 1 + \ln x}$$

$$m = f'(1) = 1 + \ln 1 = 1$$

$$y - y_0 = m(x - x_0)$$

$$y - \boxed{0} = \boxed{1}(x - \boxed{1})$$

$$\boxed{y = x - 1}$$

Exp (Cost) Total Cost $C(x) = 18{,}250 + 615\ln(4x+10)$

① Find marginal cost when 100 units are produced.

② what does ① mean.

$$\overline{MC} = C'(x) = 0 + 615\,\frac{4}{4x+10}$$

$$\overline{MC} = C'(x) = \frac{2460}{4x+10}$$

$$\overline{MC}(100) = C'(100) = \frac{2460}{4(100)+10} = \frac{246\cancel{0}}{41\cancel{0}} = \$\,6$$

② ... approximated cost

(2) $\overline{MC}(100) = \$6$ means the approximated cost of producing the 101 is \$6

Exp Find y' if

(1) $y = 1 + \log_8 10^x$

$= 1 + x\log_8 10 \longrightarrow$ رقم

$y' = 0 + \log_8 10$

$y' = \log_8 10$

(2) $y = x^2 + \log_5 (x^3 + 10x) \longrightarrow \dfrac{\ln(x^3+10x)}{\ln 5}$

$y' = 2x + \dfrac{1}{\ln 5} \cdot \dfrac{3x^2 + 10}{(x^3 + 10x)}$

$$= 2x + \frac{3x^2+10}{\ln 5\,(x^3+10x)}$$

③ $y = \log_3 \sqrt{2-x^3}$ $\longrightarrow$ $\dfrac{\ln\sqrt{2-x^3}}{\ln 3}$

$$= \frac{1}{\ln 3} \cdot \frac{1}{2} \ln(2-x^3)$$

$$= \frac{1}{\ln 3}\,\frac{1}{2} \ln(2-x^3)$$

اضم

$$y' = \frac{1}{2\ln 3}\;\frac{-3x^2}{2-x^3}$$

$$= \frac{-3}{2\ln 3}\;\frac{x^2}{(2-x^3)}$$

$$= \frac{-3}{\ln 9}\;\frac{x^2}{(2-x^3)}$$