Exercises:

18.

x_i:	1	2	3	4	5
y_i:	3	7	5	11	14

$\hat{y} = 0.2 + 2.6x$

$\hat{y}(4) = 0.2 + 2.6(4)$

$\hat{y}(4) = 10.6$

a. develop a 95% CI for the expected value of y when $X = 4$.

$1 - \alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$(1-\alpha)$ CI $E(y_p) = \hat{y}_p \mp S t_{\frac{\alpha}{2}} \sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$ with $df = 3$

→ $x_p = 4 \rightsquigarrow y_p = 10.6$

→ $\bar{X} = 3$

→ $S = \sqrt{MSE} = \sqrt{4.13} = 2.033$

→ $t_{0.025}$ with $df = 3 = 3.182$

→ $\Sigma(x_i - \bar{x})^2 = (4 + 1 + 0 + 1 + 4) = 10$

$SST = (n-1)s_y^2 = 4(20) = 80$

$SSR = b_1^2(n-1)s_x^2 = 6.76(4)(2.5) = 67.6$

$SSE = SST - SSR = 12.4$

$MSE = \frac{12.4}{3} = 4.13$

→ $E(y_p) = 10.6 \mp (2.033)(3.182)\sqrt{\frac{1}{5} + \frac{(4-3)^2}{10}}$

$= 10.6 \mp 3.54$

$= (7.06, 14.14)$.

b. develop a 95% prediction interval for y when $X = 4$.

$(1-\alpha)$ PI for $y_p = \hat{y}_p \mp S\, t_{\frac{\alpha}{2}} \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$

$= 10.6 \mp (2.033)(3.182)\sqrt{1 + \frac{1}{5} + \frac{1}{10}}$

$= 10.6 \mp 7.38$

$= (3.22, 17.98)$

$b_0 = 30.33$

$b_1 = -1.88$

$S_x^2 = 7.7$

$S_y^2 = 28.7$

19.

x_i:	2	3	5	1	8
y_i:	25	25	20	30	16

$\hat{y} = 30.33 - 1.88x$

$\hat{y}(3) = 24.69$

a. estimate the standard deviation of $\hat{y}_p$ when $x = 3$.

$1-\alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

b. Develop a 95% CI for the expected value of y when $x = 3$:

$$(1-\alpha)\ CI\ E(y_p) = \hat{y}_p \mp S\, t_{\frac{\alpha}{2}} \sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{\Sigma (x_i - \bar{x})^2}}$$

$SSR = b_1^2 (n-1) S_x^2 = 108.86$

$SST = (n-1) S_y^2 = 114.8$

$SSE = 5.94$

$MSE = \frac{SSE}{n-2} = 1.98$.

- $x_p = 3 \rightsquigarrow y_p = 24.69$
- $\bar{x} = 3.8$
- $S = \sqrt{MSE} = \sqrt{1.98} = 1.41$.
- $t_{\frac{\alpha}{2}}$ with $df = 3 \rightsquigarrow t_{0.025} = 3.182$
- $\Sigma (x_i - \bar{x})^2 = (3.24 + 0.64 + 1.44 + 7.84 + 17.64) = 30.8$.

$$\rightsquigarrow (1-\alpha)CI \text{ of } E(y_p) = 24.69 \mp 1.41\,(3.182) \sqrt{\frac{1}{5} + \frac{(3-3.8)^2}{30.8}}$$

$$= 24.69 \mp 2.11$$

$$= (22.58,\ 26.8).$$