

11.1 Parametrizations of Plane Curves

Definition: If x and y are given as functions

$$x = f(t), \quad y = g(t)$$

over an interval I of t -values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a parametric curve. The equations are parametric equations for the curve

- The variable t is a parameter for the curve.
- I is the parameter interval.
- If I is a closed interval $[a, b]$, $(f(a), g(a))$ is the initial point and $(f(b), g(b))$ is the terminal point

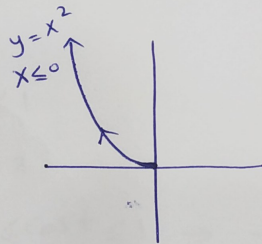
Questions: 2, 6, 10, 14, 15, 18, 20, 22, 26

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Identify the particle's path by finding a Cartesian equation ~~for~~. Graph the Cartesian equation. Indicate the portion of the graph traced by the particle and the direction of motion.

2 $x = -\sqrt{t}$, $y = t$, $t \geq 0$

$$x = -\sqrt{y} \rightarrow y = x^2 \text{ but } y \geq 0, x \leq 0$$



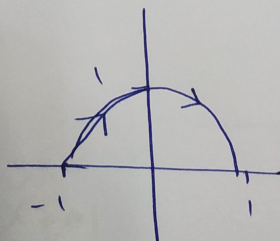
6 $x = \cos(\pi - t)$, $y = \sin(\pi - t)$, $0 \leq t \leq \pi$

$$x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1$$

$$y \geq 0$$

$$\sin(\pi - t) \geq 0$$

$$\text{for } 0 \leq t \leq \pi$$



$$\boxed{10} \quad x = 1 + \sin t, \quad y = \cos t - 2, \quad 0 \leq t \leq \pi$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$\sin^2 t = (x-1)^2 \qquad \cos^2 t = (y+2)^2$$

$$\sin^2 t + \cos^2 t = (x-1)^2 + (y+2)^2 = 1$$

a circle with center $(1, -2)$ and a radius of 1

Note that
for $0 \leq t \leq \pi$

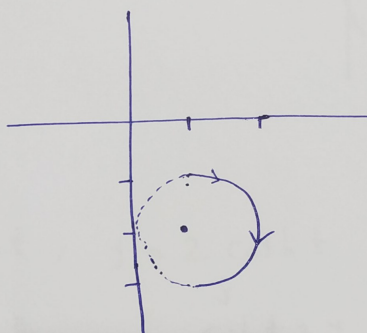
$$x \geq 0$$

$$y < 0$$

$$0 \leq \sin t \leq 1$$

$$1 \leq x \leq 2$$

$$\text{and } -1 \leq \cos t \leq 1 \rightarrow -3 \leq y \leq -1$$



$$\boxed{14} \quad x = \sqrt{t+1}, \quad y = \sqrt{t}, \quad t \geq 0$$

$$\downarrow$$

$$y^2 = t, \quad y \geq 0$$

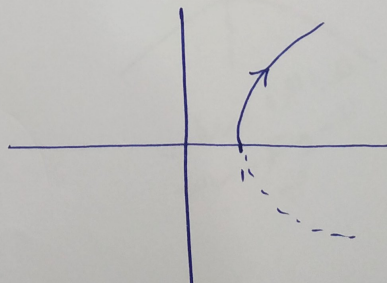
$$x = \sqrt{y^2 + 1}$$

$$x^2 = y^2 + 1$$

$$x^2 - y^2 = 1$$

a hyperbola with $y \geq 0$

$$x \geq 1$$

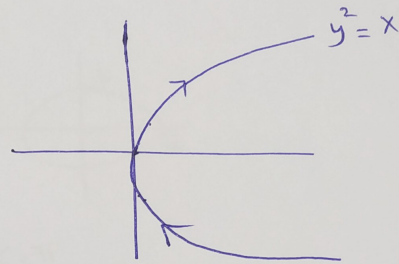


$$\boxed{15} \quad x = \sec^2 t - 1, \quad y = \tan t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

$$y^2 = \tan^2 t$$

$$x = \sec^2 t - 1 = \tan^2 t = y^2$$

$$x = y^2$$



$$\boxed{18} \quad x = 2 \sinh t, \quad y = 2 \cosh t, \quad -\infty < t < \infty$$

$$\downarrow \quad \sinh t = \frac{x}{2} \quad \downarrow \quad \cosh t = \frac{y}{2} \quad , \text{ note that } y > 0$$

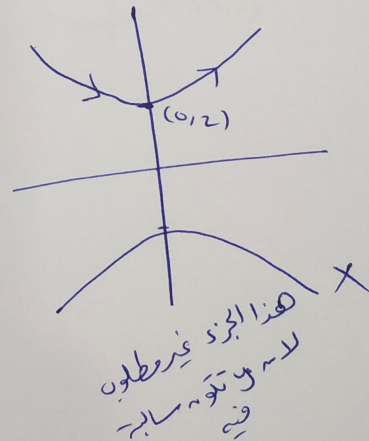
$$\cosh^2 t - \sinh^2 t = 1$$

$$\left(\frac{y}{2}\right)^2 - \left(\frac{x}{2}\right)^2 = 1$$

$$\frac{y^2}{4} - \frac{x^2}{4} = 1$$

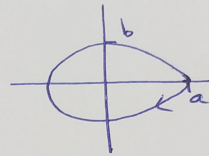
$$y^2 - x^2 = 4$$

hyperbola



20 Find parametric equations and parameter interval for the motion of a particle that starts at $(a, 0)$ and traces the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

a) once clockwise

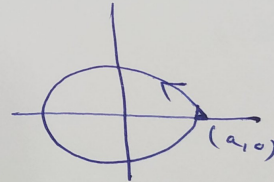


$$x = a \sin t$$

$$y = b \cos t \quad \frac{\pi}{2} \leq t \leq \frac{5\pi}{2}$$

$$\left. \begin{array}{l} a \sin t = a \rightarrow \sin t = 1 \\ b \cos t = 0 \rightarrow \cos t = 0 \end{array} \right\} t = \frac{\pi}{2}$$

b) once counterclockwise



$$x = a \cos t, \quad y = b \sin t$$

$$\left. \begin{array}{l} a \cos t = a \rightarrow \cos t = 1 \\ b \sin t = 0 \rightarrow \sin t = 0 \end{array} \right\} t = 0$$

$$0 \leq t \leq 2\pi$$

c) twice clockwise :-

$$x = a \sin t, \quad y = b \cos t \quad \frac{\pi}{2} \leq t \leq \frac{9\pi}{2}$$

d) twice counterclockwise

$$x = a \cos t, \quad y = b \sin t \quad 0 \leq t \leq 4\pi$$

[22] Find a parametrization of the curve :-

the line segment with endpoints $(-1, 3)$
and $(3, -2)$

using ~~$(-1, 3)$~~

$$\begin{aligned} x &= 3 + at \\ y &= -2 + bt \end{aligned}$$

$\rightarrow -1 = 3 + at$

$$(-1, 3)$$

$$\left. \begin{aligned} x &= -1 + at \\ y &= 3 + bt \end{aligned} \right\} \quad (-1, 3) \text{ at } t=0$$

$$(3, -2), \quad t=1$$

$$3 = -1 + a(1) \rightarrow a = 4$$

$$-2 = 3 + b(1) \rightarrow b = -5$$

$$x = -1 + 4t$$

$$y = 3 - 5t$$

26 Find a parametrization for the ray with initial point $(-1, 2)$ that passes through the point $(0, 0)$

$$\left. \begin{aligned} x &= -1 + at \\ y &= 2 + bt \end{aligned} \right\} \quad t=0$$

for $(0, 0)$ assume $t=2$

$$0 = -1 + a(2) \rightarrow a = \frac{1}{2}$$

$$0 = 2 + 2(b) \rightarrow b = -1$$

$$\begin{cases} X = -1 + \frac{1}{2}t \\ Y = 2 - t \end{cases}, t \geq 0$$