

2.1

Quadratic Equations (QE)

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The general form of any quadratic equation is

$$ax^2 + bx + c = 0 \quad \text{where } a, b, c \text{ constant, } a \neq 0$$

Exp write this quadratic equation $3x^2 - 2x = x^2 + 4x - 5$ in the general form and determine a, b, c

$$3x^2 - 2x = \cancel{x^2} + 4x - 5 \quad \Rightarrow \quad 2x^2 - 2x = \cancel{4x} - 5$$

$\begin{matrix} -x^2 & & -x^2 \\ -4x & & -4x \end{matrix}$

$$2x^2 - 6x = -5 \quad \Rightarrow \quad 2x^2 - 6x + 5 = 0$$

$\begin{matrix} +5 & +5 \end{matrix}$

$a = 2$
 $b = -6$
 $c = 5$

Exp How we solve QE: $ax^2 + bx + c = 0$?

There are two methods $\left\{ \begin{array}{l} \text{Factoring Method - FM} \\ \text{Quadratic Formula - QF} \end{array} \right.$

Remark For any real numbers a, b :

$ab = 0$ if and only if $a = 0$ or $b = 0$

Exp Solve $(2x - 4)(1 - 3x) = 0$

$$2x - 4 = 0 \quad \text{or} \quad 1 - 3x = 0$$

$$2x = 4$$

$$-3x = -1$$

$$x = 2$$

$$x = \frac{1}{3}$$

solutions

Exp Solve the following equations by factoring 60

1) $x^2 = 11x - 10$

general form:

$$x^2 - 11x + 10 = 0 \quad \checkmark$$

$$(x+a)(x+b) = 0$$

$$(x-1)(x-10) = 0$$

$$x-1=0 \quad \underline{\text{or}} \quad x-10=0$$

$$\boxed{x=1}$$

$$\boxed{x=10}$$

Check: $(1)^2 \stackrel{?}{=} 11(1) - 10$
 $1 = 1 \quad \checkmark$

$$(10)^2 \stackrel{?}{=} 11(10) - 10$$
$$100 = 110 - 10$$
$$100 = 100 \quad \checkmark$$

$$ab = 10$$

$$a+b = -11$$

$$a = -1$$

$$b = -10$$

2) $(y-3)(y+2) = -4$

general form $\Rightarrow (y-3)(y+2) = -4$

$$y^2 + 2y - 3y - 6 = -4$$

$$y^2 - y - 6 = -4$$

$$\boxed{y^2 - y - 2 = 0} \quad \checkmark$$

$$(y+a)(y+b) = 0$$

$$(y+1)(y-2) = 0$$

$$y+1=0 \quad \underline{\text{or}} \quad y-2=0$$

$$\boxed{y=-1}$$

$$\boxed{y=2}$$

check $(-1-3)(-1+2) \stackrel{?}{=} -4$
 $(-4)(1) = -4$
 $\checkmark$

$$(2-3)(2+2) \stackrel{?}{=} -4$$
$$(-1)(4) = -4$$
$$\checkmark$$

$$ab = -2$$

$$a+b = -1$$

$$a = -2$$

$$b = 1$$

$$\boxed{3} \quad \frac{x+1}{3x+6} = \frac{3}{x} + \frac{2x+6}{x(3x+6)}$$

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$$\frac{x}{x} \cdot \frac{x+1}{3x+6} = \frac{3}{x} \cdot \frac{3x+6}{3x+6} + \frac{2x+6}{x(3x+6)}$$

$$\frac{x(x+1)}{x(3x+6)} = \frac{3(3x+6)}{x(3x+6)} + \frac{2x+6}{x(3x+6)}$$

multiply by the
LCD = $x(3x+6)$

$$x(x+1) = 3(3x+6) + 2x+6$$

$$x^2 + x = \underline{9x} + \underline{18} + \underline{2x} + \underline{6}$$

$$x^2 + x = 11x + 24$$

~~$-11x$~~ ~~$-11x$~~

$$x^2 - 10x = 24$$

~~-24~~ ~~-24~~

$$\boxed{x^2 - 10x - 24 = 0} \quad \text{general form}$$

$$(x+a)(x+b) = 0$$

$$(x-12)(x+2) = 0$$

$$\left\{ \begin{array}{l} a+b = -10 \\ ab = -24 \\ \hline a = -12 \\ b = 2 \end{array} \right.$$

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$$x-12=0 \quad \text{or} \quad x+2=0$$

$$x = 12$$

✓

$$x = -2$$

x

Check $x=12$

$$\frac{12+1}{3(12)+6} \stackrel{?}{=} \frac{3}{12} + \frac{2(12)+6}{(12)(3(12)+6)}$$

$$\frac{13}{42} \stackrel{?}{=} \frac{1}{4} + \frac{\cancel{30}^5}{\cancel{12}^2(42)}$$

$$= \frac{84+20}{(8)(42)} = \frac{\cancel{104}^{13}}{\cancel{(8)}^1(42)}$$

$$\frac{13}{42} = \frac{13}{42}$$

Check $x=-2$

$$\frac{(-2)+1}{3(-2)+6} \stackrel{?}{=} \frac{3}{-2} + \frac{2(-2)+6}{(-2)(3(-2)+6)}$$

zero in denominator

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Hence, the only solution
is $\boxed{x=12}$

$$(4) \quad x^2 - 7 = 9$$

general form $\Rightarrow x^2 - 16 = 0$

$$(x+a)(x+b) = 0$$

$$(x+4)(x-4) = 0$$

$$\begin{aligned} a+b &= 0 \\ ab &= -16 \end{aligned}$$

$$\begin{aligned} a &= 4 \\ b &= -4 \end{aligned}$$

$$x+4=0 \quad \text{or} \quad x-4=0$$

$$\boxed{x = -4}$$

$$\boxed{x = 4}$$

Check

$$\begin{array}{l|l} (-4)^2 - 7 \stackrel{?}{=} 9 & (4)^2 - 7 = 9 \\ 16 - 7 \stackrel{?}{=} 9 & 16 - 7 = 9 \\ 9 = 9 \checkmark & 9 = 9 \checkmark \end{array}$$

or

$$x^2 - 7 = 9$$

$$x^2 = 16$$

$$x = \pm \sqrt{16}$$

$$= \pm 4$$

$$(5) \quad x^2 - 5 = 0 \Rightarrow x^2 = 5 \Rightarrow x = \pm \sqrt{5}$$

or $(x - \sqrt{5})(x + \sqrt{5}) = 0$

$$x - \sqrt{5} = 0 \quad \text{or} \quad x + \sqrt{5} = 0$$

$$x = \sqrt{5}$$

$$x = -\sqrt{5}$$

square
root
method

Remark The solution of $x^2 = c$ is $x = \pm \sqrt{c}$ where $c \geq 0$

$$(6) \quad 4x^2 = 9 \Rightarrow x^2 = \frac{9}{4} \Rightarrow x = \pm \sqrt{\frac{9}{4}} = \pm \frac{3}{2}$$

$$(7) \quad (3x-4)^2 = 25 \Rightarrow 3x-4 = \pm \sqrt{25} \Rightarrow 3x-4 = \pm 5$$

$$3x-4 = 5 \quad \text{or} \quad 3x-4 = -5$$

$$3x = 9$$

$$\boxed{x = 3}$$

$$3x = -1$$

$$\boxed{x = -\frac{1}{3}}$$

Now we use Quadratic Formula to solve

the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Delta = b^2 - 4ac$ is called discriminant

$$= \frac{-b \pm \sqrt{\Delta}}{2a}$$

- If $\Delta > 0$ then the equation has two distinct ^{real} roots
- If $\Delta = 0$ then the equation has exactly one real root
- If $\Delta < 0$ then the equation has no real solutions

Exp solve the following equations using the quadratic formula:

① $2x^2 - 3x = -1 \Rightarrow$ general form $2x^2 - 3x + 1 = 0$

$a=2, b=-3, c=1$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta = b^2 - 4ac$$

$$= \frac{-(-3) \pm \sqrt{1}}{2(2)} = \frac{3 \pm 1}{4}$$

$$= (-3)^2 - 4(2)(1)$$

$$= 9 - 8$$

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$$x_1 = \frac{3+1}{4} = \frac{4}{4} = 1$$

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= 1 ^{g two real roots}

$$x_2 = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

since $\Delta > 0 \Rightarrow$ there is two distinct real roots

Check $x_1 = 1 \Rightarrow 2(1)^2 - 3(1) = 2 - 3 = -1 \checkmark$

$$x_2 = \frac{1}{2} \Rightarrow 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) = \frac{2}{4} - \frac{3}{2} = \frac{1}{2} - \frac{3}{2} = \frac{-2}{2} = -1 \checkmark$$

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$$(2) \quad (x-2)^2 + 3(x-2) + \frac{9}{4} = 0$$

$$x^2 - 4x + 4 + 3x - 6 + \frac{9}{4} = 0$$

$$x^2 - x - 2 + \frac{9}{4} = 0$$

$$x^2 - x + \frac{1}{4} = 0$$

$$-2 + \frac{9}{4} = \frac{4x-2}{4 \times 1} + \frac{9}{4}$$

$$= \frac{-8}{4} + \frac{9}{4}$$

$$= \frac{1}{4}$$

$$a=1, \quad b=-1, \quad c=\frac{1}{4}$$

$$D = b^2 - 4ac = (-1)^2 - 4(1)(\frac{1}{4})$$

$$= 1 - 1$$

$$= 0 \quad \exists \text{ only one real root}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{0}}{2(1)} = \frac{1}{2} \quad \text{"x-intercept means root"}$$

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check $x = \frac{1}{2} \Rightarrow$

$$\left(\frac{1}{2} - 2\right)^2 + 3\left(\frac{1}{2} - 2\right) + \frac{9}{4} \stackrel{?}{=} 0$$

$$\left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) + \frac{9}{4} \stackrel{?}{=} 0$$

$$\frac{9}{4} - \frac{9}{2} + \frac{9}{4} \stackrel{?}{=} 0$$

$$\frac{18}{4} - \frac{9}{2} \stackrel{?}{=} 0$$

$$\frac{9}{2} - \frac{9}{2} = 0 \quad \checkmark$$