Math330

Newton and secant: Order of Convergence (P unknown)

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(Q1) Consider the equation $x = \cos x$

(a) Use Newton's method with $P_0 = 0.2$ to estimate the solution of this equation with error less that 10^{-5}

(b) Find the order of convergence and the asymptotic error constant both numerically and theoretically.

Solution: (a) $f(x) = x - \cos x$, $f'(x) = 1 + \sin x$, $f''(x) = \cos x$

Newton's iteration: $P_{n+1} = P_n - \dfrac{P_n - \cos P_n}{1 + \sin P_n}$

n	P_n	$\lvert P_n - P_{n-1}\rvert$
0	0.2	———
1	0.850777122	0.650777122
2	0.741530193	0.109246929
3	0.739086449	0.002443744
4	0.739085133	0.000001316

So $P \approx P_4 = 0.739085133$

(b) Theoretically: $f'(P) = f'(0.739085133) = 1.673612029 \neq 0$,

So P is a simple root.

Therefore, $R = 2$ and $A = \dfrac{|f''(P)|}{2|f'(P)|} = 0.220805395$

Numerically:

$\lvert E_n\rvert \approx \lvert P_n - P_{n-1}\rvert$	$\dfrac{\lvert E_{n+1}\rvert}{\lvert E_n\rvert^2}$
0.650777122	0.257955435
0.109246929	0.204756281
0.002443744	0.220365941
0.000001316	

(Q2) Consider the equation $x = \cos x$.

(a) If $P_0 = 0.5$ and $P_1 = \frac{\pi}{4}$, use the secant method to approximate the root of the equation with accuracy of 10^{-4}.

(b) Find the order of convergence and the asymptotic error constant both theoretically and numericaly.

Solution: (a) $f(x) = x - \cos x \quad , f'(x) = 1 + \sin x \quad , f''(x) = \cos x$

Secant's iteration: $P_{n+2} = P_{n+1} - \dfrac{f(P_{n+1})(P_{n+1} - P_n)}{f(P_{n+1}) - f(P_n)}$

n	P_n	$\lvert P_n - P_{n-1}\rvert$
0	0.5	——
1	0.785398163	0.285398163
2	0.736384138	0.049014025
3	0.739058138	0.002674
4	0.739085149	0.000027011338

So $P \approx P_4 = 0.739085149$

(b) Theoretically: $f'(P) = f'(0.739085149) = 1.673612041 \neq 0$,

So P is a simple root.

Therefore, $R = 1.618$ and $A = \left|\dfrac{f''(P)}{2f'(P)}\right|^{0.618} = 0.393185938$

Numerically:

$\lvert E_n\rvert \approx \lvert P_n - P_{n-1}\rvert$	$\dfrac{\lvert E_{n+1}\rvert}{\lvert E_n\rvert^{1.618}}$
0.285398163	0.372735166
0.049014025	0.351741034
0.002674	0.393005788
0.000027011338	