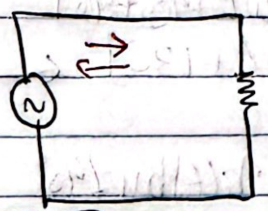
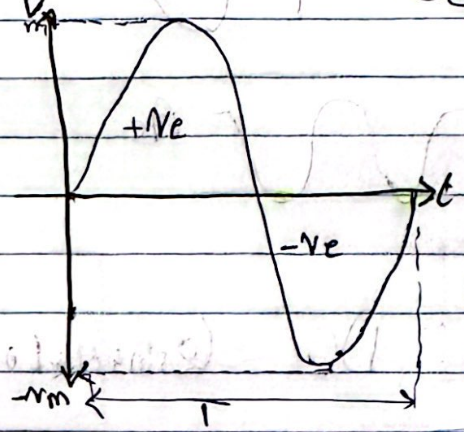
Chapter 9: AC sinosoidal steady-state analysis

التيار متغير مرة سالب ومرة موجب



$v(t) = V_m \sin \omega t$ $\quad f = \frac{1}{T}$ (Hz), $\omega = 2\pi f$ (rad/s)



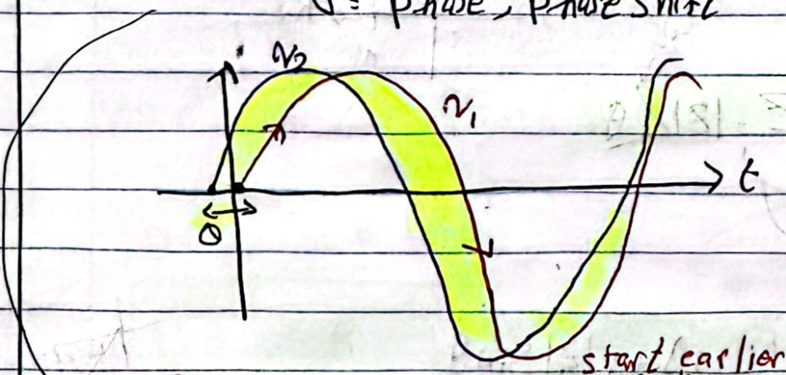
$f = \frac{1}{T}$ (Hz)

$\omega = 2\pi f$ (rad/s)

$v_1(t) = V_m \sin \omega t$

$v_2(t) = V_m \sin(\omega t + \theta)$

θ : phase, phase shift



start earlier

lead / lag

$v_2(t)$ leads $v_1(t)$ by θ

$v_1(t)$ lags $v_2(t)$ by θ

start later

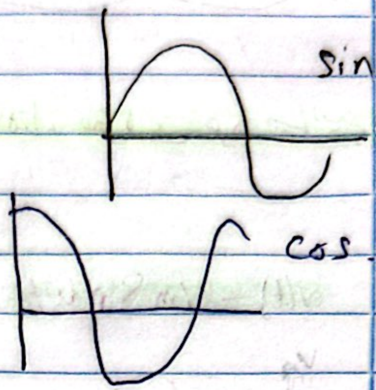
$v_1(t) = V_m \sin(\omega t - 10)$

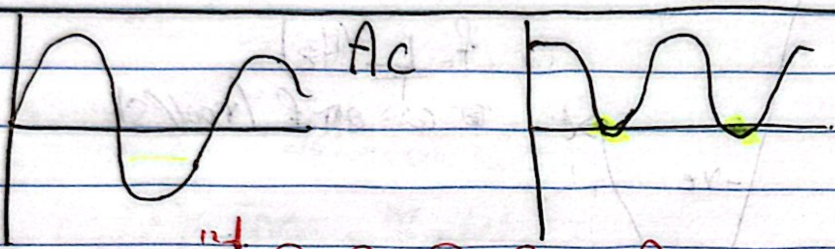
$i_1(t) = I_m \cos(\omega t + 40)$

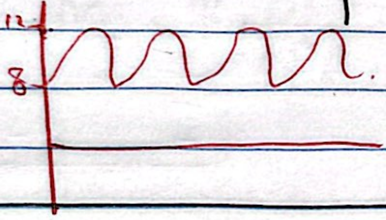
$= I_m \sin(\omega t + 40 + 90)$

$= I_m \sin(\omega t + 130^\circ)$

$i_1(t)$ leads $v_1(t)$ by 140°

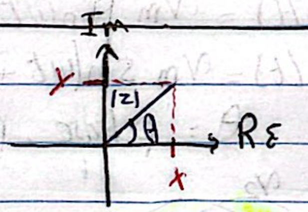






DC $(2\sin\omega t) + 10$

- Complex numbers:



$$\boxed{Z = X + jY}$$

$Z = |Z| \angle \theta$

$|Z| = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1} \frac{y}{x}$

$\rightarrow Z = x + jy = |Z| \cos\theta + j|Z| \sin\theta$

$Z = |Z| e^{j\theta}$

$\Rightarrow e^{j\theta} = \cos\theta + j \sin\theta$

$Z_1 = 4 - j2 = 4.47 \angle -26.56^\circ$

$Z_2 = 2 + j2 = 2.82 \angle 45^\circ$

بالجمع او بالطرح

$Z_1 + Z_2$

$Z_1 - Z_2$

بالضرب او القسمة

$Z_1 \times Z_2$

$\frac{Z_1}{Z_2}$

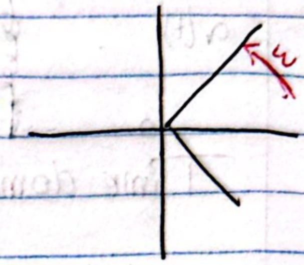
Phasor:

input $\rightarrow$ $\boxed{\text{circuit}}$ $\rightarrow$ output

$V_m \cos(\omega t + \theta_v)$ $\quad\quad$ $I_m \cos(\omega t + \theta_i)$

$\boxed{V_m \angle \theta_v}$ $\quad\quad$ $\boxed{I_m \angle \theta_i}$

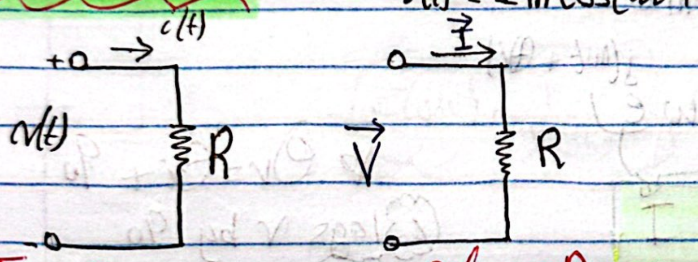
$\vec{V}$ $\quad\quad$ $\vec{I}$



* Resistor

$v(t) = V_m \cos(\omega t + \theta_i)$

$i(t) = I_m \cos(\omega t + \theta i)$



Time Domain $\quad\quad$ Phasor Domain.

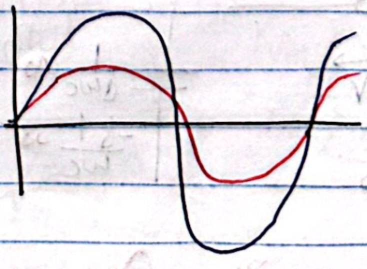
- $v(t) = R\, i(t) \longrightarrow \vec{V} = R\, \vec{I}$

$V_m e^{j(\omega t + \theta v)} = R I_m e^{j(\omega t + \theta i)}$

$V_m e^{j\theta v} = R I_m e^{j\theta i}$

$$\boxed{\vec{V} = R\, \vec{I}}$$

* $\theta_v = \theta_i$ in phase



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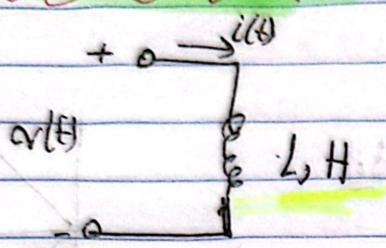
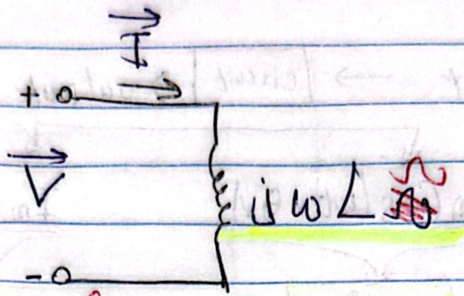
* Inductor:





Time domain — phasor domain

$$v(t) = L\frac{di(t)}{dt}$$

$$V_m e^{j(\omega t+\theta_v)} = L\frac{d}{dt} I_m e^{j(\omega t+\theta_i)}$$

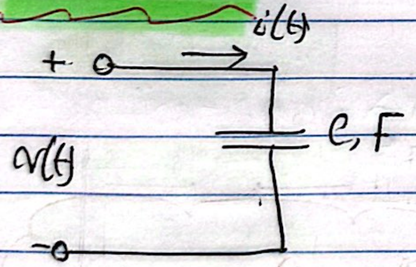
$$V_m e^{j(\omega t+\theta_v)} = L I_m j\omega e^{j(\omega t+\theta_i)}$$

$$\boxed{\vec{V} = (j\omega L)\ \vec{I}}$$

* $\theta_v = \theta_i + 90°$

i lags v by 90

* Capacitor:

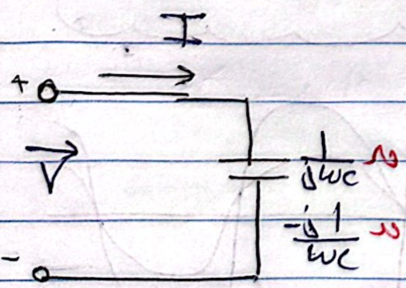


$$i(t) = C\frac{d}{dt}v(t)$$

$$\rightarrow I_m e^{j\theta_i} = C j\omega V_m e^{j\theta_v}$$

$$\boxed{\vec{I} = (j\omega C)\ \vec{V}}$$

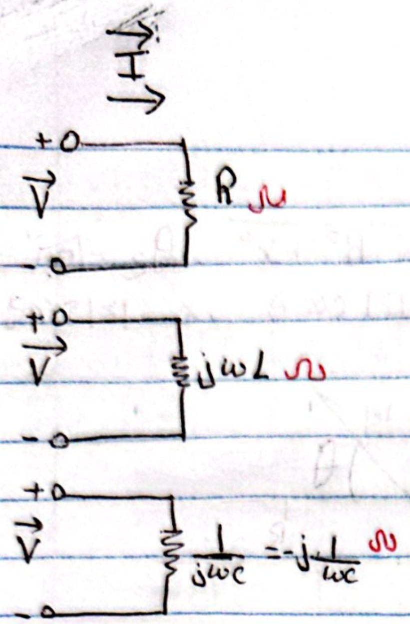
$$\boxed{\vec{V} = \frac{1}{j\omega C}\ \vec{I}}$$



$$\frac{\vec{V}}{\vec{I}} = \frac{1}{j\omega C}$$

* $\theta_i = \theta_v + 90$

& i leads v by 90



$\vec V = R\vec I$

$\vec V = (j\omega L)\vec I$

$V_s = (-j\frac{1}{\omega C})\vec I$

$\vec V = \boxed{Z(j\omega)}\,\vec I$

↳ impedance, Ω

$$\boxed{Z(j\omega) = \frac{\vec V}{\vec I}} \quad \Omega$$

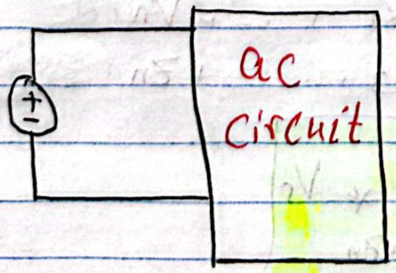
مقلوب impedance → $\boxed{Y(j\omega) = \frac{\vec I}{\vec V}}$ admittance, ℧

$℧ = \frac{1}{\Omega}$

$\therefore\ Z(j\omega) = \frac{1}{Y(j\omega)}$

Element	Impedance	admittance
R	$Z(j\omega) = R$	$Y(j\omega) = \frac{1}{R}$
C	$Z(j\omega) = \frac{1}{j\omega C}$	$Y(j\omega) = j\omega C$
L	$Z(j\omega) = j\omega C$	$Y(j\omega) = \frac{1}{j\omega L}$

Impedance: $Z(j\omega)$



$$Z(j\omega) = \frac{\vec V}{\vec I} = \frac{V_m\angle\theta_v}{I_m\angle\theta_i}$$

$$Z(j\omega) = \frac{V_m}{I_m}\angle\theta_v - \theta_i$$

$$= \underbrace{|Z|\angle\theta_z}$$

$Z(j\omega)$ is a complex number, But Not a phasor

ما بتحولها لـ sin/cos.

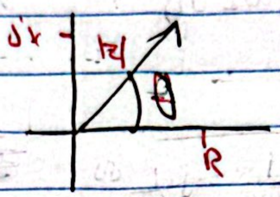
impedance

$\rightarrow Z(j\omega) = |Z| \angle \theta_Z$

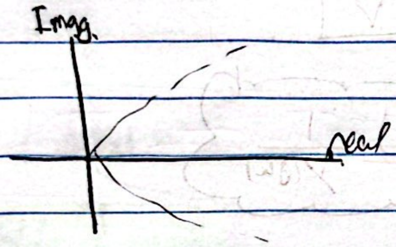
$= \underbrace{R}_{\text{resistance}} + j\underbrace{X}_{\text{reactance}}$

$|Z| = \sqrt{R^2 + X^2}$, $\theta_Z = \tan^{-1}\frac{X}{R}$

$R = |Z|\cos\theta$, $X = |Z|\sin\theta$

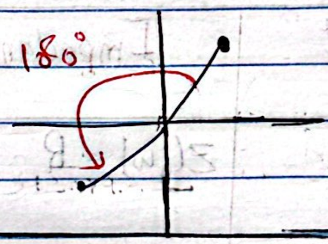


* solve all the examples (page 30 – 71) (Notes)

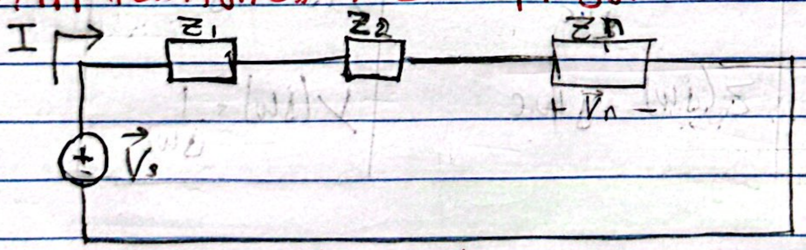


$5 + j10$ conjugate $\rightarrow 5 - j10$

$5 + j10 \rightarrow \underbrace{\times(-1)}_{\text{ضرب }-1} \rightarrow -5 - j10$ (~~±180~~) (ضرب -1)



* Application of KVL for phasor:



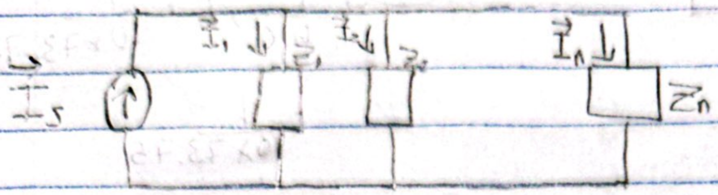
KVL: $V_s(t) = v_1(t) + v_2(t) + \dots + V_n(t)$

$\vec{V_s} = \vec{V_1} + \vec{V_2} + \dots + \vec{V_n}$

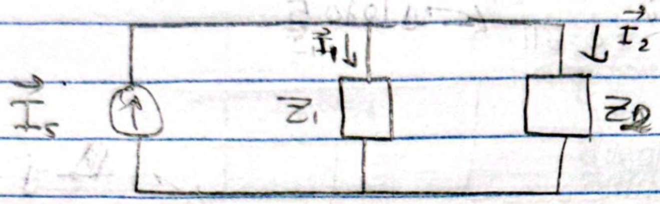
* $Z_{eq} = Z_1 + Z_2 + Z_3 + \dots + Z_n$

$$\rightarrow \boxed{\vec{V_n} = \frac{Z_n}{Z_1 + Z_2 + \dots + Z_n} * V_s}$$

* Application of KCL for Phasor:



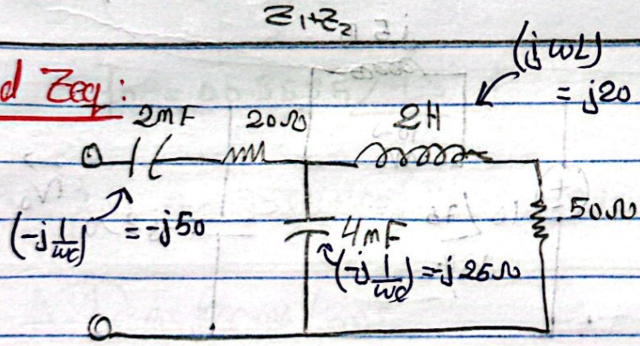
KCL: $\vec{I}_s = \vec{I}_1 + \vec{I}_2 + \dots + \vec{I}_n$



$$I_1 = \frac{Z_2}{Z_1+Z_2} * I_s$$

$$I_2 = \frac{Z_1}{Z_1+Z_2} * I_s$$

Find Zeq:



** * our reference is Cos**

إذا جاب (Sin) لازم احولها
ل (Cos)

$\omega = 10 r/s$

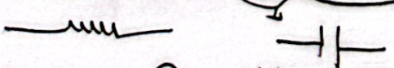
$$Z_{eq} = \left[(50 + j20) // (-j25)\right] + (20 - j50)$$

$$= \frac{(50+j20)(-j25)}{50 - j5} + 20 - j50 = \frac{-j1250 + (-j^2)(500)}{50 - j5} + 20 - j50$$

$$= \frac{1346.29 \angle -68.19^\circ}{50.25 \angle -5.71} + 20 - j50 = 26.79 \angle -62.48 + 20 - j50$$

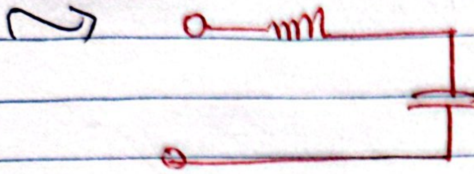
طرح الزوايا

$$= 32.37 - j73.75 \ \Omega$$



للسالب

32.37 Ω



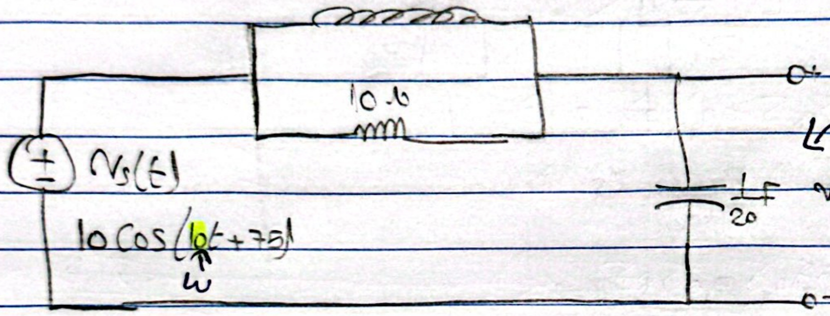
$-j\,73.75\,\Omega \rightarrow \frac{1}{\omega C}, \quad C = \frac{1}{\omega * 73.75}$

$= \frac{1}{10 \times 73.75}$

$\boxed{C = 1.35\,mF}$

※ Calculate $V_o(t)$:

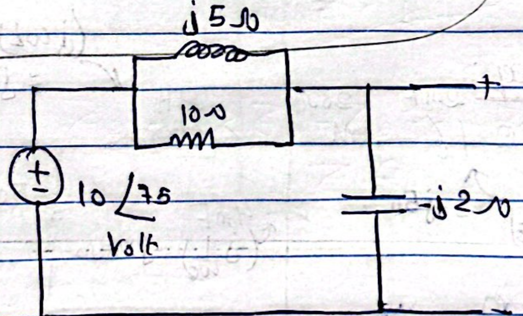
0.5H ← $j\,10 \times 0.5$



$-j\frac{1}{\omega C} = -j2$

$V_s(t)$ $10\cos(10t+75)$, ω

→ Phasor domain



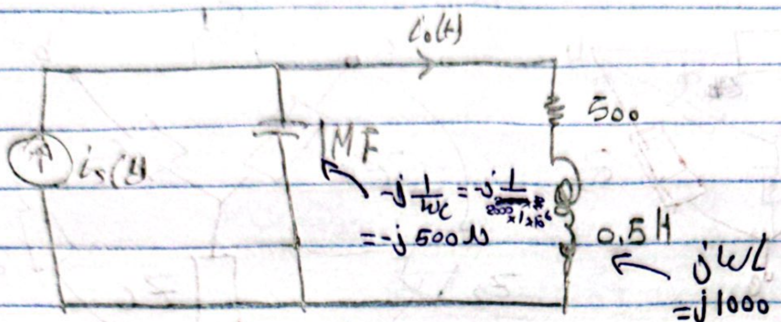
-90 $\frac{1}{2}$

$V_o = \frac{-j2}{-j2 + (10 \parallel j5)} * 10\angle 75 = \frac{(2\angle -90)(10\angle 75)}{-j2 + \frac{j50}{10+j5}}$

$= 7.071\angle -60\ V.$

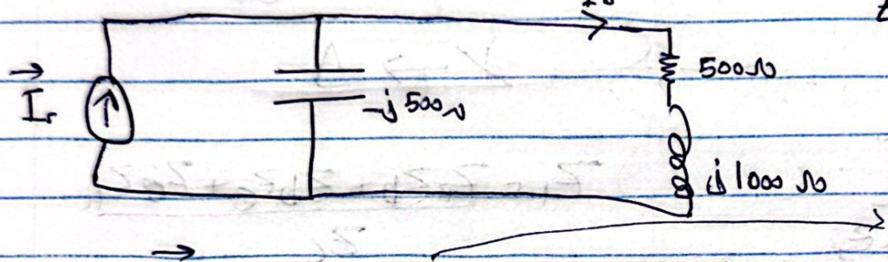
※ $V_o(t) = 7.071\cos(10t - 60^\circ)$ volt

* Calculate $i_o(t)$:



$i_s(t) = 0.05\cos(\underbrace{2000}_{w}t + 0)$, $I_s = 0.05\angle 0°$ A

t [illegible]

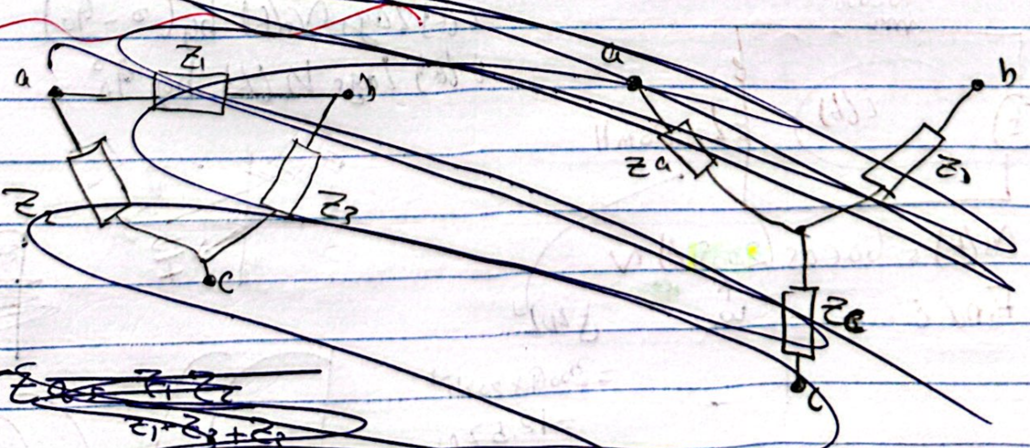


$$\vec{I_o} = \frac{-j\,500}{-j\,500 + 500 + j1000} * 0.05\angle 0° = \frac{\angle -90}{\angle 45°} \quad -90-45 = -135$$

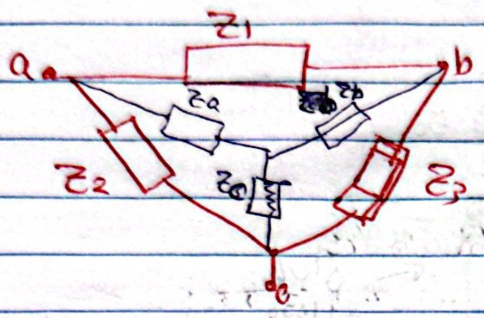
$= I_o = 0.03535\angle -135°$ A

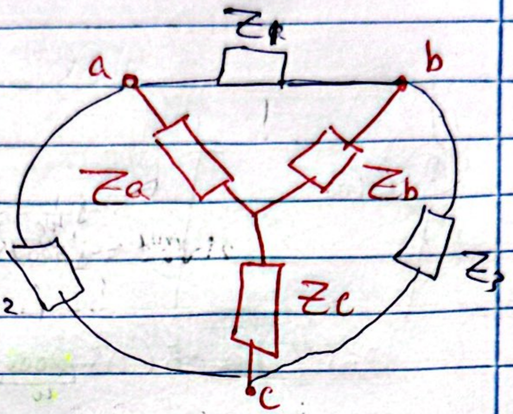
$\therefore\ i_o = 0.03535\cos(2000t + -135°)$.

* Y – Δ Transformation:



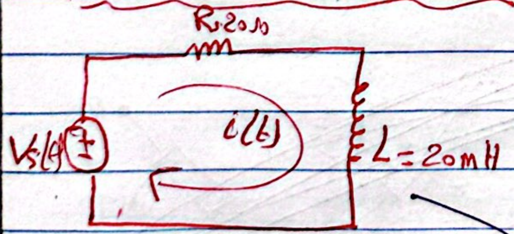
Y- Δ Transformation:





Δ → Y	Y → Δ
$Z_a = \frac{Z_1 Z_2}{Z_1+Z_2+Z_3}$	$Z_1 = \frac{Z_aZ_b+Z_bZ_c+Z_cZ_a}{Z_c}$
$Z_b = \frac{Z_1 Z_3}{Z_1+Z_2+Z_3}$	$Z_2 = \frac{Z_aZ_b+Z_bZ_c+Z_cZ_a}{Z_b}$
$Z_c = \frac{Z_2 Z_3}{Z_1+Z_2+Z_3}$	$Z_3 = \frac{Z_aZ_b+Z_bZ_c+Z_cZ_a}{Z_a}$

Series RL circuit:



$v_s(t) = 60\cos(200\pi t)$ V

ω — $+0$

Find i

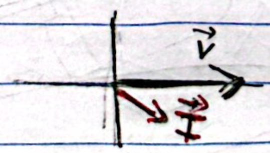
jWL

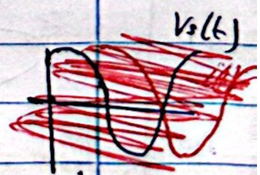
$= 200\pi \times 20\times10^{-3}$

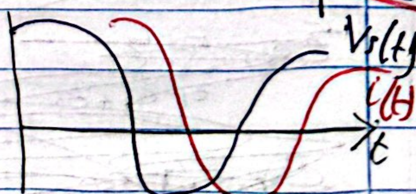
$= 12.57\Omega$

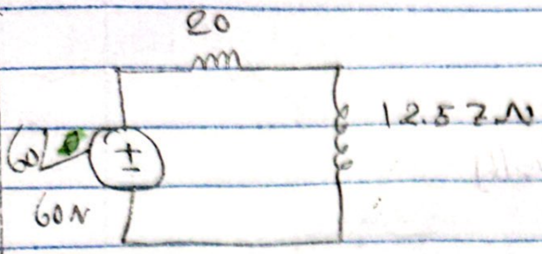
$i(t)$ Lags $V_s(t)$ by $(0 - 90)$

$i(t)$ lags $V_L(t)$ by 90°









KVL: $\vec{V_s} = \vec{V_R} + \vec{V_L}$

$$60\angle 0 = 20\vec{I} + j12.57\vec{I}$$

$$\rightarrow I = \frac{60\angle 0}{20 + j12.57} = \frac{60\angle 0}{23.6\angle 32.1}$$

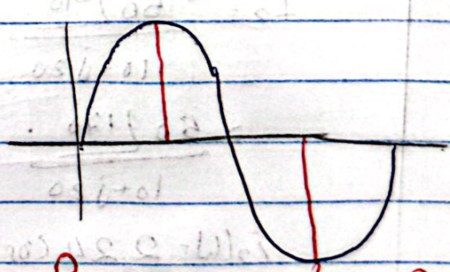
$$\vec{I} = 2.54\angle -32.1$$

$$i(t) = 2.54\cos(200\pi t - 32.1)\ \text{A}$$

$$\vec{V_L} = (j12.57)(2.54\angle -32.1) = 31.9\angle 57.9$$

$\vec{I}$ lag $\vec{V_L}$ by 90

$\vec{I}$ lag $\vec{V_s}$ by 32.1



$P = VI\cos(\theta_v - \theta_i)$

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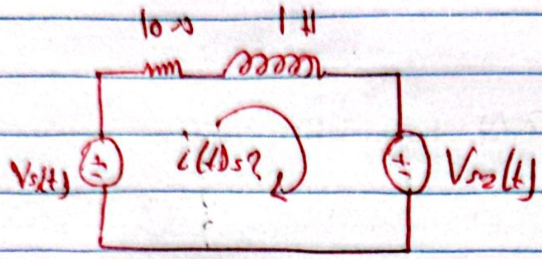
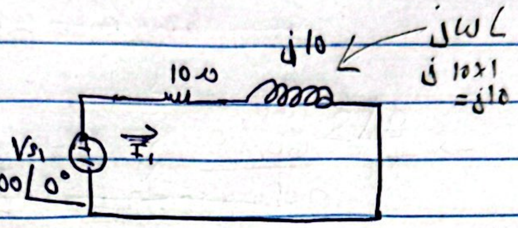


$V_{s1}(t) = 100\cos 10t$ volt , $\omega_1 = 10\,r/s$

$V_{s2}(t) = 50\cos(20t - 10°)$ volt , $\omega_2 = 20\,r/s$

→ superposition

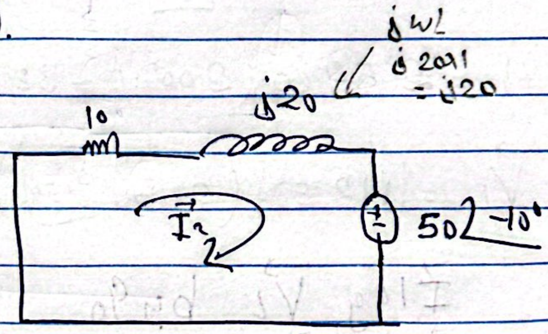
* I_1 due to $\vec{V}_{s1}(t)$



$$I_1 = \frac{100\angle 0°}{10 + j10} = 7.07\angle -45°$$

$i_1(t) = 7.07\cos(10t - 45°)$.

* I_2 due $V_{s2}(t)$



$$I_2 = \frac{-50\angle -10°}{10 + j20}$$

$$= \frac{50\angle 170}{10 + j20} = 2.24\angle 106.57°$$

$i_2(t) = 2.24\cos(20t + 106.57$

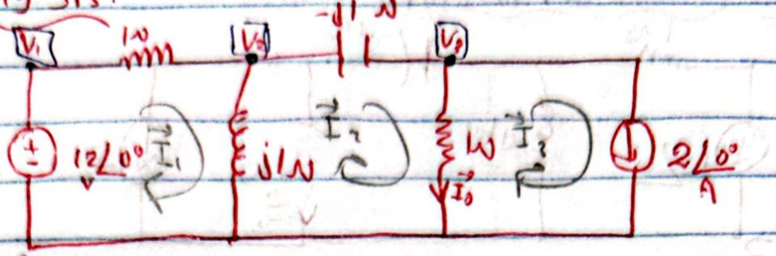
→ $i(t) = i_1(t) + i_2(t)$

$= 7.07\cos(10t - 45°)\,A + 2.24\cos(20t + 106.57°)\,A$

بجمع بس في حالة الـ Time Domain ، لانه ما بنفع نخلي الـ Polar

لانه كل واحد الها ω مختلفة

Nodal Analysis:



* Using Nodal

$-\vec{I_0} = \vec{V_3}$

$\rightarrow \boxed{V_1 = 12\angle 0}$ — ①

$\rightarrow$ KCl at Node 2:

$\frac{V_2 - V_1}{1} + \frac{\vec{V_2}}{j1} + \frac{V_2 - V_3}{-j1} = 0 \rightarrow \boxed{-V_1 + V_2 - jV_3 = 0}$ ②

KCl at Node 3:

$\rightarrow -2\angle 0 = \frac{1}{-j1}\vec{V_2} + \left(\frac{1}{-j1} + 1\right)V_3 \rightarrow \boxed{2\angle 0 = -j\vec{V_2} + (1+j)V_3}$ — ③

Solving for $V_3 \rightarrow V_3 = \left(\frac{8}{5} + j\frac{26}{5}\right)$ V

$\therefore \vec{I_0} = \frac{\vec{V_3}}{1} = \left(\frac{8}{5} + j\frac{26}{5}\right)$ A

* Using mesh Analysis. في الواجب

Find $\vec{I_0}$ using mesh:-

$\boxed{I_0 = I_2 - I_3}$ — ①

$\rightarrow$ KVL for mesh 1:

$\boxed{12\angle 0 = (1+j1)\vec{I_1} - j1\vec{I_2}}$ — ②

$\rightarrow$ KVL for mesh 2:

$0 = -j1\vec{I_1} + (1+j1-j1)\vec{I_2} - \vec{I_3} \rightarrow \boxed{0 = -j1 + I_2 - I_3}$ ③

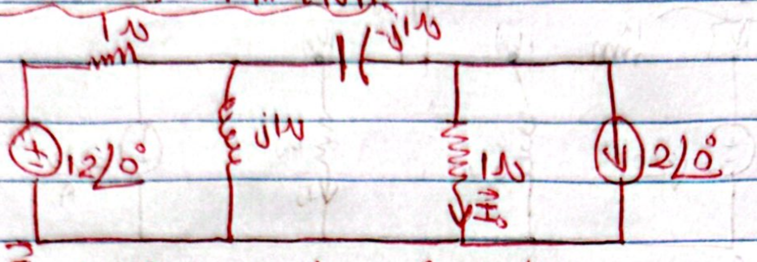
$I_3 = 2\angle 0$ A

Solving for $\vec{I_2}$ and $\vec{I_3}$ $\longrightarrow I_2 = \left(\frac{18}{5} + j\frac{26}{5}\right)$ A

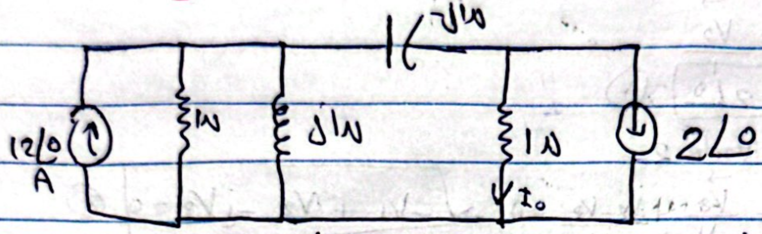
$\rightarrow I_3 = 2\angle 0$ A

$\therefore I_0 = \left(\frac{8}{5} + j\frac{26}{5}\right)$ A

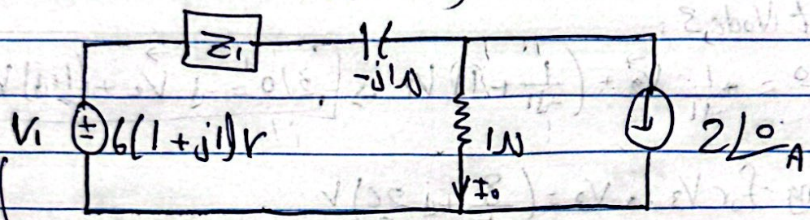
* Source Transformation:



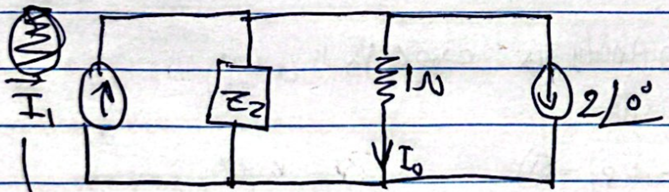
Find $\vec{I}_0$ using source transformation



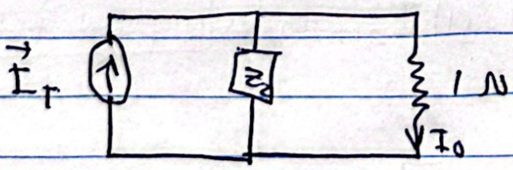
$-Z_1 = 1\Omega \parallel j1\Omega = (\frac{1}{2} + j\frac{1}{2})\Omega$



$V_1 = 12\angle 0^\circ \cdot Z = 6(1+j1)V$



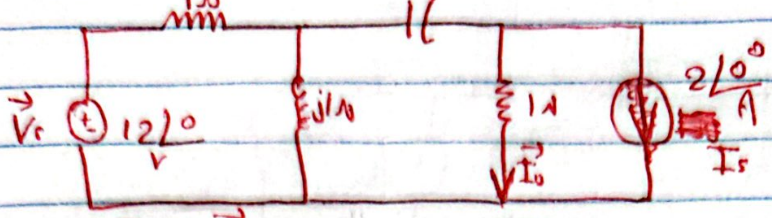
$I_1 = \frac{\vec{V}_1}{Z_2} = \frac{12(1+j1)}{1-j1}$, $Z_2 = -j1 + Z_1 = (\frac{1}{2} - j\frac{1}{2})\Omega$



$I_T = I_1 + 2\angle 0^\circ \rightarrow I_T = \left(\frac{10+j14}{1-j1}\right) A$

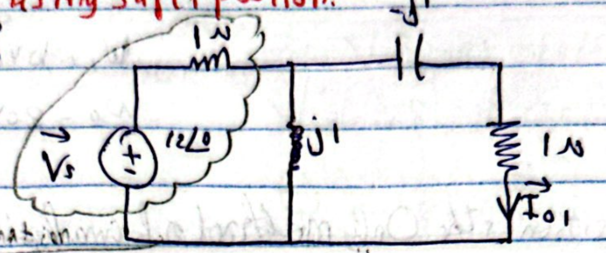
$I_0 = \frac{Z_2}{Z_2+1} \boxed{I_T = \left(\frac{8}{5} + j\frac{26}{5}\right) A}$

Superposition:

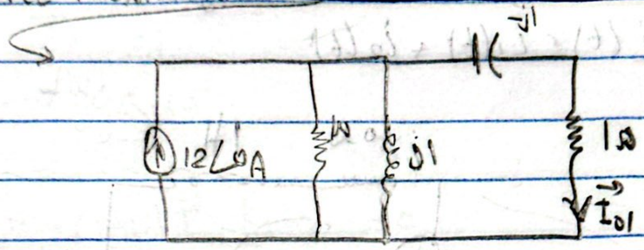


Find $\vec{I}_o$ using Superpostion:

[1] $\vec{I}_{o1}$ due to $\vec{V}_s$



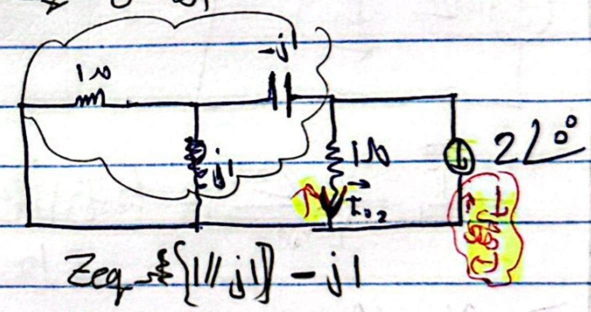
Source Transformation



$$I_{o1} = \frac{(1//j1)}{(1//j1)+(1-j1)} * 12\angle 0^\circ$$

$$= \frac{12}{1-j2} \ A$$

Zeq العكس



[2] $\vec{I}_{o2}$ due to $\vec{I}_s$

$$\vec{I}_{o2} = \frac{-Zeq}{Zeq+1} * 2$$

Zeq = $\{1//j1\} - j1$

* السالب يعني لازم اضيف

Polarity ع الزاوية 180°

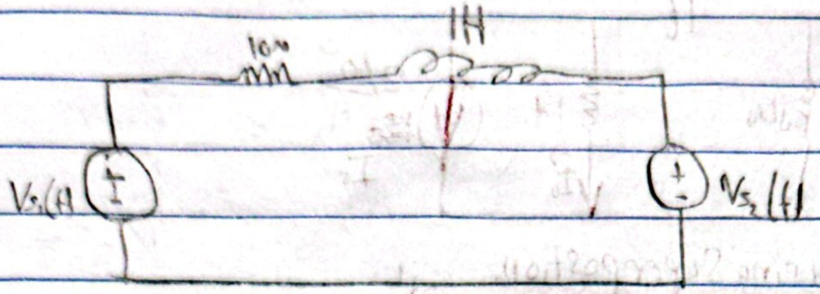
$$= \frac{Zeq}{Zeq+1} * 2\angle 180^\circ$$

$$= \frac{\{1//j1\} - j1}{(1//j1) - j1 + 1} * 2\angle 180^\circ \qquad = \frac{2}{2+j1}$$

$$\therefore I_o = \vec{I}_{o1} + \vec{I}_{o2}$$

$$= \left(\frac{8}{5} + j\frac{26}{5}\right) A.$$

The Power of Superposition



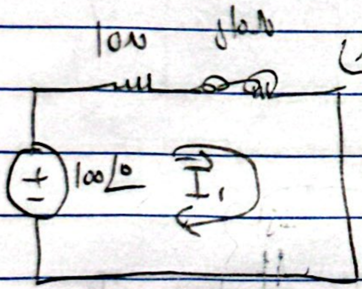
$V_{s1}(t) = 100\cos 10t$ V , $\omega_1 = 10$ V/s

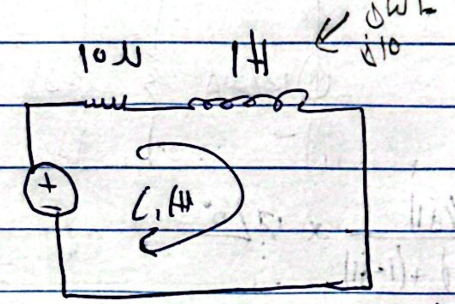
$V_{s2}(t) = 50\cos(20t - 10^\circ)$ V $\quad \omega_2 = 20$ V/s

* superposition is the Only method of analysis:-

$$i(t) = i_1(t) + i_2(t)$$

1) V_{s2} off

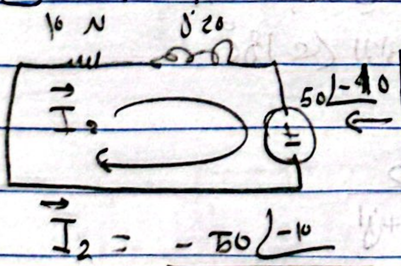


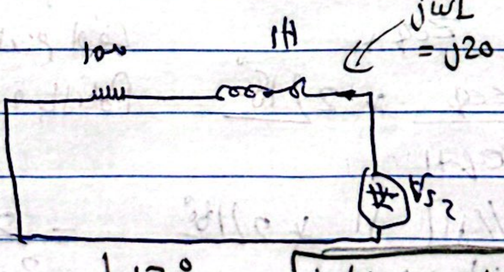


$V_{s1}(t) = 100\cos 10t$ V

$$\vec{I}_1 = \frac{100\angle 0}{10+j10} = 7.07\angle -45^\circ A \rightarrow \boxed{i_1(t) = 7.07\cos(10t - 45^\circ)\ A}$$

2) V_{s1} off



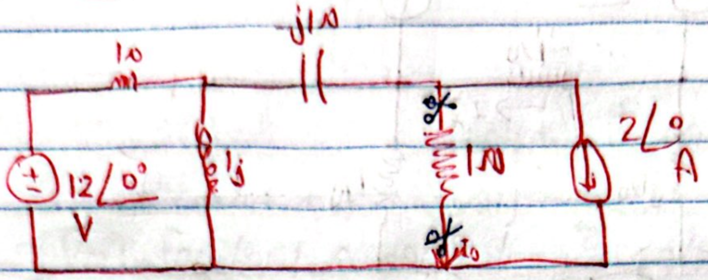


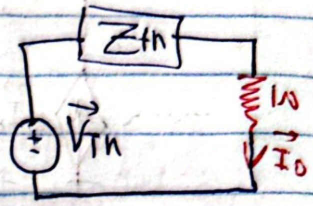
$V_{s2}(t) = 50\cos(2t - 10)$ V

$$\vec{I}_2 = \frac{-50\angle -10}{10+j20} = \frac{50\angle 170^\circ}{10+j20} \rightarrow \boxed{i_2(t) = 2.24\cos(20t + 106.57^\circ)\ A}$$

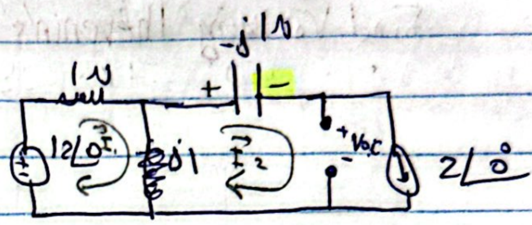
$$\therefore i(t) = 7.07\cos(10t - 45^\circ) + 2.24\cos(20t + 106.57^\circ)\ A$$

Thevenin's and Norton's Theorems:





Find $\vec{I}_o$ using Thev.



$\vec{V}_{th} = \vec{V}_{o.c}$

@ I_2 $\quad \vec{I}_2 = 2A$

$= 2 + j0$

$= 2\angle 0°$

@ I_1 $\quad -12 + (1+j1)\vec{I}_1 - (j1)2 = 0$

$\rightarrow I_1 = \dfrac{12 + j2}{1 + j1} = \boxed{7 - j5A}$

$V_{th} = \vec{V}_{-j1} + \vec{V}_{j1}$

$= -(-j1)(I_2) + (j1)(\vec{I}_1 - \vec{I}_2)$

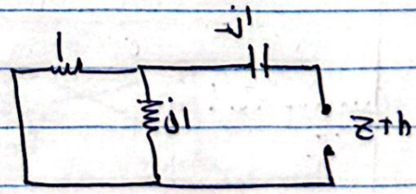
$= +j1 * 2 + j1(7 - j5 - 2)$

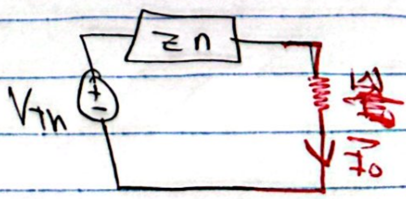
$= j2 + j5 + 5$

$= \boxed{5 + j7\ V}$

$\hookrightarrow Z_{th} = [1 // j1] + -j1$

$= \frac{1}{2} - j\frac{1}{2}$





$\vec{I}_0 = \dfrac{\vec{V}_{Th}}{Z_n + 1} = \dfrac{5 + j7}{\frac{1}{2} - j\frac{1}{2} + 1}$

$= \boxed{\dfrac{8}{5} + j\dfrac{26}{5}}$