Quadratic Equations (QE)

The general form of any quadratic equation is
$ax^2 + bx + c = 0$ where a, b, c constant, $a \neq 0$

Exp write this quadratic equation $3x^2 - 2x = x^2 + 4x - 5$ in the general form and determine a, b, c

$$3x^2 - 2x = \cancel{x^2} + 4x - 5 \quad \Rightarrow \quad 2x^2 - 2x = \cancel{4x} - 5$$
$$-x^2 \qquad \cancel{-x^2} \qquad\qquad -4x \qquad \cancel{-4x}$$

$$2x^2 - 6x = \cancel{-5} \quad \Rightarrow \quad 2x^2 - 6x + 5 = 0$$
$$+5 \qquad \cancel{+5}$$

$a = 2$
$b = -6$
$c = 5$

Exp How we solve QE : $ax^2 + bx + c = 0$?

There are two methods:
- Factoring Method - FM
- Quadratic Formula - QF

Remark For any real numbers a, b :
$ab = 0$ if and only if $a = 0$ or $b = 0$

Exp Solve $(2x-4)(1-3x) = 0$

$2x - 4 = 0$ or $1 - 3x = 0$

$2x = 4$ $\qquad$ $-3x = -1$

$\boxed{x = 2}$ $\qquad$ $\boxed{x = \frac{1}{3}}$

solutions

Exp Solve the following equations by factoring (60)

[1] $x^2 = 11x - 10$

general form: $\boxed{x^2 - 11x + 10 = 0}$ ✓

$$ab = 10$$
$$a + b = -11$$
$$a = -1$$
$$b = -10$$

$$(x + a)(x + b) = 0$$

$$(x - 1)(x - 10) = 0$$

$x - 1 = 0$ or $x - 10 = 0$

$\boxed{x = 1}$ $\boxed{x = 10}$

Check: $(1)^2 \stackrel{?}{=} 11(1) - 10$

$1 = 1$ ✓

$(10)^2 \stackrel{?}{=} 11(10) - 10$

$100 = 110 - 10$

$100 = 100$ ✓

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[2] $(y - 3)(y + 2) = -4$

general form $\Rightarrow$ $(y - 3)(y + 2) = -4$

$$y^2 + 2y - 3y - 6 = -4$$

$$y^2 - y - 6 = -4$$

$$\boxed{y^2 - y - 2 = 0}$$ ✓

$$ab = -2$$
$$a + b = -1$$
$$a = -2$$
$$b = 1$$

$$(y + a)(y + b) = 0$$

$$(y + 1)(y - 2) = 0$$

$y + 1 = 0$ or $y - 2 = 0$

$\boxed{y = -1}$ $\boxed{y = 2}$

Check $(-1 - 3)(-1 + 2) \stackrel{?}{=} -4$

$(-4)(1) = -4$ ✓

$(2 - 3)(2 + 2) \stackrel{?}{=} -4$

$(-1)(4) = -4$ ✓

3)
$$\frac{x+1}{3x+6} = \frac{3}{x} + \frac{2x+6}{x(3x+6)}$$

$$\frac{x}{x}\cdot\frac{x+1}{3x+6} = \frac{3}{x}\cdot\frac{3x+6}{3x+6} + \frac{2x+6}{x(3x+6)}$$

$$\frac{x(x+1)}{x(3x+6)} = \frac{3(3x+6)}{x(3x+6)} + \frac{2x+6}{x(3x+6)}$$

multiply by the LCD $= x(3x+6)$

$$x(x+1) = 3(3x+6) + 2x + 6$$

$$x^2 + x = 9x + 18 + 2x + 6$$

$$x^2 + x = 11x + 24$$
$$-11x \qquad \cancel{-11x}$$

$$x^2 - 10x = 24$$
$$-24 \qquad \cancel{-24}$$

$$\boxed{x^2 - 10x - 24 = 0}$$ general form

$$(x+a)(x+b) = 0$$

$$(x-12)(x+2) = 0$$

$a+b = -10$
$ab = -24$

$a = -12$
$b = 2$

$x - 12 = 0$ or $x + 2 = 0$

$x = 12$ ✓ $\qquad x = -2$ X

Check $x = 12$

$$\frac{12+1}{3(12)+6} \stackrel{?}{=} \frac{3}{12} + \frac{2(12)+6}{(12)(3(12)+6)}$$

$$\frac{13}{42} \stackrel{?}{=} \frac{1}{4} + \frac{\cancel{30}^{\,5}}{\cancel{12}_{\,2}(42)}$$

$$= \frac{84+20}{(8)(42)} = \frac{\cancel{104}^{\,13}}{\cancel{(8)}_{\,1}(42)}$$

$$\frac{13}{42} = \frac{13}{42}$$ ✓

Check $x = -2$

$$\frac{(-2)+1}{3(-2)+6} \stackrel{?}{=} \frac{3}{-2} + \frac{2(-2)+6}{(-2)(3(-2)+6)}$$

zero in denominator

Hence, the only solution is $\boxed{x = 12}$

④ $x^2 - 7 = 9$

general form $\Rightarrow x^2 - 16 = 0$

$a + b = 0$
$ab = -16$

$a = 4$
$b = -4$

$(x+a)(x+b) = 0$

$(x+4)(x-4) = 0$

$x + 4 = 0$ or $x - 4 = 0$

$\boxed{x = -4}$ $\boxed{x = 4}$

or
$x^2 - 7 = 9$
$x^2 = 16$
$x = \pm\sqrt{16}$
$= \pm 4$

Check $(-4)^2 - 7 \overset{?}{=} 9$
$16 - 7 \overset{?}{=} 9$
$9 = 9$ ✓

$(4)^2 - 7 = 9$
$16 - 7 = 9$
$9 = 9$ ✓

⑤ $x^2 - 5 = 0 \Rightarrow x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

square root method

or $(x - \sqrt{5})(x + \sqrt{5}) = 0$

$x - \sqrt{5} = 0$ or $x + \sqrt{5} = 0$

$x = \sqrt{5}$ $x = -\sqrt{5}$

Remark The solution of $x^2 = c$ is $x = \pm\sqrt{c}$ where $c \geq 0$

⑥ $4x^2 = 9 \Rightarrow x^2 = \frac{9}{4} \Rightarrow x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$

⑦ $(3x - 4)^2 = 25 \Rightarrow 3x - 4 = \pm\sqrt{25} \Rightarrow 3x - 4 = \pm 5$

$3x - 4 = 5$ or $3x - 4 = -5$

$3x = 9$ $3x = -1$

$\boxed{x = 3}$ $\boxed{x = \frac{-1}{3}}$

Now we use Quadratic Formula to solve the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{\Delta}}{2a}$$

$\Delta = b^2 - 4ac$ is called discriminant

- If $\Delta > 0$ then the equation has two distinct real roots
- If $\Delta = 0$ then the equation has exactly one real root
- If $\Delta < 0$ then the equation has no real solutions

Exp solve the following equations using the quadratic formula:

① $2x^2 - 3x = -1$ $\Rightarrow$ general form $\boxed{2x^2 - 3x + 1 = 0}$

$a = 2$, $b = -3$, $c = 1$

$$\Delta = b^2 - 4ac$$
$$= (-3)^2 - 4(2)(1)$$
$$= 9 - 8$$
$$= 1 \Rightarrow \text{two real roots}$$

since $\Delta > 0 \Rightarrow$ there is two distinct real roots

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{1}}{2(2)} = \frac{3 \pm 1}{4}$$

$$x_1 = \frac{3+1}{4} = \frac{4}{4} = 1$$

$$x_2 = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

Check $x_1 = 1 \Rightarrow 2(1)^2 - 3(1) = 2 - 3 = -1$ ✓

$x_2 = \frac{1}{2} \Rightarrow 2(\frac{1}{2})^2 - 3(\frac{1}{2}) = \frac{2}{4} - \frac{3}{2} = \frac{1}{2} - \frac{3}{2} = \frac{-2}{2} = -1$ ✓

2 $(x-2)^2 + 3(x-2) + \frac{9}{4} = 0$

$x^2 - 4x + 4 + 3x - 6 + \frac{9}{4} = 0$

$x^2 - x - 2 + \frac{9}{4} = 0$

$x^2 - x + \frac{1}{4} = 0$

$-2 + \frac{9}{4} = \frac{4 \times -2}{4 \times 1} + \frac{9}{4}$

$= \frac{-8}{4} + \frac{9}{4}$

$= \frac{1}{4}$

$a = 1 \quad , \quad b = -1 \quad , \quad c = \frac{1}{4}$

$D = b^2 - 4ac = (-1)^2 - 4(1)\left(\frac{1}{4}\right)$

$= 1 - 1$

$= 0 \quad \exists$ only one real root

$x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-(-1) \pm \sqrt{0}}{2(1)} = \frac{1}{2}$ " x-intercept means root "

Check $x = \frac{1}{2} \Rightarrow \left(\frac{1}{2} - 2\right)^2 + 3\left(\frac{1}{2} - 2\right) + \frac{9}{4} \stackrel{?}{=} 0$

$\left(\frac{-3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) + \frac{9}{4} \stackrel{?}{=} 0$

$\frac{9}{4} - \frac{9}{2} + \frac{9}{4} \stackrel{?}{=} 0$

$\frac{18}{4} - \frac{9}{2} \stackrel{?}{=} 0$

$\frac{9}{2} - \frac{9}{2} = 0$ ✓