0.2 Real Numbers $\mathbb{R}$

- In this book we will consider $\mathbb{R}$ to be our universal set U
- $\mathbb{R}$ is also called the real number line

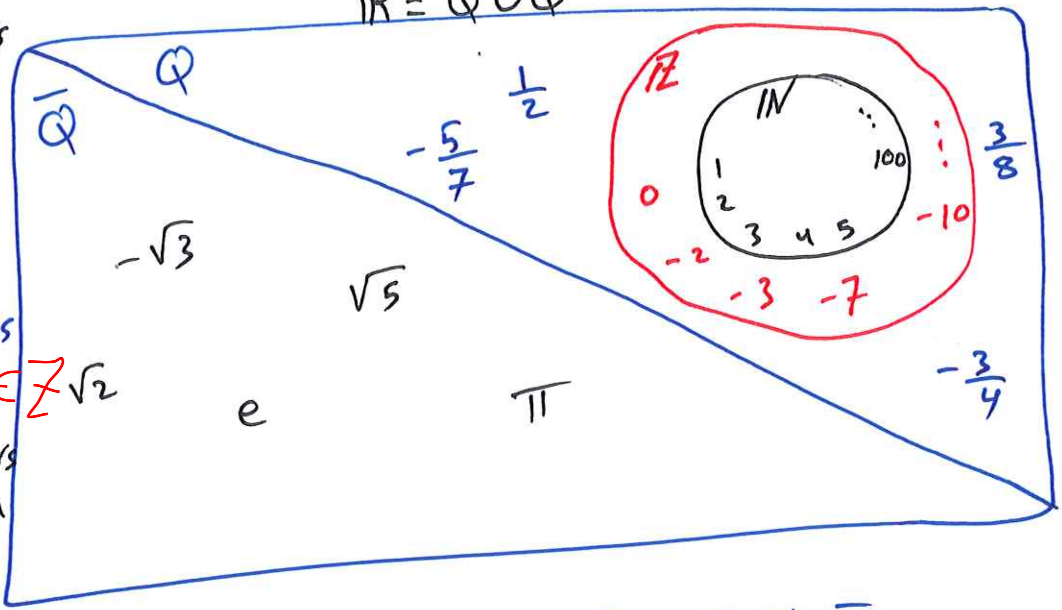
$\mathbb{R} = \mathbb{Q} \cup \overline{\mathbb{Q}}$

$\mathbb{N}$: Natural numbers

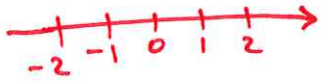
$\mathbb{Z}$: Integers

$\mathbb{Q}$: Rational numbers (All numbers that can be written as $\frac{a}{b}$, $b \neq 0$) $a, b \in \mathbb{Z}$

$\overline{\mathbb{Q}}$: Irrational numbers (can not be written as $\frac{a}{b}$)



- $\mathbb{R} = (-\infty, \infty)$ Real numbers $\Rightarrow$ $\mathbb{R} = \mathbb{Q} \cup \overline{\mathbb{Q}}$



- $\mathbb{Q}$ can have terminate decimals ($\frac{1}{2} = 0.5$) or repeate decimals ($\frac{1}{3} = 0.333\ldots$)

- $\overline{\mathbb{Q}}$ have no terminate decimals and have no repeated decimals

Properties of $\mathbb{R}$

1. $a + b = b + a$ Addition is commutative
   $ab = ba$ multiplication is commutative

2. $(a+b)+c = a+(b+c)$ Addition is associative
   $(ab)c = a(bc)$ multiplication is associative

3. $a + 0 = 0 + a = a$ The additive identity is 0

4. $a \cdot 1 = 1 \cdot a = a$ The multipcative identity is 1

[5] $a + (-a) = -a + a = 0$ The additive inverse of a is $-a$ [10]

[6] $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$ Any nonzero number a has multiplicative inverse $\frac{1}{a} = a^{-1}$

0 has no multiplicative inverse since division by 0 is undefined

[7] $a(b+c) = ab + ac$ Distribution Law: Multiplication is distributed over addition

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Exp

[1] $2+3 = 3+2 = 5$ and $(2)(3) = (3)(2) = 6$

[2] $(2+3)+4 = 2+(3+4) = 9$ and $(2)(3\cdot 4) = (2\cdot 3)(4) = 24$

[3] $8+0 = 0+8 = 8$ $\Rightarrow$ 0 is the additive identity

[4] $5\cdot 1 = 1\cdot 5 = 5$ $\Rightarrow$ 1 is the multiplicative identity

[5] $2 + (-2) = (-2)+2 = 0$ $\Rightarrow$ 2 has additive inverse -2

→ Note that the negative number is any number less than 0

→ But the negative of a number can be + or − $\Rightarrow$

$-(-2) = 2 > 0$ and $-(3) = -3 < 0$ and $-(0) = 0$

[6] $7 \cdot \frac{1}{7} = \frac{1}{7} \cdot 7 = 1$ $\Rightarrow$ 7 has multiplicative inverse $\frac{1}{7} = 7^{-1}$

[7] $2(3+4) = (2)(3) + (2)(4) = 6 + 8 = 14$ Distribution law

Inequalities and Intervals

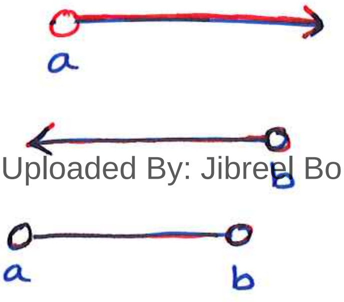
End Points

- open intervals (no end points)

$x > a$ means (a, ∞)

$x < b$ means $(-\infty, b)$

$a < x < b$ means (a, b)



- (two endpoints) Closed interval $a \leq x \leq b$ means $[a, b]$

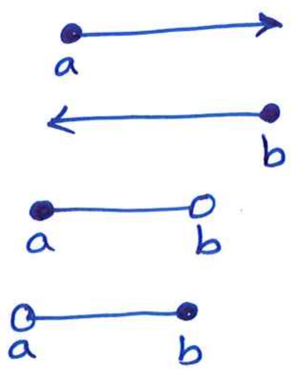
- Half-open intervals (only one endpoint)

$x \geq a$ means $[a, \infty)$

$x \leq b$ means $(-\infty, b]$

$a \leq x < b$ means $[a, b)$

$a < x \leq b$ means $(a, b]$



Exp Evaluate if possible or state meaningless:

1) $\frac{\sqrt{5}}{0}$ meaningless since division by zero is undefined (denominator is zero)

2) $\frac{0}{\sqrt{5}} = 0$

3) $\frac{\sqrt{5}}{\sqrt{5}} = 1$

4) $\frac{\sqrt{5}-\sqrt{5}}{\sqrt{5}-\sqrt{5}}$ meaningless since the denominator is zero (division by zero is undefined)

5) $(-3)^2 + 10\cdot 2 = 9 + 20 = 29$

6) $-3^2 + 10\cdot 2 = -9 + 20 = 11$

7) $\frac{(-5)(-3)-(-2)(3)}{-9+2} = \frac{15 - {}^{-}6}{-7} = \frac{15+6}{-7} = \frac{21}{-7} = -3$

Exp write inequality corresponding to the following intervals

1) $(-1, 5) \Rightarrow -1 < x < 5$

2) $(0, 7] \Rightarrow 0 < x \leq 7$

3) $[-2, \infty) \Rightarrow -2 \leq x < \infty$

4) $(-\infty, 7) \Rightarrow -\infty < x < 7$

5) $(-\infty, \infty) \Rightarrow -\infty < x < \infty$

Exp write interval corresponding to the following inequalities:

1) $-10 \leq x \leq 9 \Rightarrow$ closed interval $[-10, 9]$

2) $-4 \leq x < 11 \Rightarrow$ half-open interval $[-4, 11)$

3) $x > 0 \Rightarrow$ open interval $(0, \infty)$

4) $x \leq -2 \Rightarrow$ half-open interval $(-\infty, -2]$

5)  $\Rightarrow$ half-open interval $[0, 5)$

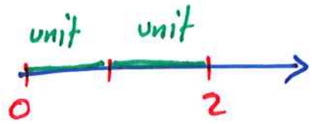
Absolute Value

- Absolute value of the number a is $|a|$ and represents the distance from origin

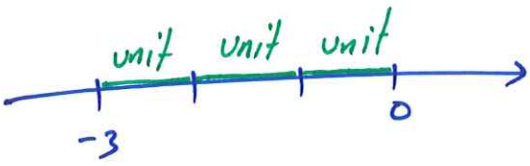
- $|a| \geq 0$ for any number $a \in \mathbb{R}$

Exp

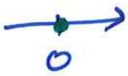
1. $|2| = 2$



2. $|-3| = 3$



3. $|0| = 0$



4. $\big|-7 - |-3|\big| = |-7-3| = |-10| = 10$

Def

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Exp

$|-3| = -(-3) = 3$ since $-3 < 0$

$|5| = 5$ since $5 \geq 0$

Operation with real numbers:

- $+2 + 3 = +5$
  $-2 - 3 = -5$
  $-\frac{1}{3} - \frac{4}{3} = \frac{-5}{3}$

- $2 - 3 = -1$
  $3 - 2 = +1$
  $-\frac{2}{3} + 1 = \frac{-2}{3} + \frac{3}{3} = \frac{1}{3}$

- $-2 - (-3) = -2 + 3 = 1$
  $14 - 8 = 6$

- $(-3)(-4) = 12$
  $\left(\frac{3}{4}\right)(2) = \frac{6}{4}$
  $\left(\frac{1}{2}\right)\left(\frac{3}{4}\right) = \frac{3}{8}$

- $(5)(-3) = -15$
  $(-2)(6) = -12$

- $(-16) \div (-2)$
  $= \frac{-16}{-2} = 8$

- $-8 \div 2 = -\frac{8}{2} = -4$

- Apriority to
  1. Brackets (parentheses)
  2. Powers ($2^3 = 2\cdot 2\cdot 2 = 8$)
  3. $\times, \div$ from left to right
  4. $+, -$ from left to right

Exp Evaluate

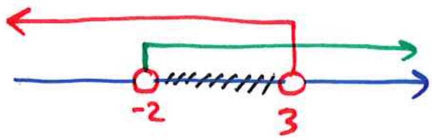
1. $-6+1 = -5$
2. $-4^2+4 = -16+4 = -12$
3. $((-4)^2-1)+3 = (16-1)+3 = 15+3 = 18$
4. $6 \div 2(2+1) = 6 \div 2(3) = 6 \div 2\cdot 3 = (6\div 2)(3) = (3)(3) = 9$
5. $9 - 2(2)(-10) = 9-(-40) = 9+40 = 49$
6. $\dfrac{(-3)^2 - 2\cdot 3 + 6}{4 - 2^2 + 3} = \dfrac{9-6+6}{4-4+3} = \dfrac{(9-6)+6}{(4-4)+3} = \dfrac{3+6}{0+3} = \dfrac{9}{3} = 3$

Exp Insert $<$ or $>$ or $=$ in $\square$:

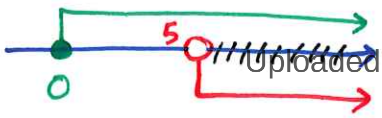
1. $-3 \ \boxed{<} \ 0$
2. $0.6666 \ \boxed{<} \ \frac{2}{3}$
3. $\pi \ \boxed{>} \ 3.14$
4. $|-4| + |7| \ \boxed{>} \ |-4+7|$
5. $\frac{1}{2} + \frac{1}{3} \ \boxed{=} \ \frac{5}{6}$

Exp Find an interval and graph for

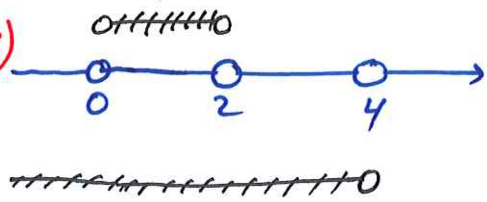
1. $(-\infty, 3) \cap (-2, \infty) = (-2, 3)$



2. $x > 5$ and $x \geq 0$ $= (5, \infty)$



3. $(-\infty, 4) \cup (0, 2) = (-\infty, 4)$



4. $x \geq 7$ or $x < 0$ $= (-\infty, 0) \cup [7, \infty)$